Chapter 4

Viscous Flow of Incompressible Fluids

CHAPTER HIGHLIGHTS

- *Dimensional analysis*
- *Buckingham's π-theorem*
- *Dimensionless numbers*
- *Flow through pipes*
- *Power transmission through pipes*
- *Water hammer in pipes*
- *Flow through syphon*
- *Laminar flow in horizontal pipes*
- *Flow of viscous fluid between two parallel plates*
- *Flow of lubricant in a journal bearing*
- *Kinetic energy correction factor (α)*
- *Momentum correction factor (β)*
- *Boundary layer theory*
- *Turbulent flow in pipes*

Dimensional Analysis

It is a mathematical technique which involves the study of dimensions for solving engineering problems. Each physical phenomenon can be expressed by an equation which relates several dimensions and non-dimensional quantities.

Buckingham's Π-Theorem

If there are n variables in a dimensionally homogenous (i.e., each additive term has the same dimensions) equation and if these variables contain m fundamentals (basic or primary) dimensions, then the variables can be arranged into $(n - m)$ dimensionless terms (or parameters) called Π-terms and the equation can be written in terms of these $(n - m)$ Π-terms.

Dimensionless Numbers

Reynolds Number (*Re*)

It is defined as the ratio of inertia force to the viscous force.

$$Re = \frac{\rho V L}{\mu}$$

Where ρ and μ are the density and viscosity of the fluid respectively. V is a characteristic velocity and L is a characteristic length.

For pipe flow, characteristic length is equal to the diameter of the pipe (D) and hence,

$$Re \text{ pipe flow} = \frac{\rho V D}{\mu}$$

Froude Number (*Fr*)

It is defined as the square root of the ratio of inertia force to the gravity force,

$$Fr = \frac{V}{\sqrt{L g}}$$

Euler Number (*Eu*)

It is defined as the square root of the ratio of inertia force to the pressure force.

$$Eu = \frac{V}{\sqrt{\frac{p}{\rho}}}$$

Where, Δp is the pressure difference

Weber Number (*We*)

It is defined as the square root of the ratio of inertia force to the surface tension force.

$$We = \sqrt{\frac{\rho V^2 L}{\sigma}} \qquad \text{i.e., } We = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

Where, σ is the surface tension.

Mach Number (*Ma*)

It is defined as the square root of the ratio of inertia force to the elastic force.

$$Ma = \frac{V}{C}$$

Where, C is the velocity of sound in the fluid.

Average Velocity (V_{avg})

It is defined as the average speed through a cross-section and is defined as:

$$V_{avg} = \frac{\int_A \rho\, u(r)\, dA}{\rho A}$$

Where ρ is the fluid density, A is the cross-sectional area, $u(r)$ is the velocity at any radius 'r' (referred to the pipe centre) the distance from the pipe centerline.

For incompressible flow in a circular pipe of radius R,

$$V_{avg} = \frac{2}{R^2}\int_O^R U(r)\, r\, dr$$

Flow Through Pipes

Critical Reynolds Numbers

1. For **flow in a circular pipe**, Reynolds number is given by,

$$Re = \frac{\rho V_{AV} D}{\mu}$$

Where

ρ = Density of fluid flowing inside the pipe

V_{avg} = Average velocity of flow inside the pipe

D = Diameter of the pipe and

μ = Dynamic viscosity of the fluid inside the pipe.

2. For **flow through ducts (or non-circular cross-section pipes)**, Reynolds number is based on the **hydraulic mean diameter** (D_m) instead of D.

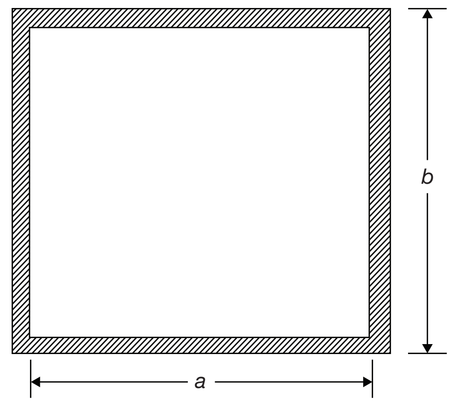
∴ For non-circular pipes,

$$R_e = \frac{\rho V_{avg} D_m}{\mu}, \text{ where}$$

$$D_m = \text{Hydraulic diameter of duct} = \frac{4A}{P_w}$$

Here, A = Area of flow and P_w = Wetted perimeter of duct

Example: For a rectangular duct of width a and height b shown in the figure,



Hydraulic mean diameter,

$$D_m = \frac{4A}{P_w} = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$$

3. For **flow over flat plate**, Reynolds number is given by

$$Re = \frac{\rho V x}{\mu}$$

Where, x = distance of the point on the plate from where the solid surface starts (measured in the direction of flow).

The Reynolds number **at and below which the flow remains laminar** (i.e., all turbulences are damped down), is called **lower critical Reynolds number**.

The Reynolds number **at and above which the flow is turbulent** (i.e., flow cannot remain laminar) is called the **upper critical Reynolds number**. In between these two critical values of Reynolds number, **flow is transitory**.

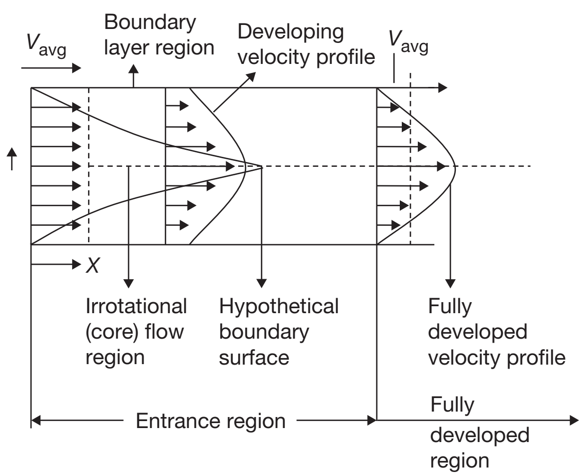
The lower critical Reynolds number and upper critical Reynolds number for various types of flows are tabulated below.

Flow Condition	Laminar	Transitional	Turbulent
Flow in circular pipes	$Re \le 2000$	$2000 < Re < 4000$	$Re \ge 4000$
Open channel flow	$Re \le 500$	$500 < Re < 1000$	$Re \ge 1000$
Flow over plate	$Re < 5 \times 10^5$	–	$Re > 5 \times 10^5$

Entrance Region and Fully Developed Flow

When a fluid enters a circular pipe at a nearly uniform velocity, a velocity gradient develops along the pipe. A boundary layer (flow region in which effects of the viscous shearing forces caused by the fluid viscosity are significant) is produced which grows in thickness to completely fill the pipe. At a point further downstream, the velocity becomes fully developed and the region from the pipe inlet to this point is called the *entrance region* whose length is called the *entrance length* '*Le*'.

The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged in the flow direction is called the '*fully developed region*'.



At the *irrotational (core) flow region,* viscous effects are negligible and velocity remains essentially constant in the radial direction. This region is separated from the boundary layer region by a hypothetical boundary surface.

The effect of the entrance region is to increase the average friction factor for the entire pipe, the increase being significant for short pipes but negligible for long pipes.

Entrance Length

In laminar flow, $\dfrac{Le}{D} \approx 0.06\, Re$

In turbulent flow, $\dfrac{Le}{D} \approx 4.4\, Re^{1/6}$

$\dfrac{Le}{D}$ is sometimes referred to as the *dimensionless entrance length*.

Entrance length for turbulent flow is much shorter than for laminar flow

Loss of Energy (or Head) in Pipes

When a fluid flows in a pipe, its motion experiences some resistance due to which the available head reduces. This loss of energy or head is classified as:

Major Energy Losses

These are energy losses due to friction and the loss of head due to friction (h_L) is calculated using Darcy–Weisbach equation given earlier.

In terms of the flow and resistance R, h_L can be written as

$$h_L = R\,Q^2$$

Flow through pipes with side tappings: Consider the flow through a pipe when a fluid is withdrawn from closely spaced side tappings along the length of the pipe as shown in the following figure.

Let the fluid be removed at a uniform rate q per unit length of the pipe. Let the volume flow rate into the pipe be Q_0 and let L and D be the length and diameter of the pipe. If f is the friction factor assumed to be constant over the length of the pipe, then

$$h_f = \frac{8 Q_0^2 f L}{\pi^2 D^5 g}\left[1 - \frac{qL}{Q_0} + \frac{1}{3}\frac{q^2 L^2}{Q_0^2}\right]$$

If the entire flow is drained off over the length L, then

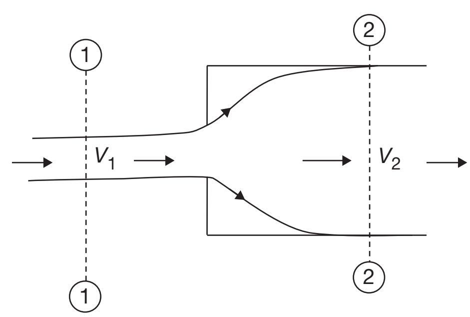
$$h_f = \frac{1}{3} f \frac{L}{D} V_0^2 \frac{1}{2g}$$

Where, $V_0 = \dfrac{Q_0}{\frac{\pi}{4} D^2}$. The above equation indicates that the loss of head due to friction over a length L of a pipe, where the entire flow is drained off uniformly from the side tappings, becomes one third of that in a pipe of same length and diameter but without side tappings.

Minor Energy Loses

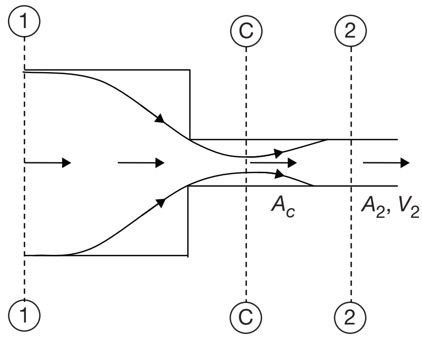
The minor energy losses include the following cases

1. **Loss of head due to sudden enlargement (h_e):**



$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

2. **Loss of head due to sudden contraction (h_c):**



$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

Where, $C_c = \frac{A_c}{A_2}$ is the *coefficient of contraction*. If the value of C_c is not known, then loss of head due to contraction may be taken as $0.5\frac{V_2^2}{2g}$.

3. **Loss of head due to obstruction in pipe (h_{obs}):**

$$h_{obs} = \left[\frac{A}{C_c(A-a)}\right]^2 \frac{V^2}{2g}$$

Where, A is the area of the pipe, a is the maximum area of obstruction and V is the velocity of liquid in the pipe.

4. **Loss of head at the entrance to pipe (h_i):**

$$h_i = 0.5\frac{V^2}{2g}$$

Where, V is the velocity of liquid in pipe.

5. **Loss of head at the exit of a pipe (h_0):**

$$h_o = \frac{V^2}{2g}$$

Where, V is the velocity at outlet of pipe.

6. **Loss of head due to bend in the pipe (h_b):**

$$h_b = \frac{KV^2}{2g}$$

Where, V is the mean velocity of flow of liquid and K is the *coefficient of bend*.

7. **Loss of head in various pipe fittings ($h_{fittings}$)**

$$h_{fittings} = \frac{KV^2}{2g}$$

Where, V is the mean velocity of flow in the pipe and K is the value of the coefficient that depends on the type of pipe fitting.

These losses (h_b and $h_{fittings}$) are sometimes expressed in terms of an equivalent length (L_e) of an unobstructed straight pipe in which an equal loss would occur for the same average flow velocity.

$$L_e = \frac{DK}{f}$$

NOTE

For a sudden expansion in a pipe flow, if D_1 and D_2 are the diameter of the pipe before and after the expansion respectively, the pressure rise is maximum when $\frac{D_1}{D_2} = \frac{1}{\sqrt{2}}$ and the maximum pressure rise would be $\frac{0.5\rho g V_1^2}{2g}$.

Equivalent Pipe

An *equivalent pipe* is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe (pipe in series) consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is known as *equivalent diameter* of the pipes in series.

Consider n pipes in series where the length, diameter and friction factor associated with the ith pipe are L_i, D_i and f_i respectively. If L, D and f are the length, diameter and friction factor associated with the equivalent pipe, then neglecting minor losses we have,

$$\frac{fL}{D^5} = \sum_{i=1}^{n} \frac{f_i L_i}{D_i^5}$$

If the friction factor f_i is equal to f, then

$$\frac{L}{D^5} = \sum_{i=1}^{n} \frac{L_i}{D_i^5}$$

The above equation is called the **Dupit's equation**.

SOLVED EXAMPLES

Example 1

A piping system consists of a pipe of length L which can be replaced by an equivalent pipe of length L_e, diameter D_e and friction factor f_e. The length of the pipe is increased by amount ΔL. The new pipe can be replaced by an equivalent pipe of length L_e, diameter $0.5\ D_e$ and friction factor $0.5\ f_e$.

If the increase in length has led to the friction factor of the new pipe being a quadruple of the old pipe, then ΔL is equal to

(A) $15L$ (B) $3L$
(C) $4L$ (D) $7L$

Solution

Let D and f be the diameter and friction factor of the old pipe.

$$\therefore \frac{L f}{D^5} = \frac{L_e f_e}{D_e^5} \quad (1)$$

For the new pipe,

$$\frac{(L+\Delta L)\times 4 f}{D^5} = \frac{L_e \times 0.5 f_e}{(0.5 D_e)^5}$$

$$= 16 \frac{L_e f_e}{D_e^5} \quad (2)$$

Substituting Eq. (1) in Eq. (2), we have

$$\frac{(L+\Delta L)\times 4 f}{D^5} = 16 \times \frac{L f}{D^5}$$

$$L + \Delta L = 4L$$

$$\Delta L = 3L$$

Hence, the correct answer is option (B).

Example 2

A piping system consists of a series of pipes in which a 20 m long pipe of diameter 250 m ($f = 0.025$), containing a value ($K = 1.0$), suddenly expands to a 500 mm diameter pipe ($f = 0.02$) of length 40 m. If the velocity of flow in the 250 mm diameter pipe is 4 m/s, then the length of an equivalent pipe ($f = 0.02$) of diameter 500 mm for the piping system would be

(A) 1090 m (B) 865 m
(C) 1065 m (D) 1050 m

Solution

Equivalent length for the 250 mm diameter pipe,

$$h_{e_1} = \frac{f_2 L_2}{D_2^5} \times \frac{D_e^5}{f_e}$$

$$= \frac{0.025\times 20}{(0.25)^5} \times \frac{(0.5)^5}{0.02} = 800\text{ m}$$

Equivalent length for the value,

$$h_{e_2} = \frac{KD}{f} = \frac{1.0\times 0.5}{0.02} = 25\text{ m}$$

Velocity of flow in the 500 mm diameter pipe

$$= \left(\frac{0.25}{0.5}\right)^2 \times 4 = 1\text{ m/s}$$

Head loss due to expansion,

$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{(4-1)^2}{2\times 9.81}$$

$$= 0.4587\text{ m}$$

Let L_{e3} be the equivalent length for the sudden expansion.

Then, $$f_e \frac{L_{e_3}}{D_e} \times \frac{V_2^2}{2g} = h_L$$

That is, $$L_{e3} = 0.4587\times 2\times 9.81\times \frac{0.5}{0.02\times (1)^2}$$

$$= 225\text{ m}$$

Equivalent length for the 500 mm diameter pipe of length 40 m.

$$L_{e4} = 40\text{ m}$$

Total equivalent length

$$= L_{e1} + L_{e2} + L_{e3} + L_{e4}$$

$$= 800 + 25 + 225 + 40 = 1090\text{ m.}$$

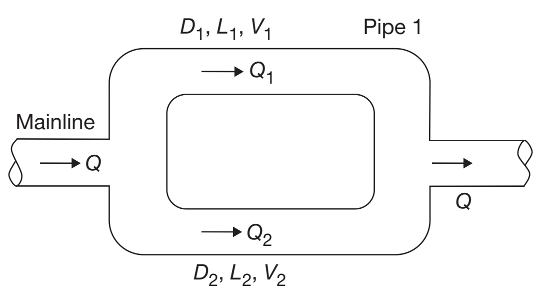
Hence, the correct answer is option (A).

