43. Bohr's Model and Physics of the Atom

Short Answer

1. Question

How many wavelengths are emitted by atomic hydrogen in visible range (380 nm – 780 nm)? In the range 50 nm to 100 nm?

Answer

We know from Rydberg's Equation, the wavelength λ of emission spectrum when electrons move from n_2 energy level to n_1 energy level is given by,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{{n_1}^2} - \frac{1}{{n^2}^2}\right)$$

where $R = 1.09677X10^7\ m^{-1}$ Z = Atomic Number

For Hydrogen atom, Z=1.

The wavelength range 380-780 nm lies in the Balmer Series.

Thus, n_1=2 for Balmer Series.

Substituting n_2=3, we get λ=656.3 nm

Substituting n_2=7, we get λ=397.0 nm

Thus, we get 5 wavelengths: n=3 to n=2; n=4 to n=2; n=5 to n=2; n=6 to n=2 and n=7 to n=2.

For other values of n_2, the range of 380-780nm is not possible.

The wavelength range 50-100 nm, within n=7 to n=2, lies in the Lyman Series.

Thus, n_1=1 for Lyman Series.

Substituting n_2=4, we get λ=97.3 nm

Substituting n_2=6, we get λ=93.8 nm

Thus, we get 3 wavelengths: n=4 to n=1; n=5 to n=1 and n=6 to n=1.

For other values of n_2 (within n=7 to n=2), the range of 50-100 nm is not possible.

2. Question

The first excited energy of a He^+ ion is the same as the ground state energy of hydrogen. Is it always true that one of the energies of any hydrogen like ion will be the same as the ground state energy of a hydrogen atom?

Answer

We know that energy E of a hydrogen or hydrogen-like species is given by,

$$E = -13.6\frac{Z^2}{n^2}\ eV$$

where Z = Atomic Number and n = Principal Quantum Number

Now, for the ground state energy of hydrogen(Z=1), Z/n= 1. Thus, E=-13.6 eV.

Also, for the first excited energy of helium ion(Z=2), Z/n= 1. Thus, E=-13.6 eV.

Thus, we have seen that in all hydrogen like species where Z=n, E=-13.6 eV.

So, for the second excited state of lithium ion(Z=2), for the third excited state of beryllium ion(Z=3), so on, we have E=-13.6 eV.

Hence, there will always be an energy level of an hydrogen like species which will be same as the ground state energy of hydrogen.

3. Question

Which wavelengths will be emitted by a sample atomic hydrogen gas (in ground state) if electrons energy 12.2 eV collide with the atoms of the gas?

Answer

We know, according to Bohr's Model, the energy ΔE released, when electrons move from n_2 energy level to n_1 energy level is given by,

$$\Delta E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

Now, given that hydrogen is in its ground state. So n_1=1.

Substituting, n_2=2, we get ΔE=10.2 eV.

Thus, the electrons will be released from n=2 level to n=1 level.

Substituting, n_2=3, we get ΔE=12.08 eV.

Thus, the electrons will be released from n=3 level to n=1 level.

This, corresponds to the Lyman Series(infrared region).

4. Question

When white radiation is passed through a sample hydrogen gas at room temperature, absorption lines an observed in Lyman series only. Explain.

Answer

We know, according to Bohr's Model, the energy ΔE released, when electrons move from n_2 energy level to n_1 energy level is given by,

$$\Delta E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

At room temperature, all electrons in a hydrogen atom are in ground state. In order to excite them to n=2 level a minimum of 10.2 eV energy is needed. This is because,

$$\Delta E = 10.2eV = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\ eV$$

White light contains radiations of energy around 10.2eV. So, it excites electrons to n=2 level. During de-excitation, electrons move from n=2 level to n=1 level. This corresponds to the Lyman Series.

Thus, absorption lines are only observed in Lyman Series.

5. Question

Balmer series was observed and analysed before the other series. Can you suggest a reason for such an order?

Answer

Balmer Series corresponds to the visible spectrum. So it is visible to the human eye.

Thus, it was observed and analyzed before the other series.

6. Question

What will be the energy corresponding to the first excited state of a hydrogen atom if the potential energy of the atom is taken to be 10 eV when the electron is widely separated from the proton? Can we still write $E_n = E_i/n^2$, or $r_n = a_0\ n^2$?

Answer

While deriving formula for energy for the Bohr's Model of an atom, we take the potential energy to be zero when the electron and proton are far apart (infinite distance). But, here the potential energy is not zero; it is 10 eV.

So the formula of energy will be modified as,

$$E = -\frac{13.6}{n^2} + 10\ eV$$

where n= Principal Quantum Number.

For the first excited state, n=2. Therefore,

$$E = -\frac{13.6}{2^2} + 10\ eV = 6.6\ eV$$

The formula of radius of orbit remains unchanged as it does not involve any concept of potential energy and can be derived just by taking into consideration the Quantization Rule and Coloumb's Law.

7. Question

The difference in the frequencies of series limit of Lyman series and Balmer series is equal to the frequency of the first line of the Lyman series. Explain.

Answer

We know from Rydberg's equation for a hydrogen atom (Z=1) the wavelength λ of emission spectrum, when electrons move from n_2 energy level to n_1 energy level, is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

where $R = 1.09777X10^7\ m^{-1}$

For the difference in the series limit (last line) of Lyman series and Balmer series.

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) - R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = R\left(1 - \frac{1}{4}\right)$$

This is because n_1=1 when we are considering Lyman Series and n_1=2 when we are considering Balmer Series.

For the first line of Lyman Series,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{(1+1)^2}\right) = R\left(1 - \frac{1}{4}\right)$$

Thus, we can see in both cases the value of 1/λ is same. So frequency (c/λ, c=speed of light=constant) will also be same for both cases.

8. Question

The numerical value of ionization energy in eV equals the ionization potential in volts. Does the equality hold if these quantities are measured in some other units?

Answer

We know that energy has dimension of

$$Energy = Charge\ X\ Potential$$

In the unit 'eV', the charge part is not multiplied with potential. The charge is kept as 'e' and its value is not incorporated. As a result, energy has the same value of potential, because only the value of potential is used and not charge when we use the unit 'eV'. Thus, energy has the same value of potential.

This is not true when we use other units of energy like joule, where the value of charge is substituted and multiplied.

9. Question

We have stimulated emission and spontaneous emission. Do we also have stimulated absorption and spontaneous absorption?

Answer

We do have stimulated absorption where atoms absorb energy and move to the excited state when electromagnetic wave of suitable frequency is incident on it.

However, there is nothing called spontaneous absorption. This is because for absorption to take place electromagnetic radiation has to be incident. Absorption is not possible without any incident of radiation, as if there is no incident energy, there is nothing for the atoms to absorb.

10. Question

An atom is in its excited state. Does the probability of its coming to ground state depend on whether the radiation is already present or not? If yes, does it also depend on the wavelength of the radiation present?

Answer

In case of stimulated emission, atoms come to ground state if there is a stimulated radiation present. In that case, it also depends on the wavelength of radiation. Only a radiation of suitable wavelength can stimulate atoms for emission.

However in case of spontaneous emission, atoms come to the ground state spontaneously. Thus, it does not depend on the presence of radiation or its wavelength.

Objective I

1. Question

The minimum orbital angular momentum to the electron in a hydrogen atom is

A. h

B. h/2

C. h/2π

D. h/λ

Answer

According to Bohr's quantization rule, orbital angular momentum

(L) of an electron is given by,

$$L = n\frac{h}{2\pi}$$

Now here, n=1,2,3… and h is the Planck’s Constant. Note that the minimum value of n is 1.

Thus, the minimum orbital angular momentum is $L = \frac{h}{2\pi}$.

Hence, option (c) is the correct answer.

2. Question

Three photons coming from excited atomic-hydrogen sample are picked up. Their energies are 12.1 eV, 10.2 eV and 1.9 eV. These photons must come from

A. a single atom

B. two atoms

C. three atoms

D. either two atoms or three atoms

Answer

We know, according to Bohr's Model, the energy ΔE released, when electrons move from n_2 energy level to n_1 energy level is given by,

$$\Delta E = 13.6 \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

Now, the photons having energies can be due to the following transitions:

From n=3 to n=1: $13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) eV = 12.1\ eV$

From n=3 to n=2: $13.6\ \left(\frac{1}{2^2} - \frac{1}{3^2}\right) eV = 1.9\ eV$

From n=2 to n=1: $13.6\ \left(\frac{1}{1^2} - \frac{1}{2^2}\right) eV = 10.2\ eV$

Now, an excited electron always comes to its ground state. Since there is only one electron in hydrogen atom, it can make transitions from n=3 to n=1 directly or from n=3 to n=2 and then finally from n=2 to n=1.

This is not possible for a single atom as only one transition is allowed. So option (a) is wrong.

