Tangents and Secants to a Circle

Exercise 9.1

Q. 1. Fill in the blanks

i. A tangent to a circle intersects it in …………….. point (s).
ii. A line intersecting a circle in two points is called a ……………..
iii. A circle can have ……………… parallel tangents at the most.
iv. The common point of a tangent to a circle and the circle is called ……………
v. We can draw ……………….. tangents to a given circle.

Answer : i. One

Property- a tangent to a circle touches it at only one point called the common point.

ii. Secant

Secant- a line that touches the circle at two different points

ii. Two

A circle can have two tangents that are parallel, these two tangents touch the circle on the opposite sides, and distance between the point of contacts is the diameter.

iv. Point of contact

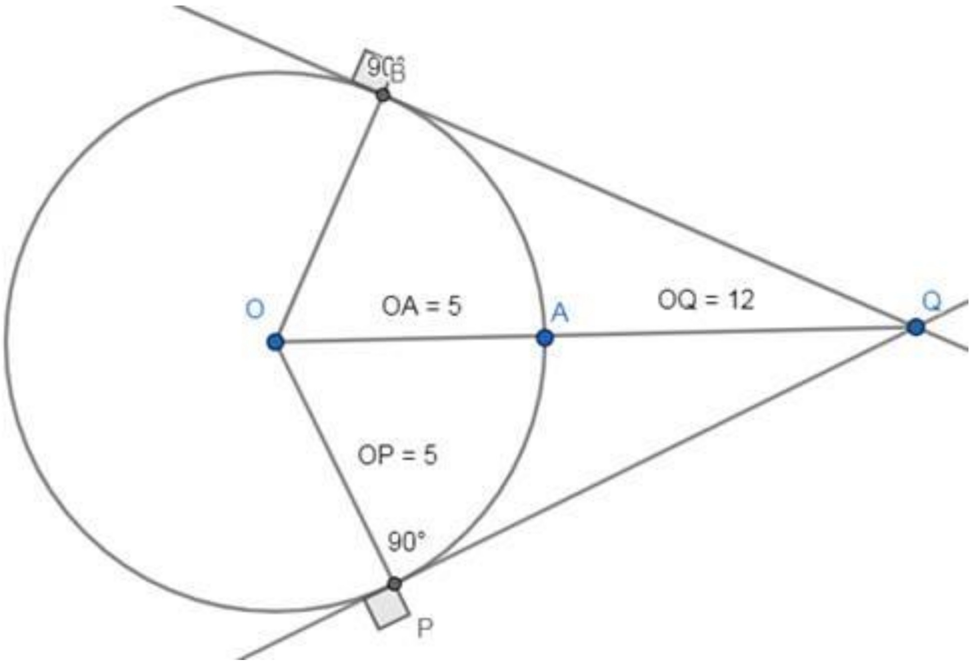
The point where tangent touches the circle is called point of contact.

v. Infinite

We can draw infinite tangents to a circle, as a circle can be assumed to be curve having infinite points and one tangent can be drawn from each point on the circle.

Q. 2. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that P OQ = 12 cm. Find length of PQ.

Answer :



We know that tangent to a circle makes a right angle with radius.

∠OPQ = 90°

Applying Pythagoras

$PQ^2 = OP^2 + OQ^2$

$PQ^2 = 5^2 + 12^2$

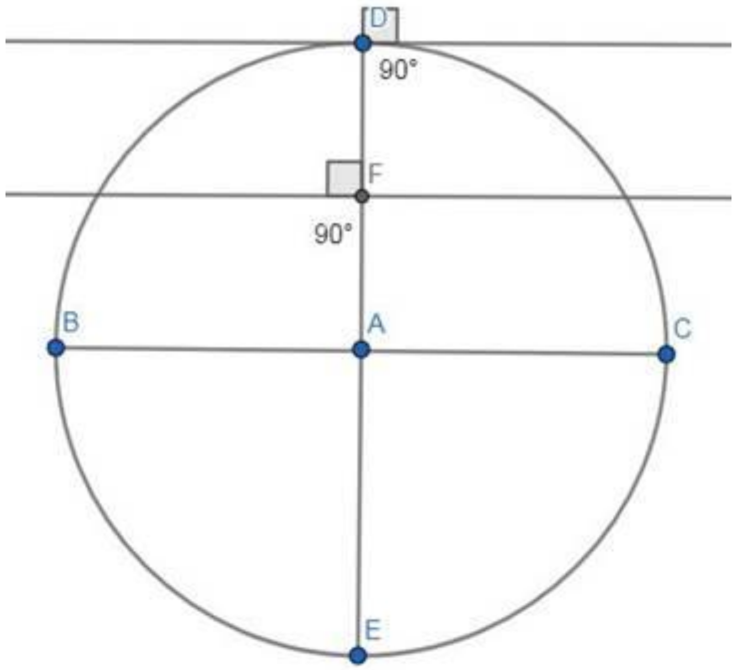
$PQ^2 = 25 + 144$

$PQ^2 = 169$

PQ = 13cm

Q. 3. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer :



Step1: Draw circle of random radius with BC as diameter.

