Assignment

Expansion of Determinants

Basic Level

1. If a, b and c are non -zero real numbers, then $\Delta=\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to **[AMU 1992; Karnataka CET 2000, 2003]**

(a) abc (b) $a^2b^2c^2$ (c) $ab+bc+ca$ (d) None of these

2. If ω is a cube root of unity and $\Delta=\begin{vmatrix} 1 & 2\omega \\ \omega & \omega^2 \end{vmatrix}$, then Δ^2 is equal to **[Rajas**

(a) $-\omega$ (b) ω (c) 1 (d) ω^2

3. The determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$ is not equal to **[MP PET 1988]**

(a) $\begin{vmatrix} 2 & 1 & 1 \\ 2 & 2 & 3 \\ 2 & 3 & 6 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 4 & 3 & 6 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 2 & 1 \\ 1 & 5 & 3 \\ 1 & 9 & 6 \end{vmatrix}$ (d) $\begin{vmatrix} 3 & 1 & 1 \\ 6 & 2 & 3 \\ 10 & 3 & 6 \end{vmatrix}$

4. If ω is the cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}=$

[MP PET 1990, 2002; Karnataka CET 1992, 93, 2002; Rajasthan PET 1985, 93, 94]

(a) 1 (b) 0 (c) ω (d) ω^2

5. If $a+b+c=0$, then the solution of the equation $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$ is **[UPSEAT 2001]**

(a) 0 (b) $\pm\frac{3}{2}(a^2+b^2+c^2)$ (c) 0, $\pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$ (d) 0, $\pm\sqrt{a^2+b^2+c^2}$

6. $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}=$

(a) $\begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$ (b) $\begin{vmatrix} y & q & b \\ x & p & a \\ z & r & c \end{vmatrix}$ (c) $\begin{vmatrix} b & y & q \\ a & p & x \\ c & z & r \end{vmatrix}$ (d) $\begin{vmatrix} y & b & q \\ z & c & r \\ x & a & p \end{vmatrix}$

7. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} =$ **[Rajasthan PET 1990, 99]**

(a) abc (b) $1/abc$ (c) $ab+bc+ca$ (d) 0

8. $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} =$ **[IIT 1980]**

(a) abc (b) $4abc$ (c) $4a^2b^2c^2$ (d) $a^2b^2c^2$

9. If $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0$, then the value of k is **[IIT 1979]**

(a) –1 (b) 0 (c) 1 (d) None of these

10. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b+c-a & c+a-b & a+b-c \end{vmatrix}$ is **[Rajas**

(a) abc (b) $a+b+c$ (c) $ab+bc+ca$ (d) None of these

11. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} =$ **[Rajasthan PET 1996]**

(a) 1 (b) 0 (c) x (d) xy

12. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} =$ **[Rajasthan PET 1990, 95]**

(a) $(a+b+c)^2$ (b) $(a+b+c)^3$ (c) $(a+b+c)(ab+bc+ca)$ (d) None of these

13. The value of the determinant $\begin{vmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix}$ is **[Rajasthan PET 1992, 96]**

(a) –75 (b) 25 (c) 0 (d) –25

14. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is **[MP PET 1993; Karnataka CET 1994; Rajasthan PET 1985]**

(a) $(a+b+c)$ (b) $(a+b+c)^2$ (c) 0 (d) $1+a+b+c$

15. $\begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix} =$ **[MP PET 1995]**

(a) 1 (b) 0 (c) $\log_a b$ (d) $\log_b a$

16. $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ is equal to **[Rajasthan PET 1995]**

(a) $3abc-a^3-b^3-c^3$ (b) $(a+b)(b+c)(c+a)$ (c) $(a-b)(b-c)(c-a)$ (d) $(a-b)(b-c)(a-c)$

17. $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} =$ **[MNR 1987]**

(a) $a(x+y+z)+b(p+q+r)+c$ (b) 0

(c) $abc + xyz + pqr$ (d) None of these

18. $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix} =$ **[IIT 1988; MP PET 1990, 91; Rajasthan PET 2002]**

(a) 0 (b) $a^3 + b^3 + c^3 - 3abc$ (c) 3 abc (d) $(a+b+c)^3$

19. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} =$ **[MP PET 1992]**

(a) $-2abc$ (b) abc (c) 0 (d) $a^2 + b^2 + c^2$

20. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} =$ **[AMU 1979; Rajasthan PET 1990; DCE 1999]**

(a) $a^3 + b^3 + c^3 - 3abc$ (b) $a^3 + b^3 + c^3 + 3abc$ (c) $(a+b+c)(a-b)(b-c)(c-a)$ (d) None of these

21. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$ **[MP PET 1991]**

(a) $3abc + a^3 + b^3 + c^3$ (b) $3abc - a^3 - b^3 - c^3$ (c) $abc - a^3 + b^3 + c^3$ (d) $abc + a^3 - b^3 - c^3$

22. If ω is a cube root of unity, then $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} =$ **[MNR 1990; MP PET 1999]**

(a) $x^3 + 1$ (b) $x^3 + \omega$ (c) $x^3 + \omega^2$ (d) x^3

23. $\begin{vmatrix} a-1 & a & bc \\ b-1 & b & ca \\ c-1 & c & ab \end{vmatrix} =$ **[Rajasthan PET 1988]**

(a) 0 (b) $(a-b)(b-c)(c-a)$ (c) $a^3 + b^3 + c^3 - 3abc$ (d) None of these

24. The value of the determinant $\begin{vmatrix} 31 & 37 & 92 \\ 31 & 58 & 71 \\ 31 & 105 & 24 \end{vmatrix}$ is **[MP PET 1992]**

(a) –2 (b) 0 (c) 81 (d) None of these

25. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix}$ is **[MNR 1991]**

(a) 20 (b) 10 (c) 0 (d) 250

26. The value of the determinant $\begin{vmatrix} 7 & 9 & 79 \\ 4 & 1 & 41 \\ 5 & 5 & 55 \end{vmatrix}$ is **[MP PET 1992]**

(a) –7 (b) 0 (c) 15 (d) 27

27. $\begin{vmatrix} 19 & 17 & 15 \\ 9 & 8 & 7 \\ 1 & 1 & 1 \end{vmatrix} =$ **[MP PET 1990]**

(a) 0 (b) 187 (c) 354 (d) 54

28. $\begin{vmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{vmatrix} =$ **[MP PET 1991]**

(a) $1+a^2+b^2+c^2$ (b) $1-a^2+b^2+c^2$ (c) $1+a^2+b^2-c^2$ (d) $1+a^2-b^2+c^2$

29. The value of the determinant $\begin{vmatrix} 2 & 8 & 4 \\ -5 & 6 & -10 \\ 1 & 7 & 2 \end{vmatrix}$ is **[MP PET 1994]**

(a) - 440 (b) 0 (c) 328 (d) 488

30. $\begin{vmatrix} 19 & 6 & 7 \\ 21 & 3 & 15 \\ 28 & 11 & 6 \end{vmatrix}$ is equal to **[Rajasthan PET 1995]**

(a) 150 (b) –110 (c) 0 (d) None of these

31. $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} =$ **[MP PET 1990]**

(a) $a^3+b^3+c^3-3abc$ (b) $3abc-a^3-b^3-c^3$

(c) $a^3+b^3+c^3-a^2b-b^2c-c^2a$ (d) $(a+b+c)(a^2+b^2+c^2+ab+bc+ca)$

32. For non-zero *a, b, c* if $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} =$ **[Kerala (Engg.) 2002]**

(a) abc (b) $\frac{1}{abc}$ (c) $-(a+b+c)$ (d) None of these

33. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ e & \pi & \sqrt{2} \\ 2 & 2 & 2 \end{vmatrix}$ is equal to **[AMU 1982]**

(a) 0 (b) e (c) π (d) $2(e-\pi+\sqrt{2})$

34. If $D = \begin{vmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{vmatrix}$, then $\begin{vmatrix} a & b & c \\ b & c & a \\ 1 & 1 & 1 \end{vmatrix}$ equals **[AMU 1988, 90, 92]**

(a) *O* (b) *D* (c) –*D* (d) None of these

35. The value of the determinant $\begin{vmatrix} a_1 & la_1+mb_1 & b_1 \\ a_2 & la_2+mb_2 & b_2 \\ a_3 & la_3+mb_3 & b_3 \end{vmatrix}$ is **[AMU 1987]**

(a) 0 (b) *l* (c) *m* (d) *lm*

36. If ω is a complex cube root of unity, then the value of the determinant $\begin{vmatrix} 1 & \omega & \omega+1 \\ \omega+1 & 1 & \omega \\ \omega & \omega+1 & 1 \end{vmatrix}$ is **[AMU 1989]**

(a) 0 (b) ω (c) 2 (d) 4

37. If $\Delta = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ 0 & 4 & 2 \end{vmatrix}$, the value of $\begin{vmatrix} 4 & 12 & 4 \\ 8 & -4 & 4 \\ 0 & 16 & 8 \end{vmatrix}$ is **[T.S. Rajendra 1990]**