This is possible for two atoms where one atom makes n=3 to n=1 transition and the other makes n=3 to n=2 and then to n=1 transitions.

This is also possible for three atoms where all the three atoms make three different transitions.

Here, only one option is correct. So among (b), (c) and (d) options, we choose (d) as it includes all the cases.

Hence, option (d) is the correct answer.

3. Question

Suppose, the electron in a hydrogen atom makes transition from n = 3 to n = 2 in 10^{-8} s. The order of the torque acting on the electron in this period, using the relation between torque and angular momentum as discussed in the chapter on rotational mechanics is

A. 10^{-34} N m

B. 10^{-24} N m

C. 10^{-42} N m

D. 10^{-8} N m

Answer

Here, from Bohr's quantization rule, we know angular momentum (L) is given by,

$$L = n\frac{h}{2\pi}$$

Where n=0,1,2….. and h=Planck's Constant.

Now, the initial angular momentum is $L_3 = \frac{3h}{2\pi}$

And, the final angular momentum is $L_2 = \frac{2h}{2\pi}$

Now from the relation between angular momentum(L) and torque(τ), we have

$$\tau = \frac{dL}{dt}$$

$$= \frac{\frac{2h}{2\pi} - \frac{3h}{2\pi}}{10^{-8}}\ Nm$$

Putting $h = 6.625\ X\ 10^{-34}\ Js$

$\tau \sim 10^{-24}\ Nm$

Hence, option (b) is the correct answer.

4. Question

In which of the following transitions will the wavelength be minimum?

A. n = 5 to n = 4

B. n = 4 to n = 3

C. n = 3 to n = 2

D. n = 2 to n = 1

Answer

We know from Rydberg's equation the wavelength λ of emission spectrum when electrons move from n_2 energy level to n_1 energy level, is given by,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

where $R = 1.09677X10^7 m^{-1}$ Z = Atomic Number

For option (a), we have $\frac{1}{\lambda} = RZ^2\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = \frac{RZ^2 9}{400}$

For option (b), we have $\frac{1}{\lambda} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{RZ^2 7}{144}$

For option (c), we have $\frac{1}{\lambda} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{RZ^2 5}{36}$

For option (d), we have $\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{RZ^2 3}{4}$

Thus, in option (d) 1/λ has the maximum value and thus λ is minimum.

Hence, option (d) is the correct answer.

5. Question

In which of the following systems will the radius of the first orbit (n = 1) be minimum?

A. Hydrogen atom

B. Deuterium atom

C. Singly ionized helium

D. Doubly ionized lithium

Answer

We know that the radius (r) of an orbit of an atom of atomic number Z, according to Bohr's Model is:

$$r = 0.529\frac{n^2}{Z}\ A^{\circ}$$

Where, n= Principal Quantum number and Z= Atomic Number.

Thus, for a fixed n (n=1), the radius of an orbit is inversely proportional to Z.

Among the options, lithium ion has the highest atomic number, Z=3.

Hence, option (d) is the correct answer.

6. Question

In which of the following systems will the wavelength corresponding to n = 2 to n = 1 be minimum?

A. Hydrogen atom

B. Deuterium atom

C. Singly ionized helium

D. Doubly ionized lithium

Answer

We know from Rydberg's equation, the wavelength λ of emission spectrum when electrons move from n_2 energy level to n_1 energy level, is given by,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

where $R = 1.09677X10^7 m^{-1}$ Z = Atomic Number

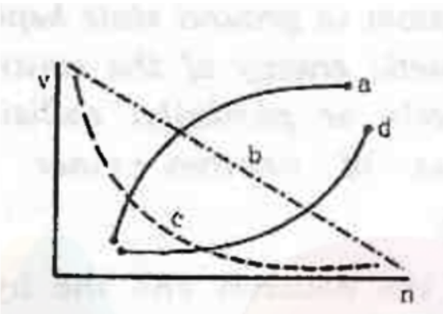
Now, here n_1=1 and n_2=2 are fixed. So wavelength is inversely proportional to Z^2.

Among the options, lithium ion has the highest atomic number, Z=3.

Hence, option (d) is the correct answer.

7. Question

Which of the following curves may represent the speed of the electron in a hydrogen atom as a function of the principal quantum number n?



Answer

The speed of an electron (v) is inversely proportional to the principal quantum number (n). Thus the v-n graph should be rectangular hyperbola.

Hence, option (c) is the correct answer.

8. Question

As one considers orbits with higher values of n in a hydrogen atom, the electric potential energy of the atom

A. decreases

B. increases

C. remains the same

D. does not increase

Answer

We know that the electric potential energy (U) of an electron of an atom of atomic number Z is given by,

$$U = -27.2\frac{Z^2}{n^2}$$

where n = Principal Quantum Number

For hydrogen atom, Z=1 is fixed. Thus, with increase in 'n', 'U' becomes less negative, which means it increases.

Hence, option (b) is the correct answer.

9. Question

The energy of an atom (or ion) in its ground state is –54.4 eV. It may be

A. hydrogen

B. deuterium

C. He^+

D. Li^{++}.

Answer

We know the energy (E) of an electron according to Bohr's Model of an atom of atomic number Z is given by,

$$E = -13.6\frac{Z^2}{n^2}\ eV$$

where n= Principal Quantum Number

For ground state, n=1 is fixed.

For hydrogen, Z=1. So E=-13.6 eV. So, option (a) is wrong.

For deuterium, Z=1. So E =-13.6 eV. So, option (b) is wrong.

For helium ion, Z=2. So E=-54.4 eV. So, option (c) maybe correct. Let's check option (d).

For lithium ion, Z=3. So E=-122.4 eV. So, option (d) is wrong.

Hence, option (c) is the correct answer.

10. Question

The radius of the shortest orbit in a one-electron system is 18 pm. It may be

A. hydrogen

B. deuterium

C. He^+

D. Li^{++}.

Answer

We know that the radius (r) of an orbit according to Bohr's Model of an atom of atomic number Z is given by,

$$r = 0.529\frac{n^2}{Z}\ A^\circ$$

where n= Principal Quantum Number

For shortest orbit, n=1 which means it is fixed.

For hydrogen, Z=1. So, r=0.529 A°. So, option (a) is wrong.

For deuterium, Z=1. So, r=0.529 A°. So, option (b) is wrong.

For helium ion, Z=2. So, r=0.2645 A°. So, option (c) is wrong.

For lithium ion, Z=3. So, r=0.176 A° ~ 18pm. So, option (d) is right.

Hence, option (d) is the correct answer.

11. Question

A hydrogen atom in ground state absorbs 10.2 eV of energy. The orbital angular momentum of the electron in increased by

A. 1.05×10^{-34} J s

B. 2.11×10^{-34} J s

C. 3.16×10^{-34} J s

D. 4.22×10^{-34} J s

Answer

We know, according to Bohr's Model, the energy ΔE released, when electrons move from n_2 energy level to n_1 energy level is given by,

$$\Delta E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$$

For hydrogen atom, Z=1 and given ΔE=10.2 and n_1=1. Let n_2=n (say).

Thus,

$$1-\frac{1}{n^2}=\frac{10.2}{13.6}$$

$$or\ n=2$$

Thus, the increase in angular momentum is given by,

$$\Delta L=\frac{2h}{2\pi}-\frac{h}{2\pi}$$

Or $L=\frac{h}{2\pi}=\frac{6.625X10^{-34}}{2\pi}=1.05X10^{-34}\,Js$

Hence, option (a) is the correct answer.

12. Question

Which of the following parameters are the same for all hydrogen like atoms and ions in their ground states?

A. Radius of the orbit

B. Speed of the electron

C. Energy of the atom

D. Orbital angular momentum of the electron

Answer

It is only angular momentum of an electron which does not depend on atomic number Z.

According to Bohr's quantization rule angular momentum (L) is,

$$L=n\frac{h}{2\pi}$$

Where n=0,1,2,3… and h= Planck's constant.

For ground state, even n=1 is fixed. Thus, L is same for hydrogen and all hydrogen like species.

Hence, option (d) is the correct answer.

13. Question

In a laser tube, all the photons

A. have same wavelength

B. have same energy

C. move in same direction

D. move with same speed.

Answer

In Laser light, photons, that is the basic quantum of light energy, travel with the same speed, which is the speed of light. It may have different wavelengths, energies and directions. But the speed of all the photons is the same and constant.

Hence, option (d) is the correct answer.

Objective II

1. Question

In a laboratory experiment on emission from atomic hydrogen in a discharge tube, only a small number of lines are observed whereas a large number of lines are present in the hydrogen spectrum of a star. This is because in a laboratory

A. the amount of hydrogen taken is much smaller than that present in the star

B. the temperature of hydrogen is much smaller than that of the star

C. the pressure of hydrogen is much smaller than that of the star.

D. the gravitational pull is much smaller than that in the star.

Answer

A material shows radiation lines when it is heated. Thus, when hydrogen gas is heated inside a discharge tube, only a small amount of radiations is observed because temperature inside the laboratory can be increased to a certain limit. As star (e.g. sun) exhibits a huge temperature, thus hydrogen emits more number of wavelengths in a star as compared to that in laboratory.

