Squares and Square Roots

Identification of Perfect Squares

Thus, perfect squares can be defined as follows.

> **“The result of the product of any natural number with itself is a perfect square or a square number”.**

In this way, we can write

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

From the above list of numbers, we can say that the numbers 1, 4, 9, 16 25, 36, 49, 64, 81, and 100 are perfect squares, as these are the squares of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 respectively.

A perfect square can also be defined as follows.

“A number x is said to be a perfect square, if x can be expressed as y^2 where y is a natural number”. Also, y^2 is read as "the square of y" or "y square".

For example, 9^2 is read as "the square of 9" or "9 square".

Let us discuss some examples based on perfect squares to understand the concept better.

Example 1:

Find the perfect square number between 30 and 85.

Solution:

The squares of 5, 6, 7, 8, 9, and 10 are as follows.

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

The perfect squares between 30 and 85 are 36, 49, 64, and 81.

Example 2:

Find the perfect squares between 70 and 90.

Solution:

We know that for three consecutive natural numbers 8, 9, and 10.

$8^2 = 64$, $9^2 = 81$, and $10^2 = 100$

64 and 100 does not lie between 70 and 90. But 81 lies between 70 and 90. Therefore, the only perfect square between 70 and 90 is 81.

Example 3:

Write how the following numbers can be read. Also, find their values.

(a) 5^2 (b) 12^2 (c) 108^2 (d) 111^2

Solution:

(a) 5^2 can be read as "the square of 5" or "5 square".

The value of 5^2 can be obtained as follows:

$5^2 = 5 \times 5 = 25$

(b) 12^2 can be read as "the square of 12" or "12 square".

The value of 12^2 can be obtained as follows:

$12^2 = 12 \times 12 = 144$

(c) 108^2 can be read as "the square of 108" or "108 square".

The value of 108^2 can be obtained as follows:

$108^2 = 108 \times 108 = 11664$

(d) 111^2 can be read as "the square of 111" or "111 square".

The value of 111^2 can be obtained as follows:

$111^2 = 111 \times 111 = 12321$

Properties of Perfect Squares

We know how to find the square of a number. On multiplying the same number two times, we obtain the square of the number. While calculating the squares of numbers, we come across various properties of perfect squares.

Here we have another interseting property about the number of digits in a square of a number.
The number of digits in a square of a number with n digits is either $2n$ −- 1 or $2n$.

Let us discuss some examples to understand the concept better.

Example 1:

Are 20124598 and 900000 perfect squares? Why?

Solution:

20124598 is not a perfect square because a perfect square never ends with 8.

900000 is also not a perfect square because it has odd number of zeroes.

Example 2:

The squares of which of the following numbers are even numbers?

9012, 3375, 1024, 378, 87

Solution:

We know that the squares of even numbers are even.

∴ Squares of 9012, 1024, and 378 are even numbers.

Example 3:

What would be the unit digit of the squares of the following numbers?

8754, 967, 35120

Solution:

The square of 8754 ends with 6.

We know that if a number ends with 4, then its square ends with 6. Therefore, the unit place digit of the square of 8754 is 6.

The square of 967 ends with 9.

We know that if a number ends with 7, then its square ends with 9. Therefore, the unit place digit of the square of 967 is 9.

The square of 35120 ends with 0.

We know that if a number ends with 0, then its square also ends with 0. Therefore, the unit place digit of the square of 35120 is 0.

Patterns In Square Numbers

Sometimes numbers show interesting patterns and this can help us in framing some useful properties. The given video will explain one such pattern related to square numbers.

Let us discuss another pattern between two consecutive square numbers.

In order to understand it, let us calculate the squares of two consecutive numbers, lets say 1 and 2.

We know, $1^2 = 1$ and $2^2 = 4$

Here, 1 and 4 are perfect squares as they are expressed as the squares of 1 and 2 respectively. Between 1 and 4, there are two numbers, 2 and 3, which are non-perfect squares.

Therefore, the number of non-perfect square numbers lying between 1^2 and 2^2 is 2.

$2^2 - 1^2 = 4 - 1 = 3$, which is 1 more than the number of non-square numbers between 1 and 4.

Therefore, we can say that the number of non-perfect square numbers between 1^2 and 2^2 is 1 less than the difference of 1^2 and 2^2.

Let us take another two consecutive numbers, lets say 4 and 5.

We know, $4^2 = 16$ and $5^2 = 25$

Here, 16 and 25 are perfect squares. The numbers between 16 and 25 such as 17, 18, 19, 20, 21, 22, 23, and 24 are non-perfect squares.

Therefore, the number of non-perfect square numbers lying between 4^2 and 5^2 is 8.