(a) 12Δ (b) $64\,\Delta$ (c) $4\,\Delta$ (d) $4^2\,\Delta$

38. If T_p, T_q, T_r are pth, qth and rth terms of an *A.P.*, then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ is equal to

(a) 1 (b) –1 (c) 0 (d) $p+q+r$

39. $\Delta = \begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bd \\ 1 & 1+ae & 1+be \end{vmatrix} =$ **[MP PET 1996]**

(a) 1 (b) 0 (c) 3 (d) $a+b+c$

40. The value of the determinant $\begin{vmatrix} 1 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 1 & 4 \end{vmatrix}$ is **[Rajasthan PET 1985]**

(a) 2 (b) –2 (c) 0 (d) 5

41. If $A = \begin{vmatrix} 0 & 3 & 4 \\ 5 & 7 & 8 \\ 0 & 6 & 8 \end{vmatrix}$, then the value of A is **[Rajasthan PET 1984]**

(a) 0 (b) 1 (c) 2 (d) 3

42. The value of $\begin{vmatrix} 1 & 2 & 4 \\ -3 & 1 & -2 \\ 2 & 2 & 4 \end{vmatrix}$ is **[Rajasthan PET 1984]**

(a) 88 (b) – 8 (c) – 40 (d) 56

43. The value of $\begin{vmatrix} 43 & 1 & 6 \\ 35 & 7 & 4 \\ 17 & 3 & 2 \end{vmatrix}$ is **[MNR 1991, 95; Rajasthan PET 1996]**

(a) 1 (b) – 1 (c) 0 (d) None of these

44. $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix} =$ **[MNR 1991]**

(a) 20 (b) 10 (c) 0 (d) 5

45. The value of $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$ is **[Rajasthan PET 1989]**

(a) –100 (b) 0 (c) 100 (d) 1000

46. The value of $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ is **[Rajasthan PET 1986]**

(a) 4 (b) 6 (c) 8 (d) 10

47. The value of the determinant $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$ is **[Rajasthan PET 1988]**

(a) $abc(a-b)(b-c)(c-a)$ (b) $(a-b)(b-c)(c-a)(a+b+c)$

(c) $(a-b)(b-c)(c-a)(ab+bc+ca)$ (d) None of these

48. The value of the determinant $\begin{vmatrix} a_1 & ma_1 & b_1 \\ a_2 & ma_2 & b_2 \\ a_3 & ma_3 & b_3 \end{vmatrix}$ is **[Rajasthan PET 1989]**

(a) 0 (b) $ma_1a_2a_3$ (c) $ma_1b_2a_2$ (d) $mb_1b_2b_3$

49. $\begin{vmatrix} 1/a & a & bc \\ 1/b & b & ca \\ 1/c & c & ab \end{vmatrix}$ equals to **[Rajasthan PET 1989]**

(a) $(a-b)(b-c)(c-a)$ (b) $a^3+b^3-c^3+3abc$ (c) 0 (d) None of these

50. $\begin{vmatrix} 2ac-b^2 & a^2 & c^2 \\ a^2 & 2ab-c^2 & b^2 \\ c^2 & b^2 & 2bc-a^2 \end{vmatrix}$ equals **[Rajasthan PET 1998]**

(a) $4abc$ (b) $-4abc$ (c) 0 (d) $(a^3+b^3+c^3-3abc)^2$

51. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 13 & 14 & 15 \end{vmatrix}$ is **[Rajasthan PET 1991]**

(a) 0 (b) 10 (c) 46 (d) 50

52. The value of the determinant $\begin{vmatrix} x & -y & z \\ -x & y & z \\ -x & -y & z \end{vmatrix}$ is **[Rajasthan PET 1991]**

(a) 0 (b) xyz (c) 2 xyz (d) 4 xyz

53. If a, b, c are all different and $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix}$ = 0, then correct statement is **[Rajasthan PET 1991]**

(a) $a+b+c=0$ (b) $ab+bc+ca=0$ (c) $a^2+b^2+c^2=bc+ca+ab$ (d) None of these

54. If $a \neq b \neq c$ and $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = 0$, then the value of $(a+b+c)$ is **[Rajasthan PET 1990]**

(a) 1 (b) 0 (c) 2 (d) $-a$

55. The value of $\begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix}$ is **[Rajasthan PET 1992; MP 1996]**

(a) 652 (b) 576 (c) 0 (d) None of these

56. If ω is a cube root of unity, then one root of the equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ is **[MNR 1990; DCE 1998]**

(a) 1 (b) ω (c) ω^2 (d) 0

57. If $\begin{vmatrix} a & -b & -c \\ -a & b & -c \\ -a & -b & c \end{vmatrix} + \lambda abc = 0$ then λ equals **[Rajasthan PET 1998]**

(a) –4 (b) 4 (c) 2 (d) None of these

58. If $a \neq b \neq c$ and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$ then **[EAMCET 1989]**

(a) $a+b+c=0$ (b) $abc = 1$ (c) $a+b+c=1$ (d) $ab+bc+ca=0$

59. $\begin{vmatrix} \sin\theta & 1 & 0 \\ 0 & \cos\phi & -\cos\theta \\ \sin\phi & 0 & 1 \end{vmatrix}$ is equal to **[Rajasthan PET 1996]**

(a) $\cos(\theta+\phi)$ (b) $\sin(\theta+\phi)$ (c) $\cos(\theta-\phi)$ (d) $\sin(\theta-\phi)$

60. $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ = (where *x, y, z* being positive) **[IIT 1993; UPSEAT 2002]**

(a) $\log_y x$ (b) $\log_z y$ (c) $\log_x z$ (d) 0

61. If $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = kxyz$, then k = **[Roorkee 1980; Rajasthan PET 1999]**

(a) 1 (b) 2 (c) 3 (d) 4

62. Let $\Delta = \begin{vmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{vmatrix}$, then Δ lies in the interval

(a) [2, 3] (b) [3, 4] (c) [1, 4] (d) (2, 4)

63. If $a \ne p, b \ne q, c \ne r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is

(a) 0 (b) 1 (c) –1 (d) 2

64. The value of the determinant $\Delta = \begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$ is **[AMU 1993]**

(a) 0 (b) $\log(xyz)$ (c) $\log(6xyz)$ (d) $6\log(xyz)$

65. If *a, b, c* are negative distinct real numbers, then the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) < 0 (b) ≤ 0 (c) > 0 (d) ≥ 0

66. If *A, B, C* are the angles of a triangle, then the value of $\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is **[Karnataka CET 2002]**

(a) $\cos A \cos B \cos C$ (b) $\sin A \sin B \sin C$ (c) 0 (d) None of these

67. $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} =$ **[Karnataka CET 1994]**

(a) $(x+p)(x+q)(x-p-q)$ (b) $(x-p)(x-q)(x+p+q)$ (c) $(x-p)(x-q)(x-p-q)$ (d) $(x+p)(x+q)(x+p+q)$

68. $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix} =$ **[Karnataka CET 1991]**

(a) 1 (b) 0 (c) –1 (d) 67

69. $\begin{vmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{vmatrix} =$ **[Karnataka CET 1991]**

(a) 4 (b) $x+y+z$ (c) xyz (d) 0

70. The value of the determinant $\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ is equal to **[Roorkee 1992]**

(a) –4 (b) 0 (c) 1 (d) 4

71. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$ **[EAMCET 1994]**

(a) 0 (b) $12\cos^2 x - 10\sin^2 x$ (c) $12\sin^2 x - 10\cos^2 x - 2$ (d) $10\sin 2x$

72. If *a, b, c* are in *A.P.*, then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is **[Rajasthan PET 1999]**

(a) $x-(a+b+c)$ (b) $9x^2+a+b+c$ (c) $a+b+c$ (d) 0

73. The value of the determinant given below $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix}$ is **[UPSEAT 2000]**

(a) 20 (b) 10 (c) 0 (d) 5

74. $\begin{vmatrix} 1 & 1 & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is not depend **[Rajasthan PET 2000]**

(a) On *x* (b) On *n* (c) Both on *x* and *n* (d) None of these

75. The value of $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$ is **[Kerala (Engg.) 2001]**

(a) 8 (b) –8 (c) 400 (d) 1

76. The value of $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ is equal to **[Kerala (Engg.) 2001]**

(a) $9a^2(a+b)$ (b) $9b^2(a+b)$ (c) $a^2(a+b)$ (d) $b^2(a+b)$

77. The value of $\begin{vmatrix} 41 & 42 & 43 \\ 44 & 45 & 46 \\ 47 & 48 & 49 \end{vmatrix} =$ **[Karnataka CET 2001]**

(a) 2 (b) 4 (c) 0 (d) 1

78. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is **[IIT Screening 2002]**

(a) 3ω (b) $3\omega(\omega-1)$ (c) $3\omega^2$ (d) $3\omega(1-\omega)$

79. If $C = 2\cos\theta$, then the value of the determinant $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix}$ is **[Orissa JEE 2002]**

(a) $\frac{\sin 4\theta}{\sin\theta}$ (b) $\frac{2\sin^2 2\theta}{\sin\theta}$ (c) $4\cos^2\theta(2\cos\theta-1)$ (d) None of these

80. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} =$ **[Rajasthan PET 2002]**

(a) $3\sqrt{3}i$ (b) $-3\sqrt{3}i$ (c) $i\sqrt{3}$ (d) 3

81. $\begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix} =$ **[Rajasthan PET 2002]**

(a) 0 (b) abc (c) $1/abc$ (d) None of these

82. $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix} =$ **[UPSEAT 2002]**

(a) 0 (b) $2abc$ (c) $a^2b^2c^2$ (d) None of these

83. The determinant $\begin{vmatrix} a & b & a-b \\ b & c & b-c \\ 2 & 1 & 0 \end{vmatrix}$ is equal to zero if a, b, c are in **[UPSEAT 2002]**

(a) G.P. (b) A.P. (c) H.P. (d) None of these

84. If $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = k(a+b+c)(a^2+b^2+c^2-bc-ca-ab)$, then k = **[Rajasthan PET 2003]**

(a) 1 (b) 2 (c) –1 (d) –2

85. The value of the determinant $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ is **[Orissa JEE 2003]**

(a) 2(10! 11!) (b) 2(10! 13!) (c) 2(10! 11! 12!) (d) 2(11! 12! 13!)