Option A. is not correct as spectral lines never depend upon the amount of material taken.

Option C. is not correct as spectral line emission is independent of pressure

Option D. is not correct as spectral lines is also independent of gravitational pull.

2. Question

An electron with kinetic energy 5eV is incident on a hydrogen atom in its ground state. The collision

A. must be elastic

B. may be partially elastic

C. must be completely inelastic

D. must be completely inelastic.

Answer

This is because we know from the special condition of elastic collision that when an object having a lesser mass strikes with an object with larger mass, the collision is elastic and the object with lesser mass moves in opposite direction with opposite velocity. Thus here the mass of electron is less as compared to that of the hydrogen and thus the collision will be elastic.

Again the energy of the first excited state of the hydrogen atom is 3.4eV and that at the ground state is 13. 6eV.So the difference becomes 10.2eV which is larger than the energy of the electron(5eV). Thus its kinetic energy remains conserved and the photon does not get absorbed.

Option B is not correct because the electron with the kinetic energy of 5eV will exchange the energy with that of the heavier mass hydrogen and thus the collision will be completely elastic.

Option C is not correct as the collision cannot be inelastic

Option D will also be not correct as the collision is not going to be inelastic.

3. Question

Which of the following products in a hydrogen atom are independent of the principal quantum number nether symbols have their usual meanings

A. vn

B. Er

C. En

D. vr

Answer

The correct answers are A. and B.

We know that,

The velocity of the electrons is given by , $v\alpha \frac{Z}{n}$ so,$vn\ \alpha\ Z$

The energy is,$E\ \alpha \frac{z^2}{n^2}$ and radius as $r\alpha \frac{n^2}{Z}$ so,$Er\ \alpha\ Z$ (α=proportionality sign)

And all the rest of the options depends on the quantum number n.

4. Question

Let A_n be the area enclosed by the nth orbit in a hydrogen atom.The graph of ln $({}^{A_n}/_{A_1})$ against ln(n)

A. will pass through the origin

B. will be a straight line with slope 4

C. will be monotonically increasing nonlinear curve

D. will be a circle

Answer: A, B

Answer

As A_n denoted the area of the nth orbit and A_1 denotes the area of the first orbit.So,area of the nth orbit will be $A_n = \pi r_n^2$ and that of the first orbit is

$A_1 = \pi r_1^2$

Thus,

$$\frac{A_n}{A_1} = \frac{\pi r_n^2}{\pi r_1^2}$$

Now, the nth radius is given by,

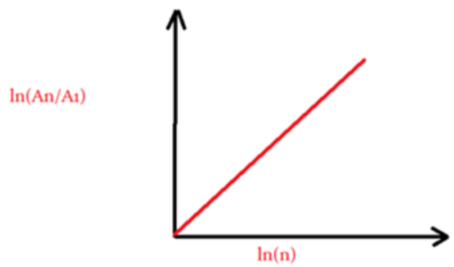
$r_n \propto n^2$

Therefore, $\frac{A_n}{A_1} = \frac{\pi r_n^2}{\pi r_1^2} = \frac{n^4}{1} = n^4$

So, $\ln(A_n/A_1) = 4\ln(n)$

On comparing the above with y=mx+c, we get

$$y = \frac{A_n}{A_1}, m = 4 \; and \; x = \ln(n)$$



Thus, Hence , we obtain that the graph will be a straight line passing through the origin with a slope of 4.

Therefore, option A and B are correct.

5. Question

Ionisation energy of a hydrogen like ion A is greater than that of another hydrogen like ion B. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum. In ground state,

A. $r_A > r_B$

B. $u_A > u_B$

C. $E_A > E_B$

D. $L_A > L_B$

Answer

As ionisation energy is the energy needed to detach an electron from the atom, thus the electrons will gain some speed when detached from the atom and therefore ionisation energy will directly depend upon the speed of the electron

Thus, option B. will be the correct answer.

Option A. is not correct as ionisation energy does not depend upon the radius of the atom

Option C. is not correct as ionisation energy is independent of the atom energy and depends upon the energy supplied to detach the electron.

Option D. is not correct as ionisation energy is independent of angular momentum.

6. Question

When a photon stimulates the emission of another photon, the two photons have

A. same energy

B. same direction

C. same phase

D. same wavelength

Answer

As the photon that stimulates another photon are identical, so they will have same energy, direction, phase and wavelength.

Exercises

1. Question

The Bohr radius is given by, $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$ .Verify that the RHS has dimensions of length.

Answer

We know that the dimensions of the following are,

$\varepsilon_0 = A^2T^2$

$h = ML^2T^{-1}$

$\pi = L^2MLT^{-2}$

$e = AT$

Thus we obtain the Bohr's radius as,

$$a_0 = \frac{A^2T^2(ML^2T^{-1})^2}{L^2MLT^{-2}M(AT)^2}$$

$$a_0 = \frac{M^2L^4T^{-2}}{M^2L^3T^{-2}}$$

Therefore, a_0 = L, which is in the dimension of length.

2. Question

Find the wavelength of the radiation emitted by hydrogen in the transitions A. n=3 to n=2, B. n=5 to n=4 and

C. n=10 to n=9.

Answer

We know that the wavelength is given by,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

As the atom is hydrogen atom, so Z=1.

A. Here, n=2 and m=3.Thus,

$$\frac{1}{\lambda} = 1.1 \times 10^7\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda} = 1.1 \times 10^7\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$Therefore, \lambda = \frac{36}{5 \times 1.1 \times 10^7} = 6.54 \times 10^{-7} = 654nm$$

B. Here, n=4, m=5.Thus,

$$\lambda = \frac{400}{1.1 \times 10^7 \times 9} = 40.404 \times 10^{-7} = 4040.4nm$$

C. Here, n=9, m=10. Thus,

$$\lambda = \frac{8100}{1.1 \times 10^7 \times 19} = 387.5598 \times 10^{-7} = 38755.9nm$$

3. Question

Calculate the smallest wavelength of radiation that may be emitted by A. hydrogen B. He^+ and C. Li^{++}

Answer

When it is referred to as the smallest wavelength, it means longest energy as because energy is inversely proportional to wavelength.

Thus, the electron will jump from ground state to the state where energy is maximum(i.e.∞)

So, the ground state is n=1 and the state of higher energy is m=∞

A. we know that,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$Thus, for\ hydrogen\ Z = 1$$

Therefore, we obtain,

$$\frac{1}{\lambda} = 1.1 \times 10^7\left(\frac{1}{1} - \frac{1}{\infty}\right), R = 1.1 \times 10^7$$

$$Thus, \lambda = \frac{1}{1.1 \times 10^7} = 0.909 \times 10^{-7} = 90.9 \times 10^{-9} \approx 91nm$$

B. As He has an atomic number 2, so Z=2.

Thus, $\frac{1}{\lambda} = 1.1 \times 10^7 \times Z^2\left(\frac{1}{1} - \frac{1}{\infty}\right)$

Therefore, we obtain,

$$\lambda = \frac{1}{1.1 \times 10^7 \times 4} = 23nm$$

C. As lithium has an atomic number of 3, so Z=3

Thus, $\frac{1}{\lambda} = 1.1 \times 10^7 \times Z^2\left(\frac{1}{1} - \frac{1}{\infty}\right)$

Therefore, we obtain,

$$\lambda = \frac{1}{1.1 \times 10^7 \times 9} = 10nm$$

4. Question

Evaluate Rydberg constant by putting the values of the fundamental constants in its expression.

Answer

Rydberg constant is given by,

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$$

As we know that R is associated with electronic transition, so all values of the fundamentals will be w.r.t the electrons.

We know,

Mass of electron,$m = 9.1 \times 10^{-31}$kg,

charge of electron, e=$1.6 \times 10^{-19} C$

Planks constant, h=$6.63 \times 10^{-34} J - s$

Velocity of light, c=$3 \times 10^8 m/s$

Permittivity,$\varepsilon_0 = 8.85 \times 10^{-12}$

Putting all these above values, we obtain the Rydberg constant to be,

$$R = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (6.63 \times 10^{-34})^3 \times 3 \times 10^8 \times (8.85 \times 10^{-12})^2}$$

$R = 1.097 \times 10^7$

5. Question

Find the binding energy of a hydrogen atom in the state n=2.

Answer

As we know that binding energy is defined as the energy released when its constituents are brought from infinity to from the system. Thus here the initial sta

te will be m=2 and this state is reached after bringing the constituent from infinity, so n=∞.