$5^2 - 4^2 = 25 - 16 = 9$, which is 1 more than the number of non-square numbers between 16 and 25.

Therefore, we can say that the number of non-perfect square numbers between 4^2 and 5^2 is 1 less than the difference of 4^2 and 5^2.

Let us now take two large numbers as 1000 and 1001. To find the number of non-perfect squares between 1000^2 and 1001^2, first of all, we have to find 1000^2 and 1001^2. Then, we take the difference between 1000^2 and 1001^2. Finally, we will subtract 1 from the difference between 1000^2 and 1001^2.

This is a time consuming process and we may make error during calculation. In order to overcome this situation, we follow another pattern.

We know, $1^2 = 1$ and $2^2 = 4$

Since we know that the number of non-perfect square numbers lying between 1^2 and 2^2 is 2,

we can write $2 = 2 \times 1$

Or 2 × the first number

Similarly, we know that the number of non-perfect square numbers lying between 4^2 and 5^2 is 8.

And, we can write $8 = 2 \times 4$

Or, 2 × the first number

Therefore, we can generalize this pattern as follows.

"There are $2n$ non-perfect square numbers (natural numbers) between the squares of the numbers n and $(n + 1)$."

Using this pattern, we can solve problems related to finding the number of non-square numbers between the squares of two consecutive numbers.

Let us discuss some examples to understand the concept better.

Example 1:

How many natural numbers are there between 258^2 and 259^2?

Solution:

If we take $n = 258$, then we have $n + 1 = 258 + 1 = 259$

We know that there are $2n$ non-perfect square numbers between n^2 and $(n + 1)^2$.

Therefore, number of natural numbers between 258^2 and $259^2 = 2n$

$= 2 \times 258$

$= 516$

Example 2:

How many non-perfect squares are there between 445^2 and 446^2?

Solution:

If we take $n = 445$, then we have $n + 1 = 445 + 1 = 446$

Here, the consecutive numbers are 445 and 446. We know that there are $2n$ non-perfect square numbers between $n^2$and $(n + 1)^2$.

∴ Number of non-perfect squares between 445^2 and $446^2 = 2n$

$= 2 \times 445$

$= 890$

Patterns Of Numbers To Determine The Square Of A Number

We know how to find the square of a number. We multiply that number two times.

The square of some numbers can be found through various patterns of numbers. Now, we will discuss some patterns to find the squares of numbers.

Let us start with the sum of first one odd number.

The first odd number is 1. Therefore, the sum of first one odd number is 1.

We can write 1 as 1^2

∴ $1 = 1^2$

Let us take the sum of first two odd numbers i.e., 1 and 3.

Sum of these numbers $= 1 + 3 = 4$

And, we can write 4 as 2^2.

∴ $1 + 3 = 2^2$

Now, let us take the sum of first three odd numbers. The first three odd numbers are 1, 3, and 5.

Therefore, sum of these numbers $= 1 + 3 + 5 = 9$

And, we can write 9 as 3^2.

$\therefore 1 + 3 + 5 = 3^2$

Continuing in this way, we obtain

$1 + 3 + 5 + 7 = 4^2$

$1 + 3 + 5 + 7 + 9 = 5^2$ and so on

On generalizing this pattern, we obtain

Sum of first *n* odd numbers = n^2

Using this pattern, we can find the square of any number. Let us find the square of 10 using this method. Square of 10 is equal to the sum of first 10 odd numbers.

$\therefore 10^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$

Here are few more properties of a square number:

- $n^2 = 2(n-1)^2 - (n-2)^2 + 2$
- $n^2 = 2\left[1 + 2 + \ldots + (n-1)\right] + n$
- $n^2 = (n-1)(n+1) + 1$

Now, let us look at another pattern.

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + 20^2 = 21^2$

In this pattern, we can observe that,

- The first and second numbers are consecutive numbers.
- The third number is the product of the first and second number.
- The fourth number is the successor of third number.

If we take the first number as 9, then

- We will have the second number as 9 + 1 = 10.
- Now, the third number is 9 × 10 = 90.
- Therefore, the fourth number is 90 + 1 = 91.

Using the given pattern, we can write

$9^2 + 10^2 + 90^2 = 91^2$

Similarly, by taking the first number as 12, we can write

$12^2 + 13^2 + 156^2 = 157^2$

In this way, we can expand a pattern through observations and hence find the square of a number related to that pattern.

