Fully Solved JCECE 2017

Engineering Entrance Exam

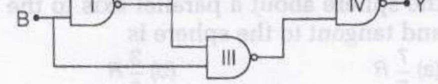
Physics

1. If the vertical component of earth's magnetic field at a place is $\sqrt{3}$ times the horizontal component, then the value of dip at that place is
   (a) 60° (b) 45° (c) 30° (d) 15°

2. An electron and a proton enter a magnetic field perpendicularly both have same kinetic energy. Which of the following is true?
   (a) Trajectory of electron is less curved
   (b) Trajectory of proton is less curved
   (c) Both trajectories are equally curved
   (d) Both move on straight line path

3. The reason a moving coil galvanometer cannot be used with an alternating current is that
   (a) the coil bends easily
   (b) the coil heats up too much
   (c) sparks can be produced
   (d) the net magnetic field produced is zero

4. Select the output Y of the combination of gates shown in figure for inputs $A = 1, B = 0$; $A = 1, B = 1$ and $A = 0, B = 0$ respectively.



   (a) (0,1, 1) (b) (1,0, 1)
   (c) (1, 1, 1) (d) (1, 0, 0)

5. The minimum number of geostationary satellites required for uninterrupted global coverage is
   (a) 3 (b) 5 (c) 7 (d) 9

6. A stationary object is released from a point P a distance $3R$ from the centre of the moon which has radius R and mass M. Which one of the following expressions gives the speed of the object on hitting the moon?
   (a) $\left(\frac{2GM}{3R}\right)^{\frac{1}{2}}$ (b) $\left(\frac{4GM}{3R}\right)^{\frac{1}{2}}$
   (c) $\left(\frac{2GM}{R}\right)^{\frac{1}{2}}$ (d) $\left(\frac{GM}{R}\right)^{\frac{1}{2}}$

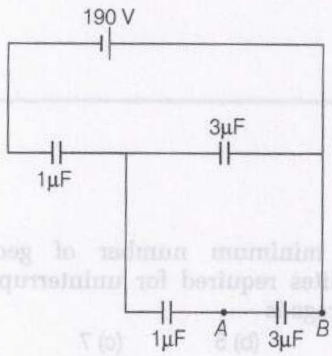
7. A stream of water flowing horizontally with a speed of 15 ms^{-1} gushes out of a tube of cross-sectional area 10^{-2}k m^2 and hits a vertical wall nearly. The force exerted on the wall by the impact of water assuming it does not rebound is
   (a) 2.25×10^3 N (b) 2.5×10^3 N
   (c) 3.0×10^3 N (d) 3.5×10^3 N

8. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3} the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is
   (a) 1 (b) 4 (c) 3 (d) 2

9. An electric field $\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N/C exists in space. The potential difference $(V_P - V_Q)$ between two points P and Q whose position vectors $\mathbf{r}_P = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\mathbf{r}_Q = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

(a) – 1V (b) 2 V (c) – 3 V (d) 4 V

10. In the circuit below, the potential difference between A and B is



(a) 10 V (b) 20 V
(c) 30 V (d) 40 V

11. A charged particle moves with a velocity v in a circular path of radius R around a long uniformly charged conductor, then

(a) $v \propto R$ (b) $v \propto \frac{1}{R}$

(c) $v \propto \frac{1}{\sqrt{R}}$ (d) v is independent of R

12. The capacitance of a parallel plate capacitor becomes $\frac{4}{3}$ times its original value. If a dielectric slab of thickness $t = \frac{d}{2}$ is inserted between the plates (where, d is the distance of separation between the plates). What is the dielectric constant of the slab?

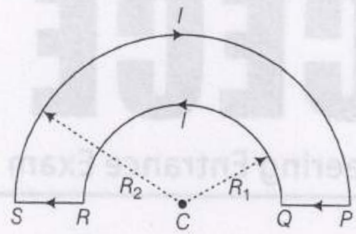
(a) $K = 2$ (b) $K = \frac{1}{2}$

(c) $K = 1$ (d) $K = \sqrt{2}$

13. A letter 'A' is constructed of a uniform wire with resistance $1.0\ \Omega\ \text{cm}^{-1}$. The sides of the letter are 20 cm and the cross-piece in the middle is 10 cm long. The apex angle is 60°. The resistance between the ends of the legs is

(a) $50.0\ \Omega$ (b) $26.7\ \Omega$
(c) $2.72\ \Omega$ (d) $34\ \Omega$

14. A wire loop $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown in figure below. The magnitude of magnetic induction at centre C is



(a) $\left(\frac{\mu_0}{4}\right) I \left[\frac{1}{R_2} - \frac{1}{R_1}\right]$

(b) $\left(\frac{\mu_0}{4}\right) I \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

(c) $\mu_0 I \left[\frac{1}{R_2} - \frac{1}{R_1}\right]$

(d) $\mu_0 I \left(\frac{1}{R_1}\right)$

15. The magnetic flux through a coil varies with time as $Q = 5t^2 + 6t + 9$. The ratio of emf at $t = 3$s to $t = 0$s will be

(a) 1 : 9 (b) 1 : 6
(c) 6 : 1 (d) 9 : 1

16. A tunnel is dug along the diameter of the earth. A mass m is dropped into it. How much time does it take to cross the earth?

(a) 169.2 min (b) 84.6 min
(c) 21.2 min (d) 42.3 min

17. A curved road of diameter 1.8 km is banked, so that no friction is required at a speed of 30m/s. What is the banking angle?

(a) 6° (b) 16° (c) 26° (d) 0.6°

18. The moment of inertia of a sphere of mass M and radius R about an axis passing through its centre is $\frac{2}{5}MR^2$. The radius of gyration of the sphere about a parallel axis to the above and tangent to the sphere is

(a) $\frac{7}{5}R$ (b) $\frac{3}{5}R$

(c) $\left(\sqrt{\frac{7}{5}}\right)R$ (d) $\left(\sqrt{\frac{3}{5}}\right)R$

19. The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of wire is

(a) $\frac{l_1 + l_2}{2}$ (b) $\sqrt{l_1 l_2}$

(c) $\frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$ (d) $\frac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$

20. In each heart beat, a heart pumps 80 ml of blood at an average pressure of 100 mm of Hg. Assuming 60 heart beats per second, the power output of the heart is

$(\rho_{Hg} = 13.6 \times 10^3 \text{ kgm}^{-3})$ $(g = 9.8 \text{ ms}^{-2})$

(a) 1.0 W (b) 1.06 W

(c) 1.12 W (d) 2.16 W

21. In Balmer series for hydrogen atom, find the energy of photon corresponding to longest wavelength.

(a) 18.9 eV (b) 3.03 eV

(c) 1.89 eV (d) 30.3 eV

22. The half life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially they have the same number of atoms. Then

(a) X and Y decay at same rate always

(b) X will decay faster than Y

(c) Y will decay faster than X

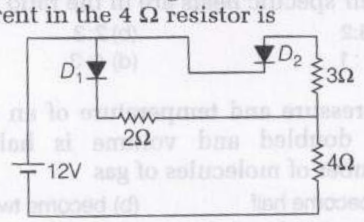
(d) X and Y have same decay rate initially

23. Cobalt-57 is radioactive, emitting β-particles. The half life for this is 270 days. If 100 mg of this is kept in an open container, then the mass of Cobalt-57 after 540 days will be

(a) 50 mg (b) $\left(\frac{50}{\sqrt{2}}\right)$ mg

(c) 25 mg (d) Zero

24. In the circuit of figure, treat diode as ideal, current in the 4 Ω resistor is



(a) 2 A (b) 3 A

(c) $\frac{12}{7}$ A (d) $\frac{30}{13}$ A

25. A travelling microscope is focussed on an ink dot. When a glass slab ($n = 1.5$) of thickness 9 cm is introduced on the dot, the travelling microscope has to be moved by

(a) 3 cm upwards (b) 5 cm upwards

(c) 3 cm downwards (d) 5 cm downwards

26. A copper wire of 3 mm^2 cross-sectional area carries a current of 5 ampere. The magnitude of the drift velocity for the electrons in the wire, (Assume copper to be monovalent, $M_{Cu} = 63.5$ kg/k mol and density of copper = 8920 kg/m^3)

(a) 0.24 m/s (b) 0.12 m/s

(c) 2.4 m/s (d) 0.06 m/s

27. Two long parallel wires placed 0.08 m apart carry currents 3A and 5A in the same direction. What is the distance from the conductor carrying larger current to the point where the resultant magnetic field is zero?

(a) 0.5 m (b) 0.04 m

(c) 0.05 m (d) 0.4 m

28. The relative permeability is represented by μ_r and the susceptibility by χ for a magnetic substance. Then for a paramagnetic substance

(a) $\mu_r > 1, \chi < 0$ (b) $\mu_r > 1, \chi > 0$

(c) $\mu_r < 1, \chi < 0$ (d) $\mu_r < 1, \chi > 0$

29. A transformer of efficiency 90% has turns ratio 1 : 10. If the voltage across the primary is 220 V and current in the primary is 0.5 A, then the current in secondary is

(a) 5.5 A (b) 5 A

(c) 4 A (d) 4.5 A

30. A 1 cm height needle is placed at a distance of 0.1 m from a convex mirror of focal length 0.05 m, then size of the image is

(a) 1 cm (b) 0.66 cm

(c) 0.33 cm (d) 0.5 cm

31. The dimensional formula for permittivity of free space (ε_0) in the equation $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ where, symbols have their usual meaning is

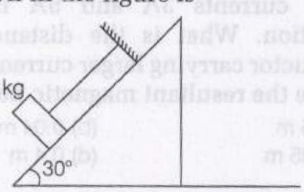
(a) $[M^1L^3A^{-2}T^{-4}]$ (b) $[M^{-1}L^{-3}T^4A^2]$

(c) $[M^{-1}L^{-3}A^{-2}T^{-4}]$ (d) $[M^1L^3T^2A^{-4}]$

32. A body moves along a straight line with acceleration $3\ ms^{-2}$ for 2s and then with acceleration $4\ ms^{-2}$ for 3s. What is his average acceleration?
(a) $3.4\ ms^{-2}$ (b) $3.5\ ms^{-2}$
(c) $3.6\ ms^{-2}$ (d) $3.7\ ms^{-2}$

33. Two bodies are projected at angles θ and $(90° - \theta)$ to the horizontal with the same speed. The ratio of their times of flight is
(a) $\sin\theta : 1$
(b) $\cos\theta : 1$
(c) $\sin\theta : \cos\theta$
(d) $\cos\theta : \sin\theta$

34. The coefficient of friction between two surfaces is $\mu = 0.8$. The tension in the string as shown in the figure is



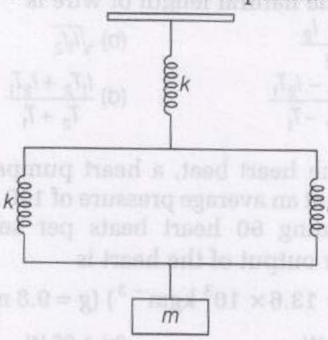
(a) 0 N (b) 6 N
(c) 4 N (d) 8 N

35. A body of density ρ and volume V is lifted through height h in a liquid of density $\sigma (< \rho)$. The increase in potential energy of the body is
(a) $V(\rho - \sigma)gh$ (b) $V\rho gh$
(c) $V\sigma gh$ (d) Zero

36. If θ is the polarising angle for two optical media, whose critical angles are C_1 and C_2, then the correct relation is
(a) $\sin\theta = \dfrac{\sin C_2}{\sin C_1}$ (b) $\theta = \dfrac{\sin C_2}{\sin C_1}$
(c) $\tan\theta = \dfrac{\sin C_1}{\sin C_2}$ (d) $\sin\theta = \dfrac{\sin C_1}{\sin C_2}$

37. Two thin lenses have a combined power of + 9 D. When they are separated by a distance of 20 cm, their equivalent power becomes $+\dfrac{27}{5}$ D, then their individual powers are
(a) 6D and 3D (b) 6D and 4D
(c) 9D and 3D (d) 9D and 6D

38. If the mass shown in figure is slightly displaced and then let go, then the system shall oscillate with a time period of



(a) $2\pi\sqrt{\dfrac{m}{3k}}$ (b) $2\pi\sqrt{\dfrac{3m}{2k}}$
(c) $2\pi\sqrt{\dfrac{2m}{3k}}$ (d) $2\pi\sqrt{\dfrac{3k}{m}}$

39. In Young's double slit experiment, using a monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ is K units. Then the intensity of light at a point where path difference is $\dfrac{\lambda}{3}$ is
(a) $\dfrac{K}{2}$ (b) $2K$ (c) $4K$ (d) $\dfrac{K}{4}$

40. The temperature coefficient of resistance of a wire is 0.00125/°C. Its resistance is 1Ω at 300 K. At what temperature, its resistance will be 2Ω?
(a) 1127 K (b) 854 K
(c) 1217 K (d) 1154 K

41. Two liquids A and B are at 32°C and 24°C. When mixed in equal masses, the temperature of mixture is found to be 28°C. Their specific heats are in the ratio
(a) 3:2 (b) 2:3
(c) 1:1 (d) 4:3

42. If pressure and temperature of an ideal gas are doubled and volume is halved, the number of molecules of gas
(a) become half (b) become two times
(c) become four times (d) remain constant

43. The rms speed of oxygen at room temperature is about 500 m/s. The rms speed of hydrogen at the same temperature is about

(a) 125 m/s
(b) 2000 m/s
(c) 8000 m/s
(d) 31 m/s

44. The distance between two points differing in phase by 60° on a wave having a wave velocity 360 m/s and frequency 500 Hz is

(a) 0.72 m (b) 0.18 m
(c) 0.12 m (d) 0.36 m

45. A particle moves according to the law $x = r\cos\frac{\pi t}{2}$. The distance covered by it in the time interval between $t = 0$ and $t = 3$s is

(a) r (b) $2r$
(c) $3r$ (d) $4r$

46. An alternating voltage $V = V_0 \sin\omega t$ is connected to a capacitor of capacity C_0 through an AC ammeter of zero resistance. The reading of ammeter is

(a) $\frac{V_0}{\sqrt{2}}$ (b) $\frac{V_0}{\omega C\sqrt{2}}$
(c) $\frac{V_0\omega C}{\sqrt{2}}$ (d) $V_0\omega C$

47. What is the required condition, if the light incident on one face of a prism, does not emerge from the other face?

(a) $n < \text{cosec}\left(\frac{A}{2}\right)$
(b) $n < \sec\left(\frac{A}{2}\right)$
(c) $n > \sec A$
(d) $n > \text{cosec}\left(\frac{A}{2}\right)$

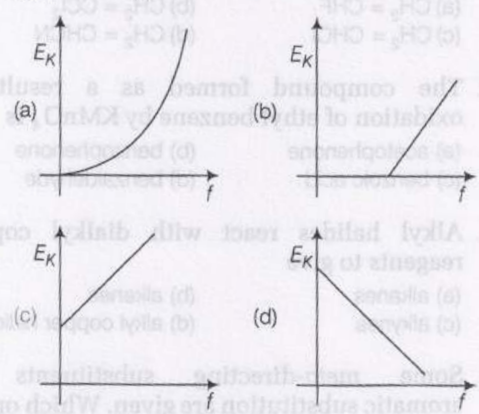
48. The critical angle for glass is 41°48′ and that for water is 48°36′. Calculate the critical angle for glass-water interface.

(a) 62°43′
(b) 34°42′
(c) 52°42′
(d) 44°42′

49. In Young's double slit experiment, one of the slits is wider than the other, so that the amplitude of light from one slit is double of that from the other slit. If I_m is the maximum intensity, what is the resultant intensity when they interfere at phase difference ϕ?