Now, the energy of hydrogen atom is given by,

$$E = \frac{-13.6}{n^2} eV$$

Where 13.6 is the binding energy of the hydrogen atom

So, if we consider transition state from infinity to n=2, we have

$$E = \frac{-13.6}{n^2} - \left(\frac{-13.6}{m^2}\right)$$

$$or\ E = 13.6\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right)$$

$$or\ E = -13.6 \times \frac{1}{4} = -3.4eV$$

6. Question

Find the radius and energy of He^+ ion in the states

A. n=1 B. n=4 C. n=10

Answer

As we know that radius of an atomic orbit is given by,

$$r = \frac{n^2 a_0}{Z}, here\ a_0 = 0.053nm$$

$$or\ r = \frac{0.053 \times n^2}{Z}$$

Again, the energy of the atomic orbit is given as,

$$E = \frac{-13.6 \times Z^2}{n^2}$$

Here Z is the atomic number of the atom. Thus Z for $He^+$is 2.

A. Here, n=1 and thus the radius and energy becomes,

$$r = \frac{0.053 \times 1^2}{2} = 0.265Å$$

And energy,$E = \frac{-13.6 \times 2^2}{1^2} = -54.4eV$

B. here n=4 and thus the radius and energy is,

$$r = \frac{0.053 \times 4^2}{2} = 4.24Å$$

And energy, $E = \frac{-13.6 \times 2^2}{4^2} = -3.4eV$

C. here n=10 and thus the radius and energy becomes,

$$r = \frac{0.053 \times 10^2}{2} = 26.5Å$$

And energy is, $E = \frac{-13.6\times2^2}{10^2} = -0.544eV$

7. Question

A hydrogen atom emits ultraviolet radiation of wavelength 102.5 nm. What are the quantum numbers of the states involved in the transition?

Answer

As we consider ground state to be n=1, so the transition involved will be from n=1 to m = (TO BE DETERMINED).

As we know from the topic of hydrogen spectra,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Here, λ=102.5nm and Z=1(as atomic number of hydrogen is 1), n=1

Thus, $\frac{1}{102.5\times10^{-9}} = 1.1 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{m^2}\right)$

$$or, \frac{10^9}{102.5} = 1.1 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{m^2}\right)$$

$$or, \frac{10^9}{102.5 \times 1.1 \times 10^7} = 1 - \frac{1}{m^2}$$

$$or, \frac{1}{m^2} = \frac{1 - 100}{102.5 \times 1.1}$$

$$or, m = 2.97 \approx 3$$

8. Question

A. Find the first excitation potential of He^+ion. B. Find the ionisation potential of Li^{++}ion.

Answer

We know the formula for energy calculation is,

$$E = \frac{-13.6 \times Z^2}{n^2}$$

So, as the He^+makes first transition,so n=1 and m=2.Thus we will obtain 2 energies at state 1 and state 2.The difference of the 2 energies will give us the excitation potential.

A.

Thus, $E_1 = \frac{-13.6\times2^2}{1^2} = -54.4eV$

$$and\ , E_2 = \frac{-13.6 \times 2^2}{2^2} = -13.6eV$$

Therefore, the excitation energy of $He^+$is,

$$E = E_2 - E_1 = -13.6 - (-54.4) = 40.8V$$

B. Ionisation potential is defined as the energy required to ionise an atom(i.e.to remove one or more electron from the valence shell).The ionisation potential of hydrogen is 13.6eV.Thus,for any other atom it will be 13.6×Z^2

Thus, for Li^{++},we have atomic number,Z=3.

So ionisation potential =13.6×9=122.4 V

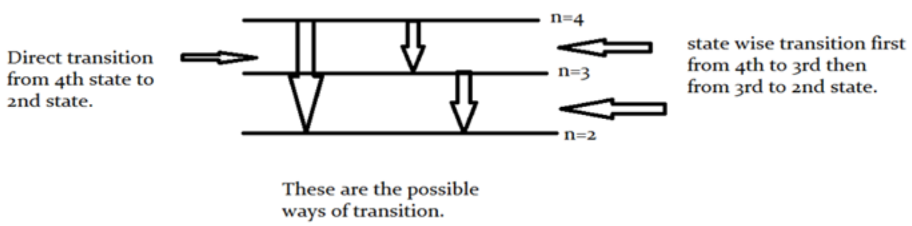
9. Question

A group of hydrogen atoms are prepared in n=4 states. List the wavelengths that are emitted as the atoms makes the transitions and return to n=2 states.

Answer

As the atom is in n=4 state, so while making a transition from n=4 to n=2 state, the transitions it will undergo are

In the first case, n=4→3→2



Thus, for the first case,

n=3 and m=4 (as the atom first makes transition from 4th state to 3rd state)

Thus the wavelength will be, $\frac{1}{\lambda} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$$or, \frac{1}{\lambda} = 1.1 \times 10^7 \times \left(\frac{1}{9} - \frac{1}{16}\right)$$

$$or, \frac{1}{\lambda} = 1.1 \times 10^7 \times \left(\frac{16-9}{144}\right) = 0.0537 \times 10^7$$

$$or, \lambda = 1910nm$$

Second case is when the atom makes transition from n=3 to m=2.

Thus,

$$\frac{1}{\lambda} = 1.1 \times 10^7 \times (\frac{1}{2^2} - \frac{1}{3^2})$$

$$or, \frac{1}{\lambda} = 1.1 \times 10^7 \times \left(\frac{1}{4} - \frac{1}{9}\right) = 1.1 \times 10^7 \times \left(\frac{9-4}{36}\right) =$$

$$or, \lambda = \frac{36 \times 10^{-7}}{5 \times 1.1} = 654nm$$

Third case is when the atom will make the transition from 4th state to directly to 3rd state. At that time, n=4 and m=2,

$$\frac{1}{\lambda} = 1.1 \times 10^7 \times \left(\frac{1}{4} - \frac{1}{16}\right)$$

$$or, \lambda = 4.87nm$$

10. Question

A positive ion having just one electron ejects it if a photon of wavelength 228Å or less is absorbed by it. Identify the ion.

Answer

As the electron is ejected, so it makes a transition from its ground state n=1 to the immediate excited state m=2.

Thus the energy associated will be,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{228} = 0.0872 \times 10^{-16} eV$$

The energy is in this term because the question says that the electron ejects when a photon of frequency 228Å is applied.

Now comparing this energy with that of the orbital energy, we get,

$$-13.6 \times Z^2 \left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 0.0872 \times 10^{-16}$$

$$or, -13.6 \times Z^2 \left(\frac{1-4}{4}\right) = 0.0872 \times 10^{-16} \times 1.6 \times 10^{-19}$$

1.6×10^{-19} is multiplied as because the energy of the photon is in electron-volt.So we multiplied the charge of the electron.

$$or, Z^2 = 5.3$$

$$or, Z = \sqrt{5.3} \approx 2.3$$

Hence, the ion may be He^+

11. Question

Find the maximum Coulomb force that can act on the electron due to the nucleus in a hydrogen atom.

Answer

Since, hydrogen atom has only one electron in its outermost shell. Thus, the distance between the nucleus and the outer electron will be the Bohr radius.

We know that the columbic force is given by, $F = \frac{q_1 \times q_2}{4\pi\varepsilon_0 r^2}$, where q1 and q2 are the magnitude of charges, r=radius(in this case is Bohr radius)

The value of $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ and the value of Bohr's radius is r=0.053Å.

The charges of proton and electron are the same but opposite in sign(i.e. 1.6×10^{-19}).But as we are taking the magnitude of the charges, so we don't require the sign involved.

Therefore, we obtain the force to be,

$$F = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(0.053 \times 10^{-10})^2} = 82.02 \times 10^{-9}$$

$$or\ , F = 8.2 \times 10^{-8} N$$

12. Question

A hydrogen atom in a state having a binding energy of 0.85eV makes transition to a state with excitation energy 10.2eV. A. Identify the quantum number n of the upper and the lower energy states involved in the transition. B. Find the wavelength of the emitted radiation.

Answer

A. We know that the binding energy is given by,

$$E = \frac{13.6 \times Z^2}{n^2} = \frac{13.6}{n_1^2}$$

Now, energy in the first state is given as, E=0.85eV

$$\text{So,} E = \frac{13.6}{n_1^2} \Rightarrow 0.85 = \frac{13.6}{n_1^2} \Rightarrow n_1^2 = \frac{0.85}{13.6} = 16 \Rightarrow n_1 = 4$$

$$\text{Again,} 10.2 = \frac{13.6}{n_2^2} \Rightarrow n_2^2 = \frac{10.2}{13.6} = 4 \Rightarrow n_2 = 2$$

Thus, the states are 4 and 2.