Step2: Draw line AD perpendicular to BC to touch the circle at D

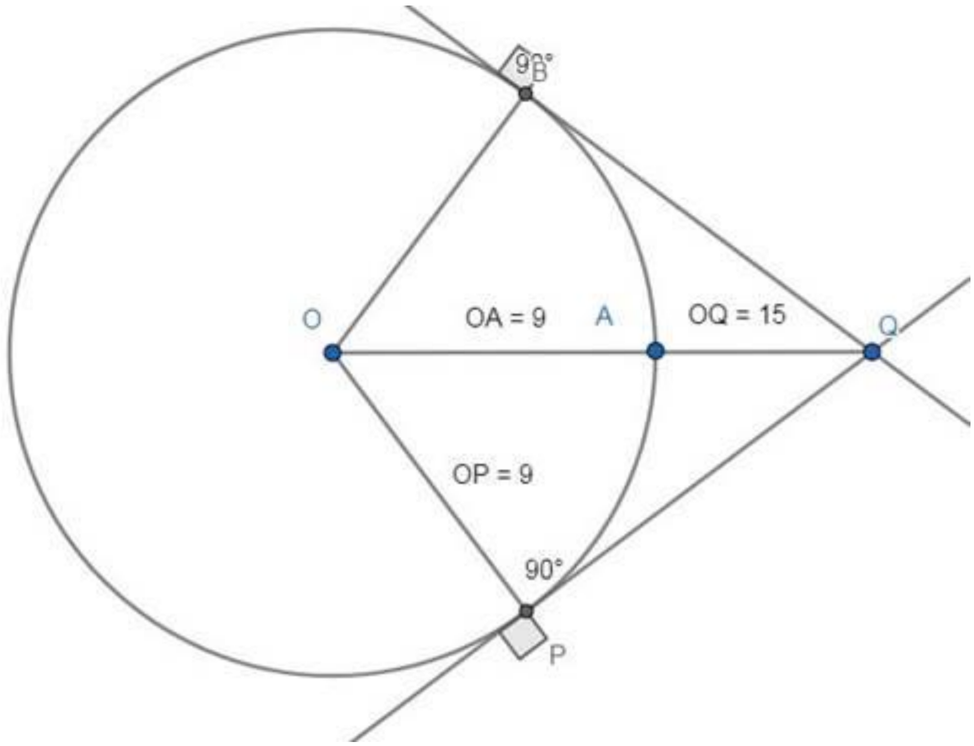
Such that ∠D = 90°

Step3: Draw line through D parallel to BC that will form a tangent

Step4: Take a random point on line AD as F and draw a line through F parallel to BC to intersect circle in two points that forms a secant.

Q. 4. Calculate the length of tangent from a point 15 cm. away from the circle of a circle of radius 9 cm.

Answer :



We know that tangent to a circle makes a right angle with radius.

$\angle OPQ = 90°$

Applying Pythagoras

$PQ^2 = OP^2 + OQ^2$

$PQ^2 = 9^2 + 15^2$

$PQ^2 = 81 + 225$

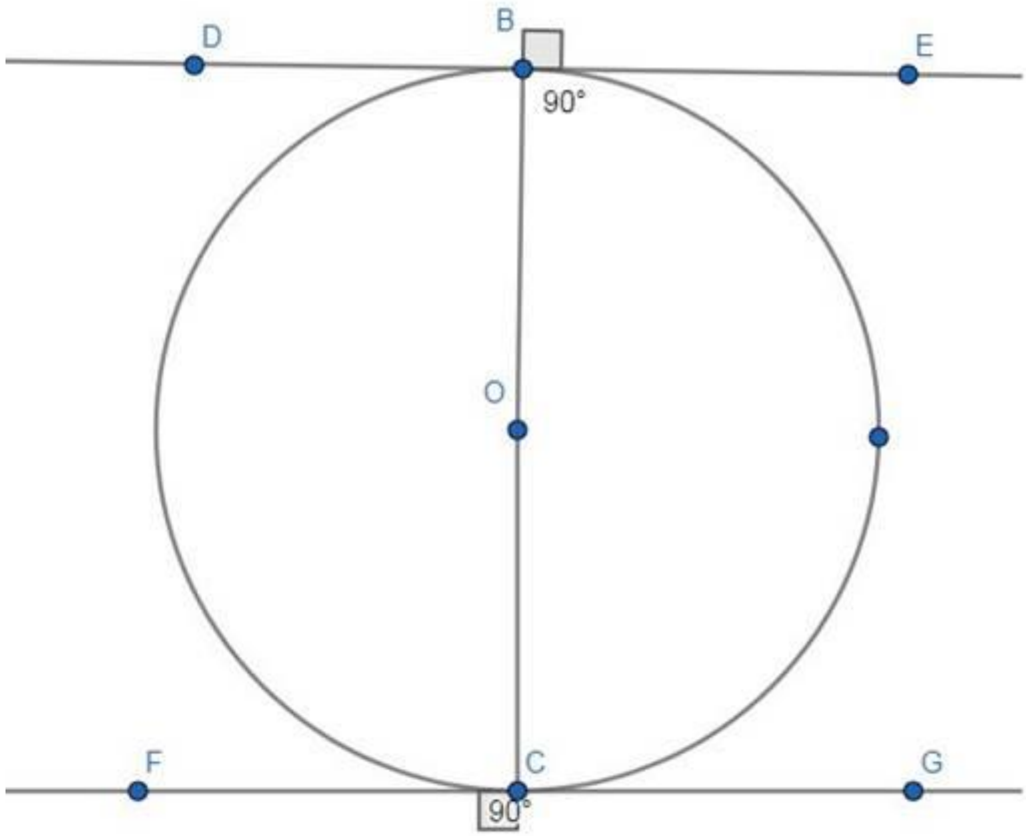
$PQ^2 = 306$

$PQ = \sqrt{306}$ cm

Length of tangent = $\sqrt{306}$ cm

Q. 5. Prove that the tangents to a circle at the end points of a diameter are parallel.

Answer :



To prove: DE ∥ FG

Proof:

We know that tangent to a circle makes a right angle with the radius.

Let DE and FG be tangent at B and C respectively.

BC forms the diameter.

∴ ∠OBE = ∠OBD = ∠OCG = ∠OCF = 90°

Also, ∠OBD = ∠OCG and ∠OBE = ∠OCF as alternate angles

∴ DE and FG make 90° to same line BC which is the diameter.

Thus DE ∥ FG

Exercise 9.2

Q. 1 A. Choose the correct answer and give justification for each.

The angle between a tangent to a circle and the radius drawn at the point of contact is

A. 60^o
B. 30^o
C. 45^o
D. 90^o

Answer : Property of tangent- tangent to a circle makes right angle with radius at point of contact

Q. 1 B. Choose the correct answer and give justification for each.

From a point Q, the length of the tangent to a circle is 24 cm. and the distance of Q from the centre is 25 cm. The radius of the circle is

A. 7 cm
B. 12 cm
C. 15 cm
D. 24.5 cm

Answer : Let O be center, OP be radius.