Pipes in Parallel

For the parallel pipe system shown below, the rate of discharge in the main line is equal to the sum of the discharge in the pipes.

That is,

$$\boxed{Q = Q_1 + Q_2}$$

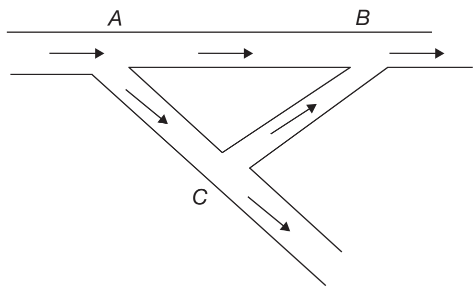


When pipes are arranged in parallel the head loss in each pipe is the same.

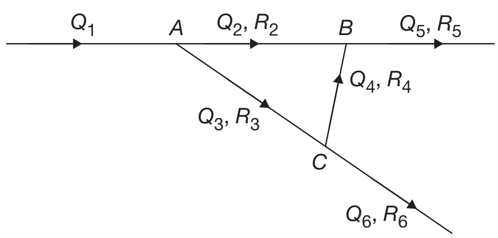
That is,

$$\boxed{\text{Loss of head in pipe 1} = \text{Loss of head in pipe 2}}$$

Pipe Network



The pipe structure shown above can be converted into a pipe network (or hydraulic circuit) with nodes (or junctions) and links. Here Q denotes the flow rate and R denotes the flow resistance.



In the above network, the algebraic sum of the flow rates at any node must be zero, i.e., the total mass flow rate towards the junction must be equal to the total mass flow rate away from it.

At a node,

$$\Sigma Q_{\text{in}} = \Sigma Q_{\text{out}}$$

Example: At node A, $Q_1 = Q_2 + Q_3$

Also in the above network, the algebraic sum of the products of the flux(Q^2) and the flow resistance (the sense being determined by the direction of flow) must be zero in any closed loop or hydraulic circuit.

In a closed loop,

$$\sum R_i \,|Q_i|\, Q_i = 0 \qquad (1)$$

Example: Considering the loop ABC, we can write

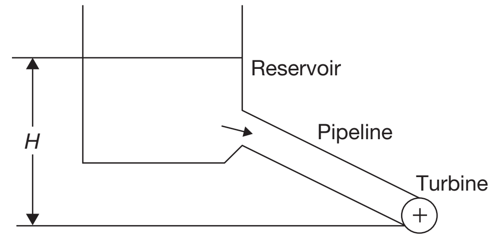
$$R_2(Q_2)^2 - R_4(Q_4)^2 - R_3(Q_3)^2 = 0$$

The term $R_4\,|Q_4|\,Q_4$ gets converted to the negative quantity $-R_4(Q_4)^2$ because in the link BC, the considered loop direction (from B to C) is opposite to the flow direction (from C to B). Eq. (1) is referred to as the *pressure equation* of the circuit. Since, $h_L = RQ^2$, Eq. (1) can be rewritten as:

$$\sum h_{L_i} = 0$$

Where the correct sign values are assigned to the h_L value.

Power Transmission through Pipes



In the above system, hydraulic power is transmitted by a pipeline (through conveyance of the liquid) to a turbine. Here, the hydrostatic head of the liquid is transmitted by the pipeline.

Potential head of liquid in the reservoir = H (difference in the liquid level in the reservoir and the turbine center).

Head available at pipe exit (or the turbine entry) = $H - h_L$ (neglecting minor losses), where h_L is loss of head in the pipeline due to friction.

Power transmitted by the pipeline (or available of the exit of the pipeline),

$$P = \rho g Q(H - h_f)$$

Efficiency of power transmission,

$$\eta = \frac{H - h_f}{H} \times 100$$

Power transmitted will be maximum when $h_f = \dfrac{H}{3}$

Maximum power transmission efficiency (or efficiency of transmission at the condition of maximum power delivered) is $\dfrac{200}{3}\%$ or 66.67%

Water Hammer in Pipes

In a long pipe, when the flow velocity of water is suddenly brought to zero (by closing a valve), there will be a sudden rise in pressure due to the momentum of water being destroyed. A pressure wave is transmitted along the pipe. A sudden pressure rise brings about the effect of a hammering action on the walls of the pipe. This phenomenon of sudden rise in pressure is known as *water hammer* or *hammer blow.*

The magnitude of pressure rise depends on:

1. Speed at which valve is closed
2. Velocity of flow

3. Length of pipe and
4. Elastic properties of the pipe material as well as that of the flowing fluid.

Let, D = Diameter of pipe
A = Area of cross-section of pipe
T = Thickness of pipe
L = Length of pipe

Time taken by the pressure wave to travel from tank to valve and valve to tank is given by, $T = \frac{2L}{C}$

Where, C is velocity of pressure wave
For gradual closure of valve $T > 2L/C$
For sudden closure of valve $T < 2L/C$

Gradual closure:

$$\frac{P}{\rho g} = \frac{LV}{gT}$$

Sudden closure and rigid pipe:

$$\frac{P}{\rho g} = \frac{VC}{g}$$, where P is pressure developed.

Sudden closure and elastic pipe:

$$\frac{P}{\rho g} = \frac{VC}{g}\sqrt{\frac{1}{1+\frac{DK}{Et}}}$$

Where
D = Diameter of pipe
K = Bulk modulus of pipe material
E = Youngs modulus of pipe material
t = Thickness of pipe

Flow Through Syphon

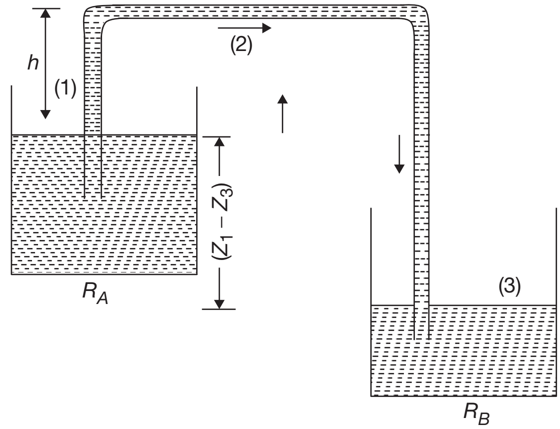
When two reservoirs, one at a higher level and another at a lower level are separated by a high level ground or hill, a long bend pipe which is used to transfer liquid from the higher altitude reservoir to the lower altitude reservoir is called a **syphon**.

Syphons are also used to:

1. Empty a channel not provided with any outlet orifice
2. To take out liquid from a tank not provided with any outlet.

A syphon used for transferring liquid from a high altitude reservoir R_A to a low altitude reservoir R_B is shown in the figure. The highest point of the syphon (2) is called the summit, while (1) and (3) are the free liquid surface in reservoir R_A and R_B respectively. The height difference between (1) and (3) is $(Z_1 - Z_3)$. Since (1) and (3) are open to atmosphere, the corresponding pressures are $p_1 = p_3 = p_a$, where p_a = atmospheric pressure.

Since (2) at a higher level than (1), pressure at (2) (i.e., p_2) is less than p_1, i.e., $p_2 < p_1$ $(p_1 = p_a)$.



Atmospheric pressure, p_a = 10.3 m of water column. Hence theoretically, for water flow, the pressure at summit p_2 can be −10.3 m of water but practically it must be between −7.6 m and −8.0 m. Hence the vertical height difference (h) between (2) and (1) must be restricted to (10.3 − 8.0 = 2.3) to (10.3 − 7.6 = 2.7) m, so that the pressure at summit (p_2) is in the range of 2.3 m to 2.7 m absolute. If the pressure at summit becomes less than this value, dissolved air and gases will come out of water and accumulate at the summit, hindering the flow of water.

If ρ is the density of liquid, V_1 = velocity of flow at (1), V_3 = velocity of flow at (3), then by applying Bernoulli's equation between points (1) and (3), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 + h_f$$

Here, h_f = head loss due to friction in syphon = $\frac{4fLV^2}{2gd}$

Where
L = Length of syphon pipe
d = Diameter of siphon pipe,
V = Average velocity of flow in the syphon pipe.
F = Friction coefficient for syphon pipe

We have, $p_1 = p_3 = p_a$ and $V_1 = V_2 = 0$. ($\therefore R_A$ and R_B are large tanks)

$$\text{Hence, } (Z_1 - Z_3) = h_f = \frac{4fLV^2}{2gd} \quad (1)$$

If $(Z_1 - Z_3)$ is known, d and L are known, then V^2 can be calculated.

Once V is known, discharge $Q = \frac{\pi}{4}d^2V$ will give the discharge through the syphon.

It must be noted that in the above calculation, we have **considered all minor losses as negligible**.

Now by applying Bernoulli's equation between points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f^1$$

Here, $V_2 = V$ (as calculated earlier)

$h_f^1 = \dfrac{4fL_1V^2}{2gd}$, where L_1 = length of syphon pipe from tank R_A to summit (2).

NOTE

$$h_f^1 = h_f \times \frac{L_1}{L}$$

Also $p_1 = p_a = 0,\ V_1 = 0$

$$\Rightarrow \frac{p_2}{\rho g} + \frac{V^2}{2g} + \frac{4fL_1V^2}{2gd} + (Z_2 - Z_1) = 0$$

i.e., $\dfrac{p_2}{\rho g} + \dfrac{V^2}{2g} + \dfrac{4fL_1V^2}{2gd} + h = 0\ (QZ_2 - Z_1 = h)$

From the above equation, minimum pressure at summit p_2 can be calculated. If minimum pressure p_2 is known, the maximum height h can be calculated.

Example 3

A large water tank empties by gravity through a syphon. The difference in levels of the high altitude and low altitude tanks is 3 m and the highest point of the siphon is 2 m above the free surface of water in the high altitude tank. The length of syphon pipe is 6 m and its bore is 25 mm. Also the length of syphon pipe from inlet to the highest point is 2.5 m. The friction coefficient for the pipe is 0.007 and all other losses are negligible. Calculate the volume flow rate of water through the syphon and the pressure head at the highest point in the pipe.

Solution

Given $Z_1 - Z_3 = 3$ m

$L = 6$ m

$d = 25$ mm $= 25 \times 10^{-3}$ m

$f = 0.007$

We have $(Z_1 - Z_3) = \dfrac{4fLV^2}{2gd}$

$$\Rightarrow V^2 = \frac{2(Z_1 - Z_3)gd}{4fL}$$

$$\Rightarrow V = \sqrt{\frac{(Z_1 - Z_3)gd}{2fL}}$$

$$= \sqrt{\frac{3 \times 9.81 \times 25 \times 10^{-3}}{2 \times 0.007 \times 6}}$$

$= 2.96$ m/s

Hence, speed of flow of water in syphon is 2.96 m/s

Q = Discharge = Area × Velocity

$$= \frac{\pi}{4}d^2V = \frac{\pi}{4} \times (25 \times 10^{-3})^2 \times 2.96$$

$= 1.453 \times 10^{-3}$ m³/s

$= 1.453$ lit/s ($\because 10^{-3}$ m³ $= 1$ litre)

∴ Volume flow rate through the syphon is 1.453 lit/s

Given, $(Z_2 - Z_1) = 2$ m

L_1 = length of pipe from inlet to summit = 2.5 m

$$\therefore h_f^1 = \frac{4fL_1V^2}{2gd}$$

$$= \frac{4 \times 0.007 \times 2.5 \times (2.96)^2}{2 \times 9.81 \times 25 \times 10^{-3}} = 1.25 \text{ m}$$

Applying Bernoulli's equation between inlet (1) and summit (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{v^2}{2g} + Z_2 + h_f^1$$

But $p_1 = 0$ ($\because$ atmospheric pressure)

$V_1 = 0$ ($\because$ large tank)

$$\Rightarrow \frac{p_2}{\rho g} + \frac{v^2}{2g} + h_f^1 + (Z_2 - Z_1) = 0$$

$$\Rightarrow \frac{p_2}{\rho g} = -(Z_2 - Z_1) - h_f^1 - \frac{v^2}{2g}$$

$$= -(2) - (1.25) - \frac{2.96^2}{2 \times 9.81}$$

$= -2 - 1.25 - 0.45$

$= -3.70$ m of water

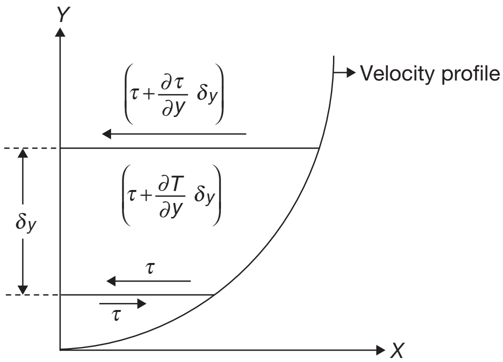
∴ Pressure head at the highest point in the syphon is –3.70 m of water (i.e., 3.7 m of water absolute).

Relationship between Shear Stress and Pressure Gradient

Let us consider a fluid element whose velocity distribution is shown in the following figure, where the shear stresses (τ) acting on the two fluid layers are also shown.

The motion of the fluid element will be resisted by shearing or frictional forces which must be overcome by maintaining a pressure gradient in the direction of flow. Here,

$$\boxed{\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x}}$$



That is, the pressure gradient $\left(\frac{\partial p}{\partial x}\right)$ in the direction of flow (steady and uniform) is equal to the shear gradient $\left(\frac{\partial \tau}{\partial y}\right)$ in the direction normal to the direction of flow. The above equation holds for all flow conditions and geometries.

Laminar Flow in Horizontal Pipes

The following discussion is based on the steady laminar incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe unless stated otherwise.

1. A fully developed laminar pipe flow is merely a balance between pressure and viscous forces. For the steady fully developed laminar flow of a fluid through a horizontal circular pipe of radius *R*, the shear stress distribution is given by

$$\boxed{\tau = \frac{-\partial p}{\partial x}\cdot\frac{r}{2}} \qquad (1)$$

Here, *x* is the distance along the pipe. The pressure gradient in the *x*-direction, $\frac{\partial p}{\partial x}$ is larger in the entrance region than in the fully developed region where it is a constant, $\frac{\partial p}{\partial x} = \frac{-\Delta p}{L}$, where Δp is the pressure drop over a flow section of length *L*.

$$\boxed{\tau = \frac{\Delta p}{L}\frac{r}{2}}$$

Few highlighting points that can be deciphered from Eq. (1) are:

(a) Flow will occur only if a pressure gradient exists in the flow.

(b) Pressure decreases in the direction of flow due to viscous effects.

(c) Shear stress varies linearly across the flow section with a value of zero at the centre of the pipe ($r = 0$) and with a maximum value $\left(= \frac{-\partial p}{\partial x}\frac{R}{2} = \frac{\Delta p}{L}\frac{R}{2}\right)$ at the pipe wall.

The shear stress at the pipe wall is called the *wall shear stress* τ_ω.

$$\boxed{\tau_\omega = \frac{\Delta p}{L}\frac{R}{2}}$$

$$\boxed{\tau = \frac{\tau_\omega r}{R}}$$

The wall shear stress is highest at the pipe inlet and it decreases gradually to the fully developed value. In a steady fully developed flow, wall shear stress remains constant. The above four equation are valid for turbulent flow also.

The equations stated in the following section rests on the following two assumptions:

(a) Fluid is Newtonian

(b) No slip of fluid particles occurs at the boundary (no-slip condition), i.e., fluid particles adjacent to the pipe will have zero velocity.

2. Velocity profile (*u*(*r*)): In a fully developed laminar flow, there is no motion in the radial direction and thus the velocity component in the direction normal to the pipe axis is everywhere zero.

For a steady fully developed pipe flow,

(a) $\frac{\partial u}{\partial x}(r,x) = 0 \Rightarrow u = u(r)$. Velocity contains only an axial component, which is a function of only the radial component.

(b) Acceleration experienced by the fluid is zero. Local acceleration is zero as the flow is steady and convective acceleration is zero as the flow is fully developed.