86. The value of $\begin{vmatrix} a^2 & -ab & -ac \\ -ab & b^2 & -bc \\ ca & bc & -c^2 \end{vmatrix}$ is **[Tamilnadu (Engg.) 2002]**

(a) $4a^2b^2$ (b) $4b^2c^2$ (c) $4c^2a^2$ (d) $4a^2b^2c^2$

87. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$ is equal to

(a) $x^2(x+3)$ (b) $3x^3$ (c) 0 (d) x^3

88. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$ (c) $x = 0, y = 3$ (d) $x = 0, y = 0$

89. The determinant $\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix} = 0$, then

(a) $d = 0$ (b) $a + d = 0$ (c) $d = 0$ or $a + d = 0$ (d) None of these

90. The value of the determinant $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$ is

(a) 0 (b) $2\sin\theta$ (c) $\sin 2\theta$ (d) None of these

Advance Level

91. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax-12$, then the value of A is **[IIT 1982]**

(a) 12 (b) 24 (c) –12 (d) –24

92. $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix} =$ **[MNR 1985; IIT 1986; MP PET 1998]**

(a) $a^2+b^2+c^2-3abc$ (b) $3ab$ (c) $3a+5b$ (d) 0

93. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} =$ **[Roorkee 1980; Rajasthan PET 1997, 99, Karnataka CET 1999, MP PET 2001]**

(a) abc (b) $2abc$ (c) $3abc$ (d) $4abc$

94. If *a, b, c* are unequal what is the condition that the value of the following determinant is zero $\Delta = \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix}$ **[IIT 1985; DCE 1999]**

(a) $1+abc=0$ (b) $(a-b)(b-c)(c-a)=0$ (c) $a+b+c+1=0$ (d) None of these

95. If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$, then $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is equal to **[Karnataka CET 1991; Rajasthan PET 2000]**

(a) 0 (b) abc (c) $-abc$ (d) None of these

96. The value of the determinant $\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$ is equal to **[AMU 1994]**

(a) 1 (b) 0 (c) 2 (d) 3

97. If $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \lambda a^2b^2c^2$ then the value of λ is **[MP PET 1999; Kurukshetra CEE 1990, 2002]**

(a) 1 (b) 2 (c) 4 (d) 3

98. The parameter, on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is

[IIT 1997]

(a) a (b) p (c) d (d) x

99. The value of the determinant $\Delta = \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$ is **[Karnataka CET 1991]**

(a) 2! (b) 3! (c) 4! (d) 5!

100. If $0 < \theta < \frac{\pi}{2}$ and $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$ than θ is equal to **[MNR 1992]**

(a) $\frac{\pi}{24}, \frac{5\pi}{24}$ (b) $\frac{5\pi}{24}, \frac{7\pi}{24}$ (c) $\frac{7\pi}{24}, \frac{11\pi}{24}$ (d) None of these

101. The value of $\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma) \\ \cos(\alpha-\beta) & 1 & \cos(\beta-\gamma) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$ is **[Rajasthan PET 2000; Pb. CET 1992]**

(a) $\begin{vmatrix} \cos\alpha & \sin\alpha & 1 \\ \cos\beta & \sin\beta & 1 \\ \cos\gamma & \sin\gamma & 1 \end{vmatrix}^2$ (b) $\begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\gamma & \cos\gamma & 0 \end{vmatrix}^2$ (c) $\begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \sin\beta & 0 & \cos\beta \\ 0 & \cos\gamma & \sin\gamma \end{vmatrix}^2$ (d) None of these

102. If *a, b, c* are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then the value of $abc(ab+bc+ca)$ is **[Kurukshetra CEE 2002]**

(a) $a+b+c$ (b) 0 (c) $a^2+b^2+c^2$ (d) $a^2-b^2+c^2$

103. $\begin{vmatrix} a^2+x^2 & ab & ca \\ ab & b^2+x^2 & bc \\ ca & bc & c^2+x^2 \end{vmatrix}$ is divisor of **[Rajasthan PET 2000]**

(a) a^2 (b) b^2 (c) c^2 (d) x^2

104. If $\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix} = 0$, then *a, b, c* are in **[AMU 2000]**

(a) A.P. (b) G.P. (c) H.P. (d) None of these

105. If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k\,abc(a+b+c)^3$, then the value of k is **[Tamilnadu (Engg.) 2001]**

(a) –1 (b) 1 (c) 2 (d) –2

106. If $a > 0$ and discriminant of $ax^2+2bx+c$ is negative, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is **[AIEEE 2002]**

(a) Positive (b) $(ac-b^2)(ax^2+2bx+c)$ (c) Negative (d) 0

107. The determinant $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} = 0$, if *a, b, c* are in **[UPSEAT 2002]**

(a) *A.P.* (b) *G.P.* (c) *H.P.* (d) None of these

108. The value of the determinant $\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$ is **[UPSEAT 2003]**

(a) $\alpha^2+\beta^2$ (b) $\alpha^2-\beta^2$ (c) 1 (d) 0

109. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix}=0$, then $\sin^2 A+\sin^2 B+\sin^2 C=$ **[Karnataka CET 2003]**

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) 1 (d) $3\sqrt{3}$

110. If $a\neq p, b\neq q, c\neq r$ and $\begin{vmatrix} p & b & c \\ p+a & q+b & 2c \\ a & b & r \end{vmatrix}=0$, then $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=$ **[EAMCET 2003]**

(a) 3 (b) 2 (c) 1 (d) 0

111. If $A=\begin{vmatrix} \sin(\theta+\alpha) & \cos(\theta+\alpha) & 1 \\ \sin(\theta+\beta) & \cos(\theta+\beta) & 1 \\ \sin(\theta+\gamma) & \cos(\theta+\gamma) & 1 \end{vmatrix}$, then **[Orissa JEE 2003]**

(a) $A=0$ for all θ (b) A is an odd function of θ (c) $A=0$ for $\theta=\alpha+\beta+\gamma$ (d)A is in

112. *l, m, n* are the *pth, qth* and *rth* term of a G.P., all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals **[AIEEE 2002]**

(a) –1 (b) 2 (c) 1 (d) 0

113. If *a, b, c* are respectively the p^{th}, q^{th}, r^{th} terms of an A.P., then $\begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix}=$ **[Kerala (Engg.) 2002]**

(a) 1 (b) –1 (c) 0 (d) *pqr*

114. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where *a, b, c* are the p^{th}, q^{th}, r^{th} terms of a H.P. is

(a) $ap+bq+cr$ (b) $(a+b+c)(p+q+r)$ (c) 0 (d) None of these

115. The value of $\begin{vmatrix} a_1x+b_1y & a_2x+b_2y & a_3x+b_3y \\ b_1x+a_1y & b_2x+a_2y & b_3x+a_3y \\ b_1x+a_1 & b_2x+a_2 & b_3x+a_3 \end{vmatrix}$ is equal to

(a) x^2+y^2 (b) 0 (c) $a_1a_2a_3x^2+b_1b_2b_3y^2$ (d) None of these

116. If α, β are non-real numbers satisfying $x^3-1=0$ then the value of $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$ is equal to

(a) 0 (b) λ^3 (c) λ^3+1 (d) None of these

117. The value of the determinant $\begin{vmatrix} {}^5C_0 & {}^5C_3 & 14 \\ {}^5C_1 & {}^5C_4 & 1 \\ {}^5C_2 & {}^5C_5 & 1 \end{vmatrix}$ is

(a) 0 (b) – (6 !) (c) 80 (d) None of these

118. $\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$ has the value

(a) 0 (b) 1 (c) $\sin A\sin B\cos C$ (d) None of these

119. The value of $\begin{vmatrix} x & x^2-yz & 1 \\ y & y^2-zx & 1 \\ z & z^2-xy & 1 \end{vmatrix}$ is

(a) 1 (b) –1 (c) 0 (d) $-xyz$

120. If $\sqrt{-1} = i$ and ω is non real cube root of unity then the value of $\begin{vmatrix} 1 & \omega^2 & 1+i+\omega^2 \\ -i & -1 & -1-i+\omega \\ 1-i & \omega^2-1 & -1 \end{vmatrix}$ is equal to