(a) $\frac{I_m}{9}\left(1 - 8\cos^2\frac{\phi}{2}\right)$
(b) $\frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$
(c) $\frac{I_m}{9}(1 - 8\cos^2\phi)$
(d) $\frac{I_m}{9}\left(1 - \sin^2\frac{\phi}{2}\right)$

50. Maximum kinetic energy of a photoelectron varies with the frequency (f) of the incident radiation as



Chemistry

1. Which of the following is fully fluorinated polymer?
(a) Neoprene (b) Teflon
(c) Thiocol (d) PVC

2. Which of the following is a polyamide?
(a) Teflon (b) Nylon-6, 6
(c) Terylene (d) Bakelite

3. Plexiglass is commercial name of
(a) glyptal
(b) polymethyl methacrylate
(c) polystyrene
(d) polyacrylonitrile

4. Among cellulose, polyvinyl chloride (PVC), nylon and natural rubber, the polymer in which intermolecular forces of attraction are weakest is
(a) nylon (b) PVC
(c) natural rubber (d) cellulose

5. The monomer used to produce orlon is
(a) $CH_2=CHF$ (b) $CH_2=CCl_2$
(c) $CH_2=CHCl$ (d) $CH_2=CHCN$

6. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is
(a) acetophenone (b) benzophenone
(c) benzoic acid (d) benzaldehyde

7. Alkyl halides react with dialkyl copper reagents to give
(a) alkanes (b) alkenes
(c) alkynes (d) alkyl copper halides

8. Some *meta*-directing substituents in aromatic substitution are given. Which one is most deactivating?
(a) $—SO_3H$ (b) —CN
(c) —COOH (d) $—NO_2$

9. When 2-butyne is treated with $Pd\text{-}BaSO_4$; the product formed will be
(a) 1-butene (b) *trans*-2-butene
(c) *cis*-2-butene (d) 2-hydroxy butane

10. Which of the following has highest knocking effect in IC engine?
(a) Branched chain paraffins
(b) Olefins
(c) Aromatic hydrocarbons
(d) Straight chain paraffins

11. Among the electrolytes Na_2SO_4, $CaCl_2$, $Al_2(SO_4)_3$ and NH_4Cl, the most effective coagulating agent for Sb_2S_3 sol is
(a) Na_2SO_4 (b) $CaCl_2$ (c) $Al_2(SO_4)_3$ (d) NH_4Cl

12. Which of the following statement is incorrect regarding physiosorption?
(a) Under high pressure it results into mutimolecular layer on adsorbent surface
(b) More easily liquefiable gases are adsorbed readily
(c) Enthalpy of adsorption ($\Delta H_{Adsorption}$) is low and positive
(d) It occurs because of van der Waal's forces

13. Gold numbers of protective colloids *A*, *B*, *C* and *D* are 0.50, 0.01, 0,10 and 0.005 respectively. The correct order of their protective powers is
(a) $A < C < B < D$ (b) $B < D < A < C$
(c) $D < A < C < B$ (d) $C < B < D < A$

14. When a sulphur sol is evaporated, sulphur is obtained. On mixing with water, sulphur sol is not formed. The sol is
(a) reversible (b) hydrophobic
(c) hydrophilic (d) lyophilic

15. The hydrocarbon which can react with sodium in liquid ammonia is
(a) styrene (b) acetylene
(c) propylene (d) pentane

16. Which of the following acids does not exhibit optical isomerism?
(a) Tartaric acid
(b) Lactic acid
(c) Maleic acid
(d) α-amino acids

17. Which of the following reactions will not result in the formation of carbon-carbon bond?

(a) Reimer-Tiemann reaction
(b) Friedel-Crafts acylation
(c) Wurtz reaction
(d) Cannizzaro reaction

18. The standard emf of galvanic cell involving 3 moles of electrons in its redox reaction is 0.59 V. The equilibrium constant for the reaction of the cell is

(a) 10^{25} (b) 10^{20}
(c) 10^{15} (d) 10^{30}

19. The potential of a hydrogen electrode at pH = 10 is

(a) 0.59 V (b) 0.00 V
(c) – 0.59 V (d) – 0.059 V

20. Which of the following electrolytic solutions has the least specific conductance?

(a) 0.002 N (b) 0.1 N
(c) 0.2 N (d) 2 N

21. Which of the following is correct statement?

(a) Acetophenone is an ether
(b) Diastase is an enzyme
(c) Cycloheptane is aromatic compound
(d) All of the above

22. Which of the following is incorrect?

(a) $FeCl_3$ is used to detect phenols
(b) Fehling's solution is used to detect glucose
(c) Tollen's reagent used to detect unsaturation
(d) $NaHSO_3$ used to detect carbonyl compound

23. Which one of the following is not a condensation polymer?

(a) Dacron
(b) Neoprene
(c) Melamine
(d) Glyptal

24. Which of the following statements is/are false?

(a) Repeat unit of natural rubber is isoprene
(b) Both starch and cellulose are made up to glucose units
(c) Artificial silk is derived from cellulose
(d) Nylon –6,6 is an elastomer

25. Bakelite is formed by the reaction of

(a) phenol and formaldehyde
(b) formaldehyde and aniline
(c) adipic acid and ethylene glycol
(d) phthalic acid and ethylene glycol

26. Dissolving 120 g of urea in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is

(a) 1.78 M
(b) 2 M
(c) 2.05 M
(d) 2.22 M

27. A 5.2 molal aqueous solution methyl alcohol, CH_3OH is supplied. What is the mole fraction of methyl alcohol in the solution?

(a) 0.05 (b) 0.10
(c) 0.18 (d) 0.086

28. 58.5 g of NaCl and 180 g of glucose were separately dissolved in 1000 mL of water. Identify the correct statement regarding the elevation of boiling point (b.p) of the resulting solutions.

(a) NaCl solution will show higher elevation of b.p
(b) Glucose solution will show higher elevation of b.p
(c) Both the solutions will show equal elevation of b.p
(d) The b.p. elevation will be shown by neither of the solutions.

29. Reaction of acetone with HCN gives

(a) substitution compound
(b) addition compound
(c) elimination product
(d) None of the above

30. Identify the correct statement.

(a) Reaction mechanisms are studied using isotopic labelling
(b) Isolation of reactive intermediates is a method of establish reaction mechanism
(c) Both (a) and (b) are correct
(d) Neither (a) nor (b) are correct

31. Glass is

(a) polymeric mixture
(b) gel
(c) super cooled liquid
(d) microcrystalline solid

32. Among the following substituted silanes the one which will give rise to cross linked silicone polymer on hydrolysis is

(a) R_3SiCl (b) R_3SiCl_2
(c) R_4Si (d) $RSiCl_3$

33. The polydispersity index of the polymer is always

(a) 1
(b) < 1
(c) 2
(d) 1 or > 1

34. Which one of the following statement is incorrect about enzyme catalysis?

(a) Enzymes are mostly proteinous in nature
(b) Enzymes are least reactive at optimum temperature
(c) Enzymes are denaturated by ultraviolet rays and at high temperature
(d) Enzyme action is specific

35. Which one of the following is an example for homogeneous catalysis?

(a) Manufacture of sulphuric acid by contact process
(b) Manufacture of ammonia by Haber's process
(c) Hydrolysis of sucrose in presence of dilute hydrochloric acid
(d) Hydrogenation of oil

36. Anisole can be prepared by the action of methyl iodide on sodium phenate. The reaction is called

(a) Wurtz reaction
(b) Williamson's reaction
(c) Fittig's reaction
(d) Etard's reaction

37. Consider the following reaction

$C_2H_5OH + H_2SO_4 \rightarrow$ Product.

Among the following, which one cannot be formed as a product under any conditions?

(a) Ethylene
(b) Ethyl hydrogen sulphate
(c) Acetylene
(d) Diethyl ether

38. From amongst the following alcohols the one that would react fastest with conc. HCl and anhy. $ZnCl_2$, is

(a) 2-methyl-propan-2-ol
(b) 2-butanol
(c) 1-butanol
(d) 2-methyl propanol

39. Hydrolysis of aromatic amide gives

(a) acids
(b) amines
(c) alcohols
(d) None of the above

40. Methyl phenyl ether can be obtained by reacting

(a) Phenolate ions and methyl iodide
(b) Bromobenzene with methoxide ions
(c) Methanol and phenol
(d) Bromobenzene and methyl iodide

41. Identify the monomers from the following.

(a) Acetic acid and benzoic acid
(b) Adipic acid and ethylene glycol
(c) Ethylene and ethanol
(d) Phthalic acid and acetic acid

42. Hydrolysis of cyanohydrin derivative produces

(a) carboxylic acids (b) alcohols
(c) aldehydes (d) ketones

43. Which of the following do not contain carbon-oxygen double bonds?

(a) Ketone (b) Esters (c) Acids (d) Ethers

44. Chloroethane reacts with X to give diethyl ether. The compound X is

(a) NaOH (b) NaOEt
(c) H_2SO_4 (d) $Na_2S_2O_3$

45. How do you distinguish chlorobenzene from benzyl chloride?

(a) $AgNO_3$ test
(b) Schiff reagent test
(c) By analysis of elemental composition
(d) By adding sodium bicarbonate

46. Acid catalysed hydration of alkenes except ethene leads to the formation of
(a) secondary of tertiary alcohol
(b) primary and secondary alcohol
(c) secondary alcohol
(d) tertiary alcohol

47. Among the following which is least acidic?
(a) Phenol (b) *o*-cresol
(c) *p*-nitrophenol (d) *p*-chlorophenol

48. An ether is more volatile than an alcohol having same molecular formula because
(a) dipolar character of ether
(b) alcohols having resonance structure
(c) intermolecular hydrogen bonding in ethers
(d) intermolecular hydrogen bonding in alcohols

49. An organic compound $A(C_4H_9Cl)$ on reaction with Na/diethyl ether gives a hydrocarbon which on monochlorination gives only one chloro derivative then, A is
(a) isobutyl chloride
(b) secondary butyl chloride
(c) tertiary butyl chloride
(d) *n*-butyl chloride

50. An oxygen containing organic compound upon oxidation forms a carboxylic acid as the only organic product with its molecular mass higher by 14 units. The organic compound is
(a) a ketone
(b) an aldehyde
(c) a primary alcohol
(d) a secondary alcohol

Mathematics

1. The distance of the point $A(a,b,c)$ from the X-axis is
(a) a
(b) $\sqrt{b^2+c^2}$
(c) $\sqrt{a^2+b^2}$
(d) a^2+b^2

2. If $\mathbf{a}\perp\mathbf{b}$ and $(\mathbf{a}+\mathbf{b})\perp(\mathbf{a}+m\mathbf{b})$, then m is
(a) 1 (b) $\frac{|\mathbf{a}|^2}{|\mathbf{b}|^2}$
(c) -1 (d) $\frac{-|\mathbf{a}|^2}{|\mathbf{b}|^2}$

3. If the points $(-1,3,2)$, $(-4,3,-2)$ and $(5,l,m)$ lie on a straight line, then l and m are
(a) 3, 10 (b) $-3, -10$
(c) $-3, 10$ (d) $3, -10$

4. The equation of a circle passing through the point $(1,1)$ and the point of intersection of the circles $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ is
(a) $4x^2+4y^2+30x-13y-25=0$
(b) $4x^2+4y^2+30x-13y+25=0$
(c) $4x^2-4y^2-30x+13y-25=0$
(d) $4x^2-4y^2+30x-13y-25=0$

5. The digit in the unit place of 7^{291} is
(a) 1 (b) 2
(c) 3 (d) 4

6. If $\cos^{-1}\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^x$, then $x^2y_2+xy_1=$
(a) n^2y (b) $-n^2y$
(c) y^2 (d) y

7. The angle between the curves $x^2+y^2=25$ and $x^2+y^2-2x+3y-43=0$ at $(-3,4)$ is
(a) $\tan^{-1}(1)$ (b) $\tan^{-1}\left(\frac{1}{68}\right)$
(c) $\frac{\pi}{2}$ (d) $\tan^{-1}\left(\frac{3}{4}\right)$

8. A man $6'$ tall moves away from a source of light $20'$ above the ground level, his rate of walking being 4 m/h. At what rate is the tip of his shadow moving?
(a) $\frac{12}{7}$ (b) $\frac{3}{7}$
(c) $\frac{40}{7}$ (d) None of these

9. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is
(a) 8 sq units (b) 4 sq units
(c) 8π sq units (d) 4π sq units

10. If the function $f(x)$ defined by $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$, then $f'(0) =$

(a) $100f'(0)$
(b) 1
(c) 100
(d) None of these

11. If $a \equiv b \pmod{m}$ and x is an integer, then which of the following is incorrect?

(a) $(a + x) \equiv (b + x) \pmod{m}$
(b) $(a - x) \equiv (b - x) \pmod{m}$
(c) $ax \equiv bx \pmod{m}$
(d) $(a \div x) \equiv (b \div x) \pmod{m}$

12. If a and b are positive integers such that $(a^2 - b^2)$ is a prime number, then

(a) $a^2 - b^2 = a + b$
(b) $a^2 - b^2 = a - b$
(c) $a^2 + b^2 = a - b$
(d) $a^2 + b^2 = a + b$

13. Which of the following is false?

(a) (N, +) is a semi-group
(b) (Z, +) is a group
(c) (N, •) is a group
(d) Set of all cube roots of unity is an abelian finite group under multiplication

14. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 4 & 5 & 1 & 6 \end{pmatrix} \in S_6$, then f^{-1} is

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 7 & 5 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 1 & 3 & 4 & 6 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 5 & 1 & 3 & 6 \end{pmatrix}$
(d) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 1 & 5 & 2 & 4 \end{pmatrix}$

15. In a group G, the equations $ax = b$ and $ya = b$ have

(a) no solutions in G
(b) infinite solutions in G
(c) unique solution in G
(d) depends on a and b

16. The equation of the tangent and normal to the ellipse $x^2 + 2y^2 + 2x - 4y - 14 = 0$ at $(2, -1)$ is

(a) $3x - 4y - 10 = 0, 4x + 3y - 5 = 0$
(b) $4x + 3y - 10 = 0, 3x + 4y - 5 = 0$
(c) $3x - 4y - 5 = 0, 4x + 3y - 10 = 0$
(d) $3x - 4y - 10 = 0, 4x - 3y - 5 = 0$

17. If the line $2x + \sqrt{6}y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$, then the point of contact is

(a) $(-4, \sqrt{6})$
(b) $(-4, -\sqrt{6})$
(c) $(4, -\sqrt{6})$
(d) $(4, \sqrt{6})$

18. The angle between two diagonals of a cube is

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(b) $\cos^{-1}\left(\frac{1}{3}\right)$
(c) $30°$
(d) $45°$

19. The equation of the plane which bisects the line joining $(3, 0, 5)$ and $(1, 2, -1)$ at right angles is

(a) $2x + y + 2z = 7$
(b) $-2x + 2y - 6z = 7$
(c) $x - y + 2z = 7$
(d) $x - y + 3z = 7$

20. The equation of the line passing through the point $(5, 3, 2)$ and perpendicular to the lines $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-4}{1}$ and $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z+1}{0}$ is

(a) $\frac{x-5}{-1} = \frac{y-3}{2} = \frac{z-2}{3}$
(b) $\frac{x+1}{5} = \frac{y-2}{3} = \frac{z-3}{2}$
(c) $\frac{x-5}{1} = \frac{y-3}{1} = \frac{z-2}{1}$
(d) $\frac{x-5}{2} = \frac{y-3}{1} = \frac{z-2}{0}$

21. Inverse of a diagonal non-singular matrix is

(a) Symmetric matrix
(b) Skew-symmetric matrix
(c) Diagonal matrix
(d) Scalar matrix

22. If the matrix $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} = A + B$, where A is symmetric and B is skew-symmetric, then B is equal to

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

23. If A is 3×4 matrix and B is a matrix such that $A'B$ and $B'A$ are both defined, then the order of B is

(a) 4×4 (b) 3×3 (c) 3×4 (d) 4×3

24. The inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(a) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ (b) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$