The velocity profile is given by,

$$\boxed{u(r) = \frac{R^2}{4\mu}\left(\frac{\Delta p}{L}\right)\left(1-\left(\frac{r}{R}\right)^2\right)} \qquad (2)$$

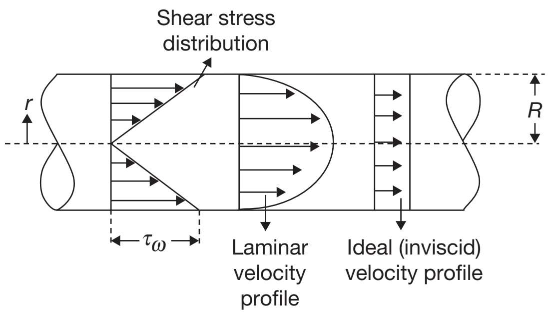
Velocity profile of a fully developed laminar flow in a pipe is parabolic while for a fully developed turbulent flow, it is much flatter.

The velocity profile has a maximum value $\left(u_{max} = \frac{R^2}{4\mu}\left(\frac{\Delta p}{L}\right)\right)$ at the pipe centerline and a minimum value (= zero) at the pipe wall.

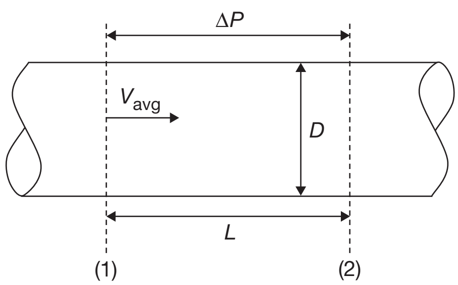
$$\boxed{u(r) = u_{max}\left(1-\left(\frac{r}{R}\right)^2\right)}$$

In a fully developed laminar pipe flow, the average velocity is one half of the maximum velocity i.e., $u_{max} = 2V_{avg}$

$$u(r) = 2V_{avg}\left(1 - \left(\frac{r}{R}\right)^2\right)$$



3. Pressure drop (ΔP):



The pressure drop (between sections 1 and 2) across a length L of a flow section in a horizontal circular pipe of diameter D,

$$\Delta P = \frac{32\mu L V_{avg}}{D^2}$$

The pressure drop for all type of flow, developed pipe flow (laminar or turbulent flows, circular or non-circular pipes, smooth or rough surfaces, horizontal or inclined pipes),

$$\Delta P = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

Where, f is the *Darcy friction factor* or *Darcy–Weisbach friction factor* or simply the *friction factor*,

$$f = \frac{8\tau_\omega}{\rho V_{avg}^2}$$

The skin friction coefficient or the *coefficient of friction* or the *Fanning friction factor* (C_f) is defined as

$$C_f = \frac{2\tau_\omega}{\rho V_{avg}^2} \qquad C_f = \frac{f}{4}$$

For a fully developed laminar flow Darcy's friction factor $f = \dfrac{64}{R_e}$ and hence the friction factor for the flow is a function of only Reynolds number and is independent of the roughness of the pipe surface.

Friction factor is maximum for a fully developed turbulent flow

4. Head loss (h_L): The pressure drop (ΔP) due to viscous effects or friction represents an irreversible pressure loss and is generally called as pressure loss due to friction (ΔP_L). Head loss (h_L) in general refers to any energy loss associated with the flow but here it is stated loss to refer to the pressure losses expressed in terms of an equivalent fluid column height.

$$h_L = \frac{\Delta P_L}{\rho_g} = f\frac{L}{D}\frac{V_{avg}^2}{2g} \qquad (3)$$

$$h_L = \frac{2\tau L}{\rho g r} = \frac{4L\tau_\omega}{\rho g D}$$

Eq. (3) is called the Darcy–Weisbach equation, is valid for laminar and turbulent flows in both circular and non-circular pipes. The head loss represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe. In Eq. (3), pressure drop is taken to be equivalent to the pressure loss and this is valid only under the assumptions by which the equivalency can be derived from Bernoulli's equation. The variable h_L is generally referred to as the head loss due to friction. It is to be noted that ΔP_L and h_L both represent losses over the length of the pipe.

For the flow of an ideal (inviscid) fluid, $h_L = 0$.

5. Required pumping power ($\mathring{W}_{pump,L}$):

The required pumping power to overcome the pressure loss,

$$\mathring{W}_{pump,L} = Q\Delta P_L = \mathring{m}gh_L$$

6. Volumetric flow rate (Q):

The average velocity for laminar flow in a horizontal circular pipe,

$$V_{avg} = \frac{\Delta PD^2}{32\mu L}$$

Volumetric flow rate for laminar flow through a horizontal pipe of diameter D and length L,

$$Q = V_{avg}A = \frac{\Delta P\pi D^4}{128\mu L}$$

The above equation is called the *Poiseuille's law*. or the *Hagen–Poiseuille equation*. The steady laminar viscous flow in a channel or tube from a region of high pressure to a region of low pressure is called *Poiseuille flow.*

$$Q = \frac{\pi}{2} U_{max}\ R^2$$

Example 4

In a horizontal circular pipe of length 20 m, a fluid (density = 850 kg/m^3, viscosity = 9 poise) flows in a steady fully developed laminar manner. If the head loss and the wall shear stress associated with the flow are 5 m and 104 N/m^2 respectively, then the Darcy friction factor for the flow is

(A) 0.5 (B) 0.0073
(C) 0.1167 (D) 1.868

Solution

$$\text{Given } L = 20 \text{ m}$$
$$\rho = 850 \text{ kg/m}^3$$
$$\mu = 0.9 \text{ Pa/s}$$
$$h_L = 5 \text{ m}$$
$$\tau_\omega = 104 \text{ N/m}^2$$

It is presumed here that all the assumptions for the pressure loss to be equal to the pressure drop are valid.

$$\therefore\ \Delta P = h_1\ \rho g = 5 \times 850 \times 9.81 = 41692.5\ P_a$$

$$\text{Now, } \tau_\omega = \frac{\Delta P}{L} \times \frac{R}{2}$$

$$\therefore \text{ Radius of the pipe, } R = \frac{2 L \tau_\omega}{\Delta P}$$

$$= \frac{2 \times 20 \times 104}{41692.5} = 0.1 \text{ m}$$

$$\text{Now, } \Delta P = \frac{32\ \mu V_{avg}\ L}{D^2}$$

$$= \frac{8 \mu V_{avg}\ L}{R^2}$$

$$\therefore\ V_{avg} = \frac{41692.5 \times (0.1)^2}{8 \times 0.9 \times 20} = 2.895 \text{ m/s.}$$

$$\text{Now, } f = \frac{8 \tau_\omega}{\rho V_{avg}^2}$$

$$= \frac{8 \times 104}{850 \times 2.895^2} = 0.1167.$$

Hence, the correct answer is option (C).

Example 5

The mass flow rate of the steady fully developed laminar flow of a fluid (density = 900 kg/m^3, viscosity = 9 poise) in a horizontal pipe of diameter 0.5 m is 212.06 kg/s. The perpendicular distance from the pipe wall at which the velocity is 0.432 m/s is

(A) 0.0236 m (B) 0.4527 m
(C) 0.0472 m (D) 0.2264 m

Solution

Given

$$\mathring{m} = 212.03 \text{ kg/s}$$
$$\rho = 900 \text{ kg/m}^3$$
$$D = 0.5 \text{ m}$$
$$V = 0.432 \text{ m/s}$$
$$\mathring{m} = \rho A V_{avg}$$
$$V_{avg} = \frac{212.06 \times 4}{900 \times \pi \times 0.5^2}$$
$$= 1.2 \text{ m/s}$$

It is known that for the flow condition given, the velocity profile of the flow is given by,

$$V = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

Here maximum velocity,

$$V_{max} = 2 \times V_{avg}$$

$$\therefore\ V = 2 \times V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$0.432 = 2 \times 1.2 \times \left(1 - \frac{r^2}{0.25^2}\right)$$

$$r = 0.2264 \text{ m}$$

∴ The perpendicular distance from the pipe wall at which the velocity is 0.432 m/s

$$= R - r = 0.25 - 0.2264$$
$$= 0.0236 \text{ m}$$

Hence, the correct answer is option (A).

Example 6

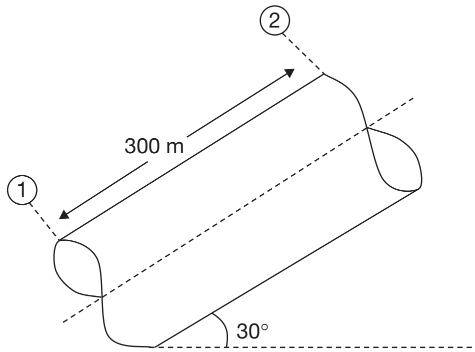
A circular pipe of diameter 0.07 m and length 300 m is inclined at an angle 30° with the horizontal. The volumetric flow rate of the steady fully developed laminar flow of the fluid (viscosity = 8 poise, density = 800 kg/m^3) in the pipe is 7 lit/s. The minimum power of a pump with efficiency 70% that can maintain this flow is

(A) 40.28 W (B) 40.28 kW
(C) 28.196 kW (D) 48.89 kW

Solution

The energy equation is given by,

$$\frac{P_1}{\rho g}+\frac{\alpha_1 v_1^2}{2g}+Z_1+h_p=\frac{P_L}{\rho g}+\alpha_2\frac{V_2^2}{2g}+Z_2+h_t+h_L \qquad (1)$$



Since no pump and turbine is involved in the flow section considered,

$$h_p = h = 0$$

The level Z_1 is considered as datum, i.e., $Z_1 = 0$. Hence, $Z_2 = L \times \sin 30 = 300 \times \frac{1}{2} = 150\text{ m}$

Although the velocity is not uniform across a pipe cross-section, the velocity profile does not change from section 1 to section 2 due to the fully developed flow.

$$\therefore\ \alpha_1 = \alpha_2$$

Now $V_1 = V_2$ (from continuity equation)

Therefore, Eq. (1) becomes,

$$\frac{P_1 - P_2}{\rho g} = 150 + h_L$$

$$= 150 + \frac{32\,\mu\,L V_{avg}}{D^2 \rho g} \quad \text{(it is assumed that } \Delta P_L - \Delta P\text{)}$$

$$= 150 + \frac{32\,\mu\,L}{D^2\,\rho g} \times \frac{Q}{\frac{\pi D^2}{4}}$$

$$= 150 + \frac{32 \times 0.8 \times 300}{0.07^2 \times 800 \times 9.81} \times \frac{0.007}{\frac{\pi \times 0.07^2}{4}}$$

$$= 513.26$$

or $$P_1 - P_2 = 513.26 \times 800 \times 9.81$$

$$= 4.028\text{ MN/m}^2$$

$$\text{Power of the pump} = \frac{Q \times (P_1 - P_2)}{\eta}$$

$$= \frac{0.007 \times 4.025 \times 10^6}{0.7}$$

$$= 40280\text{ W} = 40.28\text{ kW}.$$

Hence, the correct answer is option (B).

Example 7

The velocity distribution in a pipe is given as $u = u_{max}\left(1-\left(\frac{r}{R}\right)^3\right)$ when u_{max} is the maximum velocity at the centre of the pipe, u is the velocity at a distance r from the pipe centre line and R is the pipe radius. The ratio of the average velocity to the maximum velocity is

(A) 1 : 2 (B) 3 : 10
(C) 1 : 1 (D) 3 : 5

Solution

Given $u = u_{max}\left(1-\left(\frac{r}{R}\right)^3\right)$. Consider an elementary ring of thickness dr and at a distance r from the pipe centre. The discharge through this elementary ring is given by,

$$dQ = u \times 2\,\pi r dr$$

$$= u_{max}\left(1-\left(\frac{r}{R}\right)^3\right)2\pi r dr$$

$\therefore$ The discharge through the pipe is

$$Q = \int_O^R dQ$$

$$= \int_O^R u_{max}\left(1-\left(\frac{r}{R}\right)^3\right)2\pi r dr$$

$$= \pi\,R^2\,u_{max} \times \frac{3}{5}$$

Now, $Q = \pi R^2 u_{avg}$

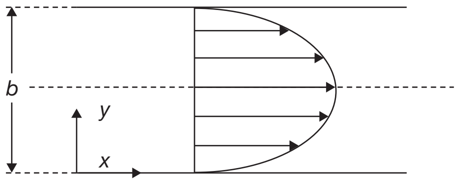
$$\Rightarrow \pi R^2 U_{avg} = \pi R^2 u_{max} \times \frac{3}{5}.$$

Hence, the correct answer is option (D).

Flow of Viscous Fluid between two Parallel Plates

Plane Poiseuille Flow

The laminar flow of a viscous fluid between two parallel plates, both of which are stationery, is called a *plane Poiseuille flow*. Consider a plane Poiseuille flow as shown in the following figure:



The velocity distribution is given by,

$$u(y) = \frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(by - y^2)$$

The discharge per unit with is given by,

$$q = -\frac{b^3}{12\mu}\frac{\partial p}{\partial x}$$

The shear stress distribution (where the fluid is a Newtonian fluid) is given by,

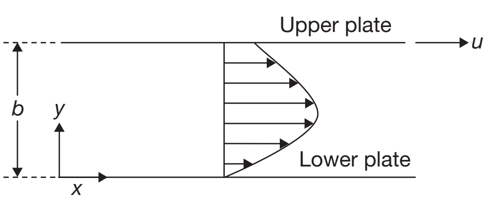
$$\tau = \frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)(b - 2y)$$

The velocity profile for a plane Poiseuille flow will be a symmetric parabolic velocity profile.

For a plane Poiseuille flow, the ratio of the average fluid velocity to the maximum fluid velocity is 2 : 3.

Couette Flow

The laminar flow of a viscous fluid between two parallel plates, one of which is moving relative to the other, is called a *Couette flow*. Consider a Couette flow where the lower plate is at rest and the upper plate moves uniformity with a constant velocity u as shown in the following figure:



The velocity distribution is given by,

$$u(y) = \frac{u}{b}y - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(by - y^2)$$

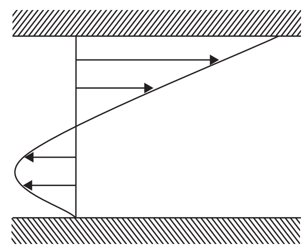
Case 1: $\frac{\partial p}{\partial x} = 0$, i.e., zero pressure gradient in the direction of motion. Then in this case, $u(y) = \frac{u\,y}{b}$ which is a linear velocity distribution. This particular case is known as *simple* or *plain* Couette flow or *simple shear flow*. This type of flow is usually used to model the lubricant motion in a journal bearing with a rotating shaft, where the velocity of the lubricant is assumed to be linear.

Case 2: $\frac{\partial p}{\partial x} < 0$, i.e., negative pressure gradient in the direction of motion. In this case, velocity is positive over the whole gap between the plates.

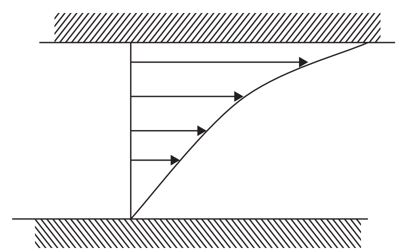
Case 3: $\frac{\partial p}{\partial x} > 0$, i.e., positive pressure gradient in the direction of motion. In this case, velocity over a portion of the gap between the plates can be negative.