(a) 1 (b) i (c) ω (d) 0

121. The value of $\begin{vmatrix} i^m & i^{m+1} & i^{m+2} \\ i^{m+5} & i^{m+4} & i^{m+3} \\ i^{m+6} & i^{m+7} & i^{m+8} \end{vmatrix}$, where $i = \sqrt{-1}$, is

(a) 1 if m is a multiple of 4 (b) 0 for all real m (c) $-i$ if m is a multiple of 3 (d) None of these

122. If the determinant $\begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is expanded in powers of $\sin x$ then the constant term in the expansion is

(a) 1 (b) 2 (c) –1 (d) None of these

123. Let $f(n) = \begin{vmatrix} n & n+1 & n+2 \\ {}^nP_n & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^nC_n & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$ where the symbols have their usual meanings. The $f(n)$ is divisible by

(a) n^2+n+1 (b) $(n+1)!$ (c) $n!$ (d) None of these

124. $\begin{vmatrix} {}^xC_1 & {}^xC_2 & {}^xC_3 \\ {}^yC_1 & {}^yC_2 & {}^yC_3 \\ {}^zC_1 & {}^zC_2 & {}^zC_3 \end{vmatrix}$ is equal to

(a) $xyz(x-y)(y-z)(z-x)$ (b) $\frac{xyz}{6}(x-y)(y-z)(z-x)$ (c) $\frac{xyz}{12}(x-y)(y-z)(z-x)$ (d) None of these

125. $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} =$ **[MNR 1985]**

(a) 2 (b) –2 (c) x^2-2 (d) None of these

Solution of Equations in the from of Determinant

Basic Level

126. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are **[IIT 1987; MP PET 2002]**

(a) –1, –2 (b) –1, 2 (c) 1, –2 (d) 1, 2

127. The roots of the equation $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ are **[MP PET 1989; Roorkee 1998]**

(a) 0, –3 (b) 0, 0, –3 (c) 0, 0, 0, –3 (d) None of these

128. If –9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then the other two roots are **[IIT 1983; MNR 1992; MP PET 1995]**

(a) 2, 7 (b) –2, 7 (c) 2, –7 (d) –2, –7

129. If $a+b+c=0$, then one root of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0$ is **[UPSEAT 2001; Karnataka CET 1993]**

(a) $x=1$ (b) $x=2$ (c) $x=a^2+b^2+c^2$ (d) $x=0$

130. If $\begin{vmatrix} 4 & 3 & 3 \\ 3 & x & 3 \\ 3 & 3 & 4 \end{vmatrix}=0$, then x equals **[Rajasthan PET 1988]**

(a) 2 (b) 3 (c) 4 (d) None of these

131. The number of roots of the equation $\begin{vmatrix} 1 & x & x+1 \\ 1 & x & x+2 \\ 1 & 0 & x+3 \end{vmatrix}=0$ is **[AMU 1998]**

(a) 1 (b) 2 (c) 3 (d) 4

132. The roots of the equation $\begin{vmatrix} 1 & 1 & 1 \\ a & x & a \\ b & b & x \end{vmatrix}=0$ are **[Rajasthan PET 1992]**

(a) 1, 1 (b) 1, *a* (c) 1, b (d) *a, b*

133. If $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix}=0$, then the values of x are **[IIT 1991]**

(a) 1, 2 (b) –1, 2 (c) –1, –2 (d) 1, –2

134. If $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}=0$, then $x=$ **[Kurukshetra CEE 1991]**

(a) 8/3 (b) 2/3 (c) 1/3 (d) None of these

135. The factors of $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$ are **[Karnataka CET 1993]**

(a) $x-a, x-b$ and $x+a+b$ (b) $x+a, x+b$ and $x+a+b$ (c) $x+a, x+b$ and $x-a-b$ (d) $x-a,\ x-b$ and $x-a-b$

136. The roots of the equation $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix}=0$ are **[Karnataka CET 1992]**

(a) 1, 2 (b) –1, 2 (c) 1, –2 (d) –1, –2

137. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix}=0$ is **[Roorkee 1991; Rajasthan PET 2001]**

(a) 6 (b) 3 (c) 0 (d) None of these

138. One of the root of the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix}=0$ is **[Tamilnadu Engg. 2002; MP PET 1988, 2002]**

(a) $-(a+b)$ (b) $-(b+c)$ (c) $-a$ (d) $-(a+b+c)$

139. If $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then **[Karnataka CET 1999]**

(a) a is one of the cube root of unity
(b) b is one of the cube root of unity
(c) $\left(\frac{a}{b}\right)$ is one of the cube root of unity
(d) $\left(\frac{a}{b}\right)$ is one of the cube root of –1

140. If $a \neq 6, b, c$ satisfy $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$, then abc = **[EAMCET 2000]**

(a) $a+b+c$
(b) 0
(c) b^3
(d) $ab+bc$

141. At what value of x, will $\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$ **[DCE 2000, 01]**

(a) $x=0$
(b) $x=1$
(c) $x=-1$
(d) None of these

142. $\begin{vmatrix} x+1 & \omega & \omega^2 \\ x+\omega & \omega^2 & 1 \\ x+\omega^2 & 1 & \omega \end{vmatrix} = 3$ is an equation of x, where ω, ω^2 are the complex cube roots of unity, what is the value of x **[DCE 2001]**

(a) 0
(b) 1
(c) –1
(d) 2

143. If 5 is one root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & -2 \\ 7 & 8 & x \end{vmatrix} = 0$, then other two roots of the equation are **[Karnataka CET 2002]**

(a) –2 and 7
(b) –2 and –7
(c) 2 and 7
(d) 2 and –7

144. Solutions of the equation $\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$ are **[AMU 2002]**

(a) $x=1,2$
(b) $x=2,3$
(c) $x=1,p,2$
(d) $x=1,2,-p$

145. If $\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix} = A_0 + A_1x + A_2x^2 + A_3x^3$, then A_1 is equal to **[AMU 2002]**

(a) abc
(b) 0
(c) 1
(d) None of these

146. If $\begin{vmatrix} x+1 & 1 & 1 \\ 2 & x+2 & 2 \\ 3 & 3 & x+3 \end{vmatrix} = 0$, then x is **[Kerala (Engg.) 2002]**

(a) 0, –6
(b) 0, 6
(c) 6
(d) –6

147. The values of x in the following determinant equation $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ **[MP PET 2003]**

(a) $x=0, x=4a$
(b) $x=0, x=a$
(c) $x=0, x=2a$
(d) $x=0, x=3a$

148. If $\begin{vmatrix} x-1 & 3 & 0 \\ 2 & x-3 & 4 \\ 3 & 5 & 6 \end{vmatrix} = 0$, then $x =$ **[Rajasthan PET 2003]**

(a) 0 (b) 2 (c) 3 (d) 1

149. If $i = \sqrt{-1}$ and $\sqrt[4]{1} = \alpha, \beta, \gamma, \delta$ then $\begin{vmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \alpha \\ \gamma & \delta & \alpha & \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ is equal to

(a) i (b) $-i$ (c) 1 (d) 0

150. If $\begin{vmatrix} a+x & a & x \\ a-x & a & x \\ a-x & a & -x \end{vmatrix} = 0$, then x is

(a) 0 (b) a (c) 3 (d) $2a$

151. The sum of two non-integral roots of $\begin{vmatrix} x & 2 & 5 \\ 3 & x & 3 \\ 5 & 4 & x \end{vmatrix} = 0$ is

(a) 5 (b) –5 (c) – 18 (d) None of these

Advance Level

152. The solution set of the equation $\begin{vmatrix} 2 & 3 & m \\ 2 & 1 & m^2 \\ 6 & 7 & 3 \end{vmatrix} = 0$ is **[AMU 1997]**

(a) (1, 2) (b) (1, –2) (c) (1, –3) (d) (0, 1)

153. If a, b, c are in A.P., then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is **[Rajasthan PET 1999]**

(a) $x-(a+b+c)$ (b) $9x^2+a+b+c$ (c) $(a+b+c)$ (d) 0

154. The value of x obtained from the equation $\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+y \end{vmatrix} = 0$ will be **[UPSEAT 1999]**

(a) 0 and $-(\alpha+\beta+\gamma)$ (b) 0 and $(\alpha+\beta+\gamma)$ (c) 1 and $(\alpha-\beta-\gamma)$ (d) 0 and $(\alpha^2+\beta^2+\gamma^2)$

155. If $ab+bc+ca=0$ and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$, then one of the value of x is **[AMU 2000]**

(a) $(a^2+b^2+c^2)^{\frac{1}{2}}$ (b) $\left[\frac{3}{2}(a^2+b^2+c^2)\right]^{\frac{1}{2}}$ (c) $\left[\frac{1}{2}(a^2+b^2+c^2)\right]^{\frac{1}{2}}$ (d) None of these