Applying Pythagoras

PQ = 24, OQ = 25

$OQ^2 = OP^2 + PQ^2$

$25^2 = OP^2 + 24^2$

$625 = OP^2 + 576$

$OP^2 = 49$

OP = 7 cm

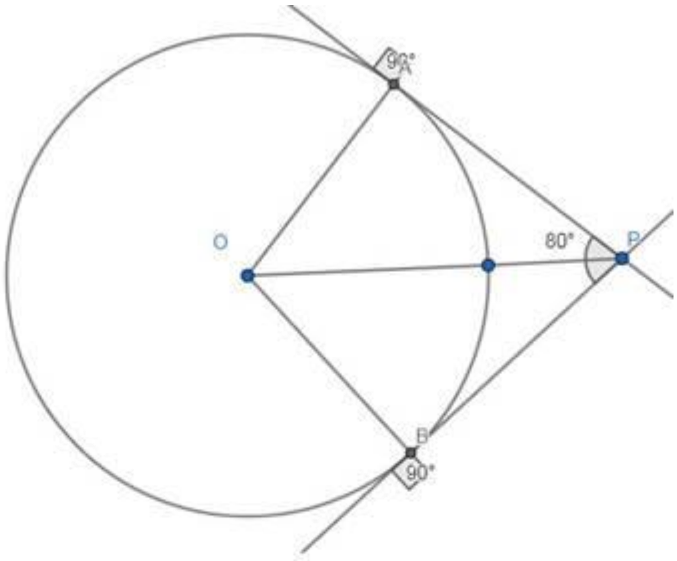
Radius of Circle = 7 cm

Q. 1 D. Choose the correct answer and give justification for each.

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80^o, then $\angle POA$ is equal to

A. 50º
B. 60º
C. 70º
D. 80º

Answer :



We know that tangent to a circle makes right angle with radius.

∴ ∠A = ∠B = 90° and ∠P = 80°

Sum of all angles of quadrilateral = 180°

∴ ∠A + ∠B + ∠P + ∠O = 360°

∴ 90° + 90° + 80° + ∠O = 360°

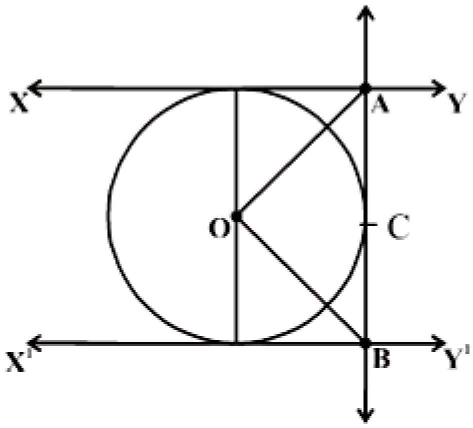
∠O = 100°

∠ AOP = ∠ BOP

∴ ∠POA = 50°

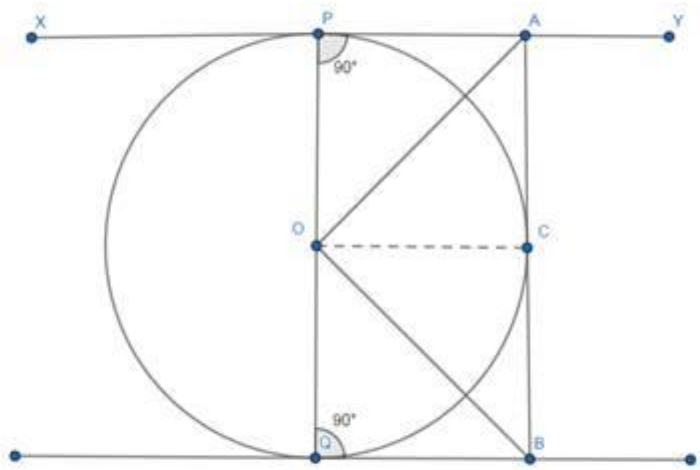
Q. 1 E. Choose the correct answer and give justification for each.

In the figure XY and X^1Y^1 are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X^1Y^1 at B then ∠AOB =



A. 80º
B. 100º
C. 90º
D. 60º

Answer :



Construct Line OC = radius

OP = OQ = OC = radius

OC∥AP and AC∥OP and

Thus AP = OP = radius

As AP = OP and ∠P = 90°

ΔOAP is isosceles triangle

∴ ∠ PAO = ∠POA = 45°

Also ∠ PAC = 90°

∠OAC = 45° ---1

Also AB is perpendicular to OC

∠OCA = 90°

In ΔAOC

∠OCA + ∠OAC + ∠COA = 180°

45 + 90 + ∠COA = 180°

∠COA = 45°

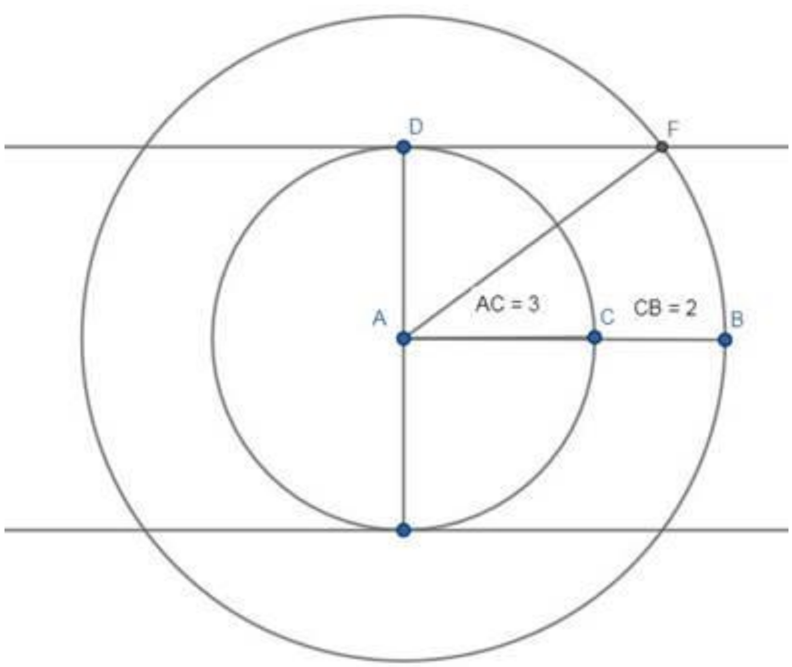
Similarly

∠BOC = 45°

∴ ∠AOB = 90°

Q. 2. Two concentric circles of radii 5 cm and 3 cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle.

Answer :



