Sample Question Paper - 6
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. Find the equations have real roots. If real roots exist, find them : $-2x^2 + 3x + 2 = 0$ **[2]**

OR

Find the value of k for which the given value is a solution of the given equation $7x^2 + kx - 3 = 0$; $x = \frac{2}{3}$

2. A 20 m deep well with diameter 7m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform. **[2]**

3. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components: **[2]**

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

4. Find the value of x for which (8x + 4), (6x - 2) and (2x + 7) are in A.P. **[2]**

5. **[2]**

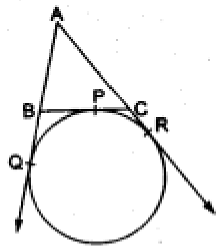
Marks	**Number of Students**	**c.f.**
0 - 10	5	5
10 - 30	15	F
30 - 60	f	50
60 - 80	8	58
80 - 90	2	60
	N = 60	$N = \Sigma f_i = 60$

Find f and F.

6. Two concentric circles are of radii 6.5 cm and 2.5 cm. Find the length of the chord of the larger circle which touches the smaller circle. **[2]**

OR

A circle is touching the side BC of $\triangle ABC$ at P and touching AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}$(perimeter of ΔABC).



Section B

7. If (m +1)th term of an A.P. is twice the (n +1)th term, prove that (3m + 1)th term is twice the (m + n + 1)th term. **[3]**

8. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of 3600 $\sqrt{3}m$, find the speed in km/hr of the plane. **[3]**

OR

A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b metres just above A is β. Prove that the height of tower is b tan α cot β.

9. From an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that. **[3]**

   i. PA.PB = PN^2 - AN^2
   ii. PN^2 - AN^2 = OP^2 - OT^2
   iii. PA.PB = PT^2

10. Solve: $\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}, x \neq \frac{4}{3}$ **[3]**

Section C

11. Draw a line segment AB of length 6.5 cm and divide it in the ratio 4:7. Measure each of the two parts. **[4]**

OR

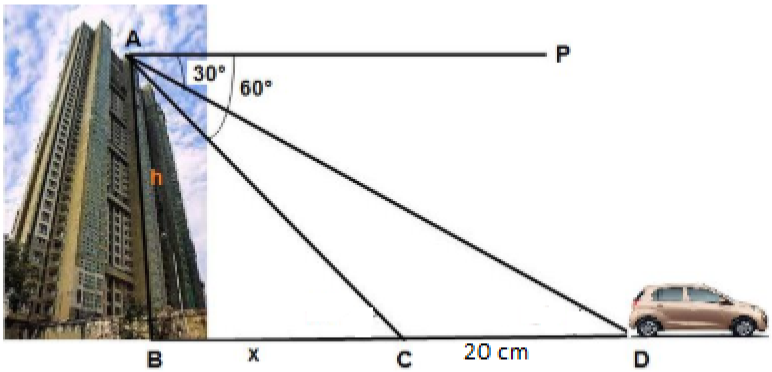
Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

12. The median of the following data is 52.5. Find the values of x and y, if the total frequency is 100. **[4]**

C.I.	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency	2	5	x	12	17	20	y	9	7	4

13. Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^o. After accelerating 20 m from point C, Vijay stops at point D to **[4]**

buy ice-cream and the angle of depression changed to 30^{o}.

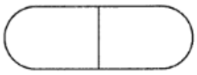


By analysing the above given situation answer the following questions:

i. Find the value of x.
ii. Find the height of the building AB.

14. Seema a class 10th student went to a chemist shop to purchase some medicine for her mother who was suffering from Dengue. After purchasing the medicine she found that the upcount capsule used to cure platelets has the dimensions as followed: **[4]**

The shape of the upcount capsule was a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm.



By reading the above-given information, find the following:

i. The surface area of the cylinder.
ii. The surface area of the capsule.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Section A

1. For real roots of quadratic equation, $b^2 - 4ac > 0$

   We have, $-2x^2 + 3x + 2 = 0$

   Now, $b^2 - 4ac > 0$

   $\Rightarrow (3)^2 - 4(-2)(2) > 0$ ($\because a = -2, b = 3, c = 2$)

   $\Rightarrow 9 + 16 > 0$

   $\Rightarrow 25 > 0$

   Now, $\sqrt{D} = 5$

   And, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm 5}{2(-2)} = \frac{-3 \pm 5}{-4}$

   $\Rightarrow x = \frac{-3+5}{-4}$ and $x = \frac{-3-5}{-4}$

   $\Rightarrow x = \frac{2}{-4}$ and $x = \frac{-8}{-4}$

   $\Rightarrow x = \frac{-1}{2}$ and 2

   Therefore, the roots of the given equation are 2 and $\frac{-1}{2}$.

   OR

   We have, $7x^2 + kx - 3 = 0$

   Since x = $\frac{2}{3}$ is the solution of the given equation

   $\therefore x = \frac{2}{3}$ satisfies the given equation

   $7(\frac{2}{3})^2 + k(\frac{2}{3}) - 3 = 0$

   $\implies \frac{28}{9} + \frac{2k}{3} - 3 = 0$

   $\implies \frac{1}{9} + \frac{2k}{3} = 0$

   $\implies \frac{2k}{3} = -\frac{1}{9} \implies k = -\frac{3}{18}$

   $\implies k = -\frac{1}{6}$

2. For well Diameter = 7 m

   $\therefore$ Radius (r) = $\frac{7}{2}$m

   Depth (h) = 20 m

   $\therefore$ Volume = $\pi r^2 h = \pi\left(\frac{7}{2}\right)^2 (20)$

   $= 245\pi \text{cm}^3$

   For platform Length (L) = 22 m

   Breadth (B) = 14 m

   Let the height of the platform be Hm.