25. If $a_1, a_2, \ldots, a_n, \ldots$ are in GP, then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$ is

(a) 0 (b) 1

(c) -1 (d) None of these

26. The value of the integral $\int e^x \{f(x) + f'(x)\}\, dx$ is

(a) $e^x f'(x) + C$ (b) $e^x f(x) + C$

(c) $\frac{e^x}{f(x)} + C$ (d) $\frac{e^x}{f'(x)} + C$

27. The value of the integral $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x}\, dx, a > 0$ is

(a) 1 (b) 0

(c) $\frac{\pi}{2}$ (d) π

28. The value of the integral $\int \frac{e^x(x^2+1)}{(x+1)^2}\, dx =$

(a) $e^x \log\left(\frac{x-1}{x+1}\right) + C$

(b) $e^x \log\left(\frac{x+1}{x-1}\right) + C$

(c) $e^x \left(\frac{x+1}{x-1}\right) + C$

(d) $e^x \left(\frac{x-1}{x+1}\right) + C$

29. The area enclosed between the parabolas $y^2 = 16x$ and $x^2 = 16y$ is

(a) $\frac{64}{3}$ sq units (b) $\frac{256}{3}$ sq units

(c) $\frac{16}{3}$ sq units (d) None of these

30. The solution of $y' = e^{x-y} + x^2 e^{-y}$ is

(a) $3(e^y - e^x) - x^3 = C$

(b) $e^y - e^x - x^3 = C$

(c) $e^y - e^x + x^3 = C$

(d) $3(e^y - e^x) + x^3 = C$

31. The characteristic equation of a matrix A is $\lambda^3 - 5\lambda^2 - 3\lambda + 21 = 0$, then $|\text{adj } A|$ is equal to

(a) 4 (b) 25 (c) 9 (d) 30

32. The area of region bounded by the lines $y = mx$, $x = 1$ and $x = 2$ and the X-axis is 7.5 sq units, then m is

(a) 2 (b) 3

(c) 4 (d) 5

33. The order and degree of $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}} = \left(\frac{d^2y}{dx^2}\right)^2$ is

(a) 2, 2 (b) 2, 4

(c) 1, 2 (d) 1, 4

34. Let A, B, C, D be the points with position vectors $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}$, $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-4\hat{\mathbf{k}}$, $-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$ respectively. If the points A, B, C, D lie on a plane, then the value of λ is

(a) 0 (b) $\frac{37}{4}$

(c) $\frac{-37}{4}$ (d) 1

35. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\mathbf{a}\times(\mathbf{a}\times\mathbf{c})+\mathbf{b}=0$, then the angle between $\mathbf{a}$ and $\mathbf{c}$ is

(a) $\frac{\pi}{6}$

(b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$

(d) Both (a) and (b)

36. The vertices of the hyperbola are at $(-5,-3)$ and $(-5,-1)$ and the extremities of the conjugate axis are at $(-7,-2)$ and $(-3,-2)$, then the equation of the hyperbola is

(a) $\frac{(y-2)^2}{1}-\frac{(x-5)^2}{4}=1$

(b) $\frac{(y+2)^2}{1}-\frac{(x+5)^2}{4}=1$

(c) $\frac{(x+5)^2}{4}-\frac{(y+2)^2}{1}=1$

(d) $\frac{(x-5)^2}{4}-\frac{(y-2)^2}{1}=1$

37. Two dices are thrown simultaneously. The probability of obtaining a total score of 5 is

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{1}{36}$

38. If A and B are events with $P(A\cup B)=\frac{3}{4}$, $P(A')=\frac{2}{3}$ and $P(A\cap B)=\frac{1}{4}$, then $P(B)$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

39. The probability that among 7 persons, no 2 were born on the same day of a week is

(a) $\frac{2}{7}$ (b) $\frac{7!}{7}$

(c) $\frac{7!}{7^7}$ (d) $\frac{2}{7^7}$

40. For the events A and B, $P(A)=\frac{3}{4}$, $P(B)=\frac{1}{5}$, $P(A\cap B)=\frac{1}{20}$, then $P\left(\frac{A}{B}\right)=\ldots\ldots$

(a) $\frac{1}{4}$ (b) $\frac{1}{15}$

(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

41. The value of $4\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}=$

(a) $\tan^{-1}\left(\frac{1}{139}\right)$

(b) $\tan^{-1}\left(\frac{1}{239}\right)$

(c) $\tan^{-1}(239)$

(d) $\tan^{-1}(139)$

42. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then the value of x

(a) 3 (b) 2

(c) 1 (d) 0

43. The general solution of $\sqrt{3}\cos x+\sin x=\sqrt{2}$, for any integer n is

(a) $n\pi+\frac{\pi}{6}\pm\frac{\pi}{4}$

(b) $n\pi-\frac{\pi}{6}\pm\frac{\pi}{4}$

(c) $2n\pi-\frac{\pi}{6}\pm\frac{\pi}{4}$

(d) $2n\pi+\frac{\pi}{6}\pm\frac{\pi}{4}$

44. The imaginary part of conjugate of $\left(\frac{1+i}{1-i}\right)^5$ is

(a) -1 (b) $-i$

(c) 1 (d) i

45. If ω is an imaginary cube root of 1, then the value of $1(2-\omega)(2-\omega^2)+2(3-\omega)(3-\omega^2)+\dots+(n-1)(n-\omega)(n-\omega^2)$ is

(a) $\frac{n(n+1)}{2}-n$

(b) $\frac{n^2(n+1)^2}{4}-n$

(c) $\frac{n(n+1)}{2}+n$

(d) $\frac{n^2(n+1)^2}{4}+n$

46. If $x+y=\tan^{-1}y$ and $y''=f(y)y'$, then $f(y)=$

(a) $\frac{1}{y}$ (b) $\frac{2}{y}$

(c) $\frac{2}{y^3}$ (d) $\frac{-2}{y^3}$

47. If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}; & \text{if } x\neq 0, \text{ then} \\ 0; & \text{if } x=0 \end{cases}$

which of the following is correct?

(a) $f(x)$ is continuous and $f'(0)$ does not exist

(b) $f(x)$ is not continuous

(c) $f(x)$ is continuous and $f'(0)$ also exists

(d) None of the above

48. If $y=\sin^{-1}\frac{1}{2}(\sqrt{1+x}+\sqrt{1-x})$, then y' is equal to

(a) $\frac{1}{2\sqrt{1-x^2}}$

(b) $\frac{-1}{2\sqrt{1-x^2}}$

(c) $\frac{1}{2\sqrt{1+x^2}}$

(d) $\frac{-1}{2\sqrt{1+x^2}}$

49. If $x^m y^n=(x+y)^{m+n}$, then y' is equal to

(a) $\frac{-y}{x}$ (b) $\frac{x}{y}$

(c) $\frac{-x}{y}$ (d) $\frac{y}{x}$

50. If $y=x^{x^{x^{\dots\infty}}}$, then y' is equal to

(a) $\frac{-y^2}{x(1-y\log x)}$

(b) $\frac{y^2}{1-y\log x}$

(c) $\frac{y^2}{x(1-y\log x)}$

(d) $\frac{-y^2}{1-y\log x}$

Answer with Solutions

Physics

1. *(a)* Here, $B_V = \sqrt{3} B_H$

We know that, angle of dip, $\theta = \tan^{-1}\left(\frac{B_V}{B_H}\right)$

$$\Rightarrow \qquad \theta = \tan^{-1}\left(\frac{\sqrt{3}\,B_H}{B_H}\right) = \tan^{-1}(\sqrt{3}) = 60^\circ$$

$\therefore$ Angle of dip, $\theta = 60^\circ$

2. *(b)* When a particle enters the magnetic field perpendicularly, then force acting on it will be,

$$\frac{mv^2}{r} = qvB$$

i.e. $\qquad r = \frac{mv}{qB} = \frac{p}{qB} \qquad [\because P = mv] \qquad \ldots(i)$

Also, kinetic energy, $K = \frac{p^2}{2m}$.

$$p^2 = 2mK$$

$$p = \sqrt{2mK} \qquad \ldots(ii)$$

Substituting the value of Eq. (ii) in Eq. (i), we get

$$r = \frac{\sqrt{2m \times K}}{qB}$$

Here, $\qquad K_p = K_e = K$ (say)

$\therefore \qquad r_p = \frac{\sqrt{2m_pK}}{qB} \qquad \ldots(iii)$

and $\qquad r_e = \frac{\sqrt{2m_eK}}{qB} \qquad \ldots(iv)$

Dividing Eq. (iii) by Eq. (iv), we get

$$\frac{r_p}{r_e} = \sqrt{\frac{m_p}{m_e}}$$

Since, $m_p >> m_e$

$\therefore \qquad r_e < r_p$

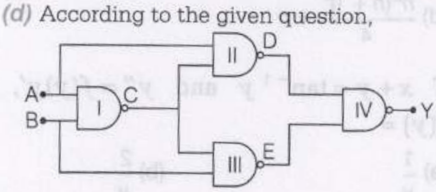
Since, the curvature $\propto \frac{1}{\text{radius } (r)}$

Hence, the trajectory of the proton in less curved.

3. *(d)* The typical coil galvanometer is based on the principle that, when a current carrying coil is placed in a magnetic field, the coil experiences a torque.

Since, the current in an AC averages out to zero i.e., the net magnetic field produced will be zero. Hence, the average torque experienced in a given time interval would be zero. Therefore, there will be no deflection.

4. *(d)* According to the given question,



Truth table

A	B	$C = \overline{A \cdot B}$	$D = \overline{A \cdot C}$	$E = \overline{C \cdot B}$	$Y = \overline{D \cdot E}$
1	0	1	0	1	1
1	1	0	1	1	0
0	0	1	1	1	0

Therefore, the output, $Y = (1, 0, 0)$

5. *(a)* World famous science fiction writer Arthur C. Clarke postulated that, 3 satellites could cover the whole world as they all are geo-stationary. Theoretically, only two are needed but this means that in some places the satellite on or near the horizon might not work too well and can cause problems. Therefore, there are usually used which are positioned in such a way that they can directly communicate with each other easily.

6. *(b)* According to the law of conservation of energy,

$$-\frac{GmM}{3R} + \frac{mv_0^2}{2} = -\frac{GmM}{R} + \frac{mv_1^2}{2}$$

where m is the mass of the object, v_0 and v_1 is the initial and final velocities of object respectively.

As, the object is stationary i.e., $v_0 = 0$.

$$\Rightarrow \qquad -\frac{GM}{3R} = -\frac{GM}{R} + \frac{v_1^2}{2}$$

$$\Rightarrow \qquad \frac{v_1^2}{2} = \frac{2GM}{3R}$$

$$\Rightarrow \qquad v_1 = 2\sqrt{\frac{GM}{3R}} = \left(\frac{4GM}{3R}\right)^{\frac{1}{2}}$$

7. *(a)* If the velocity of the stream be v (say).
In 1 second, the distance travelled = v.
$\therefore$ The volume of water hitting the wall per second, $V = va$
where, a is the cross-sectional area.

$$\Rightarrow \quad V = 15 \text{ m/s} \times 10^{-2} \text{ m}^2$$

$$[\because \text{Given, } v = 15 \text{ m/s}, a = 10^{-2} \text{ m}^2]$$

$$\Rightarrow \quad V = 15 \times 10^{-2} \text{ m}^3\text{/s}$$

Mass of water hitting the wall per second,

$$m = \text{Volume} \times \text{Density}$$
$$= 15 \times 10^{-2} \times 10^3$$
$$= 150 \text{ kg/s}$$
$$[\because \text{Density of water} = 1000 \text{kg/m}^3]$$

Initial momentum of the water hitting the wall per second = $m \times v$

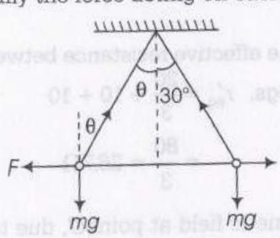
$$= 150 \times 15$$
$$= 2250 \text{ kg m/s}^2$$
$$= 2250 \text{ N}$$

Final momentum per second = 0

$\therefore$ Force exerted on the wall = Charge in the momentum of water stream per second

$$= 2250 \text{ N}$$
$$= 2.25 \times 10^3 \text{ N}$$

8. *(d)* Initially the force acting on each balls are



1. Tension (T)
2. Weight (mg)
3. Electrostatic force of repulsion F.

For its equilibrium along vertical

$$T.\cos = mg \quad \text{....(i)}$$

and along horizontal,

$$T\sin\theta = F \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\tan\theta = F/mg \quad \text{...(iii)}$$

when the balls are suspended in a liquid of density σ and dielectric constant K.

The electrostatic force will becomes $(1/K)$ times i.e. $F' = (F/K)$, while

weight $mg' = mg$ – upthrus t

$$= mg - V\sigma g \quad [\text{upthrust} = V\sigma g]$$

$$mg' = mg\left[1 - \frac{\sigma}{\rho}\right] \quad \left[\because V = \frac{\sigma}{\rho}\right]$$

For equilibrium of ball

$$\tan\theta' = \frac{F'}{mg'} = \frac{F}{mg[1-\sigma/\rho]} \quad \text{...(iv)}$$

Form Eqs. (iv) and (iii), we get

$$K = \frac{1}{\left(1 - \frac{\sigma}{\rho}\right)} = \frac{1}{\left(1 - \frac{0.8}{1.6}\right)}$$

$$= \frac{1.6}{(1.6 - 0.8)} = \frac{1.6}{0.8} = 2$$

9. *(a)* As we know,

$$\mathbf{E} = -\frac{dV}{dr}$$

$$\Rightarrow \quad \mathbf{E} \cdot dr = -dV$$

$$\Rightarrow \quad -dV = [(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot [(\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

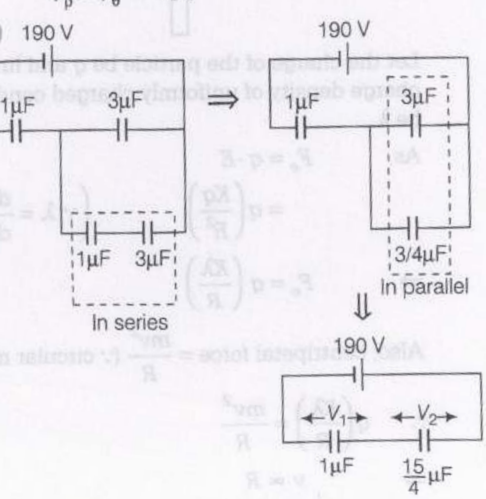
$$-(V_\rho - V_\theta) = [2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}] \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$\Rightarrow \quad -(V_\rho - V_\theta) = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= [(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})]$$

$$V_\rho - V_\theta = -1 \text{ V}$$

10. *(a)*

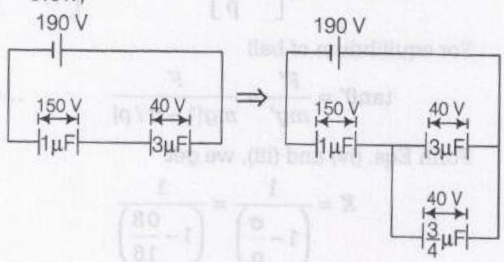


$$\therefore \quad V_1 = \left(\frac{\frac{15}{4}}{1+\frac{15}{4}}\right) \times 190$$

$$= \frac{15}{19} \times 190 = 150 \text{ V}$$

$$\Rightarrow \quad V_2 = (190 - 150) \text{ V} = 40 \text{ V}$$

Now,

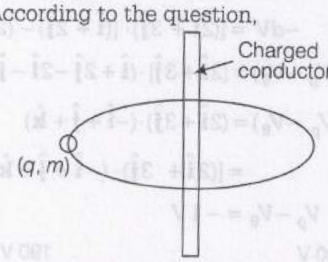


So, the potential difference between A and B will be,

$$V_{AB} = \left(\frac{1}{1+3}\right) \times 40$$

$$= \frac{1}{4} \times 40 = 10 \text{ V}$$

11. *(d)* According to the question,



Let the charge of the particle be q and linear charge density of uniformly charged conductor be λ.