AB = AF = 5cm radius of larger circle

AD = AC = 3cm radius of smaller circle

Pythagoras theorem

$AF^2 = AD^2 + DF^2$

$5^2 = 3^2 + DF^2$

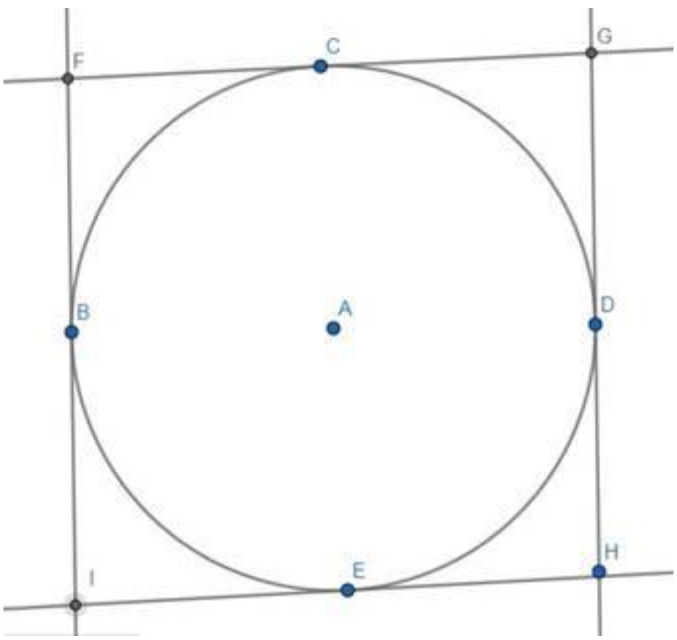
$25^2 = 9^2 + DF^2$

DF = 4 units

Length of chord = 2 × DF = 2 × 4 = 8cm

Q. 3.Prove that the parallelogram circumscribing a circle is a rhombus.

Answer :



FGHI is a parallelogram

∴ HI = FG and FI = GH----1

Also

IB = IE tangents to circle from I

And similarly

HE = HD, CG = GD and CF = BF

Adding all the equations

IE + HE + GC + CF = BF + BI + GD + DH

IH + GF = IF + GH

∴ 2HI = 2HG

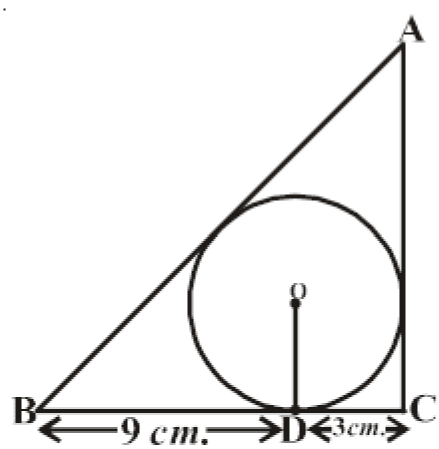
HI = HG---2

∴ GF = GH = HI = IF

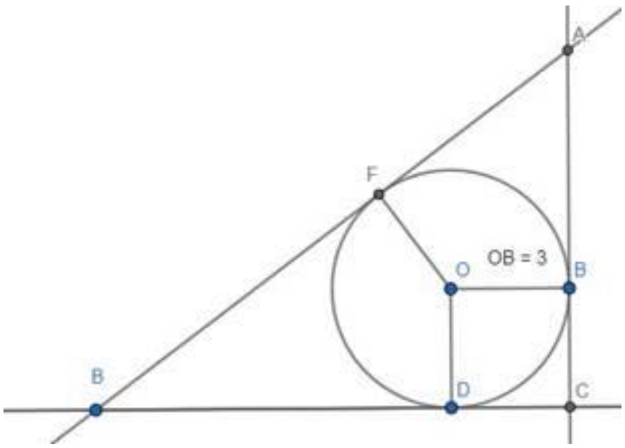
From 1 and 2

Thus FGHI is a rhombus

Q. 4. A triangle ABC is drawn to circumscribe a circle of radius 3 cm. such that the segments BD and DC into which BC is divided by the point of contact D are of length 9 cm. and 3 cm. respectively (See adjacent figure). Find the sides AB and AC.



Answer :



Construction : Draw radius OB = 3cm

Proof: AC, BC and AB are tangents

BF = BD = 9cm—tangents from B

AF = AB—tangent from A

∴ CD = CB tangents from C

OD = OB = 3cm radius

OBCD forms a square of side 3cm

OD = OB = BC = CD = 3cm

∴ ∠BCD = 90°

BD = 9cm and DC = 3cm

OB = 3cm radius of circle

Let AF = AB = x

Applying Pythagoras

AB = BC + AC

$(9 + x)^2 = (12)^2 + (3 + x)^2$

$81 + 18x + x^2 = 144 + 9 + 6x + x^2$

12x = 72

X = 6cm

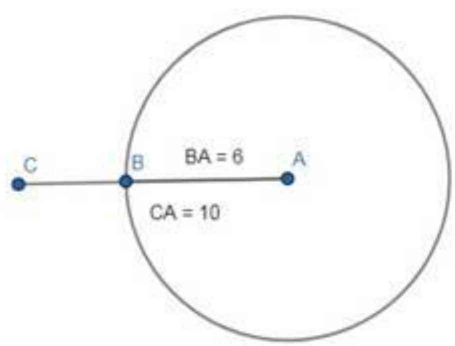
∴ AB = 9 + 6 = 15cm

AC = 6 + 3 = 9cm

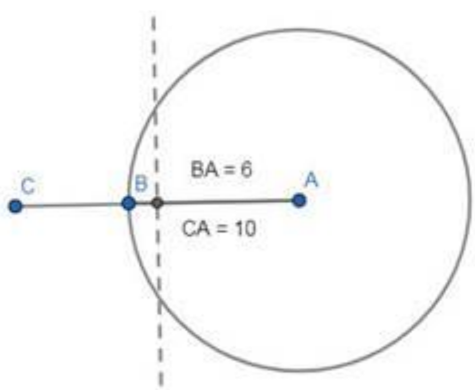
Q. 5. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Verify by using Pythogoras Theorem.