B. If we take n_1 as n and n_2 as m and thus calculate the wavelength as follows,

$$\frac{1}{\lambda} = 1.1 \times 10^7 \left(\frac{1}{4} - \frac{1}{16}\right) = 1.1 \times 10^7 \left(\frac{3}{16}\right)$$

or, λ=487nm

13. Question

Whenever a photon is emitted by hydrogen in Balmer series, it is followed by another photon in Lyman series. What wavelength does this latter photon correspond to?

Answer

It is said that the photon is emitted in Balmer series and then next in Lyman series. Balmer series correspond to n=2 state and Lyman series corresponds to n=1 state. If we take n=1 and m=2, then we obtain the wavelength as,

$$\frac{1}{\lambda} = 1.1 \times 10^7 \times \left(\frac{1}{1} - \frac{1}{4}\right) = 1.1 \times 10^7 \times \frac{3}{4}$$

$$or, \lambda = 1.215 \times 10^{-7} = 121.5 \times 10^{-9} = 122nm$$

14. Question

A hydrogen atom in state n=6 makes two successive transitions and reaches the ground state. In the first transition a photon of 1.13eV is emitted. A. Find the energy of the photon emitted in the second transition B. What is the value of n in the intermediate state?

Answer

As the hydrogen atom reaches the ground state from n=6, thus,

A. Energy at n=6, $E = \frac{-13.6 \times Z^2}{n^2} = \frac{-13.6 \times 1^2}{6^2} = -0.3777777$

Again, energy in the ground state is, $E = -13.6$ (for hydrogen atom)

Since the atom makes 2 successive transitions, so the energy in the second transition can be found out by subtracting the energy in the ground state and energy in the 6^{th} state and adding up the first transition energy.

So, $E = -13.6 - 0.3777777 + 1.13 = -12.09 = -12.1eV$

$$or, E = 12.1eV$$

(The negative sign shows that energy is being given for making the transitions from 6^{th} state to the ground state)

B. Energy in the intermediate state is given as=1.13+0.3777777=1.507eV

Thus, to calculate the n, we have, $E = \frac{13.6 \times Z^2}{n^2} = 13.6 \times \frac{1^2}{n^2}$

$$or, 1.507 = 13.6 \times \frac{1}{n^2}$$

$$or, n^2 = \frac{1.507}{13.6} = 3.03$$

$$or, n \approx 3$$

15. Question

What is the energy of a hydrogen atom in the first excited state if the potential energy is taken to be zero in the ground state?

Answer

The energy needed to take the hydrogen atom from ground state to the excited state is called excitation energy and hydrogen requires 10.2 eV of energy to get excited from the ground state. As the energy of hydrogen atom in the ground state is 13.6 eV. So, the energy needed to make the hydrogen atom reach the first excited state is,

$$E = 13.6 + 10.2 = 23.8eV$$

16. Question

A hot gas emits radiation of wavelengths 46nm,82.8nm and 103.5nm only. Assume that the atoms have only two excited states and the difference between consecutive energy levels decreases as energy is increased. Taking the energy of the highest energy state to be zero, find the energies of the ground state and the first excited state.

Answer

As the atom has only 2 excited states, thus it makes transition to n=2 from ground state. The question asks to take higher energy state to be zero, so n=2 is zero in terms of energy.

So energy of the ground state will be,

$$E = \frac{hc}{\lambda_1} = \frac{6.634 \times 10^{34} \times 3 \times 10^8}{46 \times 10^{-9}} = \frac{1242}{46} = 27eV$$

And that of the first excited state is,

$$E = \frac{hc}{\lambda_2} = \frac{6.634 \times 10^{34} \times 3 \times 10^8}{103.5 \times 10^{-9}} = 12eV$$

17. Question

A gas of hydrogen like ions is prepared in a particular excited state A. It emits photons having wavelength equal to the wavelength of the first line of the Lyman series together with photons of five other wavelengths. Identify the gas and find the principal quantum number of the state A.

Answer

The gas emits total of 6 wavelengths. Thus,

$$from\ this\ formula, we\ obtain,$$

$$\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$$

Thus the gas is in the 4^{th} state. Since the gas is hydrogen like ion and in the 4^{th} excited state, thus the gas is He^+.

18. Question

Find the maximum angular speed of the electron of a hydrogen atom in a stationary orbit.

Answer

We know,

$$mvr = \frac{nh}{2\pi}$$

Where v is the velocity is the radius and m being the mass

For angular velocity i.e. ω, we know that v=ωr.

Thus, $m\omega r^2 = \frac{nh}{2\pi} \Rightarrow \omega = \frac{nh}{2\pi mr^2} = \frac{1\times6.634\times10^{34}}{2\times3.14\times9.1\times10^{-31}\times(0.053\times10^{-10})^2}$

$$or, \omega = 4.13 \times 10^{17} rad/sec$$

19. Question

A spectroscopic instrument can resolve two nearby wavelengths λ and λ+△ λ if λ/△ λ is smaller than 8000.This is used to study the spectral lines of the Balmer series of hydrogen. Approximately how many lines will be resolved by the instrument?

Answer

The range of Balmer series is from 656.3nm to 365 nm. If the instrument can resolve up to 80000, then,

$$No. of\ wavelengths\ in\ the\ range = \frac{656.3 - 365}{8000}$$

or, No. of wavelengths = 36

20. Question

Suppose, in certain conditions only those transitions are allowed to hydrogen atoms in which the principal quantum number n changes by 2.

(a) Find the smallest wavelength emitted by hydrogen.

(b) List the wavelengths emitted by hydrogen in the visible range (380 nm to 780 nm).

Answer

(a)Given: Quantum number n changes by 2.

And as we know that for minimum wavelength $n1 = 1$ and from given condition $n2 = 3$.

Now as per Einstein-Planck equation,

$E = \frac{hc}{\lambda}$ where $E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$ and λ is wavelength.

$$\Rightarrow 13.6\left(\frac{1}{1} - \frac{1}{9}\right) eV = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.6 \times 10^{-19}} eV$$

$$\Rightarrow 13.6\left(\frac{1}{1} - \frac{1}{9}\right) eV = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{\lambda} eV$$

$$\Rightarrow \lambda = 1.027 \times 10^{-7} m = 103 nm$$

(b)Since the obtained wavelength does not lie in visible range so we will take another transition possible that is from $n = 2$ to $n = 4$.

Again using,

$E = \frac{hc}{\lambda}$ where $E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$

$$\Rightarrow 13.6\left(\frac{1}{4} - \frac{1}{16}\right) eV = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.6 \times 10^{-19}} eV$$

$$\Rightarrow \lambda = 487 nm$$

Since this wavelength lies in the range between 380nm to 780nm so this is the wavelength emitted by hydrogen in the visible range.

21. Question

According to Maxwell's theory of electrodynamics, an electron going in a circle should emit radiation of frequency equal to its frequency of revolution. What should be the wavelength of the radiation emitted by a hydrogen atom in ground state if this rule is followed?

Answer

Frequency of radiation emitted by hydrogen atom in ground state is $f = \frac{c}{\lambda}$ where c is speed and λ denotes the wavelength.

Now, if Maxwell's theory of electrodynamics is true then this frequency is equal to the frequency due to revolution in the ground state by hydrogen atom which is given as-

$f = \frac{V}{2\pi r}$ where v represents the velocity in ground state and r denotes radius in ground state of hydrogen atom.

Equating both the equations,

$$\frac{c}{\lambda} = \frac{V}{2\pi r}$$

$$\Rightarrow \lambda = \frac{2c\pi r}{V}$$

$$\Rightarrow \lambda = \frac{2 \times 3 \times 10^8 \times 3.14 \times 0.53 \times 10^{-10}}{2187 \times 10^3} m = 45.686nm$$

22. Question

The average kinetic energy of molecules in a gas at temperature T is 1.5 kT. Find the temperature at which the average kinetic energy of the molecules of hydrogen equals the binding energy of its atoms. Will hydrogen remain in molecular from at this temperature? Take k = 8.62 × 10^{-5} eV K^{-1}.

Answer

As we know Binding energy of a system is defined as the energy released when its constituents are brought from infinity to form the system.

So,

Binding energy $= E = -13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) = -13.6\left(\frac{1}{\infty} - \frac{1}{1}\right) = 13.6\ eV$

According to this question the binding energy is equal to the average kinetic energy of the molecules.

So,

$$\frac{3}{2}KT = 13.6\ eV$$

$$\Rightarrow T = \frac{13.6}{1.5 \times 8.62 \times 10^{-5}} = 1.05 \times 10^5\ K$$

No, it is not possible because according to question positive ion of hydrogen molecules exists which is practically impossible.

23. Question

Find the temperature at which the average thermal kinetic energy is equal to the energy needed to take a hydrogen atom from its ground state to n = 3 state. Hydrogen can now emit red light of wavelength 653.1 nm. Because of Maximillian distribution of speeds, a hydrogen sample emits red light at temperatures much lower than that obtained from this problem. Assume that hydrogen molecules dissociate into atoms.