Let, $$K = \frac{-b^2}{2\mu u}\left(\frac{\partial p}{\partial x}\right)$$

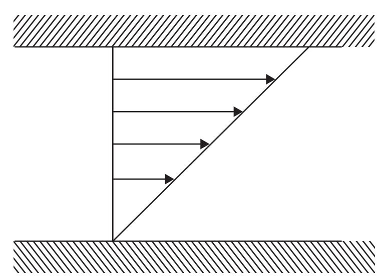
$K < -1$



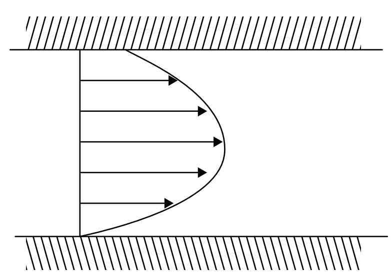
$K < 0$



$K = 0$



$K > 0$



The discharge per unit width of the plates is given by,

$$q = \frac{ub}{2} - \frac{b^3}{12\mu}\left(\frac{\partial p}{\partial x}\right)$$

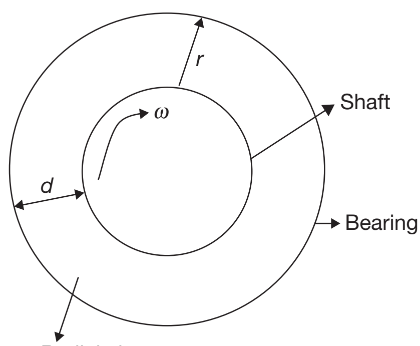
The shear stress distribution where the fluids is Newtonian is given by,

$$\tau = \mu \frac{u}{b} - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(b - 2y)$$

Flow of Lubricant in a Journal Bearing

The flow of the lubricant in a journal bearing is usually modeled as a simple (or plain) Couette flow. The velocity,

$$u(r) = \frac{ur}{d}$$



Where r is the radial distance from the outer surface of the shaft to the bearing, d is the radial clearance and u is the surface speed of the shaft. If the shaft is rotating at N rpm then,

$$U(r) = \frac{r\omega R}{d} = \frac{r 2\pi NR}{60d}$$

Where ω and R are the angular velocity and radius of the shaft respectively. The Reynolds number for the lubricant flow is defined as:

$$R_e = \frac{\rho u d}{\mu}$$

The flow condition in the bearing is said to be laminar if $R_e < 500$ and turbulent if $R_e > 500$.

Direction for solved examples 8 and 9:

A Newtonian fluid of viscosity 1 poise flows in a steady and laminar manner between two stationery parallel horizontal plates separated by a perpendicular distance of 5 mm. The pressure gradient in the horizontal direction (x-direction) is determined to be –5 kN/m^2.

Example 8

The maximum shear associated with the flow is

(A) 0 N/m^2 (B) 25 N/m^2
(C) 12.5 N/m^2 (D) 12.5×10^3 N/m^2

Solution

The shear stress distribution is

$$\tau = \frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)(b - 2y)$$

The maximum shear stress occurs at $y = 0$

$$\text{At } y = 0,\ \tau = \frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)b$$

Given, $b = 5 \times 10^{-3}$ m

$$\frac{\partial p}{\partial x} = -5 \times 10^3 \text{ N/m}^2$$

$$\therefore\ \tau = \frac{-1}{2} \times (-5 \times 10^3) \times 5 \times 10^{-3}$$

$$= 12.5 \text{ N/m}^2.$$

Hence, the correct answer is option (C).

Example 9

The maximum velocity of the fluid is

(A) 0.1563 m/s (B) 0.1042 m/s
(C) 0.0782 m/s (D) 0.1172 m/s

Solution

$$u(y) = \frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(by - y^2)$$

Since the velocity profile of this plane Poiseuille flow is a symmetric parabolic one, the maximum velocity will occur at $y = \frac{b}{2}$.

$$\therefore\ U_{max} = U\left(\frac{b}{2}\right) = \frac{-b^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$= \frac{-(5 \times 10^{-3})^2}{8 \times 0.1} \times (-5 \times 10^3).$$

$$= 0.1563 \text{ m/s}$$

Hence, the correct answer is option (A).

For the plane Poiseuille flow,

$$R_e = \frac{\rho V_{avg} b}{\mu} \text{ and friction factor,}$$

$$f = \frac{48}{R_e}$$

Example 10

A laminar flow of an oil (viscosity = 20 poise) takes place between two stationary parallel plates which are 150 mm apart. If the average velocity of flow is 1.5 m/s, then the shear stress at vertical distance of 37.5 mm from the lower plate is

(A) 40 N/m^2 (B) 160 N/m^2
(C) 90 N/m^2 (D) 60 N/m^2

Solution

Given $\mu = 2$ Pa/s, $b = 0.15$ m,

$$V_{avg} = 1.5 \text{ m/s}$$

The maximum velocity for a plane Poiseuille flow is given by,

$$V_{max} = \frac{-b^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

Also for this flow,

$$V_{avg}/V_{max} = \frac{2}{3}$$

$$\therefore V_{avg} = \frac{-b^2}{12\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$\therefore \left(\frac{\partial p}{\partial x}\right) = \frac{-1.5\times 12\times 2}{(0.15)^2} = -1600 \text{ N/m}^3$$

$$\text{Now, } \tau = \frac{-1}{2}\left(\frac{\partial p}{\partial x}\right)(b-2y)$$

$\therefore$ At $y = 0.0375$ m,

$$\tau = \frac{-1}{2}\times(-1600)\times(0.15-2\times 0.0375)$$

$$= 60 \text{ N/m}^2.$$

Hence, the correct answer is option (D).

Example 11

In a Couette flow, the two parallel identical plates are at a distance b metres apart and the upper plate moves with a constant velocity of u m/s with the lower plate stationary. The fluid flows between the plates such that the shear stress at the lower plate is zero and the discharge for this flow per unit width of the plates is given by q. If the viscosity of the fluid and the pressure gradient in the horizontal direction (x-direction) are doubled, then the discharge per unit width becomes

(A) q (B) $0.5\,q$
(C) $0.75\,q$ (D) $2\,q$

Solution

$$\text{Here, } \tau = \mu\frac{u}{b} - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(b-2y)$$

Given, at $y = 0$, $\tau = 0$

$$\Rightarrow 0 = \mu\frac{u}{b} - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)b$$

$$\text{or } \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) = \frac{u}{b^2}$$

$$\text{Now, } q = \frac{ub}{2} - \frac{b^3}{12\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$= \frac{ub}{2} - \frac{b^3}{6}\times\frac{u}{b^2}$$

$$= \frac{ub}{2} - \frac{ub}{6} = \frac{ub}{3}.$$

Hence, the correct answer is option (A).

Kinetic Energy Correction Factor (α)

Kinetic energy correction factor is defined as the ratio of the kinetic energy of flow per second based on actual velocity across a section to the kinetic energy of flow per second based on average velocity across the same section. This factor is introduced to account for the non-uniformity of the velocity profile across an inlet or outlet due to viscosity of fluid.

Kinetic energy correction factor,

$$\alpha = \frac{1}{A}\int\left(\frac{V}{V_{avg}}\right)^3 dA$$

Where

V = Local velocity at any point in the cross-section

V_{avg} = Average velocity across the cross-section.

For uniform velocity distribution, $\alpha = 1.0$

For laminar flow through a pipe, $\alpha = 2.0$

For turbulent flow through a pipe,

$\alpha = 1.01$ to 1.33.

Momentum Correction Factor (β)

The momentum correction factor is defined as the ratio of momentum of the flow per second based on the actual velocity to the momentum of the flow per second based on the average velocity across a cross-sectional area. This factor is introduced to account for the non-uniformity of the velocity across an inlet or outlet and it is defined by,

$$\beta = \frac{\int \rho\vec{v}\left(\vec{v}\cdot\vec{n}\right)dA}{\overset{\circ}{m}\,\vec{v}_{avg}}$$

If the density is uniform over the inlet or outlet and $\vec{v}$ is the same direction as $\vec{v}_{avg}$, then

$$\beta = \frac{\int \rho v(\vec{v}.\,\vec{n})dA}{\mathring{m} V_{avg}} = \frac{\int v(\vec{v}.\,\vec{n})dA}{v_{avg}^2 A}$$

If the control surface slices normal to the inlet or outlet area, i.e.,

$$(\vec{v}\cdot\vec{n})dA = vdA$$

then,

$$\boxed{\beta = \frac{1}{A}\int\left(\frac{v}{v_{avg}}\right)^2 dA}$$

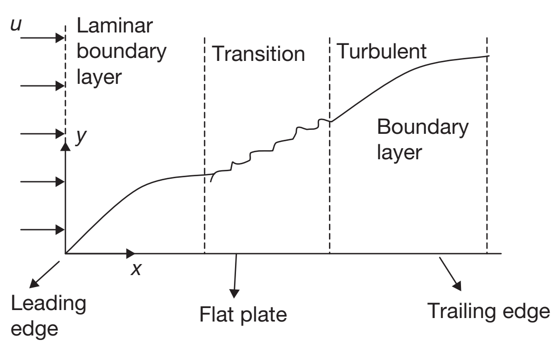
The factor β is always greater than or equal to one. For a fully developed laminar pipe flow, $\beta = \frac{4}{3}$ and for a fully developed turbulent pipe flow, $1.01 \le \beta \le 1.2$.

> At an inlet or outlet, if the flow is uniform then $\beta = 1$ and $\vec{v} = \vec{v}_{avg}$

Boundary Layer Theory

When a viscous fluid flows past a stationary solid boundary, in a small layer of fluid adjacent to the boundary, the velocity of flowing fluid increase rapidly from zero at the boundary surface and approaches the velocity of the main stream. This layer is called the *boundary layer.* A boundary layer is formed when there is relative motion between a solid boundary and the fluid in contact with it.

Boundary Layer on a Flat Plate



The above figure shows a boundary layer formed on a flat plate kept parallel to the flow of fluid of velocity u. Here u is called as the free stream velocity, sometimes denoted as u_α. The edge of the plate facing the direction of flow is called as the *leading edge* while its rear edge is called the trailing edge.

Near the leading edge of a flat plate, the boundary layer is laminar with a parabolic velocity distribution. In the turbulent boundary layer, the velocity distribution is given by the log law or Prandtl's one-seventh power law.

Characteristics of a boundary layer are:

1. The boundary layer thickness (δ) increases as the distance from the leading edge (x) decreases.
2. δ decreases as u increases.
3. δ increases as kinematic viscosity (v) increases.
4. The wall shear stress $\tau_\omega = \left(= \mu\left(\frac{\partial u}{\partial y}\right) y = 0\right)$ decreases as x increases. In the turbulent boundary layer, τ_ω shows a sudden increase and then decreases with increasing x.

> Boundary layer is laminar when $R_{ex}\left(= \frac{u\,x\,\rho}{\mu}\right) < 5\times10^5$ and turbulent when $> 5 \times 10^5$.

Boundary Layer Thickness (δ)

Boundary layer thickness is defined as that distance from the boundary in which the velocity reaches 99% of the free stream velocity ($u = 0.99\ u_\alpha$).

For greater accuracy, boundary layer thickness is defined in terms of the displacement thickness (δ^*), momentum thickness (θ) and energy thickness (δ_e).

1. **Displacement thickness (δ^*):**

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right)dy}$$

2. **Momentum thickness(θ):**

$$\boxed{\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy}$$

3. **Energy thickness (δ_e):**

$$\boxed{\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)dy}$$

NOTE

> That the difference $(U - u)$ is called the *velocity of defect.*

Shape factor:

$$\boxed{S = \frac{\delta^*}{\theta}}$$

Where, S is called the shape factor.

Energy loss: The energy loss per unit width of the plate due to the boundary layer,

$$E_L = \frac{1}{2}(\rho\,\delta_e\,u)\times u^2$$

Mass flow: The mass flow in the boundary layer at a position where the boundary thickness is δ, is given by,

$$m = \int_0^{\delta} \rho\,u dy$$

The mass entrainment (Δm) between two sections where the boundary layer thickness are δ_1 and δ_2 respectively is given by,

$$\Delta m = m_1 - m_2$$

$$\Delta m = \int_0^{\delta_1} \rho u dy - \int_0^{\delta_2} \rho u dy$$

Reynolds Number for the Plate

If L is the length of a plate, then Reynolds number for the whole plate $= \frac{\rho\,u\,L}{\mu}$. Reynolds number for the front half of the plate $= \frac{\rho\,u\,L}{2\,\mu}$.

Von Karman Momentum Equation

For a fluid flowing over a thin plate (placed at zero incidence) with a free stream velocity equal to u,

$$\frac{\tau_\omega}{\rho v^2} = \frac{d\theta}{dx}$$

The above equation is called as the Von Karman momentum equation for boundary layer flow. It is used to determine the frictional drag on a smooth flat plate for both laminar and turbulent boundary layers.

Boundary Conditions for a Velocity Distribution

The following boundary conditions must be satisfied for any assumed velocity distribution in a boundary layer over a plate:

1. At the plate surface,

$$y = 0,\ u = 0$$

2. At the outer edge of boundary layer,
   (a) $y = \delta,\ u = U$
   (b) $y = \delta,\ \frac{du}{dy} = 0$

Drag Force on the Plate

The drag force acting on a small distance d_x of a plate is given by,

$$\Delta F_D = \tau_\omega \times B \times dx$$

Where, B is the width of the plate.

Total drag force acting on a plate of length L on one side,

$$F_D = \int_0^L \Delta\,F_D = \int_0^L \tau_\omega \times B \times dx$$

Local Coefficient of Drag (C_D^*)

$$C_D^* = \frac{\tau_\omega}{\frac{1}{2}\rho\,u^2}$$

This coefficient is also sometimes called as *coefficient of skin friction*.

Average Coefficient of Drag (C_D)

$$C_D = \frac{F_D}{\frac{1}{2}\rho\,A u^2}$$

Laminar Boundary Layer over a Flat Plate

From the solution of the Blasius equation for the laminar boundary layer on a flat plate, the following results are obtained.

$$\delta = \frac{5\,x}{\sqrt{R_{e_x}}}$$

$$C_D^* = \frac{0.664}{\sqrt{R_{e_x}}}$$

$$C_D = \frac{1.328}{\sqrt{R_{e_L}}}$$

Where, $R_{e_L} = \frac{u\,L\,\rho}{\mu}$, L being the length of the plate.

Summary of Fluid Frictional Resistance

Fluid frictional resistance is the opposition force (or resistance) experienced by a fluid in motion. It exists both in streamline flow and in turbulent flow.

Fluid Friction in Streamline Flow (Laminar Flow)

1. The viscous forces predominate the inertial force in this type of flow, which occurs at low velocities.
2. Frictional resistance is proportional to the velocity of flow, contact surface area and temperature.
3. The entrance length (L_e), which is the length of pipe from its entrance to the point where flow attains fully developed profile and remains unaltered beyond that point is given by, $\boxed{L_e = 0.07\, Re\, D}$.

   Where

   Re = Reynolds's number for flow and

   D = Diameter of pipe
4. The Darcy's friction factor in smooth pipes (as per Blassius) is given by, $f = \dfrac{64}{R_e}$.

Fluid Friction in Turbulent Flow

1. As per Darcy–Weisbach equation, the head loss due to friction is:

   $$h_f = \frac{fLV^2}{2gD}$$

   Where

   L = Length of pipe

   D = Diameter of pipe

   V = Mean velocity of flow

   f = Friction factor (0.02–0.04 for metals)

   Hence, frictional resistance is proportional to square of velocity.
2. The frictional resistance does not depend upon the pressure but it varies slightly with temperature.
3. The frictional resistance is proportional to the density of the fluid.
4. The **entrance length** (L_e) ≈ $50D$. Also,

   $$\boxed{L_e = 0.7\, R_e D.}$$

   Where

   Re = Reynold's number of flow

   D = Diameter of pipe.
5. Darcy's friction factor in smooth pipes (as per Blassius) is $f = \dfrac{0.3164}{(R_e^{1/4})}$ for turbulent flow.

Variation of Pipe Roughness with Aging

The **relative smoothness** of a pipe = $\dfrac{R}{k}$.

Where

R = Radius of pipe

k = Average height of irregularities For rough pipes, friction factor depends only on $\left(\dfrac{R}{k}\right)$ and not on Reynolds number (R_e). The **relative roughness**, of pipe is $\dfrac{k}{R}$ (which is the reciprocal of the relative smoothness).

The average height of irregularities (i.e., k), which is a measure of the roughness of pipe, depends upon the age of pipe. The relation is:

$$k = k_0 + \alpha t$$

Where

k_0 = Value of pipe roughness for new pipe

t = Age of pipe (in year)

α = Constant

k = Value of pipe roughness after t years.

Turbulent Flow in Pipes

Turbulent flow is characterized by swirling regions of fluid called eddies which greatly enhance mass, momentum and heat transfer compared to laminar flow. Turbulence in a flow can be generated by:

1. Frictional forces at the boundary solid walls.
2. Flow of fluid layers, with different velocities, adjacent to one another.

Turbulence can be classified as:

1. **Wall turbulence:** Turbulence generated and continuity impacted by the boundary walls.
2. **Free turbulence:** Turbulence generated by two adjacent fluid layers in the absence of walls.
3. **Convective turbulence:** Turbulence generated at regions where there is conversion of potential energy to kinetic energy by the process of mixing.