As, $$F_e = q \cdot E$$

$$= q\left(\frac{Kq}{R^2}\right) \quad \left(\because \lambda = \frac{dq}{dL}\right)$$

$$\Rightarrow \quad F_e = q\left(\frac{K\lambda}{R}\right)$$

Also, centripetal force $= \frac{mv^2}{R}$ ($\because$ circular motion)

$$\therefore \quad q\left(\frac{K\lambda}{R}\right) = \frac{mv^2}{R}$$

$$\Rightarrow \quad v \propto R$$

This means v is independent of radius R.

12. *(a)* Let C be the original capacitance.

$\therefore$ New, capacitance, $C' = \frac{4}{3}C$

Thickness, $$t = \frac{d}{2}$$

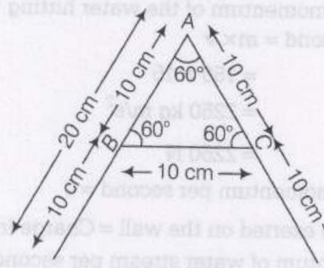
$$\Rightarrow \quad \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{4}{3}\frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad \frac{2}{\left(1 - \frac{1}{K}\right)} = \frac{4}{3}$$

$$\Rightarrow \quad K = 2$$

13. *(b)* Given, $r = 1\Omega \text{ cm}^{-1}$

Since, the angle subtended is $60°$ at the apex.



So, the resistance of each side would be 10Ω.

For ΔABC, the equivalent resistance, $r_{eq} = \frac{20}{3}$

$$= 6.67\Omega$$

Now, the effective resistance between the ends of the legs, $r'_{eq} = \frac{20}{3} + 10 + 10$

$$= \frac{80}{3} = 26.7\Omega$$

14. *(b)* Magnetic field at point C, due to wire SR and QP will be zero.

Magnetic field due to the wire QR will be,

$$B_1 = \frac{\pi}{2\pi}\left(\frac{\mu_0 I}{2R_1}\right)$$

Similarly, field at C due to arc SP,

$$B_2 = \frac{\pi}{2\pi}\left(\frac{\mu_0 I}{2R_2}\right)$$

$$\therefore \quad B = B_1 - B_2$$

$$= \frac{\mu_0 I}{4}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

15. *(c)* According to the question,

$$\phi = 5t^2 + 6t + 9$$

emf $e = -\frac{d\phi}{dt}$

$= -(10t + 6)$

At, $t = 3$ s,

$e_1 = -[10(3) + 6] = -36$ V

At, $t = 0$ s,

$e_2 = -[10(0) + 6] = -6$ V

$$\Rightarrow \frac{e_1}{e_2} = \frac{-36}{-6} = \frac{6}{1}$$

16. *(d)* Time period to cross the earth once along its diameter $= \frac{T}{2}$

$= \frac{1}{2}\left(2\pi\sqrt{\frac{R}{g}}\right)$

$= \pi\sqrt{\frac{R}{g}}$

where, R is the radius of the earth

$= 3.14 \times \sqrt{\frac{6.4 \times 10^6}{9.8}} = 2537.5$ s

$= 42.3$ min

17. *(a)* As, $\tan\theta = \frac{v^2}{rg}$

where, θ = the angle of banking.

$$\Rightarrow \tan\theta = \frac{30 \times 30}{10 \times 0.9 \times 10^3}$$

$= 0.1$

$$\Rightarrow \theta = \tan^{-1}(0.1) = 5.71 \approx 6°$$

18. *(c)* Moment of inertia of sphere about an axis through its centre, $I = \frac{2}{5}MR^2$.

Moment of inertia parallel to the above and tangent to the sphere, $I' = \frac{2}{5}MR^2 + MR^2$

$= \frac{7}{5}MR^2$

Radius of gyration, $K^2 = \frac{I'}{M} = \frac{7R^2}{5}$

$$\Rightarrow K = \sqrt{\frac{7}{5}}R$$

19. *(c)* Let the original length be l.

In first case,

$$l_1 = l + dl_1$$

Using Hooke's law, $x = \frac{T}{K}$,

where, x is the displacement.

We have, $l_1 = l + \frac{T_1}{K}$ ...(i)

Similarly, $l_2 = l + \frac{T_2}{K}$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$l = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$$

20. *(b)* Work done by the heart $= pdV$

Here, $dV = (60 \times 80 \times 10^{-6}\ \text{m}^3)$

$$p = (100 \text{ m of Hg}) \times \frac{1.01 \times 10^5 \text{ Pa}}{760 \text{ mm of Hg}}$$

$= 1.33 \times 10^4$ pa

As, power $= \frac{\text{Work done}}{\text{Time}}$

$= \frac{1.33 \times 10^4 \times 60 \times 80 \times 10^{-6}}{60}$

$= 1.06$ W

21. *(c)* For the longest wavelength in the Balmer series, $n_i = 3$ and $n_f = 2$

$$\therefore \frac{1}{\lambda} = R_H\left(\frac{1}{n_f} - \frac{1}{n_i}\right)$$

$= R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R_H}{36}$

$= \frac{5 \times 1.097 \times 10^7}{36}$

i.e. $\lambda = 6.563 \times 10^{-7}$ m $= 656.3$ nm

Energy of the photon, $E = h\nu$

$= \frac{hc}{\lambda}$ $\left[\because \nu = \frac{C}{\lambda}\right]$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{6.563 \times 10^{-7}}$

$= 3.027 \times 10^{-19}$ J $= \frac{3.027 \times 10^{-19}}{1.6 \times 10^{-19}}$

$= 1.89$ eV

22. *(b)* According to the question,

$$T_{Y_2}(X) = \tau(Y)$$

$$\Rightarrow \quad \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$$

$$\Rightarrow \quad \lambda_Y = \frac{\lambda_X}{0.693}$$

$$\Rightarrow \quad \lambda_Y > \lambda_X$$

This means, X will decay faster than Y.

23. *(c)* Here, $T_{1/2} = 270$ days.

After 540 days, it means two half lives have been completed i.e. $n = 2$.

Number of atoms, $N = \frac{m}{A}$ ...(i)

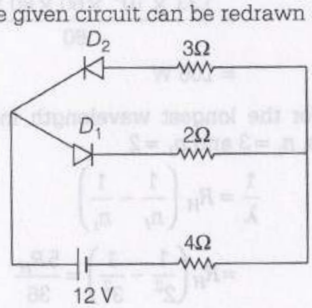
If $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad \frac{m}{m_0} = \left(\frac{1}{2}\right)^n \quad [\because \text{ from Eq. (i)}]$$

$$\Rightarrow \quad \frac{m}{100} = \frac{1}{4}$$

$$\Rightarrow \quad m = 25 \text{ mg}$$

24. *(a)* The given circuit can be redrawn as,



This means, D_2 is reversed based. Hence, this arm of the circuit will act as open arm. i.e. the close circuit will have two resistors 2Ω and 4Ω which will be in series connection.

$$\Rightarrow \quad R_{eq} = 2\Omega + 4\Omega = 6\Omega$$

So, current through 4Ω resistor will be,

$$I = \frac{V}{R} = \frac{12}{6} = 2\text{A}$$

25. *(a)* As we know,

Refractive index,

$$n = \frac{\text{Real depth } (d_{real})}{\text{Apparent depth } (d_{app})}$$

Given, $d_{\text{real}} = 9$ cm

$$\Rightarrow \quad 1.5 = \frac{9}{d_{\text{app}}}$$

$$\Rightarrow \quad d_{\text{app}} = 6 \text{ cm}$$

$\therefore$ Microscope has to be moved by $= d_{\text{real}} - d_{\text{app}}$

$$= 9 - 6 = 3 \text{ cm}$$

26. *(b)* Given, $A = 3 \times 10^{-6}$ m^2, $I = 5$ A,

$$M_{\text{Cu}} = 63.5 \text{ kg/K mol}$$

$$\rho_{\text{Cu}} = 8920 \text{ kg/m}^3$$

$$\text{Electron density} = \frac{6.023 \times 10^{23} \times 8920}{63.5} = 8.46 \times 10^{25} \text{ m}^{-3}$$

$$\therefore \text{Drift velocity, } v_d = \frac{I}{enA}$$

$$= \frac{5}{1.6 \times 10^{-19} \times 8.46 \times 10^{25} \times 3 \times 10^{-6}}$$

$$= \frac{5}{40.608} = 0.12 \text{ m/s}$$

27. *(c)* Let the distance from the conductor carrying larger current be x.

Therefore, the point at which the resultant magnetic field will be zero, will be at,

$$B_1 = B_2$$

where, B_1 and B_2 is the magnetic field produced by two parallel wires.

$$\Rightarrow \quad \frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(0.08 - x)}$$

$$\Rightarrow 5(0.08 - x) = 3x$$

$$\Rightarrow \quad x = 0.05 \text{ m}$$

28. *(b)* As a paramagnetic substance develop small magnetisation in the direction of the magnetic field. So, susceptibility $\left(\chi = \frac{M}{H}\right)$ will be small but a positive value. i.e., $\chi > 0$. Also, relative permeability, $\mu_r = 1 + \chi$, would also be positive but slightly greater than susceptibility i.e. $\mu_r > 1$.

29. *(d)* Given, $\frac{N_S}{N_P} = \frac{1}{10}$

$$V_P = 220 \text{ V}, I_P = 0.5 \text{ A}$$

We know that, $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

So, $V_S = V_P \frac{N_S}{N_P}$

$$= 220 \times \frac{1}{10} = 22 \text{ V}$$

Efficiency, $\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{I_S V_S}{I_P V_P}$

$$\frac{90}{100} = \frac{22 \times I_S}{0.5 \times 220}$$

$\Rightarrow$ $I_S = 4.5 \text{ A}$

30. *(c)* Given, $h_O = 1 \text{ cm}, = 0.01 \text{ m}, u = -0.1 \text{ m}$,

$$f = 0.05 \text{ m}$$

Using mirror formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v},$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{0.05} - \left(\frac{1}{-0.1}\right)$$

$$\frac{1}{V} = \frac{1}{0.05} + \frac{1}{0.5}$$

$\Rightarrow$ $V = 0.33 \times 10^{-1} \text{ m}$

Magnification, $m = -\frac{v}{u} = \frac{h_I}{h_O}$

$\Rightarrow$ $h_I = -\frac{v}{u} \times h_o$

$$= \frac{-0.33 \times 10^{-1}}{-0.1} \times 1 \times 10^{-2}$$

$\Rightarrow$ $0.33 \times 10^{-2} \text{ m} = 0.33 \text{ cm}$

31. *(b)* As, $F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r}\right)$

$\Rightarrow$ $\varepsilon_0 = \frac{q_1 q_2}{4\pi F r}$

$$= \frac{[\text{TA}]^2}{[\text{L}^2][\text{MLT}^{-2}]}$$

$$= [\text{M}^{-1}\text{L}^{-3}\text{T}^4\text{A}^2]$$

32. *(c)* Given, $a_1 = 3 \text{ m/s}^2, t_1 = 2\text{s}$

$$a_2 = 4 \text{ m/s}^2, t_2 = 3\text{s}$$

Average acceleration, $a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$

$$= \frac{3 \times 2 + 4 \times 3}{2 + 3}$$

$$= \frac{6 + 12}{5} = \frac{18}{5}$$

$$= 3.6 \text{ m/s}^2$$

33. *(c)* Time of flight, $T = \frac{2u \sin\theta}{g}$

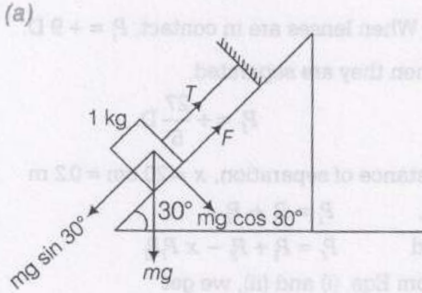
For θ, $T_1 = \frac{2u \sin\theta}{g}$ ...(i)

For $(90° - \theta)$, $T_2 = \frac{2u \sin(90° - \theta)}{g} = \frac{2u \cos\theta}{g}$ ...(ii)

dividing Eq. (i) by Eq. (ii), we get

$\therefore$ $\frac{T_1}{T_2} = \frac{2u \sin\theta}{g} \times \frac{g}{2u \cos\theta} = \frac{\sin\theta}{\cos\theta}$

34. *(a)*



From the above figure, we get

$$f = \mu N$$

$$= \mu\, mg \cos 30°$$

$$= 0.8 \times 1 \times 10 \times \frac{\sqrt{3}}{2}$$

$$= 4\sqrt{3}$$

$$= 6.92 \text{ N}$$

And $mg \sin 30° = 1 \times 10 \times \frac{1}{2} = 5 \text{ N}$

Also, $T + F = mg \sin 30°$

Since, the frictional force is more as compare to force ($mg \sin 30°$) i.e. 6.92 N > 5 N.,

Therefore, the tension in the string would be zero. No motion will take place.

35. *(a)* The weight of the body $= V\rho g$ $[\because m = \rho v]$

And upthrust $= V\sigma g$

Effective weight $= V(\rho - \sigma)g$

$\therefore$ The increase in potential energy $= V(\rho - \sigma)gh$

36. *(c)* According to the Brewster's Law

$$\mu = \tan\theta \quad \ldots(i)$$

here, μ is the relative refractive index.

$$\Rightarrow \quad \mu = \frac{\mu_{denser}}{\mu_{rarer}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \tan\theta = \frac{\mu_{denser}}{\mu_{rarer}}$$

$$\text{or} \quad \tan\theta = \frac{1}{\sin C_2} \times \frac{\sin C_1}{1}$$

$[\because \mu = \frac{1}{\sin C}$, where, C is the critical angle$]$

$$\Rightarrow \quad \tan\theta = \frac{\sin C_1}{\sin C_2}$$

37. *(a)* When lenses are in contact, $P_i = +9$ D.

When they are separated,

$$P_f = +\frac{27}{5} \text{ D.}$$

Distance of separation, $x = 20$ cm $= 0.2$ m

$$\text{So,} \quad P_i = P_1 + P_2 \quad \ldots(i)$$

$$\text{and} \quad P_f = P_1 + P_2 - x P_1 P_2 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$P_f = P_i - x P_1 P_2$$

$$\Rightarrow \quad -P_1 P_2 = \frac{1}{0.2}\left[\frac{27}{5} - 9\right]$$

$$-P_1 P_2 = \frac{-18}{0.2 \times 5}$$

$$\Rightarrow \quad P_1 P_2 = 18 \quad \ldots(iii)$$

$$\text{Also,} \quad P_1 + P_2 = 9 \quad \ldots(iv)$$

On solving Eqs. (iii) and (iv), we get

$$P_2^2 - 9P_2 + 18 = 0$$

Solving the above equations, we get

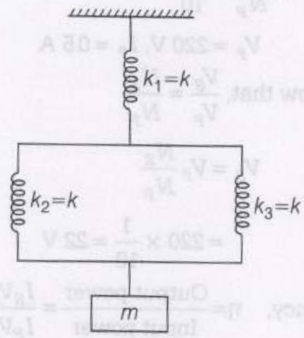
$$P_1 = 3 \text{ D}$$

and $P_2 = 6$ D

or $P_2 = 3$ D

and $P_1 = 6$ D.

38. *(b)* Consider the system as shown in figure below.



Here, k_2 and k_3 are in parallel. k_1 is in series with parallel combination of k_2 and k_3.