156. If $\Delta_1 = \begin{vmatrix} 7 & x & 2 \\ -5 & x+1 & 3 \\ 4 & x & 7 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} x & 2 & 7 \\ x+1 & 3 & -5 \\ x & 7 & 4 \end{vmatrix}$ then $\Delta_1 - \Delta_2 = 0$ for

(a) $x=2$ (b) All real x (c) $x=0$ (d) None of these

157. If $\Delta_1 = \begin{vmatrix} 10 & 4 & 3 \\ 17 & 7 & 4 \\ 4 & -5 & 7 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 4 & x+5 & 3 \\ 7 & x+12 & 4 \\ -5 & x-1 & 7 \end{vmatrix}$ such that $\Delta_1 + \Delta_2 = 0$ then

(a) $x = 5$ (b) x has no real value (c) $x = 0$ (d) None of these

158. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5 + bx^4 + cx^3 + dx^2 + \lambda x + \mu$ be an identity in x, where a, b, c, d, λ, μ are independent of x. Then the value of λ is

(a) 3 (b) 2 (c) 4 (d) None of these

159. Using the factor theorem it is found that $b+c$, $c+a$ and $a+b$ are three factors of determinant $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$. The other factor in the value of the determinant is

(a) 4 (b) 2 (c) $a+b+c$ (d) None of these

160. The roots of $\begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & v & 1 \end{vmatrix} = 0$ are independent of

(a) λ, μ, v (b) a, b (c) λ, μ, v, a, b (d) None of these

Properties of Determinants

Basic Level

161. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 5$, then $\begin{vmatrix} 3a & 3b \\ 3c & 3d \end{vmatrix} =$

(a) 15 (b) 45 (c) 405 (d) None of these

162. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$ and $\Delta' = \begin{vmatrix} 7 & 20 & 29 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$, then

(a) $\Delta' = 3\Delta$ (b) $\Delta' = \dfrac{3}{\Delta}$ (c) $\Delta' = \Delta$ (d) $\Delta' = 2\Delta$

163. If $A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$, $B = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$, $C = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$, then which relation is correct

(a) $A = B$ (b) $A = C$ (c) $B = C$ (d) None of these

164. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is equal to **[Rajasthan PET 1986]**

(a) Δ (b) $k\Delta$ (c) $3k\Delta$ (d) $k^3\Delta$

165. If the entries in a 3×3 determinant are either 0 or 1, then the greatest value of this determinant is **[AMU 1988]**

(a) 1 (b) 2 (c) 3 (d) 9

166. If the elements of the first row of the product of $\begin{vmatrix} a & b \\ -b & a \end{vmatrix}$ and $\begin{vmatrix} c & d \\ -d & c \end{vmatrix}$ are respectively A and B then the elements of the second row of the product are **[Rajasthan PET 1987]**

(a) *A, B* (b) *B, A* (c) –*B, A* (d) –*B*, –*A*

167. If for every element of any determinant of odd order $a_{ji} = -a_{ij}$, then the value of determinant is

(a) 0 (b) 1 (c) –1 (d) Δ^2

168. The sum of the products of the elements of any row of a determinant *A* with the corresponding cofactors of the same row is always equal to **[Karnataka CET 2000]**

(a) $|A|$ (b) $|A|/2$ (c) 1 (d) 0

169. If $\Delta = \begin{vmatrix} a & c & b \\ b & b & a \\ c & a & c \end{vmatrix}$ and $\Delta' = \begin{vmatrix} a^2 & c^2 & b^2 \\ b^2 & b^2 & a^2 \\ c^2 & a^2 & c^2 \end{vmatrix}$, then **[Kurukshetra CEE 1997]**

(a) $\Delta = a^2b^2c^2\Delta'$ (b) $\Delta' = a^2b^2c^2\Delta$ (c) $\Delta = abc\Delta'$ (d) None of these

170. If $\Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$, then $\begin{vmatrix} 2x+4p & p+6a & a \\ 2y+4q & q+6b & b \\ 2z+4r & r+6c & c \end{vmatrix}$ is equal to **[Rajasthan PET 1999]**

(a) 2Δ (b) 4Δ (c) 6Δ (d) Δ

171. If every element of a third order determinant of value Δ is multiplied by 5, then the value of new determinant is **[Roorkee 1993]**

(a) Δ (b) 5Δ (c) 25Δ (d) 125Δ

172. If $\Delta = \begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^p & p \\ x^{n+5} & x^{p+6} & x^{2n+5} \end{vmatrix} = 0$, then p is given by

(a) x^n (b) $(n+1)$ (c) Either *A* or *B* (d) Both *A* and *B*

173. If $\Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$, then $\begin{vmatrix} x & 2y & z \\ 2p & 4q & 2r \\ a & 2b & c \end{vmatrix}$ equals **[Rajasthan PET 1999]**

(a) Δ^2 (b) 4Δ (c) 3Δ (d) None of these

174. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$, then $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} =$ **[Tamilnadu (Engg.) 2002]**

(a) $k/6$ (b) $2k$ (c) $3k$ (d) $6k$

175. If $A = \begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$, then B is given by **[Tamilnadu (Engg.) 2002]**

(a) $B = 4A$ (b) $B = -4A$ (c) $B = -A$ (d) $B = 6A$

176. Let $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$, then $\Delta_1 \times \Delta_2$ can be expressed as the sum of how many determinants

[Tamilnadu (Engg.) 2001]

(a) 9 (b) 3 (c) 27 (d) 2

177. If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$ then

(a) $\Delta_1 + \Delta_2 = 0$ (b) $\Delta_1 + 2\Delta_2 = 0$ (c) $\Delta_1 = \Delta_2$ (d) None of these

Advance Level

178. In a third order determinant, each element of the first column consists of sum of two terms, each element of the second column consists of sum of three terms and each element of the third column consists of sum of four terms. Then it can be decomposed into n determinants, where n has the value **[Roorkee 1993]**

(a) 1 (b) 9 (c) 16 (d) 24

179. If $\Delta_1 = \begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$, then

(a) $\Delta_1 = 2\Delta_2$ (b) $\Delta_2 = 2\Delta_1$ (c) $\Delta_1 = \Delta_2$ (d) None of these

180. Consider the following statements with reference to determinants

(I) The value of determinant is unchanged if the rows and columns are interchanged

(II) If any two rows or columns of a determinant are interchanged, the sign of the determinant is changed.

(III) If any two rows or columns are identical, the value of determinant is zero

(a) I and III are correct (b) II and III are correct (c) Only I is correct (d) I, II and III are correct

181. Let a_{ij} denote the element of the i^{th} row and j^{th} column in a 3×3 determinant ($1 \le i \le 3, 1 \le j \le 3$) and let $a_{ij} = -a_{ji}$ for every i and j. Then the determinant has all the principal diagonal elements as **[AMU 1992]**

(a) 1 (b) –1 (c) 0 (d) None of these

182. If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$, then the value of $\begin{vmatrix} b_2c_3 - b_3c_2 & c_2a_3 - c_3a_2 & a_2b_3 - a_3b_2 \\ b_3c_1 - b_1c_3 & c_3a_1 - c_1a_3 & a_3b_1 - a_1b_3 \\ b_1c_2 - b_2c_1 & c_1a_2 - c_2a_1 & a_1b_2 - a_2b_1 \end{vmatrix}$ is

[Tamilnadu (Engg.) 2002]

(a) 5 (b) 25 (c) 125 (d) 0

183. Two non-zero distinct numbers *a, b* are used as elements to make determinants of the third order. The number of determinants whose value is zero for all *a, b* is

(a) 24 (b) 32 (c) $a+b$ (d) None of these

Minors and Cofactors

Basic Level

184. If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_1, B_1, C_1 etc. be the co-factors of a_1, b_1, c_1 etc., then which of the following relations is incorrect

(a) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$ (b) $b_1B_1 + b_2B_2 + c_2C_2 = \Delta$ (c) $a_3A_3 + b_3B_3 + c_3C_3 = \Delta$ (d) $a_1A_2 + b_1B_2 + c_1C_2 = \Delta$

185. If $A_1, B_1, C_1, ...$ are respectively the co-factors of the elements $a_1, b_1, c_1, ...$ of the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then $\begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} =$

(a) $a_1\Delta$ (b) $a_1a_3\Delta$ (c) $(a_1 + b_1)\Delta$ (d) None of these

186. The cofactors of 1, −2, −3 and 4 in $\begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix}$ are

(a) 4, 3, 2, 1 (b) −4, 3, 2, −1 (c) 4, −3, −2, 1 (d) −4, −3, −2, −1

187. The cofactor of 2 in $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 9 \end{vmatrix}$ is

(a) 1 (b) −5 (c) 8 (d) −8

188. If cofactor of $2x$ in the determinant $\begin{vmatrix} x & 1 & -2 \\ 1 & 2x & x-1 \\ x-1 & x & 0 \end{vmatrix}$ is zero, then x equals to