Property Values in a Turbulent Flow

At a specified location in a turbulent flow field, properties such as velocity, pressure, temperature, etc. Fluctuate with time about an average value. For a property P, the *instantaneous value* of the property ($\hat{P}$ (s, t)) at the specified location $s = f(x, y, z)$ in Cartesian coordinates) is given by,

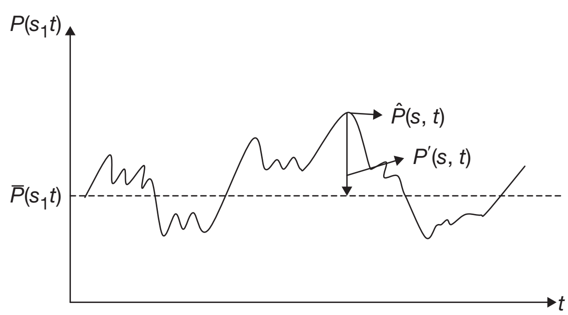
$$\boxed{\hat{P}(s,t) = \bar{P}(s,t) + P^1(s,t)}$$

Where, $\bar{P}$(s) the time average or temporal mean value and P' (s, t) is the fluctuating component. The term $\bar{P}(s)$ is a constant with respect to time.

$$\bar{P}(s) = \lim_{T\to\alpha} \frac{1}{T}\int_0^T \hat{P}(s_1\, t)\, dt$$

Where, T is the integration time over which the indicated time averaging takes place. The time average of the turbulent fluctuating component is zero, i.e.,

$$\lim_{T\to\alpha} \frac{1}{T}\int_0^T P'(s,\, t)\, d\, t = \overline{P'(s_1\, t)} = 0$$



Shear Stress in a Turbulent Flow

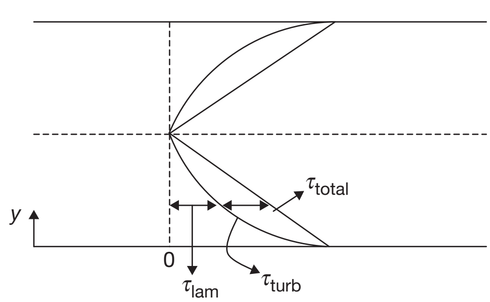
The total shear (τ_{total}) in a turbulent flow is given by,

$$\tau_{\text{total}} = \tau_{\text{lam}} + \tau_{\text{turb}}$$

Where, τ_{lam} is the *laminar shear stress* and τ_{turb} is the *turbulent shear stress.*

$$\tau_{lam} = \mu \frac{d\bar{u}}{d\,y}$$

Where, u is the x-component of the instantaneous velocity $\hat{V}$ and $\bar{u}$ is the time average (or time mean) value of u.



$$\tau_{\text{turb}} = -\rho\ \overline{u'\,v'}$$

Where, $\overline{u'v'}$ is the time average of the product of the fluctuating velocity component u' and V'. The term $\overline{u'\,v'}$ can be non-zero even if $\overline{u'} = 0$ and $\overline{v'} = 0$.

The term $\overline{u'\,v'}$ is usually found to be a negative quantity and hence shear stress is greater in turbulent flow than in laminar flow.

In laminar flow, $u' = v' = 0$ such that $\overline{u'v'} = 0$.

Terms such as $-\rho\overline{u'V'}$ or $-\rho\overline{(u')^2}$ or $-\rho\overline{V'\omega'}$ are called as *Reynolds stress* or *turbulent stresses*. Here, V and ω are the y and z components of the instantaneous velocity $\hat{V}$.

Boussinesq Approximation or Hypothesis

$$\tau_{\text{turb}} = -\rho\overline{u'V'} = \mu_t \frac{d\bar{u}}{dy}$$

Where, μ_t is the *eddy viscosity* or *turbulent viscosity.*

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{d\,\bar{u}}{dy}$$

$$\tau_{\text{total}} = \rho\,(\infty + \infty_t)\frac{d\,\bar{u}}{dy}$$

Where, $\infty_t = \dfrac{\mu_t}{\rho}$ is the *kinematic eddy viscosity* or *kinematic turbulent viscosity* or *eddy diffusivity of momentum*. Kinematic eddy viscosity depends on flow conditions and it decreases towards the wall where it becomes zero.

Prandtl's Mixing Length Theory

In this theory, the eddy viscosity is $\mu_t = \rho\, l_m^{\,2}\left|\dfrac{d\,\bar{u}}{dy}\right|$.

$$\tau_{\text{turb}} = -\rho\ \overline{U'V'} = \rho\, l_m^{\,2}\left(\frac{d\,\bar{u}}{dy}\right)^2$$

Where, l_m is the mixing length defined as the average lateral distance through which a small mass of fluid particles would move from one layer to the adjacent layer before acquiring the velocity of the new layer.

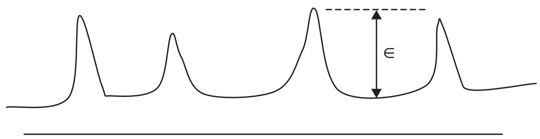
For the steady fully developed turbulent flow of a fluid in a horizontal pipe, R_e total shear stress varies linearly with the pipe radius.

$$\tau_{\text{total}} = \tau_{\omega}\frac{r}{R}$$

Where $0 \le r \le R$

At the wall, velocity gradients and thus wall shear stress are much larger for turbulent flow than for laminar flow.

Relative Roughness



The variable $\in$, referred to as *absolute roughness,* denotes the mean height of irregularities of the surface of a boundary. A boundary is generally said to be rough if the value of $\in$ is high and smooth if $\in$ is low. For a pipe, relative roughness $= \frac{\in}{D}$, where D is the diameter of the pipe.

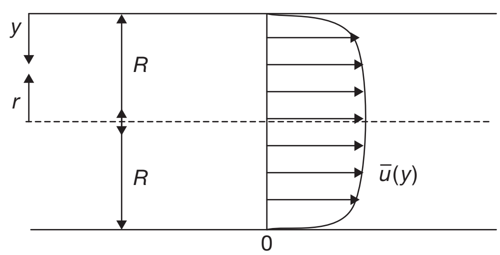
Turbulent Velocity Profile

Several empirical velocity profile units for turbulent pipe flow and among these the best known is the *power law.* Velocity profile defined as follows:

$$\frac{u}{u_{\max}} = \left(\frac{y}{R}\right)^{1/n} = \left(1 - \frac{r}{R}\right)^{1/n}$$

Where n is a constant and whole value increases as Reynolds number increases. Many turbulent flows in practice is approximated using the *one-seventh power law* velocity profile where $n = 7$. Note that the power-law velocity profile cannot be used to calculate the wall shear stress, as a velocity gradient obtained will be infinity. This law is applicable to smooth pipes.

Velocity distribution are more uniform in turbulent flow than in laminar flow.

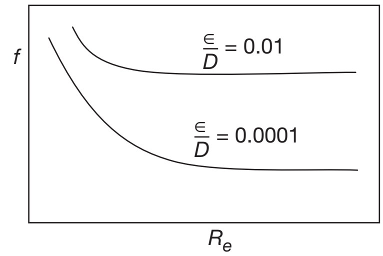


Friction Factor in Turbulent Flow

The friction factor in a fully developed turbulent pipe flow depends on the Reynolds number and the relative roughness (ε/D). The friction factor is minimum for a smooth pipe and increases with roughness. For laminar flow, the friction factor decreases as Reynolds number increases and is independent of surface roughness.

Moody Chart

It presents the Darcy friction factor for pipe flow as a function of Reynolds number and relative roughness. This chart can be used for circular pipes and non-circular (taking into consideration the hydraulic diameter) pipes.



At very large Reynolds number, R_e friction factor curves in the Moody chart are nearly horizontal and thus the friction factor are independent of the Reynolds number.

Intensity of Turbulence in a Flow

It is also called as degree of turbulence in a flow which is described by the relative magnitude of the root mean square value of the fluctuating components (u', v', and ω') with respect to the time averaged velocity ($\bar{V}$).

$$I = \frac{\sqrt{\frac{1}{3}\left(\overline{(u')^2} + \overline{(v')^2} + \overline{(\omega')^2}\right)}}{\bar{V}}$$

If the turbulence is isotropic, then $u' = v' = \omega'$.

Example 12

A liquid flows turbulently in a horizontal pipe with a pressure gradient of 3 kPa/m. The wall shear stress developed is 112.5 N/m^2. If the laminar shear stress is 10 N/m^2 at a radius of 35 mm, then the turbulent shear stress at this radius would be

(A) 52.5 N/m^2 (B) 10 N/m^2
(C) 42.5 N/m^2 (D) 95 N/m^2

Solution

Given $\frac{\Delta P}{L} = 3 \times 10^3$ Pa/m

$$\tau_\omega = 112.5 \text{ N/m}^2$$

The following equations are applicable for turbulent flows.

$$\tau_\omega = \frac{\Delta P}{L}\frac{R}{2} \quad (1)$$

$$\tau_\omega = \frac{\tau_\omega r}{R}. \quad (2)$$

From Eq. (1), we get 112.5

$$= 3 \times 10^3 \times \frac{R}{2}.$$

$\therefore$ Radius of the pipe, $R = 0.075$ m

Now, at radius $r = 0.035$ m

$$\tau = 112.5 \times \frac{0.035}{0.075} = 52.5 \text{ N/m}^2$$

Here τ is the total shear stress, i.e., $\tau_{\text{total}} = 52.5$ N/m^2

At this radius, $\tau_{lam} = 10\ N/m^2$

$$\tau_{total} = \tau_{lam} + \tau_{turb}$$

$$\tau_{total} = 52.5 - 10 = 42.5\ N/m^2$$

Hence, the correct answer is option (C).

Example 13

A fluid (density = 950 kg/m^3, viscosity = 0.1 poise) flows with an average velocity of 1 m/s in a 100 m long horizontal pipe having an absolute roughness of 0.175 mm. The magnitude of the pressure loss due to friction is obtained by multiplying the friction factor with 19 × 10^5. A set of friction factor (f) values for some given combination of Reynolds number (Re) and relative roughness (RR) values are given in the following table. The friction factor associated with the flow is

Re	*RR*	*f*
9800	0.00175	0.0338
9500	0.0035	0.0361
19000	0.00175	0.0296
19000	0.0035	0.0325

(A) 0.0338 (B) 0.0361
(C) 0.0296 (D) 0.0325

Solution

For turbulent or laminar flow, we have

$$\Delta P_L = \frac{2 f \rho \bar{V}^2 L}{D}$$

$$\text{Given, } \frac{2 \rho \bar{V}^2 L}{D} = 19 \times 10^5$$

$$\text{or } D = \frac{2 \times 950 \times 1^2 \times 100}{19 \times 10^5} = 0.1\ m$$

Given, $t = 0.175$ mm

$$\therefore \text{ Relative roughness} = \frac{t}{D} = \frac{0.175}{100}$$

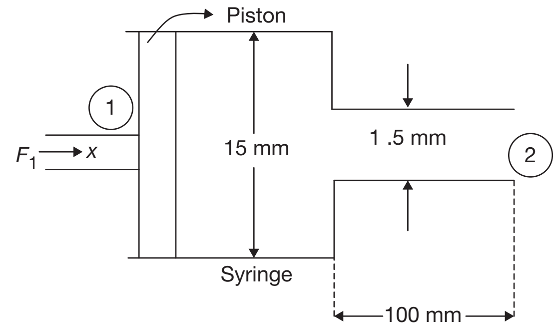
$$= 0.00175$$

$$R_e = \frac{\rho \bar{V} D}{\mu} = \frac{950 \times 1 \times 0.1}{0.01} = 9500$$

For $R_e = 9500$ and $RR = 0.00175$ friction factor $f = 0.0338$.
Hence, the correct answer is option (A).

Example 14

A force F_1 Newtons is required as the frictionless piston in a syringe to discharge 1944 mm^3/s of water through a needle as shown in the following figure. The force is determined by assuming fully developed laminar viscous flow through the needle. If ideal flow is assumed, then the force required on the piston to achieve the same discharge would be F_2 Newtons. The difference $F_1 - F_2$ neglecting losses in the syringe is equal to



(A) 0.0251 N (B) 0.2765 N
(C) 0.7856 N (D) 0.4836 N

Solution

Consider two points 1 and 2 such that both points are in the same horizontal plane and point 1 lies in the centre of the piston cross-section while point 2 lies in the centre of the needle exit cross-section.

The energy balance equation with suitable assumption can be reduced to

$$\frac{P_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + Z_1 = \frac{P_2^2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_L \quad (1)$$

Here, $Z_1 = Z_2$

$$P_1 = P_{atm} + \frac{F_1}{A_1}$$

$$P_2 = P_{atm}$$

$\alpha_1 = \alpha_2$ (uniform velocity assumed across any cross-section)

$\therefore$ Eq. (1) becomes,

$$F_1 = \left(\frac{V_2^2 - V_1^2}{2} \right) \rho A_1 + h_L A_1 \rho g$$

When ideal flow is assumed, $h_L = 0$

$$\therefore F_1 - F_2 = h_L A_1 \rho g$$

$$= f \frac{L}{D_2} \times \frac{V_2^2}{2g} \times A_1 \rho g$$

$$= f \frac{L}{D_2} \times \frac{V_2^2}{2} \times A_1 \rho$$

$$Q = 1944\ mm^3/s$$

$$= 1944 \times 10^{-9}\ m^3/s$$

$$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times (0.015)^2$$

$$= 1.767 \times 10^{-4}\ m^2$$

$$A_2 = \frac{\pi}{4} \times D_2^2 = \frac{\pi}{4} \times (0.0015)^2$$

$$= 1.767 \times 10^{-6}\ m^2$$

$$U_2 = \frac{Q}{A_2} = \frac{1944\times10^{-9}}{1.767\times10^{-6}} = 1.1 \text{ m/s}$$

Reynolds number of flow in the needle, $R_e = \frac{\rho D_2 \times V_2}{\mu}$

$$= \frac{1000\times0.0015\times1.1}{0.001} = 1650$$

$$f = \frac{64}{R_e} = \frac{64}{1650} = 0.0388$$

$$\therefore F_1 - F_2$$

$$= \frac{0.0388\times0.1}{0.0015}\times\frac{1.1^2}{2}\times1.767\times10^{-4}\times1000$$

$$= 0.2765 \text{ N}$$

Hence, the correct answer is option (B).

Example 15

Water is flowing at a volumetric flow rate of 0.08 m^3/s in a horizontal pipe of length 15 m and diameter (D) varies along its length (l) according to the linear relationship: $D = 0.25 - 0.01l$. If the friction factor is taken to be constant for the whole pipe and equal to 0.02, then the head loss due to friction in the pipe is

(A) 0.6441 m (B) 2.0611 m
(C) 10.3059 m (D) 2.5764 m

Solution

Head loss due to friction,

$$h_L = f\frac{L}{D}\frac{\bar{V}^2}{2g}$$

Where, L is the whole length of the pipe.

For a differential length of the pipe, the differential head loss due to friction can be written as:

$$dh_L = \frac{f}{D}\frac{\bar{V}^2}{2g}dl$$

$$= \frac{f}{D}\times\frac{Q^2\times16\times dl}{\pi^2\times10^4\times2g}$$

$$= \frac{.8fQ^2}{\pi^2 g}\times\frac{dl}{D^5}$$

$$= 0.08263fQ^2\frac{dl}{(0.25-0.01l)^5}$$

Integrating the above equation we have,

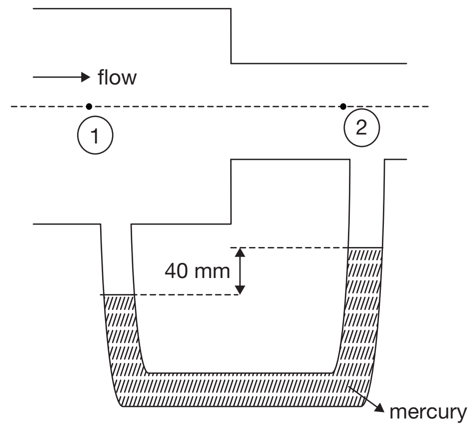
$$\int_o^{h_L} dh_L = 0.08263fQ^2\int_o^{15}\frac{dl}{(0.25-0.01l)^5}$$

That is, $h_L = 2.5764$ m

Hence, the correct answer is option (D).