Answer :

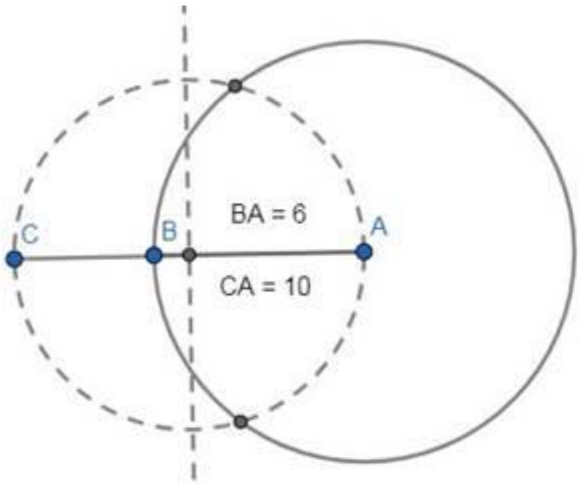
Step1: Draw circle of radius 6cm with center A, mark point C at 10 cm from center



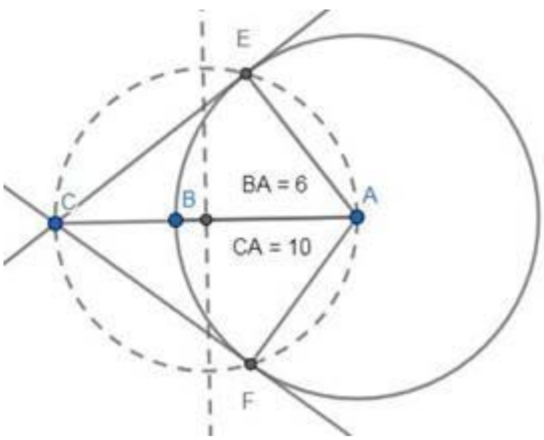
Step 2: find perpendicular bisector of AC



Step3: Take this point as center and draw a circle through A and C



Step4: Mark the point where this circle intersects our circle and draw tangents through C



Length of tangents = 8cm

AE is perpendicular to CE (tangent and radius relation)

In ΔACE

AC becomes hypotenuse

$AC^2 = CE^2 + AE^2$

$10^2 = CE^2 + 6^2$

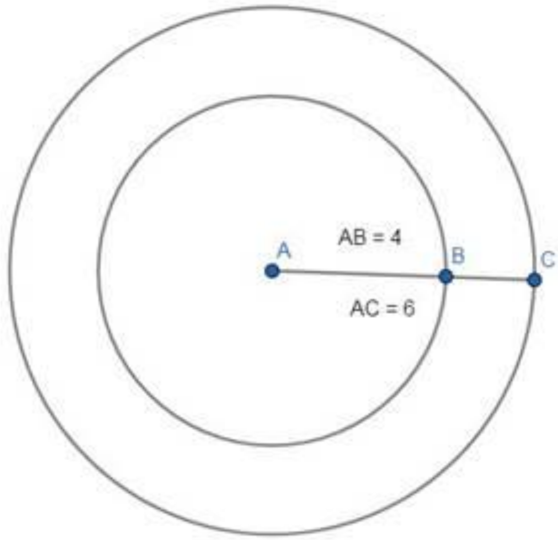
$CE^2 = 100-36$

$CE^2 = 64$

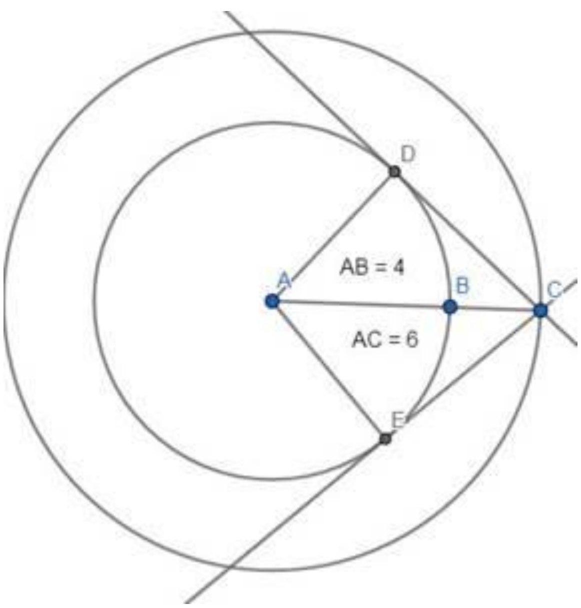
CE = 8cm

Q. 6. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Answer : Step1: Draw circles of radius 4 and 6 cm



Step 3: Draw tangent to inner circle from C



AD is perpendicular to DC- tangent and radius

In Δ ADC

AC is radius

$AC^2 = AD^2 + DC^2$

$6^2 = 4^2 + DC^2$

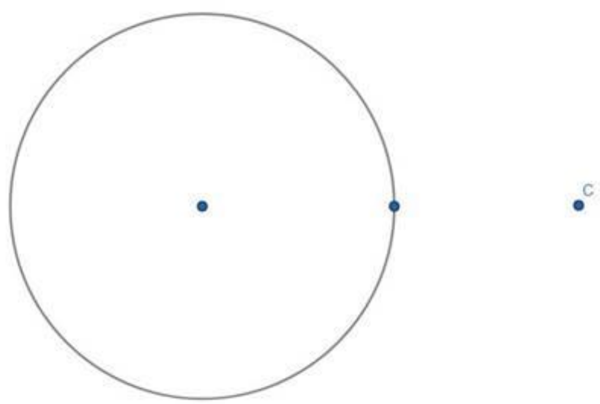
$36 = 16 + DC^2$

$DC^2 = 20$

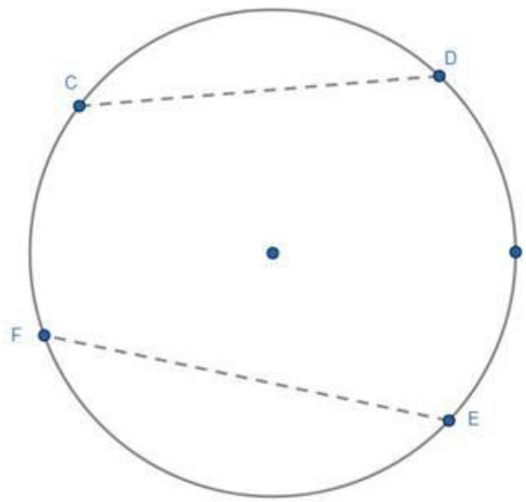
$DC = 2\sqrt{5}$ cm

Q. 7. Draw a circle with the help of a bangle, Take a point outside the circle. Construct the pair of tangents from this point to the circle measure them. Write conclusion.