(a) 0 (b) 2 (c) 1 (d) −1

189. The cofactor of element 0 in determinant $\begin{vmatrix} -1 & 0 \\ 2 & 2 \end{vmatrix}$ is

(a) −1 (b) 0 (c) 2 (d) −2

190. The cofactor of element 0 in determinant $\begin{vmatrix} -1 & 2 & 1 \\ -2 & 3 & -3 \\ 4 & 0 & -4 \end{vmatrix}$ is

(a) 2 (b) 5 (c) −5 (d) 9

191. The minors of the element of the first row in the determinant $\begin{vmatrix} 2 & -1 & 4 \\ 4 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix}$ are

(a) 2, 7, 11 (b) 7, 11, 2 (c) 11, 2, 7 (d) 7, 2, 11

Advance Level

192. The value of the determinant Δ of 3^{rd} order is 9 then the value of Δ'^2 where Δ' is a determinant formed by cofactors of the element of Δ is

(a) 9 (b) 81 (c) 729 (d) 6561

Product and Summation of Determinants

Basic Level

193. If $\Delta_1 = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}$, then $\Delta_2\Delta_1$ is equal to **[Rajasthan PET 1984]**

(a) ac (b) bd (c) $(b-a)(d-c)$ (d) None of these

194. $\begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix}$ equals

(a) $\begin{vmatrix} 2 & 0 \\ 1 & 15 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & -3 \\ 0 & 15 \end{vmatrix}$ (c) $\begin{vmatrix} -1 & 16 \\ 2 & 7 \end{vmatrix}$ (d) $\begin{vmatrix} 2 & 7 \\ -1 & 16 \end{vmatrix}$

195. The value of $\begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 3 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$ is

(a) $\begin{vmatrix} 10 & 10 \\ 4 & 12 \end{vmatrix}$ (b) $\begin{vmatrix} 10 & 10 \\ 1 & 3 \end{vmatrix}$

(c) $\begin{vmatrix} 24 & 24 \\ 1 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 24 & 24 \\ 1 & 81 \end{vmatrix}$

196. If $D_r = \begin{vmatrix} 1 & n & n \\ 2r & n^2+n+1 & n^2+n \\ 2r-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{r=1}^{n} D_r = 56$, then n equals

(a) 4 (b) 6 (c) 7 (d) 8

Advance Level

197. $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} =$ **[Tamilnadu (Engg.) 2002]**

(a) 7 (b) 10 (c) 13 (d) 17

198. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$ = square of determinant Δ of the third order then Δ is equal to

(a) $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}$ (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (c) $\begin{vmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$ (d) None of these

199. If $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$, then the value of $\sum_{r=1}^{n} D_r =$

(a) 1 (b) –1 (c) 0 (d) None of these

200. Let $D_r = \begin{vmatrix} a & 2^r & 2^{16}-1 \\ b & 3(4^r) & 2(4^{16}-1) \\ c & 7(8^r) & 4(8^{16}-1) \end{vmatrix}$, then the value of $\sum_{r=1}^{16} D_r$ is

(a) 0 (b) $a+b+c$ (c) $ab+bc+ca$ (d) None of these

201. Let m be a positive integer and $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$, $(0 \le r \le m)$, then the value of $\sum_{r=0}^{m} \Delta_r$ is given by

(a) 0 (b) m^2-1 (c) 2^m (d) $2^m \sin^2(2^m)$

202. If $U_n = \begin{vmatrix} n & 15 & 8 \\ n^2 & 35 & 9 \\ n^3 & 25 & 10 \end{vmatrix}$, then $\sum_{n=1}^{5} U_n =$

(a) 0 (b) 25 (c) 625 (d) None of these

203. If $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$, then $\sum_{a=1}^{n} \Delta_a$ is equal to

(a) 0 (b) 1 (c) $\left(\frac{n(n+1)}{2}\right)\left(\frac{a(a+1)}{2}\right)$ (d) None of these

204. If $\Delta_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} \Delta_r$ is independent of

(a) x (b) y (c) z (d) n

205. If $D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$, then $\sum_{r=1}^{n} D_r$ is equal to

(a) $\frac{1}{6}n(n+1)(2n+1)$ (b) $\frac{1}{4}n^2(n+1)^2$ (c) 0 (d) None of these

Differentiation and Integration of Determinants

Basic Level

206. Let f, g, h and k be differentiable in (a, b). If F is defined as $F(x) = \begin{vmatrix} f(x) & g(x) \\ h(x) & k(x) \end{vmatrix}$, then $F'(x)$ is given by **[SCRA 1999]**

(a) $\begin{vmatrix} f & g \\ h & k \end{vmatrix} + \begin{vmatrix} f & g \\ h' & k' \end{vmatrix}$ (b) $\begin{vmatrix} f' & g' \\ h & k \end{vmatrix} + \begin{vmatrix} f & g \\ h' & k' \end{vmatrix}$ (c) $\begin{vmatrix} f & g' \\ h & k' \end{vmatrix} + \begin{vmatrix} f' & g \\ h & k' \end{vmatrix}$ (d) $\begin{vmatrix} f & g \\ h & k' \end{vmatrix} + \begin{vmatrix} f' & g \\ h & k \end{vmatrix}$

207. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 1 & \cos x & 1 \\ 0 & 1 & \cos x \end{vmatrix}$, then $f'(\pi/3)$ equals

(a) $\frac{11\sqrt{3}}{8}$ (b) $\frac{5\sqrt{3}}{8}$ (c) $\frac{-5\sqrt{3}}{8}$ (d) None of these

208. If $\Delta = \begin{vmatrix} x^2 & \sin x \\ x & e^x \end{vmatrix}$, then $\left(\frac{d\Delta}{dx}\right)_{x=0}$ is equal to

(a) 0 (b) –1 (c) 1 (d) 2

209. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then $\lim_{x\to 0} \frac{f'(x)}{x}$ equals

(a) 0 (b) 1 (c) –2 (d) None of these

210. If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec}^2 x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$, then $\int_0^{\pi/2} f(x)dx =$

(a) $\frac{\pi}{4} + \frac{8}{15}$ (b) $\left(-\frac{\pi}{4} + \frac{8}{15}\right)$ (c) $-\left(\frac{\pi}{4} + \frac{8}{15}\right)$ (d) None of these

Advance Level

211. If $f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$, then maximum value of $f(x)$ is

(a) 0 (b) 2 (c) 4 (d) 6

212. If $\begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix} = f(x) - xf'(x)$, then $f(x)$ is equal to

(a) $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$ (b) $(x+\alpha)(x+\beta)(x+\gamma)(x+\delta)$

(c) $2(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$ (d) None of these

213. If $F(x), G(x)$ and $H(x)$ are three polynomials of degree 2, then $\phi(x)=\begin{vmatrix} F(x) & G(x) & H(x) \\ F'(x) & G'(x) & H'(x) \\ F''(x) & G''(x) & H''(x) \end{vmatrix}$ is polynomial of degree

(a) 2 (b) 3 (c) 4 (d) 0

214. Let $f(x)=\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant, then $\frac{d^3}{dx^3}[f(x)]$ at $x=0$ is **[IIT 1997]**

(a) p (b) $p+p^2$ (c) $p+p^3$ (d) Independent of p

215. If $f(x)=\begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ p & p^2 & p^3 \end{vmatrix}$, then $\frac{d^n}{dx^n}(f(x))$ at $x = 0$ is

(a) 0 (b) p (c) p^3 (d) Independent of p

216. If $f(x)=\begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$, then $f'(0)=$

(a) 4 (b) 2 (c) 3 (d) 0

System of Linear Equations

Basic Level

217. If $x+y-z=0,\ 3x-\alpha y-3z=0,\ x-3y+z=0$ has non-zero solution, then $\alpha=$ **[MP PET 1990]**

(a) –1 (b) 0 (c) 1 (d) –3

218. The number of solutions of the equations $x+4y-z=0, 3x-4y-z=0,\ x-3y+z=0$ is **[MP PET 1992]**

(a) 0 (b) 1 (c) 2 (d) Infinite

219. The following system of equations $3x-2y+z=0,\ \lambda x-14y+15z=0,\ x+2y-3z=0$ has a solution other than $x=y=z=0$ for λ equal to **[MP PET 1990]**

(a) 1 (b) 2 (c) 3 (d) 5

220. The number of solutions of equations $x+y-z=0,\ 3x-y-z=0,\ x-3y+z=0$ is **[MP PET 1992]**

(a) 0 (b) 1 (c) 2 (d) Infinite

221. If the system of following equations $2x+3y+5=0,\ x+ky+5=0,\ kx-12y-14=0$ be consistent, then k =

(a) $-2, \frac{12}{5}$ (b) $-1, \frac{1}{5}$ (c) $-6, \frac{17}{5}$ (d) $6, -\frac{12}{5}$

222. If the equation $x = ay + z,\ y = z + ax,\ z = x + y$ have non-zero solutions, then

(a) $a^2 + 1 = 0$ (b) $a^3 + 1 = 0$ (c) $a + 1 = 0$ (d) $a - 1 = 0$

223. If the system of equations $3x - y + 4z - 3 = 0$, $x + 2y - 3z + 2 = 0$, $6x + 5y + \lambda z + 3 = 0$ has infinite number of solutions, then $\lambda =$