Example 16

Water flows at the rate of 0.06 m^3/s in a pipe involving a sudden contraction where the pipe diameter decreases from 250 mm to 160 mm, as shown in the following figure. The coefficient of contraction is



(A) 0.655 (B) 0.543
(C) 0.792 (D) 0.125

Solution

$$\frac{P_1 - P_2}{\rho g} = h\left(\frac{\rho m}{\rho} - 1\right)$$

Where, ρ (density of water) = 1000 kg/m^3 and ρ_m (density of mercury) = 13600 kg/m^3 and $h = 40$ mm

$$\therefore \frac{P_1 - P_2}{\rho g} = 0.04\left(\frac{13600}{1000} - 1\right)$$

$$= 0.504$$

The energy balance with suitable assumption can be reduced to,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

Here the head loss (h_L) is equal to the head loss due to contraction,

$$h_L = h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$Z_1 = Z_2$ (as the points 1 and 2 same horizontal plane)

$$\therefore \frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g}\left(1 + \left(\frac{1}{C_c} - 1\right)^2\right) - \frac{V_1^2}{2g}$$

or $0.504 \times 2 \times 9.81$

$$= \left(\frac{0.06\times4}{\pi\times(0.16)^2}\right)^2\left(1 + \left(\frac{1}{C_c} - 1\right)^2\right) - \left(\frac{0.06\times4}{\pi\times(0.25)^2}\right)^2$$

or $C_c = 0.655$

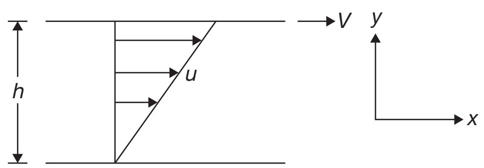
Hence, the correct answer is option (A).

Exercises

1. The kinetic energy correction factor for a fully developed laminar flow through a circular pipe is
   (A) 1.00 (B) 1.33
   (C) 2.00 (D) 1.50
2. A solid sphere (diameter 6 mm) is rising through oil (mass density 900 kg/m^3, dynamic viscosity 0.7 kg/m-s) at a constant velocity of 1 cm/s. What is the specific weight of the material from which the sphere is made? (Take $g = 9.81$ m/s^2)
   (A) 4.3 kN/m^3 (B) 5.3 kN/m^3
   (C) 8.7 kN/m^3 (D) 12.3 kN/m^3

Direction for questions 3 and 4:

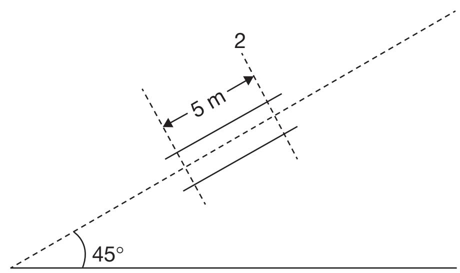
The laminar flow takes place between closely spaced parallel plates as shown in the following figure. The velocity profile is given by $u = V\frac{y}{h}$. The gap height, h, is 5 mm and the space is filled with oil (specific gravity = 0.86, viscosity $\mu = 2 \times 10^{-4}$ N-s/m^2). The bottom plate is stationary and the top plate moves with a steady velocity of $V = 5$ cm/s. The area of the plate is 0.25 m^2.



3. The rate of rotation of a fluid particle is given by
   (A) $\omega_y = 0;\ \omega_z = -\frac{V}{2h}$
   (B) $\omega_y = 0;\ \omega_z = -\frac{V}{h}$
   (C) $\omega_y = \frac{V}{h};\ \omega_z = \frac{V}{h}$
   (D) $\omega_y = \frac{V}{h};\ \omega_z = 0$
4. The power required to keep the plate in steady motion is
   (A) 5×10^{-4} W (B) 10^{-5} W
   (C) 2.5×10^{-5} W (D) 5×10^{-5} W

Direction for questions 5 and 6:

An upward flow of oil (mass density 800 kg/m^3, dynamic viscosity 0.8 kg/m-s) takes place under laminar conditions in an inclined pipe of 0.1 m diameter as shown in the figure. The pressures at sections 1 and 2 are measured as $P_1 = 435$ kN/m^2 and $p_2 = 200$ kN/m^2.



5. The discharge in the pipe is equal to
   (A) 0.100 m^3/s (B) 0.127 m^3/s
   (C) 0.144 m^3/s (D) 0.161 m^3/s
6. If the flow is reversed, keeping the same discharge, and the pressure at section 1 is maintained as 435 kN/m^2, the pressure at section 2 is equal to
   (A) 488 kN/m^2 (B) 549 kN/m^2
   (C) 586 kN/m^2 (D) 614 kN/m^2
7. For air flow over a flat plate, velocity (U) and boundary layer thickness (δ) can be expressed respectively, as

$$\frac{U}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2;\ \delta = \frac{4.64x}{\sqrt{Re_x}}$$

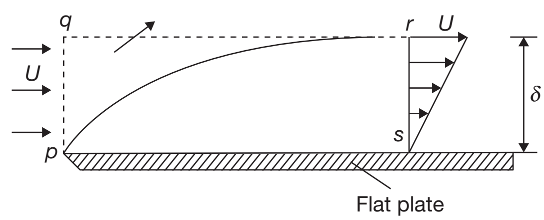
   If the free stream velocity is 2 m/s, and air has kinematic viscosity of 1.5×10^{-5} m^2/s and density of 1.23 kg/m^3, the wall shear stress at $X = 1$ m, is
   (A) 2.36×10^2 N/m^2
   (B) 43.6×10^{-3} N/m^2
   (C) 4.36×10^{-3} N/m^2
   (D) 2.18×10^{-3} N/m^2
8. A centrifugal pump is required to pump water to an open tank situated 4 km away from the location of the pump through a pipe of diameter 0.2 m having Darcy's friction factor of 0.01. The average speed of water in the pipe is 2 m/s. If it is to maintain a constant head of 5 m in the tank, neglecting other minor losses, the absolute discharge pressure at the pump exit is
   (A) 0.449 bar (B) 5.503 bar
   (C) 44.911 bar (D) 55.203 bar
9. The velocity profile in fully developed laminar flow in a pipe of diameter D is given by $u = u_0(1 - 4r^2/D^2)$, where r is the radial distance from the center. If the viscosity of the fluid is μ, the pressure drop across a length L of the pipe is:
   (A) $\frac{\mu u_0 L}{D^2}$ (B) $\frac{4\mu u_0 L}{D^2}$
   (C) $\frac{8\mu u_0 L}{D^2}$ (D) $\frac{16\mu u_0 L}{D^2}$
10. A siphon draws water from a reservoir and discharges it out at atmospheric pressure. Assuming ideal fluid

and the reservoir is large, the velocity at point P in the siphon tube is:

(A) $\sqrt{2gh_1}$ (B) $\sqrt{2gh_2}$
(C) $\sqrt{2g(h_2 - h_1)}$ (D) $\sqrt{2g(h_2 - h_1)}$

Direction for questions 11 and 12:

A smooth flat plate with a sharp leading edge is placed along a gas stream flowing at $U = 10$ m/s. The thickness of the boundary layer at section r-s is 10 mm, the breadth of the plate is 1 m (into the paper) and the density of the gas $P = 1.0$ kg/m^3. Assume that the boundary layer is thin, two-dimensional, and follows a linear velocity distribution, $u = U(y/\delta)$, at the section r-s, where y is the height from plate.



11. The mass flow rate (in kg/s) across the section q-r is:
(A) zero (B) 0.05
(C) 0.10 (D) 0.15

12. The integrated drag force (in N) on the plate, between p-s, is:
(A) 0.67 (B) 0.33
(C) 0.17 (D) zero

13. Consider an incompressible laminar boundary layer flow over a flat plate of length L, aligned with the direction of an oncoming uniform free stream. If F is the ratio of the drag force on the front half of the plate to the drag force on the rear half, then
(A) $F < 1/2$ (B) $F = 1/2$
(C) $F = 1$ (D) $F > 1$

14. While using boundary layer equations, Bernoulli's equation
(A) can be used anywhere.
(B) can be used only outside the boundary layer.
(C) can be used only inside the boundary layer.
(D) cannot be used either inside or outside the boundary layer.

15. During the measurement of viscosity of air flowing through a pipe, we use the relation $\mu = \frac{\pi d^4}{128Q}\left(-\frac{dp}{dx}\right)$ under the condition that in the measuring section
(A) there is a viscous zone near the wall and an inviscid core persists at the centre.
(B) the entire cross-section is viscous.
(C) the flow can be assumed as potential flow.
(D) the flow is irrotational.

16. If the energy grade line and hydraulic grade line are drawn for flow through an inclined pipeline the following four quantities can be directly observed:
I. Static head
II. Friction head
III. Datum head
IV. Velocity head

Starting from the arbitrary datum line, the above types of heads will be in the sequence:
(A) III, II, I, IV
(B) III, IV, II, I
(C) III, IV, I, II
(D) III, I, IV, II

17. Consider steady laminar incompressible axi-symmetric fully developed viscous flow through a straight circular pipe of constant cross-sectional area at a Reynolds number of 5. The ratio of inertia force to viscous force on a fluid particle is
(A) 5 (B) 1/5
(C) 0 (D) ∞

18. The head loss due to sudden expansion is expressed by

(A) $\frac{V_1^2 - V_2^2}{2g}$ (B) $\left[\frac{V_1 - V_2}{2g}\right]^2$

(C) $\frac{(V_1 - V_2)^2}{g}$ (D) $\frac{(V_1 - V_2)^2}{2g}$

19. The procedure to follow in solving for discharge when h_f (head loss), L (pipe length), D (inside diameter), v (kinematic viscosity) and k (wall roughness) are given, is to
(A) assume a f (friction factor), compute V, R (Reynolds number), k/D, look up for f and repeat if necessary.
(B) assume a R, compute f, check k/D, etc.
(C) assume a V, compute R, look up for f, compute V again etc.
(D) assume a O, compute V, R, look up, etc.

20. Branching pipe problems are usually solved
(A) by assuming the head loss is same through each pipe.
(B) by equivalent lengths.
(C) by assuming the elevation of the hydraulic grade line at the junction point and tying to satisfy continuity.
(D) by assuming a distribution which satisfies continuity and computing a direction.

21. The hydraulic head at a point in the soil includes_______.

22. Due to aging of a pipeline, its carrying capacity has decreased by 25%.

The corresponding increase in the Darcy Weisbach friction factor, f is …. %

23. In network of pipes
- (A) the algebraic sum of discharges around each circuit is zero.
- (B) the algebraic sum of discharges around each circuit should not be zero.
- (C) the elevation of hydraulic grade line is assumed for each junction point.
- (D) elementary circuits are replaced by equivalent pipes.

24. While deriving an expression for loss of head due to a sudden expansion in a pipe, in addition to the continuity and impulse momentum equations, one of the following assumptions is made:
- (A) head loss due to friction is equal to the head loss is eddying motion.
- (B) the mean pressure in eddying fluid is equal to the downstream pressure.
- (C) the mean pressure in eddying fluid is equal to the upstream pressure.
- (D) head lost in eddies is neglected.

25. If a single pipe of length L and diameter D is to be replaced by three pipes of same material, same length and equal diameter $d(d < D)$, to convey the same total discharge under the same head loss, then d and D are related by

(A) $D = \frac{d}{3^{1/5}}$ (B) $D = \frac{d}{2^{5/3}}$

(C) $D = \frac{d}{3^{2/3}}$ (D) $D = \frac{d}{2^{3/2}}$

26. The head loss in a sudden expansion from 6 cm diameter pipe to 12 cm diameter pipe in terms of velocity V_1 in the smaller diameter pipe is

(A) $\frac{3}{16}\cdot\frac{V_1^2}{2g}$ (B) $\frac{5}{16}\cdot\frac{V_1^2}{2g}$

(C) $\frac{7}{16}\cdot\frac{V_1^2}{2g}$ (D) $\frac{9}{16}\cdot\frac{V_1^2}{2g}$

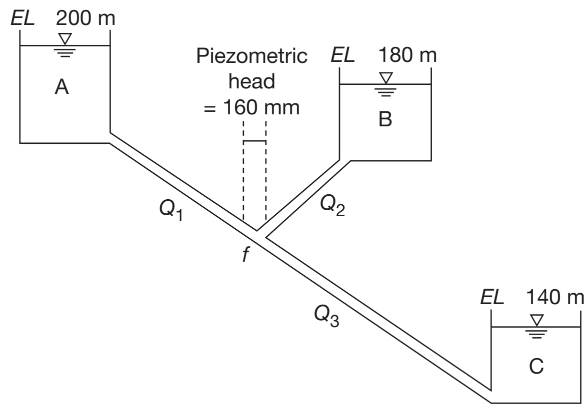
27. An inviscid, irrotational flow field of free vortex motion has a circulation constant Ω. The tangential velocity at any point in the flow field is given by Ω/r, where r, is the radial distance from the centre. At the centre, there is a mathematical singularity which can be physically substituted by a forced vortex. At the interface of the free and forced vortex motion ($r = r_c$), the angular velocity ω is given by

(A) $\frac{\Omega}{r_c^2}$ (B) $\frac{\Omega}{r_c}$

(C) Ωr_c (D) Ωr_c^2

28. A right circular cylinder, open at the top is filled with liquid of relative density 1.2. It is rotated about its vertical axis at such a speed that half the liquid spills out. The pressure at the centre of the bottom will be
- (A) zero.
- (B) one-fourth of the value when the cylinder was full.
- (C) half of the value when the cylinder was full.
- (D) not determinable from the given data.

29. Three reservoirs A, B and C are interconnected by pipes as shown in the figure. Water surface elevations in the reservoirs and the Piezometric head at the junction J are indicated in the figure.



Discharges Q_1, Q_2 and Q_3 are related as

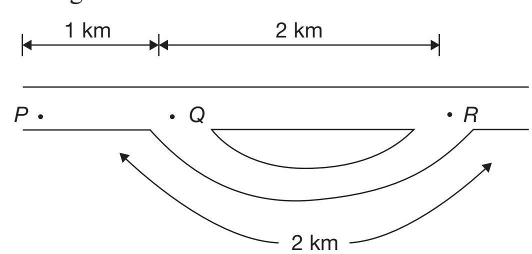
(A) $Q_1 + Q_2 = Q_3$ (B) $Q_1 = Q_2 + Q_3$

(C) $Q_2 = Q_1 + Q_3$ (D) $Q_1 + Q_2 + Q_3 = 0$

30. The comparison between pumps operating in series and in parallel is
- (A) pumps operating in series boost the discharge, whereas pumps operating in parallel boost the head.
- (B) pumps operating in parallel boost the discharge, whereas pumps operating in series boost the head.
- (C) in both cases there would be a boost in discharge only.
- (D) in both case there would be a boost in head only.

Direction for questions 31 and 32:

A pipeline (diameter 0.3 m, length 3 km) carries water from point P to point R (see figure). The piezometric heads at P and R are to be maintained at 100 m and 80 m, respectively. To increase the discharge, a second pipe is added in parallel to the existing pipe from Q to R. The length of the additional pipe is also 2 km. Assume the friction factor, $f = 0.04$ for all pipes and ignore minor losses.

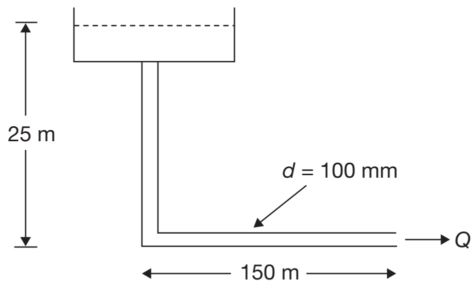


31. What is the increase in discharge if the additional pipe has same diameter (0.3 m)?

(A) 0% (B) 33%
(C) 41% (D) 67%

32. If there is no restriction on the diameter of the additional pipe, what would be the maximum increase in discharge theoretically possible from this arrangement?
(A) 0% (B) 50%
(C) 67% (D) 73%

33. A fire protection system is supplied from a water tower with a bent pipe as shown in the figure. The pipe friction f is 0.03. Ignoring all minor losses, the maximum discharge, Q, in the pipe is



(A) 31.7 lit/s (B) 24.0 lit/s
(C) 15.9 lit/s (D) 12.0 lit/s

34. In a cylindrical vortex motion about a vertical axis, r_1 and r_2 are the radial distances of two points on the horizontal plane ($r_2 > r_1$). If for a given tangential fluid velocity at r_1, the pressure difference between the points in free vortex is one-half of that when the vortex is a forced one, then what is the value of the ratio (r_2/r_1)?
(A) $\sqrt{3/2}$ (B) $\sqrt{2}$
(C) 3/2 (D) $\sqrt{3}$

35. In which one of the following cases separation of boundary layer must occur?
(A) $\frac{dp}{dx} < 0$
(B) $\frac{dp}{dx} = 0$
(C) $\frac{dp}{dx} > 0$
(D) $\frac{dp}{dx} > 0$ and the velocity profile has a point of inflection

36. A viscous fluid flows over a flat plate at zero angle of attack.

Assertion (A): The thickness of boundary layer is an ever increasing one as its distance from the leading edge of the plate increases.