Some properties related to perfect squares:

- If a number has *n* zeroes at the end, then its square ends in 2*n* zeroes.
- The remainder of a perfect square when divided by 3 is either 0 or 1, but never 2.
- The remainder of a perfect square when divided by 4 is either 0 or 1, but never 2 and 3.
- The remainder of a perfect square when divided by 8 is either 0 or 1 or 4, but never 2,3, 5, 6 or 7.
- If 1 is added to the product of four consecutive integers, then the resulting number is a perfect square.
- The square of any odd number can be expressed as the sum of two consecutive positive numbers.

Let us discuss some examples based on the determination of squares through observation of patterns.

Example 1:

Express 81 as the sum of 9 odd numbers.

Solution:

We know, $9^2 = 81$

We also know that the sum of first *n* odd numbers is n^2. Therefore, we can express 9^2, i.e., 81 as the sum of first 9 odd numbers.

Therefore, 81 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17

Example 2:

Without adding, find the sum of

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29

Solution:

Here, 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 is the sum of first 15 odd numbers.

We know that, sum of first n odd numbers = n^2

Therefore, sum of first 15 odd numbers = 15^2

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 = 225

Example 3:

Observe the following pattern and find the missing numbers.

$$11^2 = 121$$

$$101^2 = 10201$$

$$10101^2 = 102030201$$

$$1010101^2 = 1020304030201$$

$$101010101^2 = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots^2 = 10203040506050403020 1$$

Solution:

By observing the pattern, we can write

$$101010101^2 = \underline{10203040504030201}$$
$$\underline{10101010101^2} = 102030405060504030201$$

Thus, the missing numbers are 10203040504030201 and 10101010101.

Example 3:

Find the perfect square nearest to 32760.

Solution:

We can write 32760 as:
$32760 = 12 \times 13 \times 14 \times 15$
We know that, if 1 is added to the product of four consecutive integers then the resulting number is a perfect square.
The number 32760 is a product of four consecutive integers. Now, if we add 1 to 32760, we get a perfect square.
Thus, the perfect square nearest to 32760 is $32760 + 1 = 32761$.

Square Of A Number Using The Identity

We can easily find the squares of numbers such as 3, 4, 5, 6, 7, etc. by multiplying the number two times. If we find the square of 52 by multiplying it two times, then it will take a long time. In order to overcome such situations, we follow a different method to find the squares of such large numbers.

We can find the squares of such numbers very easily by using the identity $(a + b)^2$

Let us discuss some more examples based on square of numbers using this method.

Example:

Find the squares of 83 and 34.

Solution:

We know, $83 = 80 + 3$

Now, $83^2 = (80 + 3)^2$

$= (80 + 3)(80 + 3)$

$= 80(80 + 3) + 3(80 + 3)$

$= 80^2 + 80 \times 3 + 3 \times 80 + 3^2$

$= 6400 + 240 + 240 + 9$

$= 6889$

We know, $34 = 30 + 4$

Now, $34^2 = (30 + 4)^2$

$= (30 + 4)(30 + 4)$

$= 30(30 + 4) + 4(30 + 4)$

$= 30^2 + 30 \times 4 + 4 \times 30 + 4^2$

$= 900 + 120 + 120 + 16$

$= 1156$

Squares Of Numbers Ending With 5

What is the square of 75?

We can find the square of 75 by multiplying it two times.

Therefore, $75^2 = 75 \times 75 = 5625$

Now, we can write 5625 as

$5625 = 56 \times 100 + 25\times1$

Or, $5625 = (7 \times 8) \times 100 + 25$

Therefore, we can write

$75^2 = (7 \times 8) \times 100 + 25$

Let us find the square of another number that ends with 5, lets say 55.

Now, $55^2 = 55 \times 55$

$= 3025$

We can write 3025 as

$3025 = 30 \times 100 + 25 \times 1$

or $3025 = (5 \times 6) \times 100 + 25$

Therefore, we can write

$55^2 = (5 \times 6) \times 100 + 25$

Observing the above two examples, we can write a general formula to determine the square of a number that ends with 5, let say $a5$.

$(m5)^2 = \{m \times (m + 1) \times 100\} + 25$**, where** m **is the number excluding unit place digit**

We use this formula to find the square of any number that ends with 5.

Using this formula, let us find the square of 135.

In 135, the unit place digit is 5. Therefore, the number excluding unit place digit 5 is 13.

Therefore, $135^2 = 13 \times (13 + 1) \times 100 + 25$

$= 18200 + 25$

$= 18225$

If we find the square of 135 by multiplying it two times, then we will obtain the same answer.

$135^2 = 135 \times 135 = 18225$

But in multiplication, we will take a longer time as compared to the previous method. Therefore, we use the above method for finding the square of a number that ends with 5 rather than multiplying it two times.

Let us discuss one more example based on this method.