Effective spring constant of the combination is

$$k_{eff} = \frac{(k_2 + k_3)k_1}{k_1 + k_2 + k_3}$$

$$= \frac{2k \times k}{3k} = \frac{2}{3}k$$

$\therefore$ Time period of oscillation

$$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$$

$$= 2\pi\sqrt{\frac{m}{\frac{2}{3}k}}$$

$$= 2\pi\sqrt{\frac{3m}{2k}}$$

39. *(d)* Phase difference corresponding to path difference λ is

$$\Delta\phi = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

Intensity at the point having this phase difference 2π is

$$I = 4I_0 \cos^2\frac{\phi}{2}$$

$$= 4I_0\left(\cos\frac{2\pi}{2}\right)^2$$

$$= 4I_0(-1)^2 = 4I_0$$

Here, I_0 is intensity of individual source.

It is given that

$$I = 4I_0 = K$$

$$\Rightarrow \quad I_0 = \frac{K}{4} \quad \ldots(i)$$

Phase difference corresponding to path difference of $\frac{\lambda}{3}$ is

$$\Delta\phi' = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

Thus, intensity of light corresponding to this phase difference of $\frac{2\pi}{3}$ is

$$I' = 4I_0 \times \left(\cos\frac{2\pi}{3}\right)^2$$

$$= K \times \left[-\frac{1}{2}\right]^2 \quad \text{[from Eq. (i)]}$$

$$= \frac{K}{4}$$

40. *(a)* It is given that, temperature coefficient of resistance,

$$\alpha = 0.00125/°C$$

$$R_1 = 1\Omega, T_1 = 300\text{ K}$$

$$R_2 = 2\Omega, T_2 = ?$$

Now as we know that,

$$R_2 = R_1[1 + \alpha\Delta T]$$

$$\Rightarrow 2 = 1[1 + 0.00125 \times (T_2 - T_1)]$$

$$\Rightarrow 2 = 1 + 125 \times 10^{-5} \times (T_2 - 300)$$

$$\Rightarrow \frac{1 \times 10^5}{125} = T_2 - 300 \Rightarrow T_2 = 1100\text{ K}$$

The closest option is (a).

41. *(c)* Let specific heats of liquids A and B are respectively S_A and S_B.

It is given that

$$T_A = 32°C, T_B = 24°C$$

and $T_{mix} = 28°\text{ C}$

When these liquids are mixed, then

heat lost by A = heat gained by B

$$\Rightarrow mS_A\Delta T = mS_B\Delta T'$$

$$\Rightarrow mS_A(T_A - T_{mix}) = mS_B(T_{mix} - T_B)$$

$$\Rightarrow S_A(32 - 28) = S_B(28 - 24)$$

$$\Rightarrow S_A \times 4 = S_B \times 4$$

$$\Rightarrow S_A : S_B = 1 : 1$$

42. *(b)* Applying ideal gas equation, we have

$$pV = nRT$$

where, p = pressure of the gas
V = volume of the gas
n = number of moles
R = gas constant
T = temperature of the gas

$$\therefore \quad n_1 = \frac{p_1V_1}{RT_1}$$

and $$n_2 = \frac{p_2V_2}{RT_2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{p_2V_2}{p_1V_1} \times \frac{T_1}{T_2} = \left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right)\left(\frac{T_1}{T_2}\right)$$

$$\frac{n_2}{n_1} = (2)\left(\frac{1}{2}\right)(2)$$

$$n_2 = 2n_1$$

43. *(b)* RMS speed of gas is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where, R = gas constant
T = absolute temperature in Kelvin
M = molecular mass of the gas

$$\therefore \frac{(v_{rms})\text{ oxygen}}{(v_{rms})\text{ hydrogen}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}}$$

$$= \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$\Rightarrow \frac{500}{(v_{rms})\text{ hydrogen}} = \frac{1}{4}$$

$$\Rightarrow (v_{rms})\text{ hydorgen} = 2000\text{ m/s}$$

44. *(c)* Let the distance between the two points is Δx.

It is given that,

Phase difference, $\Delta\phi = 60° = \frac{\pi}{3}$ rad.

Since, $$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

$$\Rightarrow \frac{\pi}{3} = \frac{2\pi}{\lambda} \times \Delta x \quad \text{...(i)}$$

Also we know that,

$$f = \frac{v}{\lambda}$$

$$\Rightarrow \lambda = \frac{v}{f}$$

$$\Rightarrow \lambda = \frac{360}{500} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\pi}{3} = 2\pi \times \frac{500}{360} \times \Delta x$$

$$\frac{360}{6} \times \frac{1}{500} = \Delta x$$

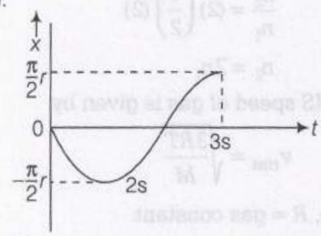
$$\Rightarrow \Delta x = 0.12\text{ m}$$

45. (c) Given, $x = r\cos\frac{\pi t}{2}$

$$\Rightarrow \quad v = \frac{dx}{dt}$$

$$= -\frac{\pi}{2} r \sin\frac{\pi t}{2}$$

Graphically it can be represented as shown in figure.



Distance covered by the particle

= Magnitude of area of $x - t$ curve between 0 to 2s and 2 to 3s

$$= \int_0^2 vdt + \int_2^3 vdt$$

$$= \left| -\int_0^2 \frac{\pi}{2} r \sin\frac{\pi}{2} tdt \right| + \left| \int_2^3 -\frac{\pi}{2} r \sin\frac{\pi}{2} tdt \right|$$

$$= \left| \frac{-\pi}{2} r \int_0^2 \sin\frac{\pi t}{2} dt \right| + \left| -\frac{\pi}{2} r \int_2^3 \sin\frac{\pi}{2} tdt \right|$$

$$= \left| \left[\frac{\pi r}{2} \frac{\cos\frac{\pi t}{2}}{\frac{\pi}{2}} \right]_0^2 \right| + \left| \left[\frac{\pi r}{2} \frac{\cos\frac{\pi}{2} t}{\frac{\pi}{2}} \right]_2^3 \right|$$

$$= |r(\cos\pi - \cos 0)| + \left| r\left(\cos\frac{3\pi}{2} - \cos\pi\right) \right|$$

$$= r|-1-1| + r|0-(-1)| = 2r + r = 3r$$

46. (c) Given, AC voltage is

$$V = V_0 \sin\omega t$$

Capacitance, $C = C_0$

RMS value of the voltage,

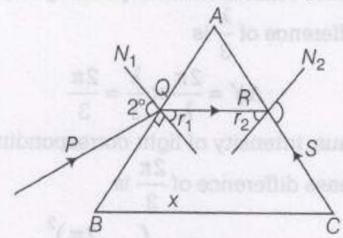
$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

AC ammeter reads rms value

$$\therefore \quad I = \frac{V_{\text{rms}}}{X_C} = \frac{V_0}{\sqrt{2}} \times \frac{1}{\frac{1}{\omega C}}$$

$$= \frac{V_0}{\sqrt{2}} \times \omega C = \frac{V_0 \omega C}{\sqrt{2}}$$

47. (d) Consider the situation shown in figure.



Light rays PQ is incident on the face AB. It goes along QR and then along RS.

Here,

i = angle of incidence at the face AB

r_1 = angle of refraction through AB

r_2 = angle of incidence at AC

n = refractive index of the material of the prims.

For the prism, we have

$$r_1 + r_2 = A \quad \text{...(i)}$$

Now for light not to emerge from the face AC,

$$r_2 > C$$

Similarly for no emergence from the face AB,

$$r_1 > C$$

$$\Rightarrow \quad r_1 + r_2 > 2C$$

Here, C is critical angle.

$$\Rightarrow \quad A > 2C \quad [\because r_1 + r_2 = A]$$

$$\Rightarrow \quad \frac{A}{2} > C$$

$$\Rightarrow \quad \sin\frac{A}{2} > \sin C$$

$$\Rightarrow \quad \sin\frac{A}{2} > \frac{1}{n}$$

$$\Rightarrow \quad n > \frac{1}{\sin\frac{A}{2}}$$

$$\Rightarrow \quad n > \text{cosec}\left(\frac{A}{2}\right)$$

48. (a) Critical angle of glass-water interface is given by

$$\sin C_{g/w} = \frac{1}{\frac{\mu_g}{\mu_w}} = \frac{\mu_w}{\mu_g} \quad \text{...(i)}$$

Also we can write,

For glass, $\sin C_g = \frac{1}{\mu_g}$

for water, $\sin C_w = \dfrac{1}{\mu_w}$

$$\therefore \quad \sin C_{g/w} = \frac{\sin C_g}{\sin C_w}$$

$$= \frac{\sin 41°48'}{\sin 48°36'}$$

To convert minute into degree: Minute is multiplied by $1/60°$.

Now, we get

$$\frac{\sin 41.8°}{\sin 48.6°} = 0.89$$

$$\Rightarrow \quad C_{g/w} = 62.87°$$

$$\approx 62.52'$$

Hence, closest option is (a).

49. *(b)* Let intensity of two sources in YDSE be I_1 and I_2. Assume phase difference between the sources is ϕ.

Resultant intensity in YDSE is given by

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos\phi \quad \ldots\text{(i)}$$

It is given that, $a_1 = 2a_2$

$$\Rightarrow \quad a_1^2 = 4a_2^2$$

$$\Rightarrow \quad I_1 = 4I_2 \quad [\because I \propto a^2] \quad \ldots\text{(ii)}$$

Now from Eqs. (i) and (ii), we have

$$I = 4I_2 + I_2 + 2 \times 2 \times I_2 \cos\phi$$

$$I = 5I_2 + 4I_2 \cos\phi \quad \ldots\text{(iii)}$$

Given,

$$I_m = I_{\max}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2$$

$$\Rightarrow \quad I_m = (2\sqrt{I_2} + \sqrt{I_2})^2$$

$$I_m = 9I_2$$

$$I_2 = \frac{I_m}{9} \quad \ldots\text{(iv)}$$

Now from Eqs. (iii) and (iv), we get

$$I = 5 \times \left(\frac{I_m}{9}\right) + 4 \times \frac{I_m}{9}\cos\phi$$

$$= \frac{4I_m}{9} + \frac{4I_m}{9}\cos\phi + \frac{I_m}{9}$$

$$= \frac{4I_m}{9}[1 + \cos\phi] + \frac{I_m}{9}$$

$$= \frac{4I_m}{9}\left[2\cos^2\frac{\phi}{2}\right] + \frac{I_m}{9}$$

$$[\because 1 + \cos\phi = 2\cos^2\phi/2]$$

$$= \frac{I_m}{9}\left[8\cos^2\frac{\phi}{2} + 1\right]$$

$$= \frac{I_m}{9}\left[1 + 8\cos^2\frac{\phi}{2}\right]$$

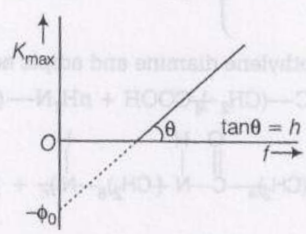
50. *(b)* From Einstein's photoelectric equation, we have

$$K_{\max} = hf - \phi_0 \quad \ldots\text{(i)}$$

Here, ϕ_0 = work function of the surface

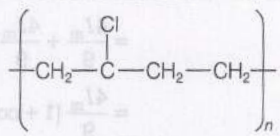
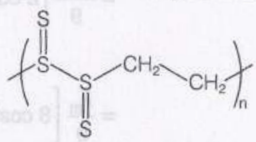
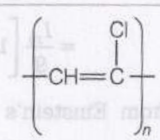
= constant

From Eq. (i), graph of $K_{\max}$ v/s f is

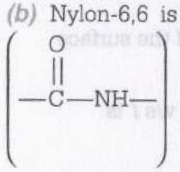


Chemistry

1. *(b)* Among the given options only teflon has fully fluorinated monomer and hence the fully fluorinated polymer.

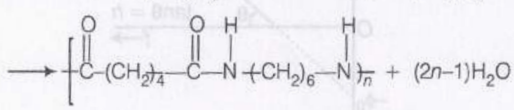
Polymer	Structure
Neoprene	
Teflon	$-(CF_2-CF_2)_n-$
Thiocol	
PVC	

2. *(b)* Nylon-6,6 is polyamide as it has an amide

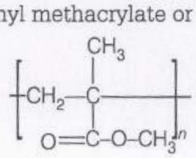


linkage. It is a polymer of hexamethylene diamine and adipic acid

$$n\text{HOOC}-(CH_2)_4-\text{COOH} + nH_2N-(CH_2)_6-NH_2$$



3. *(b)* Polymethyl methacrylate or PMMA



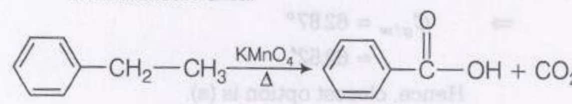
is commercially known as plexiglass. It is used to manufacture window pane, toys etc. Glyptal is polymer of ethylene glycol and phthalic acid. It used in making of paints and lacquers.

Polystyrene is the polymer of styrene and it is used as insulator, toys, household articles. Polyacrylonitrile (PAN) is commercially known as orlon and is used in making of synthetic fibres and synthetic wool.

4. *(c)* The intermolecular forces of attraction are weakest in natural rubber. It is a randomly oriented amorphous polymer without much cross-linking. As these can be easily stretched out, these are also called elastomers.

5. *(d)* The monomer of orlon, also called polyacrylonitrile (PAN) is acrylonitrile ($CH_2=CHCN$).

6. *(c)* Ethyl benzene, like, methyl benzene (toluene), xylene and phenol, gets oxidised by $KMnO_4$ to form benzoic acid.



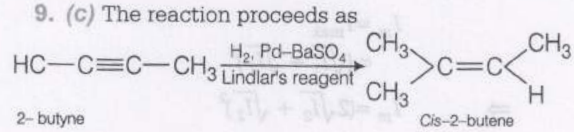
7. *(a)* Alkyl halides when reacts with dialkyl copper reagents (like R_2CuLi) then alkane is formed and this process is called Corey-House synthesis.

$$R'-X + Li^+[R-Cu-R]^- \rightarrow R-R' + RCu + Li^+X^-$$

8. *(d)* Deactivating strength of *meta*-directing substituents (or electron with drawing group -EWG) decreases with decrease in $-I$ effect, which follows the order as

$$R_4N^{\oplus} > -NO_2 > -CN > -SO_3H > -COOH > -F > -Cl > -I > -OR > -OH > -NH_2 > -C_6H_5$$

9. *(c)* The reaction proceeds as



10. *(d)* The tendency to knock decrease in the following order.

Straight chain alkanes > branched chain alkanes > olefins > cycloalkanes > aromatic hydrocarbons.

11. *(c)* More is the number of dissociated ions formed by an electrolyte, more is the coagulating effect of the electrolyte. Hence, $Al_2(SO_4)_3$ is the correct answer as it form 5(2 + 3) ions.

12. *(c)* When an atom or molecule get adsorbed on the surface of an adsorbent a low amount of heat is released. Hence, enthalpy of adsorption is low (20 − 40 kJ/mol) but have negative value.

13. *(a)* More is the gold number less is the protective power of colloids. Hence, the correct order is as follows

$$A < C < B < D$$
$$0.50 \quad 0.10 \quad 0.01 \quad 0.005$$

14. *(b)* Sulphur sol being hydrophobic, is formed by the oxidation of H_2S gas as follows

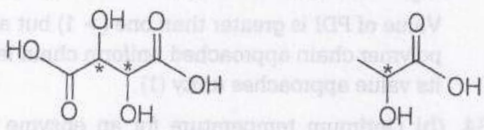
$$2H_2S + O_2 \longrightarrow 2H_2O + 2S$$

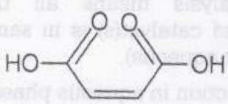
Direct addition of water does not form sol due to its hydrophobic nature.