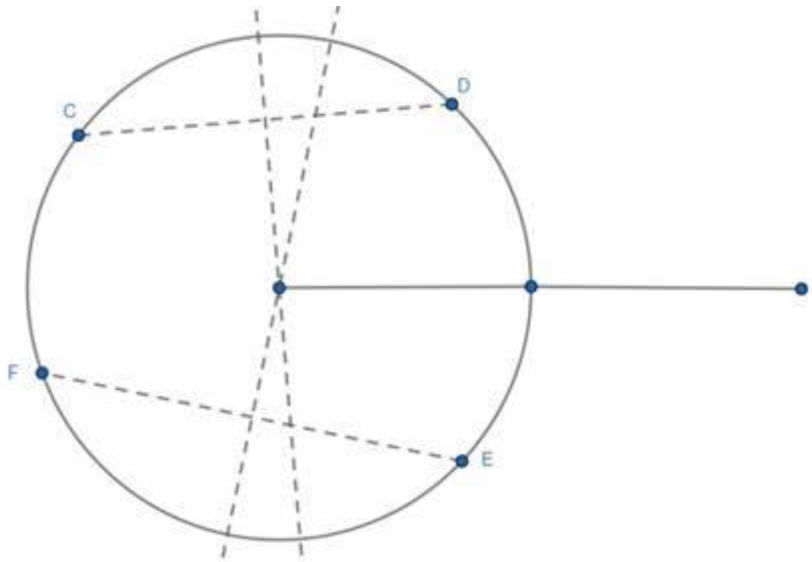
Answer : Step1: Draw a circle with bangle, its center is not known



Step2: to find the center draw 2 random chords C and FE



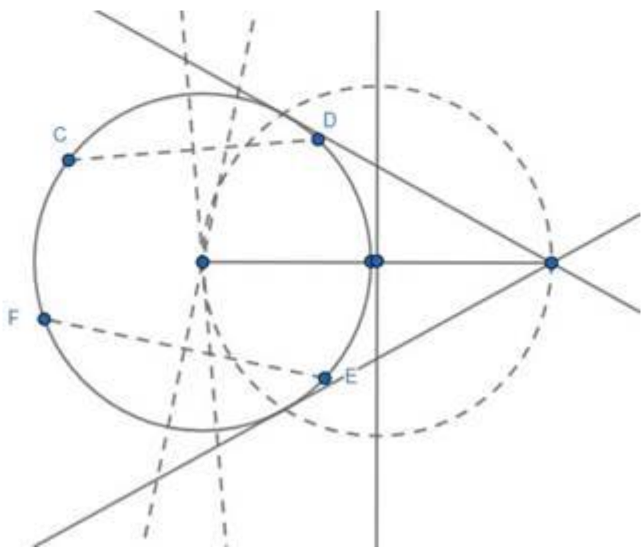
Step 3: Find the perpendicular bisectors of these chords, the points where they intersect is the center of circle



Step 4:

Draw a line from center to a point outside it.

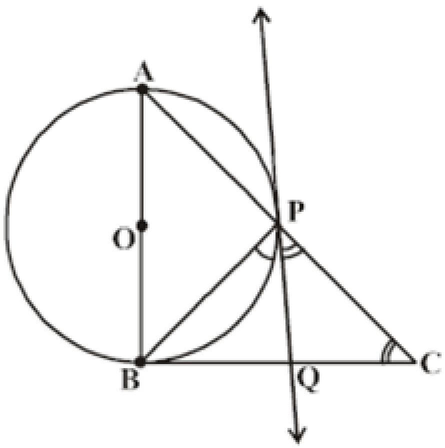
Step 5: draw its perpendicular bisector



Step 6: Cut arcs over the circle with this point as center and point outside it as radius

Step 7: Draw tangent to the circle where the arcs cut the circle

Q. 8. In a right triangle ABC, a circle with a side AB as diameter is drawn to intersect the hypotenuse AC in P. Prove that the tangent to the circle at P bisects the side BC.



Answer : ΔABC is right angled triangle

∠ ABC = 90°

Drawn with AB as diameter that intersects AC at P, PQ is the tangent to the circle which intersects BC at Q.

Join BP.

PQ and BQ are tangents from an external point Q.

∴ PQ = BQ ---1 tangent from external point

⇒ ∠PBQ = ∠BPQ = 45° (isosceles triangle)

Given that, AB is the diameter of the circle.

∴ ∠APB = 90° (Angle subtended by diameter)

∠APB + ∠BPC = 180° (Linear pair)

∴ ∠BPC = 180° – ∠APB = 180° – 90° = 90°

Consider ΔBPC,

∠BPC + ∠PBC + ∠PCB = 180° (Angle sum property of a triangle)

∴ ∠PBC + ∠PCB = 180° – ∠BPC = 180° – 90° = 90° ---2

$\angle BPC = 90°$

$\therefore \angle BPQ + \angle CPQ = 90°$ ...3

From equations 2 and 3, we get

$\angle PBC + \angle PCB = \angle BPQ + \angle CPQ$

$\Rightarrow \angle PCQ = \angle CPQ$ (Since, $\angle BPQ = \angle PBQ$)

Consider ΔPQC,

$\angle PCQ = \angle CPQ$

$\therefore PQ = QC$ ---4

From equations 1 and 4, we get

$BQ = QC$

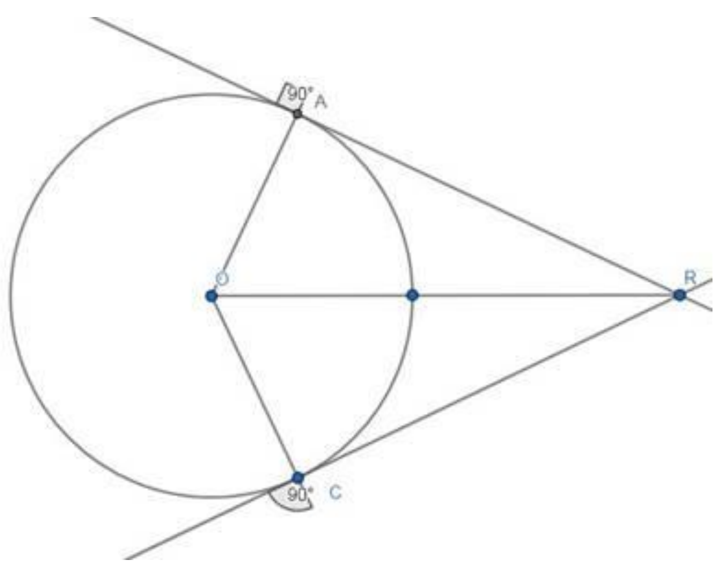
Therefore, tangent at P bisects the side BC.