Example 1:

Find the squares of 95 and 305 without actually multiplying them two times.

Solution:

1. Square of 95

The number obtained by excluding the unit place digit of 95 is 9.

Therefore, $95^2 = 9 \times (9 + 1) \times 100 + 25$

$= 9000 + 25$

$= 9025$

2. Square of 305

The number obtained by excluding the unit place digit of 305 is 30.

Therefore, $305^2 = 30 \times (30 + 1) \times 100 + 25$

$= 93000 + 25$

$= 93025$

Pythagorean Triplets

If you are asked to find a Pythagorean triplet whose one member is 24, then how would you find such a triplet?

We can find it very easily, if we know about the general form of Pythagorean triplet.

In this way, we form Pythagorean triplets from a given number. Let us discuss some more examples based on Pythagorean triplets.

Example 1:

Check whether 7, 24, and 25 forms a Pythagorean triplet or not.

Solution:

$7^2 = 49$

$24^2 = 576$

$25^2 = 625$

Now, $49 + 576 = 625$

$\therefore 7^2 + 24^2 = 25^2$

Here, the square of a number is equal to the sum of the squares of the other two numbers. Therefore, we can say that 7, 24, and 25 forms a Pythagorean triplet.

Example 2:

Write a Pythagorean triplet whose least member is 20.

Solution:

20 is a member of Pythagorean triplet. Therefore, we can express 20 as $2m$.

Take $2m = 20$, we obtain $m = 10$

Now, $m^2 - 1 = 10^2 - 1 = 99$

$\therefore m^2 + 1 = 10^2 + 1 = 101$

Therefore, 20, 99, and 101 is the Pythagorean triplet whose least member is 20.

Square Root of Numbers

What is the square of 6?

We can find the square of 6 by multiplying 6 two times.

Therefore, we can write

$6^2 = 6 \times 6$

$\therefore 6^2 = 36$

Hence, we can say that 36 is the square of 6.

We can write the expression, **"36 is the square of 6"** in another way as **"the square root of 36 is 6"**.

Mathematically, we write it as $\sqrt{36} = 6$.

Here, the symbol $\sqrt{\ }$ represents the **square root.**

Similarly, for $8^2 = 64$, we write $\sqrt{64} = 8$ i.e., the square root of 64 is 8.

For $9^2 = 81$, we write $\sqrt{81} = 9$ i.e., the square root of 81 is 9.

From the above observations, we can say that **finding square root is the inverse operation of squaring.**

Note: We know that, $5^2 = 25$ and $(-5)^2 = 25$. Therefore, we can say that the square roots of 25 are 5 and −5. However, we generally take the positive value while writing square root of a number. Therefore, we write $\sqrt{25} = 5$.

Let us solve another example to understand the concept better.

Example:

Which of the following statement is/are correct?

(a) 49 is the square root of 7.

(b) 12 is the square root of 144.

(c) 1 is the square root of 1.

Solution:

(a) Since $7^2 = 49$. Therefore, 7 is the square root of 49.

Thus, the given statement is incorrect.

(b) Since $12^2 = 144$. Therefore, 12 is the square root of 144.

Thus, the given statement is correct.

(c) Since $1^2 = 1$. Therefore, 1 is the square root of 1.

Thus, the given statement is correct.

Square Root Of A Number By Repeated Subtraction Method

We know what is the square root of a number. However, we do not know how to find the square root of a given number.

Let us learn a method to find the square root of a given number. This method is known as **repeated subtraction method**.

By this technique, we can also check whether a number is a perfect square or not. In case the number is not a perfect square, we will not obtain 0 at any step.

Let us solve another example to understand the concept better.

Example:

Find the square root of 49 using repeated subtraction method.

Solution:

To find the square root of 49, successive odd numbers starting from 1 have to be subtracted from 49.

By doing so, we obtain

(1) 49 − 1 = 48 (2) 48 − 3 = 45 (3) 45 − 5 = 40 (4) 40 − 7 = 33

(5) 33 − 9 = 24 (6) 24 − 11 = 13 (7) 13 − 13 = 0

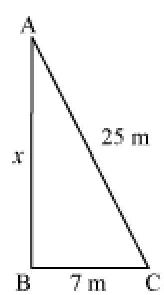
From this calculation, we obtain 0 in 7^{th} step.

$\therefore \quad \sqrt{49} = 7$

Prime Factorisation Method of Finding Square Roots

A park is in the shape of a right triangle. The length of the hypotenuse and a side of this park are 25 m and 7 m respectively. What is the length of the third side?

Let us consider the following triangle ABC as the right triangular park whose hypotenuse AC = 25 m and side BC = 7 m.