Answer

Energy needed to take hydrogen atom from ground to second exited state is given by

$$E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)eV = 13.6\left(\frac{1}{1} - \frac{1}{9}\right)eV = 12.08\ eV$$

This energy is equal to the average thermal kinetic energy.

$\frac{3}{2}KT = 12.08\ eV$ where value of K=$8.62 \times 10^{-5} eV/k$

$$\Rightarrow T = \frac{12.08}{1.5 \times 8.62 \times 10^{-5}} = 9.4 \times 10^4\ kelvin$$

24. Question

Average lifetime of a hydrogen atom excited to n = 2 state is 10^{-8} s. Find the number of revolutions made by the electron on the average before it jumps to the ground state.

Answer

As we know that number of revolution is equal to average lifetime divided by time period. So first we calculate time period.

Now, from Bohr's model we can say that

$$frequency = \frac{me^4}{4\varepsilon^2 n^3 h^3}$$

Where n=2, m=$9.1 \times 10^{-31} kg$ and h= 6.63×10^{-34}

Time period $= \frac{1}{f} = \frac{4\varepsilon^2 n^3 h^3}{me^4}$

$$\Rightarrow \frac{4 \times 8.85^2 \times 2^3 \times 6.63^3}{9.1 \times 1.6^4} \times 10^{-19} sec$$

$$\Rightarrow 1.224 \times 10^{-15} sec$$

Number of revolution $= \frac{Average\ lifetime}{Time\ period}$

$$= \frac{10^{-8}}{1.224 \times 10^{-15}} = 8.16 \times 10^6\ revolutions$$

25. Question

Calculate the magnetic dipole moment corresponding to the motion of the electron in the ground state of a hydrogen atom.

Answer

We know that dipole moment $\mu = niA$

As hydrogen atom is in ground state, so $n = 1$.

And we know that $i = \frac{q}{t}$ $and\ t = \frac{1}{f}$

Adding both the equations we get

$i = qf$

Hence, $\mu = niA = qfA$

Now from Bohr's model

$$f = \frac{me^4}{4\varepsilon^2 n^3 h^3}$$

And area is equal to $\pi r^2 n^2$.

So, $\mu = qfA = e \times \frac{me^4}{4\varepsilon^2 n^3 h^3} \times \pi r^2 n^2$

$$= \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^5 \times 3.14 \times (0.53 \times 10^{-10})^2}{4 \times (8.85 \times 10^{-12})^2 \times (6.64 \times 10^{-34})^3 \times 1}$$

$= 9.17 \times 10^{-24} A/m^2$

26. Question

Show that the ratio of the magnetic dipole moment to the angular momentum (ℓ = mvr) is a universal constant for hydrogen like atoms and ions. Find its value.

Answer

We know that magnetic dipole moment is $\mu = niA$

And we know that $i = \frac{q}{t}$ $and\ t = \frac{1}{f}$

Adding both the equations we get

$i = qf$

Hence, $\mu = niA = qfA$

Now from Bohr's model

$$f = \frac{me^4}{4\varepsilon^2 n^3 h^3}$$

And area is equal to $\pi r^2 n^2$.

So, $\mu = qfA = e \times \frac{me^4}{4\varepsilon^2 n^3 h^3} \times \pi r^2 n^2$

Now, angular momentum $l = mvr = \frac{nh}{2\pi}$

Now $\frac{\mu}{l} = e \times \frac{me^4}{4\varepsilon^2 n^3 h^3} \times \pi r^2 n^2 \times \frac{2\pi}{nh}$

$$= \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^5 \times 3.14^2 \times (0.53 \times 10^{-10})^2}{2 \times (8.85 \times 10^{-12})^2 \times (6.64 \times 10^{-34})^4 \times 1^2}$$

$= 8.73 \times 10^{10}\ C/kg$

Since this term is independent of Z so it is universal constant.

27. Question

A beam of light having wavelengths distributed uniformly between 450 nm to 550 nm passes through a sample of hydrogen gas. Which wavelength will have the least intensity in the transmitted beam?

Answer

As we know that energy associated with a wavelength λ

Is equal to $\frac{1242}{\lambda}\ eV$ where λ is in nm and this is derived from $E = hc/\lambda$.

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.6 \times 10^{-19}} eV$$
$$= \frac{1242}{\lambda} eV \text{ where } \lambda \text{ is in nm.}$$

So,

Energy associated with 450nm wavelength $= \frac{1242}{450}\ eV = 2.76eV$

Energy associated with 550nm wavelength $= \frac{1242}{550}\ eV = 2.26eV$

Since the light is coming in the visible region.

So, we have n1=2 and n2=3,4,5,6....and so on.

Now energy corresponding to change in these transition state is

E_2-E_3= $13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) eV = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 1.9eV$

E_2-E_4= $13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) eV = 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = 2.55eV$

E_2-E_5= $13.6\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right)eV = 13.6\left(\frac{1}{4}-\frac{1}{25}\right) = 2.85eV$

As E_2-E_4 is in range between the two energy produced by two given wavelengths so wavelength corresponding to this energy is absorbs.

$$E = \frac{1242}{\lambda} eV \ where \ \lambda \ is \ in \ nm.$$

$$\lambda = \frac{1242}{2.55} nm = 487.05nm$$

28. Question

Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms falls on helium ions in n = 1 and n = 2 states. What are the possible transitions of helium ions as they absorb energy from the radiation?

Answer

Radiation coming from change in transition state of hydrogen atom

$$E = 13.6\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right) = 13.6\left(\frac{1}{1}-\frac{1}{4}\right) = 10.2eV$$

Now we have to check transition possible for n=1 and n=2 states separately.

From n = 1 to 2 for helium ions(Z=2).

$$E1 = Z^2 \times 13.6\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right) = 4 \times 13.6\left(\frac{1}{1}-\frac{1}{4}\right) = 40.8eV$$

It is not possible as it is greater than E.

From n=1 to 3

$$E2 = Z^2 \times 13.6\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right) = 4 \times 13.6\left(\frac{1}{1}-\frac{1}{9}\right) = 48.3eV$$

It is also not possible.

Now as we can see energy is increasing (E2>E1) we can say that no transition is possible from n=1.

From n=2 to 3

$$E3 = Z^2 \times 13.6\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right) = 4 \times 13.6\left(\frac{1}{4}-\frac{1}{9}\right) = 7.56eV$$

This is possible as E3<E.

From n=2 to 4

$$E4 = Z^2 \times 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) = 4 \times 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = 10.2eV$$

It is also possible as E4=E.

Now as energy reached maximum limit so no further transition changes is possible.

Hence E3(n=2 to 3) and E4(n=2 to 4) are only possible transition change.

29. Question

A hydrogen atom in ground state absorbs a photon of ultraviolet radiation of wavelength 50 nm. Assuming that the entire photon energy is taken up by the electron, with what kinetic energy will the electron be ejected?

Answer

According to the question

Work function=Energy required to remove electron from ground state to n=∞

Work function = w

$$= E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) = 13.6\left(\frac{1}{1} - \frac{1}{\infty}\right) = 13.6eV$$

Now applying photoelectric effect.

$$\frac{hc}{\lambda} = w + KE$$

And

$$\frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.6 \times 10^{-19}} eV$$
$$= \frac{1242}{\lambda} eV \text{ where } \lambda \text{ is in nm.}$$

$$KE = \frac{hc}{\lambda} - w = \frac{1242}{50} - 13.6 = 24.84 - 13.6 = 11.24eV$$

30. Question

A parallel beam of light of wavelength 100 nm passes through a sample of atomic hydrogen gas in ground state.

(a) Assume that when a photon supplies some of its energy to a hydrogen atom, the rest of the energy appears as another photon moving in the same direction as the incident photon. Neglecting the light emitted by the excited hydrogen atoms in the directions of the incident beam. What wavelengths may be observed in the transmitted beam?

(b) A radiation detector is placed near the gas to detect radiation coming perpendicular to the incident beam. Find the wavelengths of radiation that may be detected by the detector.

Answer

Given: wavelength=100nm

Energy associated with this wavelength

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda \times 1.6 \times 10^{-19}} eV$$
$$= \frac{1242}{\lambda} eV \; where \; \lambda \; is \; in \; nm.$$

$E = \frac{1242}{\lambda(nm)}$ (substituting the vaue of c and plank's constant)

$= 12.42eV$

(a)Suppose E_n be the energy in the nth orbit. Let consider all possible change in transitions state.