Q. 9. Draw a tangent to a given circle with center O from a point 'R' outside the circle. How many tangent can be drawn to the circle from that point?

Hint : The distance of two points to the point of contact is the same.

Answer : Let O be center of circle and R be a point outside it.

Draw tangent to circle at A and B through R

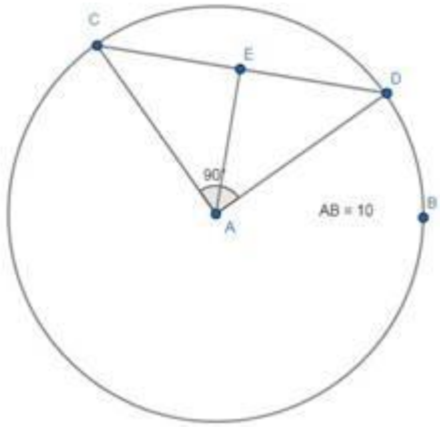


Exercise 9.3

Q. 1. A chord of a circle of radius 10 cm. subtends a right angle at the centre. Find the area of the corresponding :

(use π = 3.14)
i. Minor segment
ii. Major segment

Answer :



i. Let Major segment A_1 minor segment be A_2

$\angle A = 90°$

$$\frac{\pi r^2}{360°} = \frac{area\ of\ sector\ ACD}{90°}$$

$$\frac{\pi \times 10^2}{360°} = \frac{area\ of\ sector\ ACD}{90°}$$

$$\frac{\pi \times 10^2}{360°} \times 90° = area\ of\ sector\ ACD$$

$$area\ of\ sector\ ACD = \frac{\pi \times 10^2}{4}$$

$$area\ of\ sector\ ACD = \frac{3.14 \times 10^2}{4}$$

area of sector ACD = $78.5cm^2$

Pythagoras

$AC^2 = CD^2 + AD^2$

$AC^2 = 10^2 + 10^2$

$AC^2 = 200$

$AC = 10\sqrt{2}$

Height of triangle =

$\text{Cos } 45° = \frac{AE}{AD}$

$$\frac{AE}{AD} = \frac{1}{\sqrt{2}}$$

$AE = \frac{10}{\sqrt{2}}$

Area of minor segment = Area of sector ACD – Area ΔACD

$= 78.5 - \frac{1}{2} \times 10\sqrt{2} \times \frac{10}{\sqrt{2}}$

= 78.5-50

$= 28.5 \text{ cm}^2$

ii. Major segment = Area of circle – minor segment

$= \pi \times r^2$ –minor segment

$= \pi \times 10^2 - 28.5$

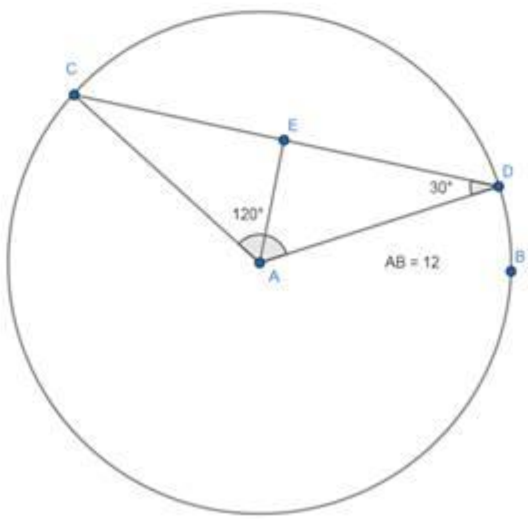
= 314-28.5

$= 285.5 cm^2$

Q. 2. A chord of a circle of radius 12 cm. subtends an angle of 120º at the centre. Find the area of the corresponding minor segment of the circle

(use π = 3.14 and √3 = 1.732)

Answer :



Let Major segment A_1 minor segment be A_2

$$\frac{\pi r^2}{360°} = \frac{area\ of\ sector\ ACD}{120°}$$

$$\frac{\pi \times 12^2}{360°} = \frac{area\ of\ sector\ ACD}{120°}$$

$$\frac{\pi \times 12^2}{360°} \times 120° = area\ of\ sector\ ACD$$

$$area\ of\ sector\ ACD = \frac{\pi \times 12^2}{3}$$

$$area\ of\ sector\ ACD = \frac{3.14 \times 12^2}{3}$$

area of sector ACD = 150.72 sq.cm

$\text{Sin } 30° = \frac{AE}{AD}$

$$\frac{AE}{AD} = \frac{1}{2}$$

AE = 6cm

$\text{Cos } 30° = \frac{DE}{AD}$

$$\frac{DE}{AD} = \frac{\sqrt{3}}{2}$$

DE = 1.732 × 6

DE = 10.392cm

CD = 2 × 10.392 = 20.784cm

Area of minor segment = Area of sector ACD – Area ΔACD

$= 150.72 - \frac{1}{2} \times 20.784 \times 6$

= 1550.72-62.352

= 88.44 cm^2

Q. 3. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm. sweeping through an angle of 115º. Find the total area cleaned at each sweep of the blades.