Let AB be *x.*

Using Pythagoras theorem for triangle ABC,

$AB^2 + BC^2 = AC^2$

$x^2 + (7)^2 = (25)^2$

$x^2 + 49 = 625$

$x^2 = (625 - 49)$

$x^2 = 576 \text{ m}^2$

To find the value of x, we require a number whose square is 576.

For this, we will find the square root of 576. Mathematically, we write it as

$$x = \sqrt{576} \text{ m}$$

Here, $\sqrt{\ }$ represents the square root. To find the square root of 576, we follow a method, which is known as prime factorization method.

In this way, we can find the square root of a given number by prime factorization method and solve problems related to it.

Let us discuss some more examples to understand the concept better.

Example 1:

Find the square roots of the following numbers.

(i) 324

(ii) 676

(iii) 1225

(iv) 3136

Solution:

(i) The prime factorization of 324 is

$$\begin{array}{r|l}
2 & 324 \\ \hline
2 & 162 \\ \hline
3 & 81 \\ \hline
3 & 27 \\ \hline
3 & 9 \\ \hline
3 & 3 \\ \hline
 & 1
\end{array}$$

$\therefore 324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$\therefore \sqrt{324} = 2 \times 3 \times 3 = 18$

(ii) The prime factorization of 676 is

$$\begin{array}{r|l}
2 & 676 \\ \hline
2 & 338 \\ \hline
13 & 169 \\ \hline
13 & 13 \\ \hline
 & 1
\end{array}$$

$\therefore 676 = \underline{2 \times 2} \times \underline{13 \times 13}$

$\therefore \sqrt{676} = 2 \times 13 = 26$

(iii) The prime factorization of 1225 is

$$\begin{array}{r|l}
5 & 1225 \\ \hline
5 & 245 \\ \hline
7 & 49 \\ \hline
7 & 7 \\ \hline
 & 1
\end{array}$$

$\therefore 1225 = \underline{5 \times 5} \times \underline{7 \times 7}$

$\therefore \sqrt{1225} = 5 \times 7 = 35$

(iv) The prime factorization of 3136 is

$$\begin{array}{r|l}
2 & 3136 \\ \hline
2 & 1568 \\ \hline
2 & 784 \\ \hline
2 & 392 \\ \hline
2 & 196 \\ \hline
2 & 98 \\ \hline
7 & 49 \\ \hline
7 & 7 \\ \hline
 & 1
\end{array}$$

$$\therefore\ 3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$$

$$\therefore \sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$$

Example 2:

Is 504 a perfect square? If not,

(i) find the smallest number multiplied to this number, so that the product would be a perfect square

(ii) find the smallest number by which 504 must be divided, so that the quotient is a perfect square

Solution:

The prime factorization of 504 is

$$\begin{array}{r|l}
2 & 504 \\ \hline
2 & 252 \\ \hline
2 & 126 \\ \hline
3 & 63 \\ \hline
3 & 21 \\ \hline
7 & 7 \\ \hline
 & 1
\end{array}$$

$$504 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times 7$$

Here, the prime factors 2 and 7 do not occur in pair. Therefore, 504 is not a perfect square.

(i) In order to obtain a perfect square, each factor of 504 must be paired. Therefore, we have to make pairs of 2 and 7. For this, 504 should be multiplied by 2 × 7 i.e., 14.

$$\text{Now,} \quad 504 \times (2 \times 7) = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$
$$504 \times (14) = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Therefore, 14 should be multiplied with 504 to make it a perfect square.

(ii) In order to obtain a perfect square, each factor of 504 must be paired. Therefore, 504 should be divided by 2 × 7 i.e., 14.

$$\text{Now,} \quad 504 \div (2 \times 7) = \underline{2 \times 2} \times \underline{3 \times 3}$$
$$504 \div (14) = \underline{2 \times 2} \times \underline{3 \times 3}$$

Therefore, 504 should be divided by 14 so that the quotient is a perfect square.

Example 3:

Find the smallest perfect square which is a multiple of the numbers 8, 9, and 20.

Solution:

To find the smallest number which is a multiple of 8, 9, and 20, first of all, we have to find the least common multiple (L.C.M.) of these numbers and then, we find the required perfect square.