Energy absorbed in transition state in n=1 to n=2

$$E = 13.6\left(\frac{1}{1} - \frac{1}{4}\right) = 10.2eV$$

Energy left$= 12.42eV - 10.2eV = 2.22eV$

Energy left$= \frac{1242}{\lambda(nm)}$ as we know that

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda \times 1.6 \times 10^{-19}} eV$$
$$= \frac{1242}{\lambda} eV \; where \; \lambda \; is \; in \; nm.$$

$$So, 2.22eV = \frac{1242}{\lambda(nm)}$$

$\Rightarrow \lambda = 560nm$

Energy absorbed in transition state in n=1 to n=3

$$E = 13.6\left(\frac{1}{1} - \frac{1}{9}\right) = 12.1eV$$

Energy left$= 12.42eV - 12.1eV = 0.32eV$

Energy left$= \frac{1242}{\lambda(nm)}$

$$\Rightarrow 0.32eV = \frac{1242}{\lambda(nm)}$$

$\Rightarrow \lambda = 3880nm$

Energy absorbed in transition state in n=3 to n=4

$$E = 13.6\left(\frac{1}{9} - \frac{1}{16}\right) = 0.65eV$$

Energy left$= 12.42eV - 0.65eV = 11.77eV$

Energy left$= \frac{hc}{\lambda}$

$$\Rightarrow 11.77eV = \frac{1242}{\lambda(nm)}$$

$$\Rightarrow \lambda = 105nm$$

(b)According to the question if hydrogen atom is radiated perpendicularly then only absorbed energy to change the transition state is taken into consideration.

For n=1 to n=2

$$E = 10.2eV$$

And

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda \times 1.6 \times 10^{-19}} eV$$
$$= \frac{1242}{\lambda} eV \text{ where } \lambda \text{ is in } nm.$$

$$\Rightarrow \frac{1242}{\lambda(nm)} = 10.2eV$$

$$\Rightarrow \frac{1242}{\lambda} = 10.2$$

$$\Rightarrow \lambda = 121nm$$

For n=1 to n=3

$$E = 12.1eV$$

$$\Rightarrow \frac{1242}{\lambda(nm)} = 12.1eV$$

$$\Rightarrow \frac{1242}{\lambda} = 12.1$$

$$\Rightarrow \lambda = 103nm$$

For n=3 to n=4

$$E = 0.65eV$$

$$\Rightarrow \frac{1242}{\lambda(nm)} = 0.65eV$$

$$\Rightarrow \frac{1242}{\lambda} = 0.65$$

$$\Rightarrow \lambda = 1911nm$$

Hence three wavelength obtained are 103nm,121nm,1911nm.

31. Question

A beam of monochromatic light of wavelength λ ejects photoelectrons from a cesium surface (ϕ = 1.9 eV). These photoelectrons are made to collide with hydrogen atoms is ground state. Find the maximum value of λ for which

(a) hydrogen atoms may be ionized,

(b) hydrogen atoms may get excited from the ground state to the first excited state and

(c) the excited hydrogen atoms may emit visible light.

Answer

Since we have to find maximum λ so energy of ionization should be maximum.

So n_1=1 and n_2=∞

$E = 13.6\left(\frac{1}{1} - \frac{1}{\infty}\right) = 13.6eV$ and we know that

$$\frac{hc}{\lambda} - work\ function = E$$

$$\frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda \times 1.6 \times 10^{-19}} eV$$
$$= \frac{1242}{\lambda} eV\ where\ \lambda\ is\ in\ nm.$$

$$\frac{1242}{\lambda(nm)} - 1.9 = 13.6\text{eV}$$

$$\Rightarrow \frac{1242}{\lambda} = (1.9 + 13.6)\ \text{eV}$$

$$\Rightarrow \lambda = 80\text{nm}$$

(b)To change the transition state from n=1 to n=2

$$E = 13.6\left(\frac{1}{1} - \frac{1}{4}\right) = 10.2eV$$

$$\frac{1242}{\lambda(nm)} - 1.9 = 10.2\text{eV}$$

$$\Rightarrow \frac{1242}{\lambda} = (1.9 + 10.2)\ \text{eV}$$

$$\Rightarrow \lambda = 102\ \text{nm}$$

(c) To get the light in visible region the change in transition state should be in Balmer series (n=2 to n=3)

But here atom is in ground state.

So, n=1 to n=3

Hence,

$$E = 13.6\left(\frac{1}{1} - \frac{1}{9}\right) = 12.09eV$$

$$\frac{1242}{\lambda(nm)} - 1.9 = 12.09\text{eV}$$

$$\Rightarrow \frac{1242}{\lambda} = (1.9 + 12.09)\ \text{eV}$$

$$\Rightarrow \lambda = 89\ \text{nm}$$

32. Question

Electrons are emitted from an electron gun at almost zero velocity and are accelerated by an electric filed E through a distance of 1.0 m. The electrons are now scattered by an atomic hydrogen sample in ground state. What should be the minimum value of E so that red light of wavelength 656.3 nm may be emitted by the hydrogen?

Answer

As the red light has wavelength 656.3nm which is in Balmer Series so for minimum energy change in state should be from n=2 to n=3. But right now hydrogen atom is in ground state.

Hence n=1 to n=3

So

$$E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) = 13.6\left(\frac{1}{1} - \frac{1}{9}\right) = 12.09eV$$

Hence minimum electric field required = 12.09 v/m

33. Question

A neutron having kinetic energy 12.5 eV collides with a hydrogen atom at rest. Neglect the difference in mass between the neutron and the hydrogen atom and assume that the neutron does not leave its line of motion. Find the possible kinetic energies of the neutron after the even.

Answer

According to the question as masses of hydrogen atom and neutron are equal and line of motion does not change, hence this is a type of elastic collision in one line/dimension.

Hence the velocity of two particles interchanged as per elastic collision principle.

So neutron will come to rest. Hence its kinetic energy is zero.

34. Question

A hydrogen atom moving at speed v collides with another hydrogen atom kept at rest. Find the minimum value of v for which one of the atoms may get ionized. The mass of a hydrogen atom = 1.67×10^{-27} kg.

Answer

According to problem this is a type of collision so we can conserve momentum and energy.

Let take v_1 and v_2 be two velocity of hydrogen after collision.

From conservation of momentum we have

$$mv = mv1 + mv2$$

$$\Rightarrow v = v1 + v2$$

From conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^{\ 2} + \frac{1}{2}mv_2^{\ 2}\text{+E}$$

Where E is ionization energy and is equal to 13.6eV.

Solving this equation

$$v^2 = v_1^{\ 2} + v_2^{\ 2} + \frac{2E}{m}$$

Putting value of $v = v1 + v2$

$$(v1 + v2)^2 = v_1^{\ 2} + v_2^{\ 2} + \frac{2E}{m}$$

$$v_1v_2 = \frac{E}{m}$$

And we know that

$$(v_1 + v_2)^2 = (v1 + v2)^2 - 4v_1v_2$$

Substituting $v_1 v_2$ *and* v from above expressions.

$$(v_1 + v_2)^2 = v^2 - 4\frac{E}{m}$$

Now for minimum v, $v_1 = v_2$

Then

$$v^2 = 4\frac{E}{m}$$

$$v = \sqrt{4\frac{E}{m}}$$

$$= \sqrt{\frac{4 \times 13.6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} \text{m/s}$$

$$= 7.2 \times 10^4 \text{m/s}$$

35. Question

A neutron moving with a speed v strikes a hydrogen atom in ground state moving towards it with the same speed. Find the minimum speed to the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron = mass of hydrogen = 1.67×10^{-27} kg.

Answer

For inelastic collision to take place between neutron and hydrogen of same mass the sum of initial kinetic energy should be greater than or equal to amount of energy absorbed for change in transition to first exited state.

Energy for first exited state $E = 13.6\left(\frac{1}{1} - \frac{1}{4}\right) = 10.2eV$

Sum of kinetic energy= K.E of neutron+ K.E of hydrogen atom

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2$ (as both has same velocity)

$$= mv^2$$

So for minimum speed

$$mv^2 = E$$

$$v = \sqrt{\frac{E}{m}}$$

$$= \sqrt{\frac{10.2 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} \text{m/s}$$

$= 3.13 \times 10^4 \text{m/s}$

36. Question

When a photon is emitted by a hydrogen atom, the photon carries a momentum with it.

(a) Calculate the momentum carried by the photon when a hydrogen atom emits light of wavelength 656.3 nm

(b) With what speed does the atom recoil during this transition? Take the mass of the hydrogen atom = 1.67×10^{-27} kg.

(c) Find the kinetic energy of recoil of the atom.

Answer

(a)As per $P = \frac{h}{\lambda}$

$= \frac{6.63 \times 10^{-34}}{656.3 \times 10^{-9}} = 10^{-27} \text{kgm/s}$

(b)As $P = mv$

$10^{-27} = 1.67 \times 10^{-27} \times v$

$v = 0.6 \text{m/s}$

(c) K.E$= \frac{1}{2} mv^2$

$= \frac{1}{2} \times 1.67 \times 10^{-27} \times 0.6^2$ J

$= 0.3 \times 10^{-27} J = 1.9 \times 10^{-9} eV$

37. Question

When a photon is emitted from an atom, the atom recoils. The kinetic energy of recoil and the energy of the photon come from the difference in energies between the states involved in the transition. Suppose, a hydrogen atom changes its state from n = 3 to n = 2. Calculate the fractional change in the wavelength of light emitted, due to the recoil.