(a) 7 (b) −7 (c) 5 (d) −5

224. The system of equations $4x - 5y - 2z = 2,\ 5x - 4y + 2z = 3$ and $2x + 2y + 8z = 1$ is

(a) Consistent (unique solution) (b) Inconsistent

(c) Consistent (infinite solutions) (d) None of these

225. The system of equations $\lambda x + y + z = 0$, $-x + \lambda y + z = 0$, $-x - y + \lambda z = 0$, will have a non-zero solution if real values of λ are given by **[IIT 1984]**

(a) 0 (b) 1 (c) 3 (d) $\sqrt{3}$

226. The equations $x + 2y + 3z = 1, x - y + 4z = 0$ and $2x + y + 7z = 1$ have **[Karnataka CET 1992]**

(a) Only one solution (b) Only two solutions (c) No solution (d) Infinitely many solutions

227. The number of solutions of $2x + y = 4,\ x - 2y = 2,\ 3x + 5y = 6$ is **[Karnataka CET 1991]**

(a) 0 (b) 1 (c) 2 (d) Infinitely many

228. The existence of unique solution of the system $x + y + z = b,\ 2x + 3y - z = 6,\ 5x - y + az = 10$ depends on **[Kurukshetra CEE 2**

(a) b only (b) a only (c) a and b (d) Neither a nor b

229. The value of k for which the set of equations $3x + ky - 2z = 0$, $x + ky + 3z = 0$ and $2x + 3y - 4z = 0$ has a non-trivial solution is

[Kurukshetra CEE 1996]

(a) 15 (b) 16 (c) 31/2 (d) 33/2

230. $x + y + z = 6, x - y + z = 2$ and $2x + y - z = 1$ then x, y, z are respectively

(a) 3, 2, 1 (b) 1, 2, 3 (c) 2, 1, 3 (d) None of these

231. Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the system has **[Roorkee 1990]**

(a) More than two solutions (b) One trivial and one non-trivial solutions

(c) No solution (d) Only trivial solution (0, 0, 0)

232. If $2x + 3y - 5z = 7$, $x + y + z = 6$, $3x - 4y + 2z = 1$, then $x =$ **[MP PET 1987]**

(a) $\begin{vmatrix} 2 & -5 & 7 \\ 1 & 1 & 6 \\ 3 & 2 & 1 \end{vmatrix} \div \begin{vmatrix} 7 & 3 & -5 \\ 6 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} -7 & 3 & -5 \\ -6 & 1 & 1 \\ -1 & -4 & 2 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & -5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{vmatrix}$

(c) $\begin{vmatrix} 7 & 3 & -5 \\ 6 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & -5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{vmatrix}$ (d) None of these

233. The system of equations $x + y + z = 2$, $3x - y + 2z = 6$ and $3x + y + z = -18$ has **[Kurukshetra CEET 2002]**

(a) A unique solution (b) No solution

(c) An infinite number of solutions (d) Zero solution as the only solution

234. If $2x+3y+4z=9,\ 4x+9y+3z=10,\ 5x+10y+5z=11$ then find the value of x **[UPSEAT 2002]**

(a) $\begin{vmatrix} 9 & 3 & 4 \\ 10 & 9 & 3 \\ 11 & 10 & 5 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & 4 \\ 4 & 9 & 3 \\ 5 & 10 & 5 \end{vmatrix}$ (b) $\begin{vmatrix} 9 & 4 & 3 \\ 10 & 3 & 9 \\ 11 & 5 & 10 \end{vmatrix} \div \begin{vmatrix} 2 & 3 & 4 \\ 4 & 9 & 3 \\ 5 & 10 & 5 \end{vmatrix}$

(c) $\begin{vmatrix} 9 & 4 & 9 \\ 10 & 3 & 3 \\ 11 & 5 & 10 \end{vmatrix} \div \begin{vmatrix} 3 & 2 & 4 \\ 9 & 4 & 3 \\ 10 & 5 & 5 \end{vmatrix}$ (d) None of these

235. Equations $x+y=2,\ 2x+2y=3$ will have **[UPSEAT 1999]**

(a) Only one solution (b) Many finite solutions (c) No solution (d) None of these

236. If the system of equations $x-ky-z=0,\ kx-y-z=0$ and $x+y-z=0$ has a non zero solution, then the possible value of k are

[IIT Screening 2000]

(a) –1, 2 (b) 1, 2 (c) 0, 1 (d) –1, 1

237. The system of equations $x_1-x_2+x_3=2$, $3x_1-x_2+2x_3=-6$ and $3x_1+x_2+x_3=-18$ has **[AMU 2001]**

(a) No solution (b) Exactly one solution (c) Infinite solutions (d) None of these

238. The existence of the unique solution of the system $x+y+z=\lambda,\ \ 5x-y+\mu z=10,\ \ 2x+3y-z=6$ depends on

[Kurukshetra CEE 2002]

(a) μ only (b) λ only (c) λ and μ both (d) Neither λ nor μ

239. The number of solutions of the following equations $x_2-x_3=1$, $-x_1+2x_3=-2$, $x_1-2x_2=3$ is **[MP PET 2000]**

(a) Zero (b) One (c) Two (d) Infinite

240. The number of values of k for which the system of equations $(k+1)x+8y=4k,\ \ kx+(k+3)y=3k-1$ has infinitely many solutions, is **[IIT Screening 2002]**

(a) 0 (b) 1 (c) 2 (d) Infinite

241. For what value of λ, the system of equations $x+y+z=6,\ \ x+2y+3z=10,\ \ x+2y+\lambda z=12$ is inconsistent **[AIEEE 2002]**

(a) $\lambda=1$ (b) $\lambda=2$ (c) $\lambda=-2$ (d) $\lambda=3$

242. The values of the *x, y, z* in order, of the system of equations $3x+y+2z=3,\ \ 2x-3y-z=-3,\ \ x+2y+z=4$ are **[MP PET**

(a) 2, 1, 5 (b) 1, 1, 1 (c) 1, –2, –1 (d) 1, 2, –1

243. The number of solutions of the system of equations $2x+y-z=7,\ \ x-3y+2z=1,\ \ x+4y-3z=5$ is **[EAMCET 2003]**

(a) 3 (b) 2 (c) 1 (d) 0

244. The system of equations $x+y+z=6,\ \ x+2y+3z=10,\ \ x+2y+\lambda z=\mu$ has no solution for **[Orissa JEE 2003]**

(a) $\lambda\neq 3,\ \mu=10$ (b) $\lambda=3,\ \mu\neq 10$ (c) $\lambda\neq 3,\ \mu\neq 10$ (d) None of these

245. The system of equations $ax+4y+z=0,\ bx+3y+z=0,\ \ cx+2y+z=0$ has non-trivial solution if *a, b, c* are in

(a) AP (b) GP (c) HP (d) None of these

246. The system of equations $2x-y+z=0, x-2y+z=0, \lambda x-y+2z=0$ [has infinite number of nontrivial solutions for]

(a) $\lambda=1$ (b) $\lambda=5$ (c) $\lambda=-5$ (d) No real value of λ

247. The system of equations $2x+3y=8,\ \ 7x-5y+3=0,\ \ 4x-6y+\lambda=0$ is solvable if λ is

(a) 6 (b) 8 (c) –8 (d) –6

248. The system of the equations $x+2y+3z=4,\ \ 2x+3y+4z=5,\ \ 3x+4y+5z=6$ has

(a) Infinitely many solutions (b) No solution (c) Unique solution (d)None

249. If the equations $x=ay+z,\ \ y=az+x$ and $z=ax+y$ are the consistent having non-trivial solution, then

(a) $a^3=1$ (b) $a^3+1=0$ (c) $a+1=0$ (d) None of these

Advance Level

250. The value of a for which the system of equations $a^3x+(a+1)^3y+(a+2)^3z=0$, $ax+(a+1)y+(a+2)z=0$ and $x+y+z=0$ has a non-zero solution is **[Pb. CET 2000]**

(a) –1 (b) 0 (c) 1 (d) None of these

251. The values of λ which make the system of linear equations $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ to have the so **[Kurukshetra CEE 1991]**

(a) $\lambda=3$ or 4 (b) $\lambda=0$ or ∞

(c) $\lambda=1$ or 2 (d) $\lambda=K$, where K is an arbitrary constant

252. If the equations $x=cy+bz,\ y=az+cx$ and $z=bx+ay$ have a solution other than $x=0,\ y=0$ and $z=0$, then *a, b* and *c* are connected by the relation **[MP PET 1998]**

(a) $a^3+b^3+c^3+3abc=0$ (b) $a^2+b^2+c^2+2abc=1$ (c) $a^2+b^2+c^2-bc-ca-ab=0$ (d) None of these