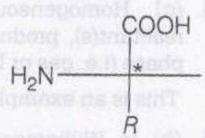
15. *(b)* Reaction of acetylene with sodium in liquid ammonia proceeds as

$$H—C\equiv C—H + 2Na \xrightarrow{\text{Liq. } NH_3} CH_2 = CH_2 + 2NaNH_2$$

16. *(c)* Structure of these acids are







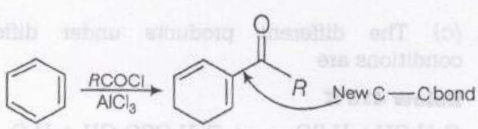
α = amino acid

All of these acids except maleic acid have at least one chiral carbon (*) hence these are optically active but maleic acid is not.

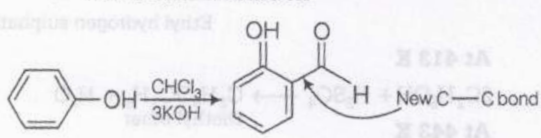
17. *(d)* Wurtz reaction

$$2R—X + 2Na \longrightarrow R\overset{\text{New C — C bond}}{—}R + 2Na^+X^-$$

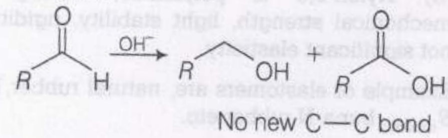
Friedel-Craft acylationation



Reimer Tiemann reaction



Cannizzaro reaction



18. *(d)* $E^\circ_{cell} = \frac{0.059}{n} \log K_C$

$$\Rightarrow \quad 0.59 = \frac{0.059}{3} \log K_C \Rightarrow \log K_C = 30$$

$$\Rightarrow \quad K_C = \text{antilog } 30 = 10^{30}$$

19. *(c)* We have,

$$pH = \log [H^+]$$

or $\quad 10 = \log [H^+]$

For hydrogen electrode,

$$K_C = [H^+]$$

and $\log K_C = \log [H^+] = 10$

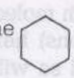
$$\therefore \quad E^\circ_{cell} = -\frac{0.059}{n} \times \log [H^+] = -\frac{0.059}{1} \times 10$$

$$= -0.59 \text{ V}$$

20. *(a)* Specific conductance of a solution is directly proportional to its concentration.

21. *(b)* Acetophenone $\left(CH_3—\overset{O}{\overset{\|}{C}}—CH_3 \right)$ is a ketone.

Diastase is an enzyme that converts starch into maltose.

Cycloheptane  is a cyclic aliphatic compound.

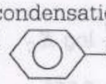
22. *(c)* Tollen's reagent is used to detect the presence of aldehyde.

23. *(b)* Few examples are

Addition Polymer	Condensation Polymer
Polythene	Nylon-6, 6
Polyprene	Nylon-6
Neoprene	Terylene, glyptal
Polyvinyl chloride (PVC)	Bakelite
Teflon etc	Melamine etc

24. (d) Nylon-6,6 is polyamide, having high mechanical strength, light stability, rigidity but not significant elasticity.

Example of elastomers are, natural rubber, Buna-S_{rubber}, buna-N rubber etc.

25. (a) Bakelite is a condensation polymer formed of monomers phenol  OH or (C_6H_5OH)

and formaldehyde (HCHO).

26. (c) Total mass of = 120 + 1000 = 1120 g

Density of solution = 1.15 g/mL

$$\text{Volume of solution} = \frac{\text{mass}}{\text{density}} = \frac{1120}{1.15} = 973.15 \text{mL}$$

$$\text{Molarity} = \frac{W \times 100^0}{\text{m} \times \text{volume (in mL)}}$$

$$= \frac{120 \times 1000}{60 \times 973.15} = 2.05 \text{ M}$$

27. (d) Relation between mole fraction (χ) of solute and molality (m) is

$$\chi = \frac{m M_1}{(1000 + mM_1)}$$

Here, M_1 = molecular mass of the solvent.

$$\therefore \quad \chi = \frac{5.2 \times 18}{1000 + 5.2 \times 18}$$

$$= \frac{93.6}{1093.6}$$

$$= 0.086$$

28. (a) In case of NaCl, each molecule will dissociate to form two entities (ions) hence, its van't Hoff factor will increase and so will be its colligative properties. This will result in boiling of solution at higher temperature.

29. (b) $CH_3-\overset{\overset{\large O}{||}}{C}-CH_3 + H-CN \longrightarrow CH_3-\underset{\underset{\large CN}{|}}{\overset{\overset{\large OH}{|}}{C}}-CH_3$

Acetone, Hydrogen cyanide

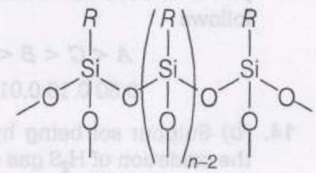
30. (c) Both the statements are true.

31. (c) Glass is referred to as super cooled liquid as it does not exist in crystalline form but in amorphous form, in which molecules are allowed to move very slowly.

32. (d) Preparation silicone takes place as

$$RSiCl_3 + 3H_2O \longrightarrow RSi(OH)_3 + 3HCl$$

$$nRSi(OH)_3 \xrightarrow{(n-1)H_2O}$$



Silicone

33. (d) Polydispersity index (PDI) is the measure of the distribution of molecular mass in a given polymer sample. It is calculated as

$$\text{PDI} = \frac{M_w}{M_n}$$

Where, M_w is the weight average molecular weight and

M_n is the number average molecular weight.

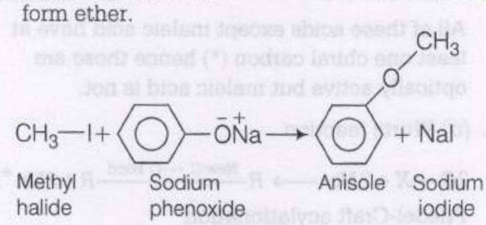
Value of PDI is greater than one (> 1) but as the polymer chain approached uniform chain length its value approaches unity (1).

34. (b) Optimum temperature for an enzyme is the temperature at which it is most active.

35. (c) Homogeneous catalysis means all the reactant(s), product(s) and catalyst(s) is in same phase (i.e. gas or liquid or aqueous).

This is an example of reaction in aqueous phase.

36. (b) In Williamson's synthesis alkyl halide and sodium alkoxide (or sodium phenoxide) react to form ether.



37. (c) The different products under different conditions are

Below 413 K

$$C_2H_5OH + H_2SO_4 \longrightarrow C_2H_5OSO_2OH + H_2O$$

(or $C_2H_5 \cdot HSO_4$)

Ethyl hydrogen sulphate

At 413 K

$$2C_2H_5OH + H_2SO_4 \longrightarrow C_2H_5OC_2H_5 + H_2O$$

Diethyl ether

At 443 K

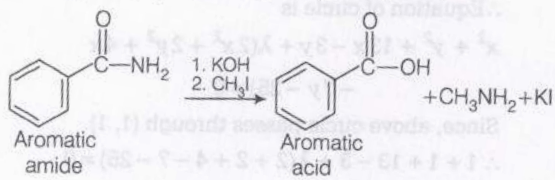
$$C_2H_5OH + H_2SO_4 \longrightarrow CH_2 = CH_2 + H_2O$$

Ethylene

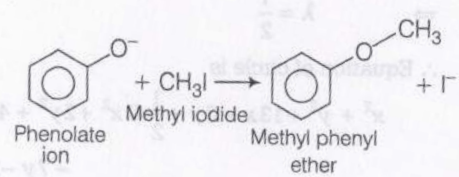
38. *(a)* Reaction with conc. HCl and anhy. $ZnCl_2$ with alcohol is the Lucas test for differentiation between 1°, 2° and 3° alcohols.

It has no effect on primary alcohol in secondary alcohols give turbidity slowly while tertiary alcohols (like 2-methyl-propan-2-ol) gives turbidity immediately.

39. *(a)* Hydrolysis of amide forms acids as

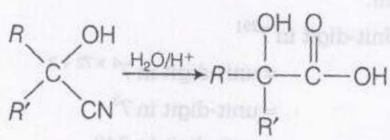


40. *(a)*



41. *(b)* Adipic acid (a dioic acid) and ethylene glycol (adiol) polymerise together to form a polyester called polyethlyene adipate (PEA).

42. *(a)* Hydrolysis of cyanohydrin derivative produces carboxylic acid as



43. *(d)* Compounds like aldehydes, ketones, carboxylic acids, esters etc have a carbonyl ($>C=O$) group which consists a carbon-oxygen double bond. While compounds like ethers, alcohols, phenols have a carbon-oxygen single (C— O) bond.

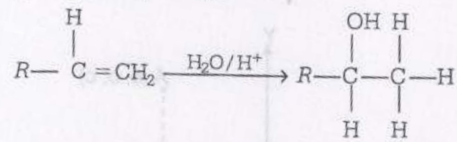
44. *(b)* $C_2H_5—Cl + \overset{+}{Na}\overset{-}{O}C_2H_5$
Chloro ethane Sodium ethanoate

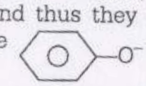
$\longrightarrow C_2H_5—O—C_2H_5 + Na^+\overset{-}{Cl}$
Diethyl ether Sodium chloride

45. *(a)* The $AgNO_3$ test is used to detect halide ions.

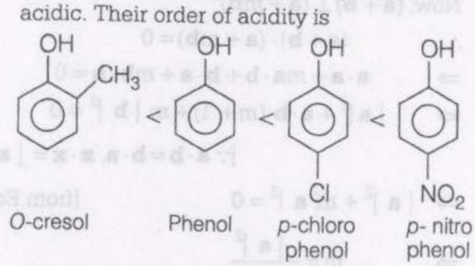
$$R—X + AgNO_3 + H_2O \longrightarrow R—OH + \overset{+}{Ag}X\downarrow + HNO_3$$

46. *(a)* Ethene being a 2-carbon compound always forms primary alcohol

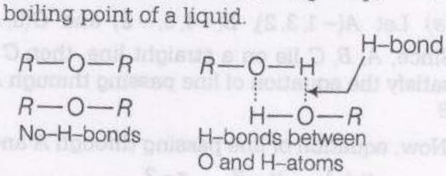


47. *(b)* Nitro (—NO_2) and chloro (— Cl) group has negative inductive effect (—NO_2 having more than — Cl) and thus they pull electron density from phenolate  ring making release of proton easier and the compound more acidic.

On the other hand methyl (— CH_3) group has positive inductive effect making cresol less acidic. Their order of acidity is



48. *(d)* Intermolecular H-bonding helps to increase boiling point of a liquid.



49. *(c)* The compound can be tertiary butyl chloride only since this compound has primary hydrogen atoms only.

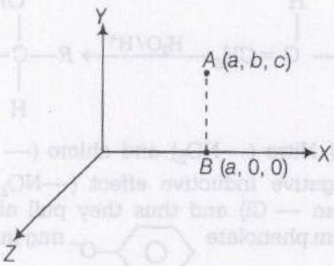
50. *(c)* Primary alcohol upon oxidation forms carboxylic acid while oxidation of secondary alcohol forms ketone.

e.g $CH_3OH \xrightarrow{[O]}$ H— COOH
(Molecular weight = 32) (Molecular weight = 46)
(Higher by 14 units)

Mathematics

1. *(b)* Distance of the point $A(a, b, c)$ from the X-axis

$$= \sqrt{(a-a)^2 + (b-0)^2 + (c-0)^2}$$
$$= \sqrt{b^2 + c^2}$$



2. *(d)* We have, $\mathbf{a} \perp \mathbf{b}$

$\therefore \quad \mathbf{a} \cdot \mathbf{b} = 0 \quad$ ...(i)

Now, $(\mathbf{a} + \mathbf{b}) \perp (\mathbf{a} + m\mathbf{b})$

$\therefore \quad (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + m\mathbf{b}) = 0$

$\Rightarrow \quad \mathbf{a} \cdot \mathbf{a} + m\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} + m\mathbf{b} \cdot \mathbf{b} = 0$

$\Rightarrow \quad |\mathbf{a}|^2 + \mathbf{a} \cdot \mathbf{b}\,(m+1) + m\,|\mathbf{b}|^2 = 0$

$[\because \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}, \mathbf{x} \cdot \mathbf{x} = |\mathbf{x}|^2]$

$\Rightarrow \quad |\mathbf{a}|^2 + m|\mathbf{a}|^2 = 0 \quad$ [from Eq. (i)]

$\Rightarrow \quad m = -\frac{|\mathbf{a}|^2}{|\mathbf{b}|^2}$

3. *(a)* Let $A(-1, 3, 2)$, $B(-4, 3, -2)$ and $C(5, l, m)$.

Since, A, B, C lie on a straight line, then C must satisfy the equation of line passing through A and B.

Now, equation of line passing through A and B is

$$\frac{x+1}{-4+1} = \frac{y-3}{3-3} = \frac{z-2}{-2-2}$$

$\Rightarrow \quad \frac{x+1}{-3} = \frac{y-3}{0} = \frac{z-2}{-4}$

Now, point C satisfy above equation.

$\therefore \quad \frac{5+1}{-3} = \frac{l-3}{0} = \frac{m-2}{-4}$

$\Rightarrow \quad -2 = \frac{l-3}{0} = \frac{m-2}{-4}$

$\therefore \quad l - 3 = 0 \Rightarrow l = 3$

and $\quad m - 2 = 8 \Rightarrow m = 10$

4. *(a)* Equation of circle passing through the points of intersection of the circles.

$S_1 : x^2 + y^2 + 13x - 3y = 0$ and

$S_2 : 2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is given by

$S : S_1 + \lambda S_2 = 0$

$\therefore$ Equation of circle is

$$x^2 + y^2 + 13x - 3y + \lambda(2x^2 + 2y^2 + 4x - 7y - 25) = 0$$

Since, above circle passes through (1, 1).

$\therefore 1 + 1 + 13 - 3 + \lambda(2 + 2 + 4 - 7 - 25) = 0$

$12 + \lambda(-24) = 0$

$\Rightarrow \quad \lambda = \frac{1}{2}$

$\therefore$ Equation of circle is

$$x^2 + y^2 + 13x - 3y + \frac{1}{2}(2x^2 + 2y^2 + 4x - 7y - 25) = 0$$

$\Rightarrow \quad 2x^2 + 2y^2 + 26x - 6y + 2x^2 + 2y^2 + 4x - 7y - 25 = 0$

$\Rightarrow \quad 4x^2 + 4y^2 + 30x - 13y - 25 = 0$

5. *(c)* After every fourth power the unit digit repeats itself.

$\therefore$ Unit-digit in 7^{291}

$= \text{unit-digit in } 7^{4 \times 72 + 3}$

$= \text{unit-digit in } 7^3$

$= \text{unit-digit in } 343$

$= 3$

6. *(*)* We have,

$$\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)$$

On differentiating with respect to x, we get

$$\frac{-b}{\sqrt{b^2 - y^2}} \frac{y_1}{b} = \frac{1}{(x/n)} \frac{1}{n}$$

$$-xy_1 = \sqrt{b^2 - y^2} \quad \text{...(i)}$$

On differentiating again with respect to x_1, we get

$$-[y_1 + xy_2] = \frac{1}{2\sqrt{b^2 - y^2}}(-2yy_1)$$

$$-[y_1 + xy_2] = -\frac{yy_1}{\sqrt{b^2 - y^2}}$$

$$-[y_1 + xy_2] = \frac{y}{x}$$

$$x^2 y_2 + xy_1 = -y$$

7. *(b)* The equations of the circles are

$$x^2 + y^2 = 25 \quad \ldots(i)$$

$$x^2 + y^2 - 2x + 3y - 43 = 0 \quad \ldots(ii)$$

The centres of these circles are $C_1(0, 0)$ and $C_2\left(1, -\frac{3}{2}\right)$ respectively.