Reason (R): In practice, 99 per cent of the depth of the boundary layer is attained within a short distance from the leading edge.

(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

37. Velocity distribution in a boundary layer flow over a plate is given by $(u/u_m) = 1.5\eta$ where, $\eta = y/\delta$; y is the distance measured normal to the plate; δ is the boundary layer thickness; and u_m is the maximum velocity at $y = \delta$. If the shear stress τ, acting on the plate is given by $\tau = K(\mu\, u_m)/\delta$, where μ is the dynamic viscosity of the fluid, then K takes the value of
(A) 0 (B) 1
(C) 1.5 (D) None of these

38. A flat plate is kept in an infinite fluid medium. The fluid has a uniform free-stream velocity parallel to the plate. For the laminar boundary layer formed on the plate, pick the correct option matching List I and List II.

	List I		List II
a.	Boundary layer thickness	1.	Decreases in the flow direction
b.	Shear stress at the plate	2.	Increases in the flow direction
c.	Pressure gradient along the plate	3.	Remains unchanged

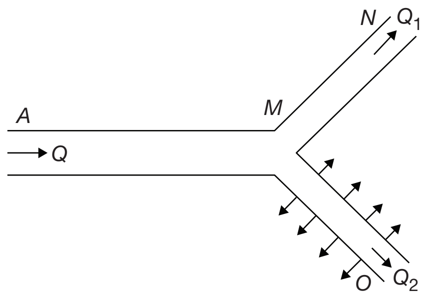
Codes:

	a	b	c		a	b	c
(A)	1	2	3	(B)	2	2	2
(C)	1	1	2	(D)	2	1	3

39. The thickness of the laminar boundary layer on a flat plate at a point A is 2 cm and at a point B, 1 m downstream of A, is 3 cm. What is the distance of A from the leading edge of the plate?
(A) 0.50 m (B) 0.80 m
(C) 1.00 m (D) 1.25 m

40. A fluid (density = 900 kg/m^3 and viscosity = 3×10^{-3}kg/ms) flows upwards between two inclined parallel identical plate at a volumetric rate of 3 lit/s per unit width in metres of the plates. The plates are inclined at an angle of 30° with the horizontal and the plates are 20 mm wide apart. The pressure difference between two sections that are 15 metres apart is
(A) 66218 N/m^2 (B) 66420 N/m^2
(C) 203 N/m^2 (D) 132638 N/M^2

41. In a horizontal plane, water flows through a pipe of 200 mm diameter and 20 km length. At a point M, as shown in the following figure, the pipe is branched off into two identical parallel pipes of diameter 100 mm and length 10 km. The friction factor for all pipes is to be taken to be equal to 0.015. If in the pipe MQ, water is completely drained off from closely spaced side tappings at a constant rate of 0.01 lit/s per metre length of the pipe, then the discharge in $MN(Q_1)$ is



(A) 0.1577 m^3/s (B) 0.0577 m^3/s
(C) 0.1 m^3/s (D) 0.0264 m^3/s

42. An oil (viscosity = 0.8 kg/ms and density = 1400 kg/m^3) flows in a laminar manner between two parallel inclined plates 15 mm apart and inclined at 45° to the horizontal. The pressure at two points 1.5 m vertically apart are 100 kN/m^2 and 300 kN/m^2. If the upper plate moves at a velocity of 2.5 m/s but in a direction opposite to the flow, then the velocity of the flow at a distance of 5 mm from the lower plate is
(A) 2.51 m/s (B) 1.23 m/s
(C) 2.42 m/s (D) 1.58 m/s

43. The velocity distribution in the boundary layer over the face of a spillway was observed to have the form:

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{0.22}$$

At a certain section AA', the boundary layer thickness was estimated to be 70 mm. If the energy loss per metre length of the spillway is 325.64 kN-m/s, then the free stream velocity of the section AA' is
(A) 28 m/s (B) 35 m/s
(C) 21 m/s (D) 207 m/s

44. A old pipeline which has relative roughness of K/D = 0.005 operates at a Reynold's number which is sufficiently high for the flow to be beyond the range of viscous influence and the corresponding f = 0.03. If through further aging, the relative roughness is doubled and the corresponding f = 0.0375, the power increase required to maintain the same rate of flow would be about
(A) 25% (B) 50%
(C) 75% (D) 100%

45. The Prandtl mixing length for turbulent flow through pipes is
(A) dependent on shear stress at the wall.
(B) a universal constant.
(C) zero at the pipe wall.
(D) independent of radial distance from pipe axis.

46. Cavitation is caused by
(A) high velocity. (B) low pressure.
(C) high pressure. (D) high temperature.

47. A flat plate kept at zero incidence in a stream of fluid with uniform velocity develops a turbulent boundary layer over the whole of the plate. If the average coefficient of drag for the whole plate having a turbulent boundary layer is given by $C_D = \frac{0.072}{(Re_L)^{0.2}}$, then the ratio of the drag force on the rear half of the plate to the drag force on the front half of the plate is
(A) 1.349 (B) 0.4256
(C) 0.7411 (D) 0.5743

48. A ship with hull length of 100 m is to run with a speed of 10 m/s. For dynamic similarity, the velocity for a 1: 25 model of the ship in a towing tank should be
(A) 2 m/s (B) 10 m/s
(C) 20 m/s (D) 25 m/s

49. Consider the following statements:
I. Complete similarity between model and prototype envisages geometric and dynamic similarities only.
II. Distorted models are necessary where geometric similarity is not possible due to practical reasons.
III. In testing of model of a ship, the surface tension forces are generally neglected.
IV. The scale effect takes care of the effect of dissimilarity between model and prototype.
(A) I and III (B) I, II and IV
(C) II and III (D) II and IV

50. Match List I with List II and select the correct answer from the given codes:

	List I (Forces)		List II (Dimensionless number)
a.	Gravity force	1.	Weber number
b.	Pressure force	2.	Mach number
c.	Surface tension	3.	Froude's Number
d.	Elastic force	4.	Euler's number

Codes:

	a	b	c	d		a	b	c	d
(A)	1	2	4	3	(B)	4	3	2	1
(C)	3	4	1	2	(D)	2	1	3	4

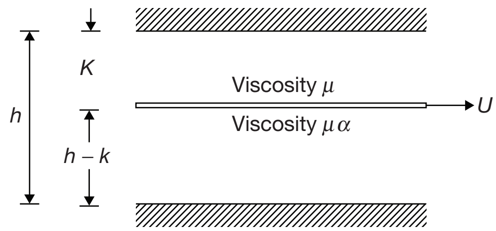
Direction for questions 51 and 52:

A liquid of viscosity 0.8 and specific gravity 1.3 flows through a circular pipe of 100 mm diameter. Maximum shear stress at the pipe wall is 220 N/m^2.

51. Pressure gradient of the flow in N/m^2 per m is
(A) −6800
(B) −8800
(C) 6800
(D) 8800

52. Average velocity of flow is
(A) 2.6 m/s
(B) 2.9 m/s
(C) 3.2 m/s
(D) 3.4 m/s

Direction for questions 53 and 54:



A large thin plate is pulled at a constant velocity U through a narrow gap of height h. On one side of the plate is filled with oil of viscosity μ and the other side oil of viscosity $\alpha\mu$, where α is a constant.

53. Total drag force on the plate is

(A) $A\mu U\left(\frac{1}{k}+\frac{\alpha}{h-k}\right)$

(B) $A\mu U\left(k+\frac{h-k}{\alpha}\right)$

(C) $\frac{\mu U}{A}\left(\frac{1}{k}+\frac{\alpha}{h-k}\right)$

(D) $\frac{\mu U}{A}\left(k+\frac{h-k}{\alpha}\right)$

54. Value of k such that the drag force is minimum is

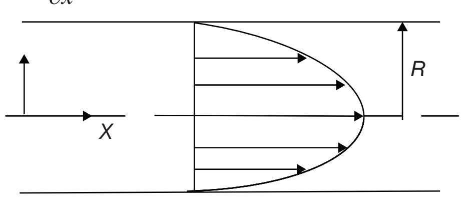
(A) $\frac{\sqrt{h}}{1+\alpha}$ (B) $\frac{\sqrt{h}}{1-\alpha}$

(C) $\frac{h}{1-\sqrt{\alpha}}$ (D) $\frac{h}{1+\sqrt{\alpha}}$

55. The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression:

$$u(r)=\frac{-R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left(1-\frac{r^2}{R^2}\right)$$

Where $\frac{\partial p}{\partial x}$ is a constant.



The average velocity of fluid in the pipe is

(A) $\frac{-R^2}{8\mu}\left(\frac{dp}{dx}\right)$

(B) $\frac{-R^2}{4\mu}\left(\frac{dp}{dx}\right)$

(C) $\frac{-R^2}{2\mu}\left(\frac{dp}{dx}\right)$

(D) $\frac{-R^2}{\mu}\left(\frac{dp}{dx}\right)$

56. Match the following:

	List I		List II
P.	Compressible flow	**1.**	Nusselt number
Q.	Boundary layer flow	**2.**	Reynold's number
R.	Pipe flow	**3.**	Skin friction coefficient
S.	Heat convection	**4.**	Mach number

Codes:

	P	Q	R	S
(A)	3	1	4	2
(B)	3	4	2	1
(C)	4	3	2	1
(D)	2	1	3	4

57. In laminar flow through a pipe, the pressure drop per unit length of pipe is given by

(A) $\frac{32\mu\bar{u}}{D}$ (B) $\frac{16\mu\bar{u}}{D^2}$

(C) $\frac{128\mu Q}{\pi D^4}$ (D) $\frac{128\mu Q}{\pi D^2}$

58. A pipe 300 m long slopes down at 1 in 100 and tapers from 600 mm diameter to 300 mm diameter. Oil is passing through the pipe at a rate of 90 litres per second. Specific gravity of oil is 0.8. If the pressure gauge at the higher end reads 60 kN/m^2, pressure at the lower end of the pipe (in kN/m^2) is______.

(A) 89.2 (B) 88.4
(C) 82.3 (D) 82.9

59. A pipe of 240 mm diameter and 12,000 m length is laid at a slope of 1 in 150. An oil of specific gravity 0.85 is pumped up at a rate of 0.02 m^3/s. If the coefficient of friction is 0.0266, power (in kW) required to pump the oil is______.

(A) 24.3 (B) 23.2
(C) 22.1 (D) 25.4

60. Crude oil is pumped through a 150 mm diameter smooth pipe which is subjected to seasonal changes in temperature from 0°C to 38°C. At maximum temperature, power required to maintain a flow of 30 lit/s is 2.3 kN per 300 m. If kinematic viscosity of crude oil at 0°C is 2.8 stokes, the power requirement per 300 m (in kW) to maintain same rate of flow is______(specific gravity of crude oil = 0.9).

(A) 5.64 (B) 4.92
(C) 5.28 (D) 5.47

61. Water flows through a pipe of 250 mm diameter.The coefficient of friction between water and pipe surface is 0.04. There is a shear stress of 0.15 kN/m^2 at a distance of 40 mm from the pipe axis. Shear stress at the pipe wall (in kN/m^2) is______.

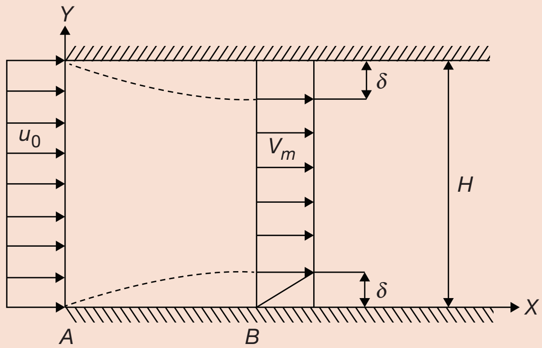
(A) 0.423 (B) 0.486
(C) 0.468 (D) 0.438

Previous Years' Questions

1. Flow rate of a fluid (density = 1000 kg/m^3) in a small diameter tube is 800 mm^3/s. the length and the diameter of the tube are 2 m and 0.5 mm, respectively. The pressure drop in 2 m length is equal to 2.0 MPa. The viscosity of the fluid is **[GATE, 2007]**

(A) 0.025 N-s/m^2 (B) 0.012 N-s/m^2
(C) 0.00192 N-s/m^2 (D) 0.00102 N-s/m^2

2. Consider a steady incompressible flow through a channel as shown below.



The velocity profile is uniform with a value of u_0 at the inlet section A. The velocity profile at section B downstream is

$$u = \begin{cases} V_m \dfrac{y}{\delta}, & 0 \le y \le \delta \\ V_m, & \delta \le y \le H-\delta \\ V_m \dfrac{H-y}{\delta}, & H-\delta \le y \le H \end{cases}$$

The ratio $\dfrac{p_A - p_B}{\frac{1}{2} pu_0^2}$ (where p_A and p_B are the pressures at section A and B, respectively, and ρ is the density of the fluid) is **[GATE, 2007]**

(A) $\dfrac{1}{(1-(\delta/H))^2} - 1$ (B) $\dfrac{1}{[1-(\delta/H)]^2}$

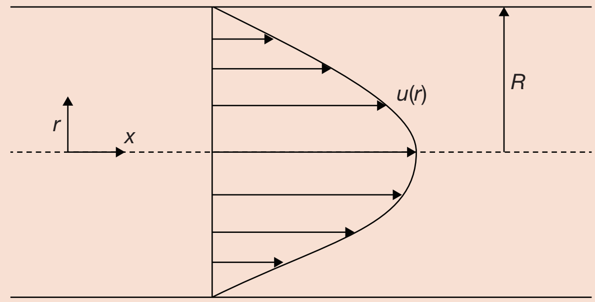
(C) $\dfrac{1}{(1-(2\delta/H))^2} - 1$ (D) $\dfrac{1}{1+(\delta/H)}$

3. Water at 25°C is flowing through a 1.0 km long GI pipe of 200 mm diameter at the rate of 0.07 m^3/s. If value of Darcy friction factor for this pipe is 0.02 and density of water is 1000 kg/m^3, the pumping power (in kW) required to maintain the flow is **[GATE, 2008]**

(A) 1.8 (B) 17.4
(C) 20.5 (D) 41.0

4. The velocity profile of a fully developed laminar flow in a straight circular pipe, as shown in the figure, is given by the expression $u(r) = -\dfrac{R^2}{4\mu}\left(\dfrac{dp}{dx}\right)\left(1-\dfrac{r^2}{R^2}\right)$.

where $\dfrac{dp}{dx}$ is a constant. The average velocity of fluid in the pipe is **[GATE, 2008]**



(A) $-\dfrac{R^2}{8\mu}\left(\dfrac{dp}{dx}\right)$ (B) $-\dfrac{R^2}{4\mu}\left(\dfrac{dp}{dx}\right)$

(C) $-\dfrac{R^2}{2\mu}\left(\dfrac{dp}{dx}\right)$ (D) $-\dfrac{R^2}{\mu}\left(\dfrac{dp}{dx}\right)$

5. The maximum velocity of a one-dimensional incompressible fully developed viscous flow, between two fixed parallel plates, is 6 m/s. The mean velocity (in m/s) of the flow is **[GATE, 2008]**