253. If $a=\frac{x}{y-z}$, $b=\frac{y}{z-x}$ and $c=\frac{z}{x-y}$ where *x, y, z* are not all zero, then $ab+bc+ca=$

(a) 0 (b) 1 (c) –1 (d) 2

254. Let *a, b, c* be positive real numbers. the following system of equations in *x, y,* and *z*

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has **[IIT 1995]**

(a) No solution (b) Unique solution (c) Infinitely many solutions (d)Finitely many solutions

255. The system of the linear equations $x+y+z=2,\ 2x+y-z=3,\ 3x+2y+kz=4$ has a unique solution if **[EAMCET 1994; DCE 2000]**

(a) $k\neq 0$ (b) $-1<k<1$ (c) $-2<k<2$ (d) $k=0$

256. If $a>b>c$ and the system of equations $ax+by+cz=0$, $bx+cy+az=0$, $cx+ay+bz=0$ has a non-trivial solution then both the roots of the quadratic equation at^2+bt+c are

(a) Real (b) Of opposite sign (c) Positive (d) Complex

257. If $x^ay^b=e^m$, $x^cy^d=e^n$, $\Delta_1=\begin{vmatrix} m & b \\ n & d\end{vmatrix}$, $\Delta_2=\begin{vmatrix} a & m \\ c & n\end{vmatrix}$, $\Delta_3=\begin{vmatrix} a & b \\ c & d\end{vmatrix}$, then the values of *x* and *y* are respectively **[Tamilnadu (Engg.) 2002]**

(a) Δ_1/Δ_3 and Δ_2/Δ_3 (b) Δ_2/Δ_1 and Δ_3/Δ_1 (c) log (Δ_1/Δ_3) and $\log(\Delta_2/\Delta_3)$ (d) e^{Δ_1/Δ_3} and e^{Δ_2/Δ_3}

258. If the equations $a(y+z)=x$, $b(z+x)=y$, $c(x+y)=z$, where $a\neq-1,\ b\neq-1,\ c\neq-1$, admit of nontrivial solutions then $(1+a)^{-1}+(1+b)^{-1}+(1+c)^{-1}$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

259. If the system of equations $ax+by+c=0$, $bx+cy+a=0$ and $cx+ay+b=0$ has a solution then the system of equations

$(b+c)x+(c+a)y+(a+b)z=0$

$(c+a)x+(a+b)y+(b+c)z=0$

$(a+b)x+(b+c)y+(c+a)z=0$, has

(a) Only one solution (b) No solution (c) Infinite number of solutions (d) None of these

Some Special Determinants

Basic Level

260. The value of $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is

(a) $abc + 2fgh - af^2 - bg^2 - ch^2$ (b) $abc + fgh - af^2 - bg^2 - ch^2$

(c) $2abc + fgh - af^2 - bg^2 - ch^2$ (d) None of these

261. The value of $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ is

(a) 0 (b) abc (c) $(a-b)(b-c)(c-a)$ (d) None of these

262. The value of the determinant $\begin{vmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{vmatrix}$ is

(a) fgh (b) $-fgh$ (c) 0 (d) $1/fgh$

263. The value of an even order skew symmetric determinant is

(a) 0 (b) Perfect square (c) ± 1 (d) None of these

264. The value of an odd order skew symmetric determinant is

(a) Perfect square (b) Negative

(c) ± 1 (d) 0

265. In a skew-symmetric matrix, the diagonal elements are all **[MP PET 1987]**

(a) Different from each other (b) Zero (c) One (d)None

Advance Level

266. The value of $\begin{vmatrix} 0 & 2+3i & \frac{3}{2}-5i \\ -2+3i & 0 & 7-4i \\ -\frac{3}{2}-5i & -7-4i & 0 \end{vmatrix}$ is

(a) Purely real (b) Purely imaginary (c) Non real complex (d) None of these

Miscellaneous Problems

Basic Level

267. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$, then the two triangles whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) and (a_1, b_1), (a_2, b_2), (a_3, b_3), are

(a) Congruent (b) Similar (c) Equal in area (d) None of these

Advance Level

268. If *A, B* and *C* are the angles of a triangle and

$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$, then the triangle must be

(a) Equilateral (b) Isosceles (c) Any triangle (d) Right angled.

269. If $f(x) = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g = \begin{vmatrix} x^2-2x+3 & 7x+2 & x+4 \\ 2x+7 & x^2-x+2 & 3x \\ 3 & 2x-1 & x^2-4x+7 \end{vmatrix}$, then $g =$

(a) –200 (b) 100 (c) 112 (d) –108

270. If *a, b, c* are the sides of a ΔABC and *A, B, C* are respectively the angles opposite to them, then $\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos(B-C) \\ c\sin A & \cos(B-C) & 1 \end{vmatrix}$ is equal to

(a) $\sin A - \sin B \sin C$ (b) *abc* (c) 1 (d) 0

271. If $\Delta = \begin{vmatrix} \frac{1}{z} & \frac{1}{z} & -\frac{(x+y)}{z^2} \\ -\frac{(y+z)}{x^2} & \frac{1}{x} & \frac{1}{x} \\ -\frac{y(y+z)}{x^2 z} & \frac{x+2y+z}{xz} & -\frac{y(x+y)}{xz^2} \end{vmatrix}$, then Δ is independent of

(a) *x* (b) *y* (c) *z* (d) $\Delta = 0$

272. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x)$ is equal to

(a) *ax* (b) $ax(2a+3x)$ (c) $ax(2+3x)$ (d) None of these

273. If *f(x)* is a polynomial satisfying $f(x) = \frac{1}{2}\begin{vmatrix} f(x) & f\left(\frac{1}{x}\right) - f(x) \\ 1 & f\left(\frac{1}{x}\right) \end{vmatrix}$ and $f(2) = 17$, then the value of $f(5)$ is

(a) 126 (b) 626 (c) –124 (d) 624

274. If $\Delta = \begin{vmatrix} \tan x & \tan(x+h) & \tan(x+2h) \\ \tan(x+2h) & \tan x & \tan(x+h) \\ \tan(x+h) & \tan(x+2h) & \tan x \end{vmatrix}$, then $\lim_{h\to 0} \frac{\Delta}{h^2}$ is equal to

(a) $\tan x \sec^4 x$ (b) $9\tan x \sec^2 x$ (c) $\tan x \sec^4 x$ (d) $9\tan x \sec x$

275. If $x^2+y^2+z^2 = 1$, then $\Delta = \begin{vmatrix} x^2+(y^2+z^2)\cos\theta & xy(1-\cos\theta) & xz(1-\cos\theta) \\ xy(1-\cos\theta) & y^2+(z^2+x^2)\cos\theta & yz(1-\cos\theta) \\ zx(1-\cos\theta) & yz(1-\cos\theta) & z^2+(x^2+y^2)\cos\theta \end{vmatrix}$ is independent of

(a) *x, y, z* (b) *y* only (c) *z* only (d) θ only

276. If $x+y+z=\pi$, then the value of the determinant $\begin{vmatrix} \sin(x+y+z) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$ is **[Pb. CET 1999]**

(a) 0 (b) $2\sin B\tan A\cos C$ (c) 1 (d) None of these

277. If $[a]$ denotes the greatest integer less than or equal to a and $-1 \le x < 0,\ 0 \le y < 1,\ 1 \le z < 2$, then $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is equal to

(a) $[x]$ (b) $[y]$ (c) $[z]$ (d) None of these

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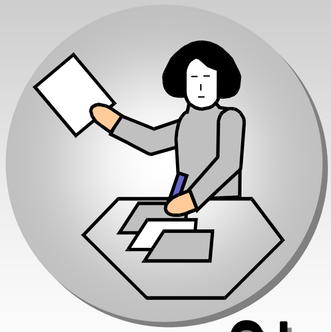

Answer Sheet

Determinants — ***Assignment (Basic and Advance Level)***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	b	a	b	c	a	d	c	d	d	d	b	d	c	b	c	b	a	c	c
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	d	d	b	c	b	a	a	b	c	b	d	a	b	a	d	b	c	b	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
a	b	c	c	b	a	c	a	c	d	a	d	a	b	c	d	b	b	d	d
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
d	d	d	a	c,d	c	b	b	d	d	a	d	c	b	b	b	c	b	d	a
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a	a	a	c	c	d	a	d	c	a	b	d	d	a	b	b	c	b	c	c
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
b	a	d	b	c	c	b	d	a	b	d	d	c	c	b	b	d	a	c	d
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
b	c	c	c	b	b	b	a	d	d	a	d	b	b	a	b	c	d	d	c
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
a	b	d	a	b	a	d	d	d	a	b	c	d	a	a	b	a	a	a	b
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	c	d	d	b	c	a	a	d	a	d	d	b	d	b	c	a	d	b	d
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
c	b	b	d	a	a	c	c	d	c	b	d	b	d	b	c	b	a	c	a
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
a	d	a	a,b,c,d	c	b	b	a	c	d	d	a	d	d	a,d	b	d	b	d	d
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
c	c	d	b	a	d	b	b	d	b	a	c	a	a	c	d	c	a	a	b
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
d	d	d	b	a	c	b	a	d	a	c	b	c	b	a	b	d	a	b	a
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**			
a	c	b	d	b	b	c	b	d	d	a,b,c,d	b	b	b	a	a	c			