Now,

$$\begin{array}{r|l} 2 & 8,\ 9,\ 20 \\ \hline 2 & 4,\ 9,\ 10 \\ \hline 2 & 2,\ 9,\ 5 \\ \hline 5 & 1,\ 9,\ 5 \\ \hline 3 & 1,\ 9,\ 1 \\ \hline 3 & 1,\ 3,\ 1 \\ \hline & 1,\ 1,\ 1 \end{array}$$

$$\text{Now, L.C.M. of 8, 9, and 20} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$
$$\text{Prime factorization of } 360 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times 5$$

It can be seen that 2 and 5 are not in pairs. Therefore, 360 is not a perfect square. In order to obtain a perfect square, each factor of 360 must be paired. Therefore, we have to make pairs of 2 and 5. Therefore, 360 should be multiplied by 2 × 5 i.e., 10.

Hence, required perfect square = 360 × 10 = 3600

Example 4:

A farmer had 11300 plants. He planted them in such a way that each row contains as many plants as two times the number of rows. At last, he found that 50 plants were not planted. Find the number of rows and the number of plants in each row.

Solution:

Let the number of rows be x.

Number of plants in each row = $2x$

Therefore, number of plants planted by the farmer = $(x)(2x) = 2x^2$

The farmer had 11300 numbers of plants. 50 plants were not planted among these.
Therefore, number of plants planted by the farmer = 11300 − 50 = 11250

$$\text{Now, } 2x^2 = 11250$$
$$\Rightarrow x^2 = \frac{11250}{2} = 5625$$
$$\therefore x = \sqrt{5625}$$

The prime factorization of 5625 is

$$\begin{array}{r|l} 3 & 5625 \\ \hline 3 & 1875 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\text{Now, } 5625 = \underline{3\times3}\times\underline{5\times5}\times\underline{5\times5}$$
$$\therefore\ x = \sqrt{5625} = 3\times5\times5 = 75$$
$$2x = 2\times75 = 150$$

Therefore, the number of rows is 75 and the number of plants in each row is 150.

Square Roots of Perfect Squares by Division Method

What is the square root of 6241?

We know how to find the square root of 6241 by prime factorization method. For this, first of all, we have to find the prime factors of 6241.

- The smallest prime number is 2. But 6241 is not divisible by 2. Therefore, we can say that 2 is not a prime factor of 6241.
- Similarly, we will find that the prime numbers 3, 5, 7, 11, 13, 17, 19, etc. are not factors of 6241.

Actually 6241 is the square of 79 and it is a prime number. We have already taken a lot of time to find the prime factors of 6241 and have not been able to obtain it. If we keep on doing this to obtain such a large prime number, 79, then it will be a time consuming process and there might be some errors in our calculation.

In order to overcome such situations, we follow another method to find the square root of a number. This method is known as **division method**.

Using division method, we can also find the square root of a decimal. Let us learn this method and find the square root of a decimal number, lets say 51.84.

In this way, we can find the square root of a number by division method and solve problems related to it.

Let us discuss some examples to understand the concept better.

Example 1:

Find the least number that must be added to 7523 so as to obtain a perfect square. Also, find the square root of the perfect square so obtained.

Solution:

By finding the square root of 7523 by division method, we obtain

$$\begin{array}{r|l} & \quad 86 \\ \hline 8 & \overline{75}\,\overline{23} \\ & -64 \\ \hline 166 & \;\;1123 \\ & -\;996 \\ \hline & \;\;\;127 \end{array}$$

When we found $\sqrt{7523}$ by long division method, we obtained the remainder as 127.

This shows, $86^2 < 7523$

The next perfect square to 86^2 is 87^2.

Since, $87^2 = 7569$

Therefore, 7569 is the perfect square which is greater than 7523.

7569 – 7523 = 46

Therefore, 46 should be added to 7523 so that the result is a perfect square.

Now, $\sqrt{7569} = 87$

Therefore, the square root of the perfect square is 87.

Example 2:

Find the least number that must be subtracted from 3241 so as to obtain a perfect square. Also, find the square root of the perfect square so obtained.

Solution:

By finding the square root of 3241 by division method, we obtain

$$\begin{array}{r|l} & \quad 56 \\ \hline 5 & \overline{32}\,\overline{41} \\ & -25 \\ \hline 106 & \quad 741 \\ & -636 \\ \hline & \quad 105 \end{array}$$

When we found $\sqrt{3241}$ by long division method, we obtained 105 as the remainder.

This shows that 56^2 is less than 3241 by 105.

This means, if we subtract 105 from 3241, then the remainder we obtain is a perfect square.

Therefore, required perfect square = 3241 – 105 = 3136

Now, $\sqrt{3136} = 56$

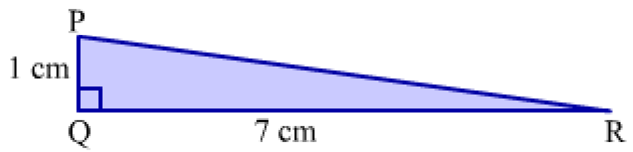
Therefore, the square root of the perfect square is 56.