Answer

Let E be the energy due to change in transition state and E_r be the recoil energy, $\Delta\lambda = \lambda' - \lambda \textit{ be change in wavelength.}$

So, $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$

And similarly from question $\lambda' = \frac{hc}{E - E_r}$

And $E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 1.89eV = 3.024 \times 10^{-19}J$

$$E_r = E - \frac{1}{2}mv^2$$

$$E_r = 3.024 \times 10^{-19}J - \frac{1}{2} \times 9.1 \times 10^{-31}\left[\left(\frac{2.18}{2}\right)^2 - \left(\frac{2.18}{3}\right)^2\right] \times 10^{12}\,J$$

$$E_r = 3.024 \times 10^{-19}J - 3.0225 \times 10^{-19}J$$

Fractional change in wavelength

$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \frac{E_r}{E - E_r} = 10^{-9}$ (after substituting the values and considering approximation.)

38. Question

The light emitted in the transition n = 3 to n = 2 in hydrogen is called H_α light. Find the maximum work function a metal can have so that H_α light can emit photoelectrons from it.

Answer

Maximum work function of a metal so that it can emit H_α light will be equal to energy absorbed by these H_α light to change the transition state from n=1 to n=2

$$E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 1.89eV$$

So maximum work function can be $1.89eV$.

39. Question

Light from Balmer series of hydrogen is able to eject photoelectrons from a metal. What can be the maximum work function of the metal?

Answer

Maximum work function of the metal will be equal to the maximum energy liberated in Balmer Series to change the transition state.

And for maximum energy in Balmer Series

$$n_1 = 2 \text{ and } n_2 = \infty$$

$$E = 13.6\left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) = 13.6\left(\frac{1}{4} - \frac{1}{\infty}\right) = 3.4eV$$

So maximum work function can be $3.4eV$.

40. Question

Radiation from hydrogen discharge tube falls on a cesium plate. Find the maximum possible kinetic energy of the photoelectrons. Work function of cession is 1.9 eV.

Answer

Given: Work function of cesium is 1.9 eV.

Radiation from hydrogen discharge tube is equal to the photon energy in ground state and is equal to 13.6eV.

Now applying photoelectric effect.

Maximum K.E = Energy of photon – Work function

$= 13.6eV - 1.9eV$

$= 11.7eV$

41. Question

A filter transmits only the radiation of wavelength greater than 440 nm. Radiation from a hydrogen discharge tube goes through such a filter and is incident on a metal of work function 2.0 eV. Find the stopping potential which can stop the photoelectrons.

Answer

Given: Threshold wavelength $\lambda = 440\text{nm}$

Work function $\phi = 2eV$

Applying photoelectric effect equation

$E - \phi = \text{K.E} = \text{e}V_0$

$and\ we\ know\ that\ E = \frac{1242}{\lambda} eV\ where\ \lambda\ is\ in\ nm$ from

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.6 \times 10^{-19}} eV$$
$$= \frac{1242}{\lambda} eV\ where\ \lambda\ is\ in\ nm.$$

$$\Rightarrow \left(\frac{1242}{440} - 2\right) eV = eV_0$$

$V_0 = 0.82 volt$

42. Question

The earth revolves round the sun due to gravitational attraction. Suppose that the sun and the earth are point particles with their existing masses and that Bohr's quantization rule for angular momentum is valid in the case of gravitation.

(a) Calculate the minimum radius the earth can have for its orbit.

(b) What is the value of the principal quantum number n for the present radius? Mass of the earth = 6.0 × 10^{24} kg, mass of the sun = 2.0 × 10^{30} kg, earth-sun distance =1.5 × 10^{11} m.

Answer

Given: Mass of the earth = m_e = 6.0 × 10^{24} kg

Mass of the sun = m_s = 2.0 × 10^{30} kg

earth-sun distance = r =1.5 × 10^{11} m

Applying Bohr's quantization rule for angular momentum

$$m_e vr = \frac{nh}{2\pi}$$

And due to gravitational force balancing centripetal force.

$$\frac{Gm_s m_e}{r^2} = \frac{m_e v^2}{r}$$

Using both the equations we get

$$r = \frac{n^2 h^2}{4\pi^2 G m_s{}^2 m_e{}^2}$$

And $v = \frac{2\pi G m_s m_e}{nh}$

(a) For minimum value of r, n=1

$$r = \frac{1^2 \times (6.63 \times 10^{-34})^2}{4 \times 3.14^2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}} = 2.3 \times 10^{-138} m$$

(b) For present radius

$$n = \sqrt{\frac{4\pi^2 r G m_s{}^2 m_e{}^2}{h^2}}$$

After substituting the values as above

$$n = 2.5 \times 10^{74}$$

43. Question

Consider a neutron and an electron bound to each other due to gravitational force. Assuming Bohr's quantization rule for angular momentum to be valid in this case, derive an expression for the energy of the neutron-electron system.

Answer

Applying Bohr's quantization rule for angular momentum

$$m_e vr = \frac{nh}{2\pi}$$

And due to gravitational force balancing centripetal force.

$$\frac{Gm_s m_e}{r^2} = \frac{m_e v^2}{r}$$

Using both the equations we get

$$r = \frac{n^2 h^2}{4\pi^2 G m_s{}^2 m_e{}^2}$$

And $v = \frac{2\pi G m_s m_e}{nh}$

Now substituting these values in

$$K.E = \frac{1}{2} m_e v^2 = \frac{1}{2} m_e \left(\frac{2\pi G m_s m_e}{nh}\right)^2 = \frac{2\pi^2 G^2 m_s{}^2 m_e{}^2}{n^2 h^2}$$

$$P.E = -\frac{Gm_s m_e}{r} = -\frac{4\pi^2 G^2 m_s{}^2 m_e{}^2}{n^2 h^2}$$

Total energy = K.E + P.E

$$= -\frac{2\pi^2 G^2 m_s{}^2 m_e{}^2}{n^2 h^2}$$

44. Question

A uniform magnetic field B exists in a region. An electron projected perpendicular to the field goes in a circle. Assuming Bohr's quantization rule for angular momentum, calculate

(a) the smallest possible radius of the electron

(b) the radius of the nth orbit and

(c) the minimum possible speed of the electron.

Answer

Applying Bohr's quantization rule

$$mvr = \frac{nh}{2\pi}$$

And as electron is projected perpendicular to the magnetic field it will follow

$$r = \frac{mv}{qB}$$

Using both the equations and q=e

$$eBr^2 = \frac{nh}{2\pi}$$

$$r = \sqrt{\frac{nh}{2\pi eB}}$$

(a)Smallest possible radius is when n=1

$$r = \sqrt{\frac{h}{2\pi eB}}$$

(b)Radius in nth orbit

$$r = \sqrt{\frac{nh}{2\pi eB}}$$

(c)Using $mvr = \frac{nh}{2\pi}$ again and substituting the value of r from above

$$v = \sqrt{\frac{nheB}{2\pi m^2}}$$

For minimum possible value n=1

$$v = \sqrt{\frac{heB}{2\pi m^2}}$$

45. Question

Suppose in an imaginary world the angular momentum is quantized to be even integral multiples of h/2π. What is the longest possible wavelength emitted by hydrogen atoms in visible range in such a world according to Bohr's model?

Answer

According to question if even quantum numbers are only allowed as there is even multiple of $h/2\pi$ then longest wavelength is possible for n=2 to n=4

Energy in this path

$$E = 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = 2.55eV$$

and we know that

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda} J = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{\lambda \times 1.6 \times 10^{-19}} eV$$

$$= \frac{1242}{\lambda} eV \; where \; \lambda \; is \; in \; nm.$$

$$\frac{1242}{\lambda} = 2.55$$

$$\lambda = 487nm$$

46. Question

Consider an excited hydrogen atom in state n moving with a velocity v (v << c). It emits a photon in the direction of its motion and changes its state to a lower state m. Apply momentum and energy conservation principles to calculate the frequency v of the emitted radiation. Compare this with the frequency v_0 emitted if the atom were at rest.

Answer

As per the question v is frequency of emitted radiation and

v_0 is frequency if atom is at rest and velocity of photon is u.

But as u << c so velocity of emitted photon is same as that of

Hydrogen atom =u.

Now applying Doppler's effect formula

$$v = v_0 \left(\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}} \right)$$

As u << c $\Rightarrow \frac{u}{c} \ll c$

Hence

$$v = v_0 \left(1 + \frac{u}{c}\right)$$