Given point is $P(-3, 4)$.

$$\therefore \quad C_1C_2 = \frac{\sqrt{13}}{2}, C_1P = 5, C_2P = \frac{\sqrt{185}}{2}$$

$$\therefore \cos \angle C_1PC_2 = \frac{C_1P^2 + C_2P^2 - C_1C_2^2}{2C_1P \cdot C_2P}$$

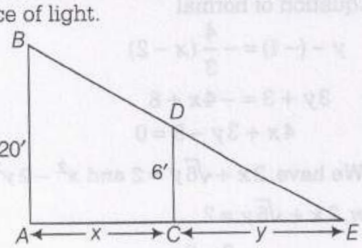
$$= \frac{25 + \frac{185}{4} - \frac{13}{4}}{2 \times 5 \times \frac{\sqrt{185}}{2}}$$

$$= \frac{68}{5\sqrt{185}}$$

$$\therefore \tan \angle C_1PC_2 = \frac{1}{68}$$

$$\Rightarrow \quad \angle C_1PC_2 = \tan^{-1}\left(\frac{1}{68}\right)$$

8. *(a)* Let $CE = y$ be the length of the shadow of the man at time t when he is x feet away from the source of light.



It is given that $\frac{dx}{dt} = 4$ m/h

Clearly, $\Delta ABE \sim \Delta CDE$

$$\Rightarrow \quad \frac{AB}{CD} = \frac{AE}{CE}$$

$$\Rightarrow \quad \frac{20}{6} = \frac{x + y}{y}$$

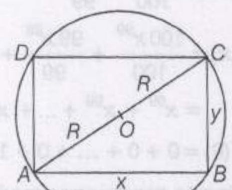
$$\Rightarrow \quad 20y = 6x + 6y$$

$$\Rightarrow \quad 14y = 6x$$

$$\Rightarrow \quad y = \frac{6}{14}x = \frac{3}{7}x$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{3}{7}\frac{dx}{dt} = \frac{3}{7} \times 4 = \frac{12}{7} \text{ m/h}$$

9. *(a)* Let A be the area of rectangle $ABCD$ whose sides are x and y.



$$\therefore \quad A = xy$$

$$= x\sqrt{4R^2 - x^2} \quad [\because x^2 + y^2 = 4R^2]$$

$$= x\sqrt{4(2)^2 - x^2} \quad [\because R = 2 \text{ units}]$$

$$= x\sqrt{16 - x^2}$$

$$\therefore \quad \frac{dA}{dx} = 1\sqrt{16 - x^2} + x \cdot \frac{1}{2\sqrt{16 - x^2}}(-2x)$$

$$= \sqrt{16 - x^2} - \frac{x^2}{\sqrt{16 - x^2}}$$

For maximum, $\frac{dA}{dx} = 0$

$$\therefore \quad \sqrt{16 - x^2} = \frac{x^2}{\sqrt{16 - x^2}}$$

$$\Rightarrow 16 - x^2 = x^2 \Rightarrow x = 2\sqrt{2} \text{ units}$$

Again, $$\frac{d^2A}{dx^2} = \frac{1}{2\sqrt{16 - x^2}}(-2x) - \left[\frac{2x\sqrt{16 - x^2} - \frac{x^2}{2\sqrt{16 - x^2}}(-2x)}{(16 - x^2)}\right]$$

$$\therefore \quad \left.\frac{d^2A}{dx^2}\right|_{2\sqrt{2}} = -\text{ve}$$

$\therefore$ Maximum area $= x\sqrt{16 - x^2}$

$$= 2\sqrt{2}\sqrt{16 - (2\sqrt{2})^2}$$
$$= 2\sqrt{2}\sqrt{16 - 8}$$
$$= 2\sqrt{2}\sqrt{8}$$
$$= 2\sqrt{2} \times 2\sqrt{2}$$
$$= 8 \text{ sq units}$$

10. *(b)* We have,

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$$

$$\therefore \quad f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \dots + \frac{2x}{2} + 1 + 0$$
$$= x^{99} + x^{98} + \dots + x + 1$$

$$\therefore \quad f'(0) = 0 + 0 + \dots + 0 + 1 = 1$$

11. *(d)* According to congruence modulo,

if $\quad a \equiv b \pmod{m}$

$\Rightarrow m$ divides $a - b \Rightarrow \frac{m}{a-b}$

(a) $(a + x) \equiv (b + x) \pmod{m}$

$\Rightarrow \frac{m}{(a+x)-(b+x)} \Rightarrow \frac{m}{a-b}$ which is correct.

(b) $(a - x) \equiv (b - x) \pmod{m}$

$\Rightarrow \frac{m}{(a-x)-(b-x)}$

$\Rightarrow \frac{m}{(a-b)}$ which is correct.

(c) $ax \equiv bx \pmod{m}$

$\Rightarrow \frac{m}{ax - bx} \Rightarrow \frac{m}{x(a-b)}$

$\Rightarrow m$ divides $x(a-b)$, since x is an integer, which is correct.

(d) $(a \div x) \equiv (b \div x) \pmod{m}$

$\Rightarrow \frac{m}{(a/x - b/x)} \Rightarrow \frac{mx}{(a-b)}$

$\Rightarrow mx$ divides $a - b$ which is not true.

12. *(a)* Suppose $a = 3$ and $b = 2$,

then $a^2 - b^2 = (3)^2 - (2)^2$

$= 9 - 4 = 5$ which is prime

Now, $\quad a + b = 3 + 2 = 5$

$\therefore \quad a^2 - b^2 = a + b$

13. *(c)* We know that multiplicative in these of natural number does not exist.

i.e. $\quad a \in N \not\Rightarrow a^{-1} \in N$

14. *(b)* Given $f = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 3\,2\,4\,5\,1\,6 \end{pmatrix}$

$$\Rightarrow \quad f = (1\,3\,4\,5)$$
$$\Rightarrow \quad f^{-1} = (1\,3\,4\,5)^{-1}$$
$$= (5\,4\,3\,1)$$
$$= \begin{pmatrix} 5\,4\,3\,1 \\ 4\,3\,1\,5 \end{pmatrix}$$
$$= \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 5\,2\,1\,3\,4\,6 \end{pmatrix}$$

15. *(c)* If $ax = b$ and $ya = b$, then it has unique solution in G.

16. *(a)* Given equation of ellipse is

$$x^2 + 2y^2 + 2x - 4y - 14 = 0$$

On differentiation both sides, we get

$$2x + 4yy' + 2 - 4y' = 0$$
$$\Rightarrow \quad (4y - 4)y' = -(2x + 2)$$
$$\Rightarrow \quad y' = \frac{-2(x+1)}{4(y-1)}$$
$$\therefore \quad \left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{-2(2+1)}{4(-1-1)} = \frac{-6}{-8} = \frac{3}{4}$$

$\therefore$ Equation of tangent

$$y - (-1) = \frac{3}{4}(x - 2)$$
$$\Rightarrow \quad 4y + 4 = 3x - 6 \Rightarrow 3x - 4y - 10 = 0$$

$\therefore$ Equation of normal

$$y - (-1) = -\frac{4}{3}(x - 2)$$
$$\Rightarrow \quad 3y + 3 = -4x + 8$$
$$\Rightarrow \quad 4x + 3y - 5 = 0$$

17. *(c)* We have, $2x + \sqrt{6}y = 2$ and $x^2 - 2y^2 = 4$

Now, $2x + \sqrt{6}y = 2$

$$\Rightarrow \quad y = \frac{2 - 2x}{\sqrt{6}}$$

Putting, $y = \frac{2-2x}{\sqrt{6}}$ in the equation of hyperbola, we get

$$x^2 - 2\left[\frac{2 - 2x}{\sqrt{6}}\right]^2 = 4$$

$$\Rightarrow \quad x^2 - 2\left(\frac{2(1-x)}{\sqrt{6}}\right)^2 = 4$$

$$\Rightarrow \quad x^2 - \frac{8}{6}(1-x)^2 = 4$$

$$\Rightarrow \quad 6x^2 - 8(1-x)^2 = 24$$

$$\Rightarrow \quad 6x^2 - 8[1 + x^2 - 2x] = 24$$

$$\Rightarrow \quad 6x^2 - 8 - 8x^2 + 16x = 24$$

$$\Rightarrow \quad -2x^2 + 16x - 32 = 0$$

$$\Rightarrow \quad x^2 - 8x + 16 = 0$$

$$\Rightarrow \quad (x-4)^2 = 0$$

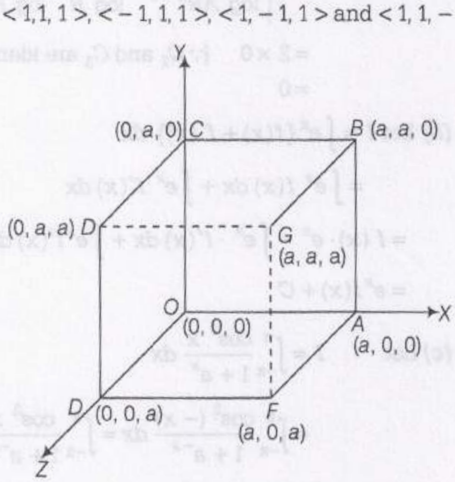
$$\Rightarrow \quad x = 4$$

$$\therefore \quad y = \frac{2 - 2\times 4}{\sqrt{6}} = \frac{2-8}{\sqrt{6}} = -\sqrt{6}$$

$\therefore$ Point of contact is $(4, -\sqrt{6})$.

18. *(b)* Let the side of the cube be a. Then, dr' of the diagonals will be

$< 1, 1, 1 >, < -1, 1, 1 >, < 1, -1, 1 >$ and $< 1, 1, -1 >$



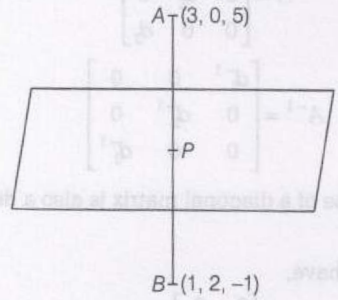
$\therefore$ Angle between any two diagonal will be

$$\cos\theta = \frac{1\times(-1) + 1\times 1 + 1\times 1}{\sqrt{1^2+1^2+1^2}\sqrt{(-1)^2+1^2+1^2}}$$

$$= \frac{1}{3}$$

$$\therefore \quad \theta = \cos^{-1}\frac{1}{3}$$

19. *(d)* Since, the plane bisects the line segment AB at right angle.



$$\therefore \quad P = \left(\frac{3+1}{2}, \frac{0+2}{2}, \frac{5-1}{2}\right) = (2, 1, 2)$$

and dr' of normal of the plane

$$= < 3-1, 0-2, 5-(-1) >$$

$$= < 2, -2, 6 >$$

$$= < 1, -1, 3 >$$

$\therefore$ Equation of plane will be

$$1(x-2) - 1(y-1) + 3(z-2) = 0$$

$$\Rightarrow \quad x - 2 - y + 1 + 3z - 6 = 0$$

$$\Rightarrow \quad x - y + 3z = 7$$

20. *(a)* Let the dr' of the required line be $< a, b, c >$.

Since, the line is perpendicular to the given lines

$$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-4}{1}$$

and $$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z+1}{0}$$

$$\therefore \quad a\times 1 + b\times(-1) + c\times(1) = 0$$

and $$a\times 2 + b\times 1 + c\times 0 = 0$$

$$\therefore \quad a - b + c = 0$$

and $$2a + b + 0c = 0$$

$$\therefore \quad \frac{a}{0-1} = \frac{-b}{0-2} = \frac{c}{1+2}$$

$$\Rightarrow \quad \frac{a}{-1} = \frac{b}{2} = \frac{c}{3}$$

$$\therefore \quad < a, b, c > = < -1, 2, 3 >$$

$\therefore$ Equation of line will be

$$\frac{x-5}{-1} = \frac{y-3}{2} = \frac{z-2}{3}$$

21. *(c)* Let $A=\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

Then, $A^{-1}=\begin{bmatrix} d_1^{-1} & 0 & 0 \\ 0 & d_2^{-1} & 0 \\ 0 & 0 & d_3^{-1} \end{bmatrix}$

∴ Inverse of a diagonal matrix is also a diagonal matrix.

22. *(b)* We have,

$$A+B=\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \quad \text{...(i)}$$

Also, A is symmetric and B is skew-symmetric.

$$\therefore \quad A'+B'=\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}'$$

$$\Rightarrow \quad A-B=\begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \quad \text{...(ii)}$$

$[\because A'=A, B'=-B]$

On subtracting Eq. (ii) from Eq. (i), we get

$$2B=\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$\Rightarrow \quad B=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

23. *(b, c)* Given order of A in 3×4.

Therefore order of A' is 4×3.

As $A'B$ is defined, so number of rows in matrix B is 3.

Also $B'A$ is defined, so column in matrix B' should be 3 i.e number of rows in matrix B is 3.

Hence, the order of matrix B is 3×4 or 3×3.

24. *(a)* Since, given matrix is a diagonal matrix.

$$\therefore \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}^{-1}=\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

25. *(a)* We have, $a_1, a_2, a_3, \ldots, a_n \ldots$ are in GP.

Let the first term and common ratio of the GP be A and R respectively.

$$\therefore \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$=\begin{vmatrix} \log AR^{n-1} & \log AR^{n} & \log AR^{n+1} \\ \log AR^{n+2} & \log AR^{n+3} & \log AR^{n+4} \\ \log AR^{n+5} & \log AR^{n+6} & \log AR^{n+7} \end{vmatrix}$$

On applying $C_2 \to C_2-C_1, C_3 \to C_3-C_1$, we get

$$=\begin{vmatrix} \log AR^{n-1} & \log R & \log R^2 \\ \log AR^{n+2} & \log R & \log R^2 \\ \log AR^{n+5} & \log R & \log R^2 \end{vmatrix}$$

$$=\begin{vmatrix} \log AR^{n-1} & \log R & 2\log R \\ \log AR^{n+2} & \log R & 2\log R \\ \log AR^{n+5} & \log R & 2\log R \end{vmatrix}$$

$$=2\begin{vmatrix} \log AR^{n-1} & \log R & \log R \\ \log AR^{n+2} & \log R & \log R \\ \log AR^{n+5} & \log R & \log R \end{vmatrix}$$

$=2\times0$ $[\because C_2$ and C_3 are identical$]$

$=0$

26. *(b)* Let $I=\int e^x\{f(x)+f'(x)\}\,dx$

$$=\int e^x f(x)\,dx+\int e^x f'(x)\,dx$$

$$=f(x)\cdot e^x-\int e^x\cdot f'(x)\,dx+\int e^x f'(x)\,dx+C$$

$$=e^x f(x)+C$$

27. *(c)* Let $I=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}\,dx$ ...(i)

$$=\int_{-\pi}^{\pi}\frac{\cos^2(-x)}{1+a^{-x}}\,dx=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^{-x}}\,dx$$

$$I=\int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{a^x+1}\,dx \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_{-\pi}^{\pi}\cos^2 x\,dx$$

$$=2\int_0^{\pi}\cos^2 x\,dx \quad [\because f(-x)=f(x)]$$

$$=2\times2\int_0^{\pi/2}\cos^2 x\,dx$$

$[\because f(\pi-x)=f(x)]$

$$= 4\int_0^{\pi/2} \frac{1+\cos 2x}{2}\, dx$$

$$= 2\left[x + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{\pi}{2}\right] = \pi$$

$$\therefore \quad I = \frac{\pi}{2}$$

28. *(d)* Let $I = \int e^x \frac{(x^2+1)}{(x+1)^2}\, dx$

$$= \int e^x \left[\frac{(x+1)^2 - 2x}{(x+1)^2}\right] dx$$

$$= \int e^x\, dx - 2\int e^x \frac{x}{(x+1)^2}\, dx$$

$$= e^x - 2\int e^x \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] dx$$

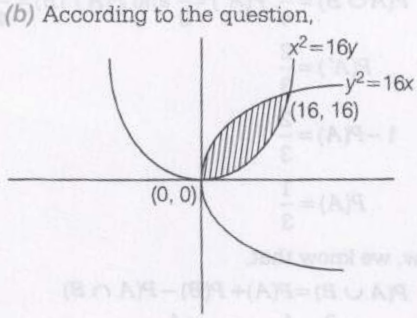
$$= e^x - 2\frac{e^x}{x+1} + C$$

$$[\because \int e^x \{f(x) + f'(x)\}\, dx = e^x f(x) + C]$$

$$= e^x \left[\frac{x+1-2}{x+1}\right] + C$$

$$= e^x \left(\frac{x-1}{x+1}\right) + C$$

29. *(b)* According to the question,



$$\therefore \text{Required area} = \int_0^{16} \left(\sqrt{16x} - \frac{x^2}{16}\right) dx$$

$$= \left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{16\times 3}\right]_0^{16}$$

$$= \frac{8}{3}(16)^{\frac{3}{2}} - \frac{(16)^3}{48}$$

$$= \frac{8}{3}\times 64 - \frac{256}{3}$$

$$= \frac{512}{3} - \frac{256}{3} = \frac{256}{3} \text{ sq units}$$

30. *(a)* We have,

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^x \cdot e^{-y} + x^2 e^{-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$\Rightarrow \quad e^y dy = (e^x + x^2)\, dx$$

On integrating both the sides, we get

$$\int e^y dy = \int (e^x + x^2)\, dx$$

$$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + C'$$

$$\Rightarrow \quad e^y - e^x = \frac{x^3}{3} + C'$$

$$\Rightarrow \quad 3(e^y - e^x) - x^3 = 3C'$$

$$\Rightarrow \quad 3(e^y - e^x) - x^3 = C \qquad [\because 3C' = C]$$

31. *(a)* Let roots of given characteristic equation are $\lambda_1, \lambda_2, \lambda_2$.