Square Roots of Non-perfect Square Numbers

Square Roots of Non-perfect Square Numbers

We have studied about the square roots of numbers that are perfect squares. But what about non-perfect square numbers? Can we find their square roots as well? If yes, how?

Have a look at the given triangle.



In ΔPQR, PQ = 1 cm, QR = 7 cm, ∠Q= 90°.

By Pythagoras theorem:

$(PR)^2 = (PQ)^2 + (QR)^2$

$\Rightarrow (PR)^2 = (1)^2 + (7)^2$

$\Rightarrow (PR)^2 = 1 + 49$

$\Rightarrow (PR)^2 = 50$

$\Rightarrow PR = \sqrt{50}$ cm

Thus, the length of line segment PR is $\sqrt{50}$ cm.

It can be observed that 50 is a non-perfect square number. Therefore, to find the length of PR, we need to find the square root of the non-perfect square number 50.

Let us try to calculate the square root of 50 using division method.

We can write 50 as 50.000000… to simplify our division.

The method of finding the square root of decimal numbers is same as that of integers.

Firstly, the digits in the integer part are paired off starting from the right and then the digits in the decimal part (i.e. .0000…) are paired off starting from the left.

	7.071
7	$\overline{50}.\overline{00}\ \overline{00}\ \overline{00}\ldots$
+ 7	– 49
1407	1 00 00
+ 7	– 98 49
14141	1 51 00
+ 1	–1 41 41
	9 59

The value of $\sqrt{50}$ has been calculated up to the third decimal place. As 50 is not a perfect square, we will get more decimal places if we continue the process. So, this is a never ending process.

∴ $\sqrt{50}$ = 7.071...

Here, the dots on the right side of the decimal number denote the fact that there will be an infinite number of digits.

Similarly, with the help of this method, we can find the square roots of other decimal numbers.

For example, let us find the square root of 231.79.

Here, 231.79 is paired as $\overline{2}\,\overline{31}.\overline{79}$.

	15.2
1	$\overline{2}\ \overline{31}.\ \overline{79}$
+ 1	–1
25	1 31
+ 5	– 1 25
302	6 79
+ 2	– 6 04
	75

∴ $\sqrt{231.79}$ = 15.2

Approximate value of the square root of a non-perfect square number

There are many problems in cases where square roots of non-perfect square numbers are involved. Since their values are non-terminating decimal numbers, we cannot consider them to simplify problems and thus, we take their approximate values.

For example, in ΔPQR (discussed above), the length of hypotenuse PR, when measured using a ruler, is obtained to be 7.1 cm. So, we can say that 7.1 is the approximate value of $\sqrt{50}$.

Using division method, the approximate value of the square root of a non-perfect square number can be calculated up to one, two, three or any number of decimal places, as shown below:

• Find the square root of the given number using division method up to one place more than the required decimal place.

For example, to determine the approximate value of $\sqrt{27}$ up to the second decimal place, the value of $\sqrt{27}$ is found up to the third decimal place.

• If the digit at the additional decimal place is 5 or more than 5, then increase the digit at the previous place by 1 and drop the digit at the additional place and all further places (if any).

For example, the value of $\sqrt{27}$ cup to the third decimal place is obtained as 5.196. Here, the digit at the third place, i.e. 6, is greater than 5. Thus, we drop 6 and increase the digit before it, i.e. 9, by 1. Hence, we get the approximate value of $\sqrt{27}$ up to the second decimal place as 5.20.

• If the digit at the additional decimal place is less than 5, then we drop that digit (and all further digits) by keeping the remaining number as it is.

For example, the value of $\sqrt{50}$ up to third decimal place is obtained as 7.071. Here, the digit at the third place, i.e. 1, is less than 5. Thus, we drop 1 and keep the remaining number as it is. Hence, the value of $\sqrt{50}$ up to the second decimal place is obtained as 7.07.

Similarly, the approximate value of the square roots of non-perfect square numbers up to the required decimal place can be obtained.

Let us go through some examples for a better understanding of the concept.

Example 1:

Find the square root of 3 up to three decimal places using division method.

Solution:

The square root of 3 up to three decimal places can be calculated using division method as follows:

$$
\begin{array}{r|l}
 & 1.732 \\
\hline
1 & \overline{3}.\overline{00}\ \overline{00}\ \overline{00} \\
+1 & -1 \\
\hline
27 & 2\ 00 \\
+7 & -1\ 89 \\
\hline
343 & \ \ 11\ 00 \\
+3 & \ \ -10\ 29 \\
\hline
3462 & \ \ \ \ 71\ 00 \\
+2 & \ \ \ \ -69\ 24 \\
\hline
 & \ \ \ \ \ \ 1\ 76
\end{array}
$$

$\therefore \sqrt{3} = 1.732$

Example 2:

Find the square root of 58.315 up to two decimal places.