(use π = 22/7)

Answer : Radius = 25cm

Angle = 115°

$$\frac{\pi r^2}{360°} = \frac{area\ of\ sector\ ACD}{115°}$$

$$\frac{\pi \times 25^2}{360°} = \frac{area\ of\ sector\ ACD}{115°}$$

$$\frac{\pi \times 25^2}{360°} \times 115° = area\ of\ sector\ ACD$$

$$area\ of\ sector\ ACD = \frac{\pi \times 25^2 \times 115}{360}$$

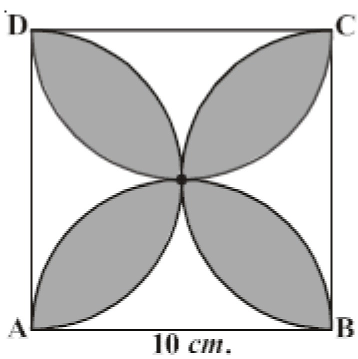
Area of sector ACD = 627.22cm2

For 2 such wipers: 2 × 627.22

Area = 1254.45 cm2

Q. 4. Find the area of the shaded region in figure, where ABCD is a square of side 10 cm. and semicircles are drawn with each side of the square as diameter

(use π = 3.14)



Answer : Consider midpoint side AB, it forms Smaller square of size 5cm

For this smaller square

Area of shaded region

= Area of 1st quadrant + area of 2nd quadrant –area of square

Area of both quadrants is same

∴ Area of shaded region

= 2 × Area of quadrant - area of square

$$= 2 \times \frac{\theta}{360^\circ} \times \pi \times r^2 - r \times r$$

$$= 2 \times \frac{90}{360} \times \pi \times 5 - 5 \times 5$$

$$= 2 \times \frac{1}{4} \times \pi \times 5^2 - 5 \times 5$$

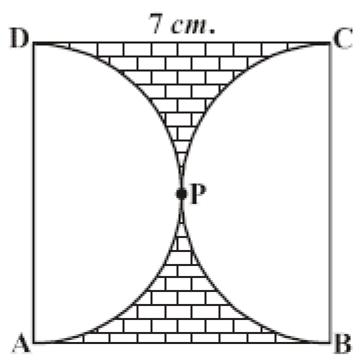
= 14.25 cm^2

Area of total shaded region = 4 × 14.25

= 57 cm^2

Q. 5. Find the area of the shaded region in figure, if ABCD is a square of side 7 cm. and APD and BPC are semicircles.

(use π = 22/7)



Answer : Area of shaded region = Area of square – area of 2 semicircles

Area of square = 7 × 7 = 49 sq.cm

$$\text{Area of semicircles} = 2 \times \frac{\pi \times r^2}{2}$$

= $\pi \times r^2$

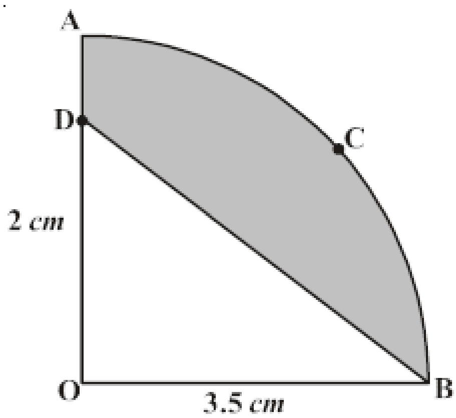
= $\pi \times 3.5^2$

= 38.5 sq.cm

Area of shaded region = 49-38.5 = 10.5 cm^2

Q. 6. In figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm., find the area of the shaded region.

(use π = 22/7)



Answer : Area of Shaded region = Area of sector OABC-Area of ΔDOB

OB = OA = 3.5cm

$$\frac{\pi r^2}{360°} = \frac{area\ of\ sector\ OABC}{90°}$$

$$\frac{\pi \times 3.5^2}{360°} = \frac{area\ of\ sector\ OABC}{90°}$$

$$\frac{\pi \times 3.5^2}{360°} \times 90° = area\ of\ sector\ OABC$$

$$area\ of\ sector\ OABC = \frac{22 \times 3.5^2}{7 \times 4}$$

area of sector OABC = 9.625 cm^2

$$\text{Area of } \Delta DOB = \frac{1}{2} \times 3.5 \times 2$$

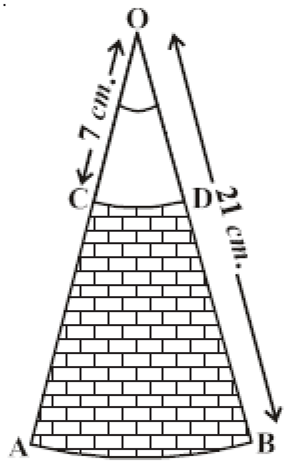
= 3.5 cm^2

Area of Shaded region = 9.625-3.5

= 6.125 cm^2

Q. 7. AB and CD are respectively arcs of two concentric circles of radii 21 cm. and 7 cm. with centre O (See figure). If ∠AOB = 30°, find the area of the shaded region.

(use π = 22/7)



Answer : Area of shaded region = Area(Sector OAB)-Area(Sector OCD)

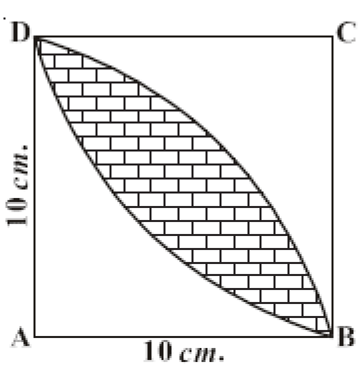
$$= \frac{\pi \times 21^2}{360°} \times 30 - \frac{\pi \times 7^2}{360°} \times 30$$

$$= \frac{22 \times 21^2}{7 \times 12} - \frac{22 \times 7^2}{7 \times 12}$$

= 102.67 cm^2

Q. 8. Calculate the area of the designed region in figure, common between the two quadrants of the circles of radius 10 cm. each.

(use π = 3.14)



Answer : Area of shaded region

= Area of 1^{st} quadrant + area of 2^{nd} quadrant –area of square

Area of both quadrants is same

∴ Area of shaded region

= 2 × Area of quadrant - area of square

$$= 2 \times \frac{\theta}{360^{\circ}} \times \pi \times r^2 - r \times r$$

$$= 2 \times \frac{90}{360} \times \pi \times 10^2 - 10 \times 10$$

$$= 2 \times \frac{1}{4} \times \pi \times 10^2 - 10 \times 10$$

= 57.07 cm^2