Then, $(\lambda_1\lambda_2\lambda_3) = \frac{-2}{(1)} = -2$

We know that, the product of the roots of a square matrix of order n is equal to the determinant of the matrix.

$$\therefore \quad |A| = \lambda_1\lambda_2\lambda_3 = -2$$

$$\therefore \quad |\text{adj } A| = |A|^{3-1} = (A)^2 = (-2)^2 = 4$$

32. *(d)* According to the question,

$$\int_1^2 mx\, dx = 7.5$$

$$\Rightarrow \quad \left[m\frac{x^2}{2}\right]_1^2 = \frac{15}{2}$$

$$\Rightarrow \quad 2m - \frac{m}{2} = \frac{15}{2}$$

$$\Rightarrow \quad \frac{3m}{2} = \frac{15}{2}$$

$$\Rightarrow \quad m = 5$$

33. *(b)* We have, $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}}=\left(\frac{d^2y}{dx^2}\right)^2$

$$\Rightarrow 1+\left(\frac{dy}{dx}\right)^2=\left(\frac{d^2y}{dx^2}\right)^4$$

$\therefore$ Order $=2$ and degree $=4$

34. *(c)* We have, $A(3,-2,-1)$, $B(2,3,-4)$, $C(-1,2,2)$ and $D(4,5,\lambda)$. Since A, B, C, D are coplanar.

$$\therefore \begin{vmatrix} 4-3 & 5+2 & \lambda+1 \\ -1-3 & 2+2 & 2+1 \\ 2-3 & 3+2 & -4+1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 1 & 7 & \lambda+1 \\ -4 & 4 & 3 \\ -1 & 5 & -3 \end{vmatrix}=0$$

$$\Rightarrow 1(-12-15)-7(12+3)+(\lambda+1)(-20+4)=0$$

$$\Rightarrow -27-105-16\lambda-16=0$$

$$\Rightarrow 16\lambda=-148$$

$$\Rightarrow \lambda=\frac{-148}{16}=\frac{-37}{4}$$

35. *(d)* We have, $|\mathbf{a}|=1$, $|\mathbf{b}|=1$ and $|\mathbf{c}|=2$.

Also, $\mathbf{a}\times(\mathbf{a}\times\mathbf{c})+\mathbf{b}=0$

$$\Rightarrow (\mathbf{a}\cdot\mathbf{c})\,\mathbf{a}-(\mathbf{a}\cdot\mathbf{a})\,\mathbf{c}+\mathbf{b}=0$$

$$\Rightarrow (\mathbf{a}\cdot\mathbf{c})\mathbf{a}-\mathbf{c}+\mathbf{b}=0 \quad [\because \mathbf{a}\cdot\mathbf{a}=|\mathbf{a}|^2=1]$$

$$\Rightarrow (\mathbf{a}\cdot\mathbf{c})\mathbf{a}-\mathbf{c}=-\mathbf{b}$$

$$\Rightarrow |(\mathbf{a}\cdot\mathbf{c})\mathbf{a}-\mathbf{c}|^2=|\mathbf{b}|^2$$

$$\Rightarrow |(\mathbf{a}\cdot\mathbf{c})\mathbf{a}|^2+|\mathbf{c}|^2-2\{(\mathbf{a}\cdot\mathbf{c})\mathbf{a}\cdot\mathbf{c}\}=|\mathbf{b}|^2$$

$$\Rightarrow (\mathbf{a}\cdot\mathbf{c})^2|\mathbf{a}|^2+|\mathbf{c}|^2-2(\mathbf{a}\cdot\mathbf{c})(\mathbf{a}\cdot\mathbf{c})=|\mathbf{b}|^2$$

$$\Rightarrow (\mathbf{a}\cdot\mathbf{c})^2\{|\mathbf{a}|^2-2\}+|\mathbf{c}|^2=|\mathbf{b}|^2$$

$$\Rightarrow -(\mathbf{a}\cdot\mathbf{c})^2+4=1$$

$$\Rightarrow (\mathbf{a}\cdot\mathbf{c})^2=3$$

$$\Rightarrow \mathbf{a}\cdot\mathbf{c}=\pm\sqrt{3}$$

$$\Rightarrow |\mathbf{a}|\,|\mathbf{c}|\cos\theta=\pm\sqrt{3}$$

$$\Rightarrow \cos\theta=\pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta=\frac{\pi}{6}$$

or $\frac{5\pi}{6}$

36. *(b)* We have, vertices $(-5,-3)$ and $(-5,-1)$ and extremities of the conjugate axis are at $(-7,-2)$ and $(-3,-2)$.

$$\therefore \text{Centre of hyperbola}=\left(\frac{-5-5}{2},\frac{-3-1}{2}\right)=(-5,-2)$$

Now, length of transverse axis $=2b$

$$\Rightarrow 2b=\sqrt{(-5+5)^2+(-1+3)^2}=2$$

$$\Rightarrow b=1$$

and length of conjugate axis $=2a$

$$\Rightarrow 2a=\sqrt{(-3+7)^2+(-2+2)^2}=4$$

$$\Rightarrow a=2$$

$\therefore$ Equation of hyperbola is

$$\frac{(y+2)^2}{1}-\frac{(x+5)^2}{4}=1$$

37. *(a)* Total number of outcomes $=6\times6=36$

$\therefore n(S)=36$

Let $E=$ The event showing sum 5

$\therefore E=\{(1,4),(2,3),(3,2),(4,1)\}$

$\therefore n(E)=4$

$\therefore$ Required probability $=P(E)$

$$=\frac{n(E)}{n(S)}=\frac{4}{36}=\frac{1}{9}$$

38. *(b)* We have,

$$P(A\cup B)=\frac{3}{4},\ P(A')=\frac{2}{3} \text{ and } P(A\cap B)=\frac{1}{4}$$

$$\because P(A')=\frac{2}{3}$$

$$\Rightarrow 1-P(A)=\frac{2}{3}$$

$$\Rightarrow P(A)=\frac{1}{3}$$

Now, we know that,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow \frac{3}{4}=\frac{1}{3}+P(B)-\frac{1}{4}$$

$$\Rightarrow P(B)=\frac{3}{4}-\frac{1}{3}+\frac{1}{4}=\frac{9-4+3}{12}=\frac{8}{12}=\frac{2}{3}$$

39. (*c*) Required probability $= \frac{{}^7P_7}{7^7}$

$$= \frac{7!}{7^7}$$

40. (*a*) We have,

$$P(A) = \frac{3}{4},\ P(B) = \frac{1}{5},\ P(A \cap B) = \frac{1}{20}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\left(\frac{1}{20}\right)}{\left(\frac{1}{5}\right)}$$

$$= \frac{5}{20} = \frac{1}{4}$$

41. (*b*) $4\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}$

$$= 2\left[2\tan^{-1}\left(\frac{1}{5}\right)\right] - \frac{\pi}{4}$$

$$= 2\tan^{-1}\left[\frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2}\right] - \frac{\pi}{4}$$

$$= 2\tan^{-1}\left[\frac{\frac{2}{5}}{\frac{24}{25}}\right] - \frac{\pi}{4}$$

$$= 2\tan^{-1}\frac{5}{12} - \frac{\pi}{4}$$

$$= \tan^{-1}\left[\frac{2\times\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2}\right] - \frac{\pi}{4}$$

$$= \tan^{-1}\left[\frac{\left(\frac{5}{6}\right)}{\left(\frac{144-25}{144}\right)}\right] - \frac{\pi}{4}$$

$$= \tan^{-1}\frac{120}{119} - \frac{\pi}{4}$$

$$= \tan^{-1}\frac{120}{119} - \tan^{-1} 1$$

$$= \tan^{-1}\left(\frac{\frac{120}{119}-1}{1+\frac{120}{119}\times 1}\right)$$

$$= \tan^{-1}\left(\frac{1}{239}\right)$$

42. (*a*) We have,

$$\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \operatorname{cosec}^{-1}\left(\frac{5}{4}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \sec^{-1}\left(\frac{5}{4}\right) \quad \left[\because \sin^{-1} a + \cos^{-1} a = \frac{\pi}{2}\right]$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right) \quad \left[\because \sec^{-1} a = \cos^{-1}\frac{1}{a}\right]$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\sqrt{1-\left(\frac{4}{5}\right)^2} \quad [\because \cos^{-1} x = \sin^{-1}\sqrt{1-x^2}]$$

$$\Rightarrow \sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$$

$$\Rightarrow \frac{x}{5} = \frac{3}{5}$$

$$\Rightarrow x = 3$$

43. (*d*) We have,

$$\sqrt{3}\cos x + \sin x = \sqrt{2}$$

$$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}$$

$$\Rightarrow x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4} \quad [\because \cos x = \cos y \Rightarrow x = 2n\pi \pm y]$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{6} \pm \frac{\pi}{4}$$

44. *(c)* Let $Z=\left(\frac{1+i}{1-i}\right)^5$

$$=\left[\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right]^5$$
$$=\left[\frac{1+2i+i^2}{1-i^2}\right]^5$$
$$=\left[\frac{1+2i-1}{1-(-1)}\right]^5$$
$$=\left[\frac{2i}{2}\right]^5$$
$$=i^5$$
$$=i^4\cdot i$$
$$=i$$

$\therefore \quad \text{Im}(Z)=1$

45. *(b)* We have,

$$T_n=(n-1)(n-\omega)(n-\omega^2)$$
$$=(n-1)(n^2-n\omega^2-n\omega+\omega^3)$$
$$=(n-1)(n^2-n(\omega+\omega^2)+\omega^3)$$
$$=(n-1)(n^2+n+1)$$
$$[\because \omega^3=1, 1+\omega+\omega^2=0]$$
$$=n^3-1$$

$\therefore \quad S_n=\Sigma T_n$
$$=\Sigma(n^3-1)$$
$$=\frac{n^2(n+1)^2}{4}-n$$

46. *(c)* We have,

$$x+y=\tan^{-1}y$$
$$\Rightarrow \quad 1+y'=\frac{1}{1+y^2}\cdot y'$$
$$\Rightarrow \quad 1=y'\left[\frac{1}{1+y^2}-1\right]$$
$$\Rightarrow \quad 1=y'\left[\frac{-y^2}{1+y^2}\right]$$
$$\Rightarrow \quad y'=-\left(\frac{y^2+1}{y^2}\right)$$
$$\Rightarrow \quad y'=-\left[1+\frac{1}{y^2}\right]$$
$$\Rightarrow \quad y''=-\left[\frac{-2}{y^3}y'\right]$$
$$\Rightarrow \quad y''=\frac{2}{y^3}y'$$

47. *(a)* We have,

$$f(x)=\begin{cases} x\cdot e^{-\left(-\frac{1}{x}+\frac{1}{x}\right)} & x<0 \\ x\cdot e^{-\left(\frac{1}{x}+\frac{1}{x}\right)} & x>0 \\ 0 & x=0 \end{cases}$$

$$\begin{cases} x &, x<0 \\ x\cdot e^{-\frac{2}{x}} &, x>0 \\ 0 &, x=0 \end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x)=\lim_{x\to 0} x=0$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0} xe^{-\frac{2}{x}}=0\times 0=0$$
$$f(0)=0$$
$$\therefore \quad \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

So, $f(x)$ is continuous at $x=0$.

Also, (LHD at $x=0$) $=\left(\frac{d}{dx}(x)\right)_{x=0}=1$

(RHD at $x=0$) $=\left(\frac{d}{dx}\left(xe^{-\frac{2}{x}}\right)\right)_{x=0}$

$$=\left(e^{-\frac{2}{x}}+\frac{2e^{-\frac{2}{x}}}{x}\right)_{x=0}$$

= Does not exist.

Thus, $f(x)$ is everywhere continuous but not differentiable at $x=0$.

48. *(b)* We have,

$$y=\sin^{-1}\frac{1}{2}(\sqrt{1+x}+\sqrt{1-x})$$

Put $\quad x=\cos 2\theta$

$$\therefore \quad y = \sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{2}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{2}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta\right)$$

$$= \sin^{-1}\sin\left(\frac{\pi}{4} + \theta\right)$$

$$= \frac{\pi}{4} + \theta$$

$$\therefore \quad y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x \qquad [\because x = \cos 2\theta]$$

$$\therefore \quad y' = -\frac{1}{2\sqrt{1-x^2}}$$

49. *(d)* We have,

$$x^m y^n = (x+y)^{m+n}$$

Taking log both the sides, we get

$m \log x + n \log y = (m+n) \log (x+y)$

On differentiating both the sides, we get

$$\frac{m}{x} + \frac{n}{y} y' = \frac{m+n}{x+y}(1+y')$$

$$\Rightarrow \frac{m}{x} + \frac{n}{y} y' = \frac{m+n}{x+y} + \frac{m+n}{x+y} y'$$

$$\Rightarrow \left(\frac{n}{y} - \frac{m+n}{x+y}\right) y' = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\Rightarrow \left[\frac{nx+ny-my-ny}{y(x+y)}\right] y' = \frac{mx+nx-mx-my}{(x+y)x}$$

$$\Rightarrow \left[\frac{nx-my}{y}\right] y' = \left[\frac{nx-my}{x}\right]$$

$$\Rightarrow \quad y' = \frac{y}{x}$$

50. *(c)* We have,

$$y = x^{x^{x^{\cdots\infty}}}$$

Let $\quad y = x^y$

On taking log both the sides, we get

$$\log y = y \log x$$

On differentiating both the sides w.r.t. x, we get

$$\frac{1}{y} y' = y' \log x + \frac{y}{x}$$

$$\Rightarrow \quad y' = yy' \log x + \frac{y^2}{x}$$

$$\Rightarrow \quad xy' = xy \log x \, y' + y^2$$

$$\Rightarrow \quad xy'(1 - y\log x) = y^2$$

$$\Rightarrow \quad y' = \frac{y^2}{x(1 - y\log x)}$$