(A) 2 (B) 3
(C) 4 (D) 5

6. The flow of water (mass density = 1000 kg/m^3 and kinematic viscosity = 10^{-6} m^2/s) in a commercial pipe, having equivalent roughness k_s as 0.12 mm, yields an average shear stress at the pipe boundary = 600 N/m^2. The value of k_s/δ' (δ' being the thickness of laminar sub-layer) for this pipe is **[GATE, 2008]**

(A) 0.25 (B) 0.50
(C) 6.0 (D) 8.0

7. Oil flows through a 200 mm diameter horizontal cast iron pipe (friction factor, $f = 0.0225$) of length 500 m. The volumetric rate is 0.2 m^3/s. The head loss (in m) due to friction is (assume $g = 9.81$ m/s^2) **[GATE, 2009]**

(A) 116.18 (B) 0.116
(C) 18.22 (D) 232.36

8. An incompressible fluid flows over flat plate with zero pressure gradient. The boundary layer thickness is 1 mm at a location where the Reynolds number is 1000. If the velocity of the fluid alone is increased by a factor of 4, then the boundary layer thickness at the same location, in mm will be **[GATE, 2009]**

(A) 4 (B) 2
(C) 0.5 (D) 0.25

9. Water flows through a 100 mm diameter pipe with a velocity of 0.015 m/s. If the kinematic viscosity of water is 1.13×10^{-6} m^2/s, the friction factor of the pipe material is **[GATE, 2009]**

(A) 0.0015 (B) 0.032
(C) 0.037 (D) 0.048

10. For steady, fully developed flow inside a straight pipe of diameter D, neglecting gravity effects, the pressure drop Δp over a length L and the wall shear stress τ_w are related by **[GATE, 2010]**

(A) $\tau_w = \dfrac{\Delta pD}{4L}$ (B) $\tau_w = \dfrac{\Delta pD}{4L^2}$

(C) $\tau_w = \dfrac{\Delta pD}{2L}$ (D) $\tau_w = \dfrac{4\Delta pD}{D}$

11. Water flows through a pipe having an inner radius of 10 mm at the rate of 36 kg/hr at 25°C. The viscosity of water at 25°C is 0.001 kg/ms. The Reynolds number of the flow is ______. **[GATE, 2011]**

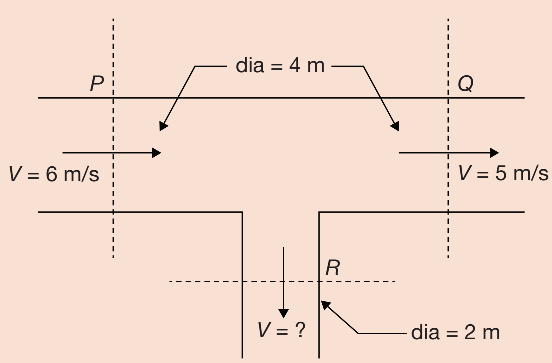
12. For a fully developed flow of water in a pipe having diameter 10 cm, velocity 0.1 m /s and kinematic viscosity 10^{-5} m^2/s, the value of Darcy friction factor is ______. **[GATE, 2011]**

13. A single pipe of length 1500 m and diameter 60 cm connects two reservoirs having a difference of 20 m in their water levels. The pipe is to be replaced by two pipes of the same length and equal diameter '*d*' to convey 25% more discharge under the same head loss. If the friction factor is assume to be the same for all the pipes, the value of '*d*' is approximately equal to which of the following options? **[GATE, 2011]**

(A) 37.5 cm (B) 40.0 cm
(C) 45.0 cm (D) 50.0 cm

14. Water flows through a 10 mm diameter and 250 m long smooth pipe at an average velocity of 0.1 m/s. The density and the viscosity of water are 997 kg/m^3 and 855×10^{-6} N. s/m^2, respectively. Assuming fully–developed flow, the pressure drop (in Pa) in the pipe is ______. **[GATE, 2012]**

15. The circular water pipes shown in the figure are flowing full. The velocity of flow (in m/s) in the branch pipe '*R*' is **[GATE, 2012]**



(A) 3 (B) 4
(C) 5 (D) 6

16. Consider laminar flow of water over a flat plate of length 1 m. If the boundary layer thickness at a distance of 0.25 m from the leading edge of the plate is 8 mm, the boundary layer thickness (in mm), at a distance of 0.75 m, is ______. **[GATE, 2013]**

17. A 2 km pipe of 0.2 m diameter connects two reservoirs. The difference between the water levels in the reservoir is 8 m. The Darcy-Weisbach friction factor of the pipe is 0.04. Accounting for frictional entry and exit losses, the velocity in the pipe in (m/s) is **[GATE, 2013]**

(A) 0.63 (B) 0.35
(C) 2.52 (D) 1.25

18. With reference to a standard Cartesian (x, y) plane, the parabolic velocity distribution profile of fully developed laminar flow in x-direction between two parallel, stationary and identical plates that are separated by distance, h, is given by the expression

$$u = -\frac{h^2}{8\mu}\frac{dp}{dx}\left[1-4\left(\frac{y}{h}\right)^2\right]$$

In this equation, the $y = 0$ axis lies equidistant between the plates at a distance $h/2$ from the two plates, p is the pressure variable and μ is the dynamic viscosity term. The maximum and average velocities are, respectively **[GATE, 2014]**

(A) $u_{max} = -\dfrac{h^2}{8\mu}\dfrac{dp}{dx}$ and $u_{average} = \dfrac{2}{3}u_{max}$

(B) $u_{max} = \dfrac{h^2}{8\mu}\dfrac{dp}{dx}$ and $u_{average} = \dfrac{2}{3}u_{max}$

(C) $u_{max} = -\dfrac{h^2}{8\mu}\dfrac{dp}{dx}$ and $u_{average} = \dfrac{3}{8}u_{max}$

(D) $u_{max} = \dfrac{h^2}{8\mu}\dfrac{dp}{dx}$ and $u_{average} = \dfrac{3}{8}u_{max}$

19. Consider the turbulent flow of a fluid through a circular pipe of diameter, D. Identify the correct pair of statements.

I. The fluid is well-mixed
II. The fluid is un-mixed
III. $Re_D < 2300$
IV. $Re_D > 2300$ **[GATE, 2014]**

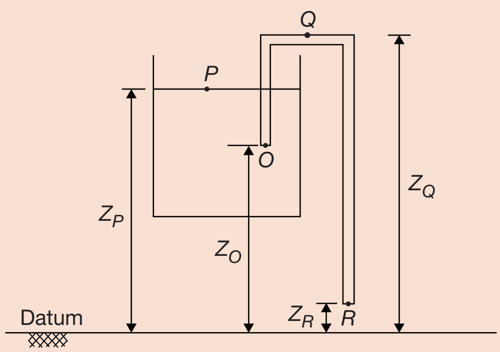
(A) I, III (B) II, IV
(C) II, III (D) I, IV

20. An incompressible homogeneous fluid is flowing steadily in a variable diameter pipe having the large and small diameters as 15 cm and 5 cm, respectively. If the velocity at a section at the 15 cm diameter portion of the pipe is 2.5 m/s, the velocity of the fluid (in m/s) at a section falling in 5 cm portion of the pipe is _______. **[GATE, 2014]**

21. An incompressible fluid is flowing at a steady rate in a horizontal pipe. From a section, the pipe divides into two horizontal parallel pipes of diameters d_1 and d_2 (where $d_1 = 4d_2$) that run for a distance of L each and then again join back to a pipe of the original size. For both the parallel pipes, assume the head loss due to friction only and the Darcy-Weisbach friction factor to be the same. The velocity ratio between the bigger and the smaller branched pipes is _______. **[GATE, 2014]**

22. A straight 100 m long raw water gravity main is to carry water from an intake structure to the jack well of a water treatment plant. The required flow through this water main is 0.21 m^3/s. Allowable velocity through the main is 0.75 m/s. Assume $f = 0.01$, $g = 9.81$ m/s^2. The minimum gradient (in cm/100 m length) to be given to this gravity main so that the required amount of water flows without any difficulty is ________. **[GATE, 2014]**

23. A siphon is used to drain water from a large tank as shown in the figure below. Assume that the level of water is maintained constant. Ignore frictional effect due to viscosity and losses at entry and exist. At the exit of the siphon, the velocity of water is **[GATE, 2014]**



(A) $\sqrt{2g(Z_Q - Z_R)}$ (B) $\sqrt{2g(Z_P - Z_R)}$

(C) $\sqrt{2g(Z_0 - Z_R)}$ (D) $\sqrt{2g\,Z_Q}$

24. A fluid of dynamic viscosity 2×10^{-5} kg/ ms and density 1 kg/m^3 flows with an average velocity of 1 m/s through a long duct of rectangular (25 mm × 15 mm) cross-section. Assuming laminar flow, the pressure drop (in Pa) in the fully developed region per metre length of the duct is _______. **[GATE, 2014]**

25. A circular pipe has a diameter of 1 m, bed slope of 1 in 1000, and Manning's roughness coefficient equal to 0.01. It may be treated as an open channel flow when it is flowing just full, i.e., the water level just touches the crest. The discharge in this condition is denoted by Q_{full}. Similarly, the discharge when the pipe is flowing half-full, i.e., with a flow depth of 0.5 m, is denoted by Q_{half}. The ratio $\frac{Q_{full}}{Q_{half}}$ is **[GATE, 2015]**

(A) 1 (B) $\sqrt{2}$

(C) 2 (D) 4

26. Consider fully developed flow in a circular pipe with negligible entrance length effects. Assuming the mass flow rate, density and friction factor to be constant, if the length of the pipe is doubled and the diameter is halved, the head loss due to friction will increase by a factor of **[GATE, 2015]**

(A) 4 (B) 16

(C) 32 (D) 64

27. The Blaussius equation related to boundary layer theory is a **[GATE, 2015]**

(A) third-order linear partial differential equation.
(B) third-order non-linear partial differential equation.
(C) second-order non-linear ordinary differential equation.
(D) third-order non-linear ordinary differential equation.

28. For flow through a pipe of radius R, the velocity and temperature distribution are as follows: $u(r, z) = C_1$, and $T(r, x) = C_2\left[1-\left(\frac{r}{R}\right)^3\right]$, where C_1 and C_2 are constants.

The bulk mean temperature is given by $T_m = \frac{2}{U_m R^2}\int_0^R u(r, x)T(r, x)\, r dr$, with U_m being the mean velocity of flow. The value of T_m is **[GATE, 2015]**

(A) $\frac{0.5C_2}{U_m}$ (B) $0.5C_2$

(C) $0.6C_2$ (D) $\frac{0.6C_2}{U_m}$

29. Air (ρ = 1.2 kg/m^3 and kinematic viscosity, $d = 2 \times 10^{-5}$ m^2/s) with a velocity of 2 m/s flows over the top surface of a flat plate of length 2.5 m. If the average value of friction coefficient is $C_f = \frac{1.328}{\sqrt{Re_x}}$, the total drag force (in N) per unit width of the plate is _______. **[GATE, 2015]**

30. Within a boundary layer for a steady incompressible flow, the Bernoulli's equation **[GATE, 2015]**

(A) holds because the flow is steady.
(B) holds because the flow is incompressible.
(C) holds because the flow is transitional.
(D) does not hold because the flow is frictional.

31. The head loss for a laminar incompressible flow through a horizontal circular pipe is h_1. Pipe length and fluid remaining the same, if the average flow velocity doubles and the pipe diameter reduces to half its previous value, the head loss is h_2. The ratio h_2/h_1 is **[GATE, 2015]**

(A) 1 (B) 4
(C) 8 (D) 16

32. For a fully developed laminar flow of water (dynamic viscosity 0.001 Pa-s) through a pipe of radius 5 cm, the axial pressure gradient is –10 Pa/m. The magnitude of axial velocity (in m/s) at a radial location of 0.2 cm is ______. **[GATE, 2015]**

33. Couette flow is characterized by **[GATE, 2015]**
(A) steady, incompressible, laminar flow through a straight circular pipe.
(B) fully developed turbulent flow through a straight circular pipe.
(C) steady, incompressible, laminar flow between two fixed parallel plates.
(D) steady, incompressible, laminar flow between one fixed plate and the other moving with a constant velocity.

34. Three parallel pipes connected at the two ends have flow-rates Q_1, Q_2 and Q_3 respectively, and the corresponding frictional head losses are h_{L1}, h_{L2}, and h_{L3} respectively. The correct expressions for total flow rate (Q) and frictional head loss across the two ends (h_L) are **[GATE, 2015]**
(A) $Q = Q_1 + Q_2 + Q_3$; $h_L = h_{L1} + h_{L2} + h_{L3}$
(B) $Q = Q_1 + Q_2 + Q_3$; $h_L = h_{L1} = h_{L2} = h_{L3}$
(C) $Q = Q_1 = Q_2 = Q_3$; $h_L = h_{L1} + h_{L2} + h_{L3}$
(D) $Q = Q_1 = Q_2 = Q_3$; $h_L = h_{L1} = h_{L2} = h_{L3}$

35. Two reservoirs are connected through a 930 m long, 0.3 m diameter pipe, which has a gate valve. The pipe entrance is sharp (loss coefficient = 0.5) and the valve is half-open (loss coefficient = 5.5). The head difference between the two reservoirs is 20 m. Assume the friction factor for the pipe as 0.03 and g = 10 m/s^2. The discharge in the pipe accounting for all minor and major losses is ________. **[GATE, 2015]**

36. The drag force, F_D, on a sphere kept in a uniform flow field depends on the diameter of the sphere, D; flow velocity, V, fluid density, ρ; and dynamic viscosity, μ. Which of the following options represents the non-dimensional parameters which could be used to analyze this problem? **[GATE, 2015]**

(A) $\dfrac{F_D}{VD}$ and $\dfrac{\mu}{\rho VD}$ (B) $\dfrac{F_D}{\rho VD^2}$ and $\dfrac{\rho VD}{\mu}$

(C) $\dfrac{F_D}{\rho V^2 D^2}$ and $\dfrac{\rho VD}{\mu}$ (D) $\dfrac{F_D}{\rho V^3 D^3}$ and $\dfrac{\mu}{\rho VD}$

37. A nozzle is so shaped that the average flow velocity changes linearly from 1.5 m/s at the beginning to 15 m/s at its end in a distance of 0.375 m. The magnitude of the convective acceleration (in m/s^2) at the end of the nozzle is _________. **[GATE, 2015]**

38. A pipe of 0.7 m diameter has a length of 6 km and connects two reservoirs A and B. The water level in reservoir A is at an elevation 30 m above the water level in reservoir B. Halfway along the pipe line, there is a branch through which water can be supplied to a third reservoir C. The friction factor of the pipe is 0.024. The quantity of water discharged into reservoir C is 0.15 m^3/s. Considering the acceleration due to gravity as 9.81 m/s^2 and neglecting minor losses, the discharge (in m^3/s) into the reservoir B is _______. **[GATE, 2015]**

Answer Keys

Exercises

1. C **2.** A **3.** A **4.** C **5.** B **6.** D **7.** C **8.** B **9.** D **10.** C
11. B **12.** C **13.** D **14.** B **15.** B **16.** D **17.** A **18.** D **19.** A **20.** D
21. Pressure head as well as datum head **22.** 77% **23.** A **24.** C **25.** A **26.** D **27.** A
28. A **29.** A **30.** B **31.** C **32.** D **33.** D **34.** B **35.** D **36.** D **37.** C
38. D **39.** B **40.** B **41.** B **42.** C **43.** B **44.** A **45.** A **46.** B **47.** C
48. A **49.** C **50.** C **51.** B **52.** D **53.** A **54.** D **55.** A **56.** C **57.** C
58. D **59.** C **60.** D **61.** C

Previous Years' Questions

1. C **2.** A **3.** B **4.** A **5.** C **6.** D **7.** A **8.** C **9.** D **10.** A
11. 635 to 638 **12.** 0.06 to 0.07 **13.** D **14.** 6800 to 6900 **15.** B **16.** 13.5 to 14.2
17. A **18.** A **19.** D **20.** 22.5 **21.** 2 **22.** 4.8 **23.** B **24.** 1.7 to 2 **25.** C **26.** D
27. D **28.** C **29.** 0.0158 to 0.0162 **30.** D **31.** C **32.** 6.2 to 6.3 **33.** D **34.** B
35. **0.**1413 m^3/s **36.** C **37.** 540 **38.** **0.**5716