Solution:

The square root of 58.315 up to two decimal places can be calculated using division method as follows:

$$
\begin{array}{r|l}
 & 7.63 \\
\hline
7 & \overline{58}.\overline{31}\ \overline{50} \\
+7 & -49 \\
\hline
146 & 9\ 31 \\
+6 & -8\ 76 \\
\hline
1523 & \ \ 55\ 50 \\
+3 & \ \ -45\ 69 \\
\hline
 & \ \ \ \ 9\ 81
\end{array}
$$

$\therefore \sqrt{58.315} = 7.63$

Example 3:

Find the approximate value of the square root of 31.25 up to the first decimal place by division method.

Solution:

Since we have to find the approximate value of the square root of 31.25 up to the first decimal place, we must find its square root up to the second decimal place.

Let us write 31.25 as 31.2500 and find its square root.

	5.59
5	$\overline{31}.\overline{25}\ \overline{00}$
+5	−25
105	6 25
+5	−5 25
1109	1 00 00
+9	−99 81
	19

$\therefore \sqrt{31.25} = 5.59$

In 5.59, the digit at the second place after decimal, i.e. 9, is greater than 5. Thus, we drop 9 and increase the digit before it, i.e. 5, by 1.

Hence, the approximate value of $\sqrt{31.25}$ up to the first decimal place is 5.6.

Estimation Of Square Root Of Numbers

Sohan has a square land of area 450 m^2. Can we calculate the length of one side of the land?

Let us consider that the length of one side of the square land be *x.*

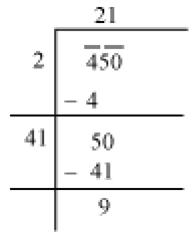
$\therefore$ Area of the land = (side)^2 = x^2

However, it is given that, area of the land = 450 m^2

$\therefore x^2 = 450$ m^2

or $x = \sqrt{450}$ m

Let us find the square root of 450 by division method.



Here, the remainder is 9. Therefore, 450 is not a perfect square.

450 is not a perfect square. This can also be checked by prime factorising it.

450 = 2 × 3 × 3 × 5 × 5. The number 450 has three distinct prime factors, 2, 3 and 5. The prime factors 3 occurs two times and 5 also occurs two times but the 2 occurs only once. Hence, we cannot pair the prime factors of 450. Thus, 450 is not a perfect square.

A number fails to be a perfect square, if in its prime factorisation some prime factors may not occur even number of times.

Now, it is very difficult to find the square root of 450. Therefore, we cannot find the side of the square. To find the square root in such situations, we follow another method which is known as **estimation method**.

Let us discuss this method and find the side of the square.

We know that, $21^2 = 441$ and $22^2 = 484$

Now, $441 < 450 < 484$

$21^2 < 450 < 22^2$

Here, we can see that 441 is much closer to 450 than 484.

Therefore, $\sqrt{450}$ is approximately equal to 21.

This means, the length of the side of the square is approximately 21 m.

Here, we can see that the side of the square is not exactly 21 m. But it is closer to 21 m. This method of calculating square root is known as estimation method.

Let us discuss one more example based on this concept to understand it better.

Example 1:

Estimate $\sqrt{161}$.

Solution:

Let us find $\sqrt{161}$ by division method.

$$\begin{array}{r|l} & \quad 12 \\ \hline 1 & \overline{1}\,\overline{61} \\ & -1 \\ \hline 22 & \ \ 61 \\ & -44 \\ \hline & \ \ 17 \end{array}$$

We know that, $12^2 = 144$ and $13^2 = 169$

Now, $144 < 161 < 169$

$12^2 < 161 < 13^2$

$\therefore 12 < \sqrt{161} < 13$

Here, we can see that 161 is much closer to 169 than 144.

Therefore, $\sqrt{161}$ is approximately equal to 13.

Therefore, the estimated value of $\sqrt{161}$ is 13.

Example 2:
If the area of a square is 105 cm^2, then what is its side rounded to the nearest integer?

Solution:
Since $A = (\text{side})^2$, we have $(\text{side})^2 = 105\ cm^2$.
But $100 < 105 < 121$ and 100 is nearer to 105 than 121. Hence, the nearest integer to $\sqrt{105}$ is $\sqrt{100} = 10$.
Thus, the side of the square rounded to the nearest integer is 10 cm.