   Then, volume of the platform

   $= LBH = 22 \times 14 \times H = 308\text{Hm}^3$

   According to the question,

   308H = 245π

   $\Rightarrow H = \frac{245\pi}{308} \Rightarrow H = \frac{245 \times 22}{308 \times 7} \Rightarrow H = 2.5$

   Hence, the height of the platform is 2.5 m.

3. Here, the maximum class frequency is 61, and the class corresponding to this frequency is 60-80. So, the modal class is 60-80.

   Therefore h = 20, l = 60, $f_1 = 61$, $f_0 = 52$, $f_2 = 38$

   $Mode = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h = 60 + \left[\frac{61-52}{2(61) - 52 - 38}\right] \times 20 =$

   $60 + \left[\frac{9}{122-90}\right] \times 20 = 60 + \frac{180}{32} = 60 + 5.625 = 65.625$

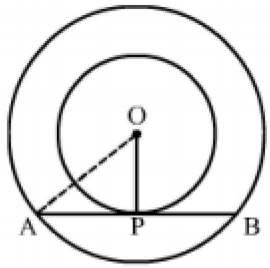
   Therefore, the modal lifetime of the components is 65.625 hours.

4. Here we are given that 8x+4,6x-2 and 2x+7 are in AP
Here
$a_1=8x+4, a_2=6x-2$ and $a_3=2x+7$
Then common difference $d = a_2 - a_1 = a_3 - a_2$
$\Rightarrow$(6x - 2) - (8x + 4) = (2x + 7) - (6x - 2)
$\Rightarrow$6x - 2 - 8x - 4 = 2x + 7 - 6x + 2
$\Rightarrow$-2x - 6 = -4x + 9
$\Rightarrow$-2x + 4x = 9 + 6
$\Rightarrow$2x = 15
$\Rightarrow x = \frac{15}{2}$

5.

Marks	Number of Students	c.f.
0 - 10	5	5
10 - 30	15	$15+5=20=F$
30 - 60	$50-20=30=f$	50
60 - 80	8	58
80 - 90	2	60
	N = 60	N = Σf_i = 60

$f = 30 \; and \; F = 20$

6. 

We know that the radius and tangent are perpendicular at their point of contact
In right Triangle AOP
$AO^2 = OP^2 + PA^2$
$\Rightarrow (6.5)^2 = (2.5)^2 + PA^2$
$\Rightarrow PA^2 = 36$
$\Rightarrow$ PA = 6cm
Since, the perpendicular drawn from the center bisects the chord.
PA = PB = 6cm
Now, AB = AP = PB = 6 + 6 = 12cm
Hence, the length of the chord of the larger circle is 12 cm.

OR

We know that the lengths of tangents drawn from an external point to a circle are equal.
AQ = AR, ...(i) [tangents from A]
BP = BQ ...(ii) [tangents from B]
CP = CR ... (iii) [tangents from C]
Perimeter of $\triangle ABC$
= AB + BC + AC
= AB + BP + CP + AC
= AB + BQ + CR + AC [using (ii) and (iii)]
= AQ + AR
= 2AQ [using (i)
$\therefore \quad AQ = \frac{1}{2}(\text{ perimeter of } \triangle ABC)$

Section B

7. Given,

$a_{m+1} = 2a_{n+1}$

$\Rightarrow$ a + (m + 1 - 1)d = 2[a + (n + 1 - 1)d]

$\Rightarrow$ a + md = 2[a + nd]

$\Rightarrow$ a + md = 2a + 2nd

$\Rightarrow$ md - 2nd = 2a - a

$\Rightarrow$ md - 2nd = a ....(i)

To prove:

$a_{3m+1} = 2a_{m+n+1}$

Proof:

LHS

= a_{3m+1}

= a + (3m + 1 - 1)d

= a + 3md

= md - 2nd + 3md [From (i)]

= 4md - 2nd

RHS

= $2a_{m+n+1}$

= 2[a + (m + n + 1 -1)d]

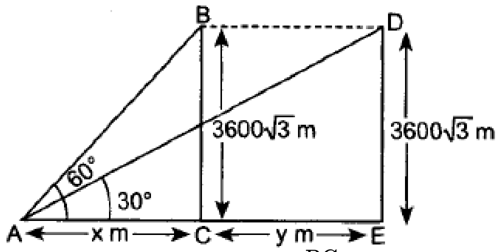
= 2[a + md - nd]

= 2[md - 2nd + md + nd] [From (i)]

= 2[2md - nd]

= 4md - 2nd

Hence, LHS = RHS

8. 

In rt. $\triangle$ACB, $\tan 60^\circ = \frac{\text{BC}}{\text{AC}}$

$\sqrt{3} = \frac{3600\sqrt{3}}{x}$

$x = 3600\ m$

Now, In right AED,

$\tan 30^\circ = \frac{\text{DE}}{\text{AE}}$

$\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{3600+y}$

$3600 + y = 10800$

$y = 7200m$

$BD = CE$

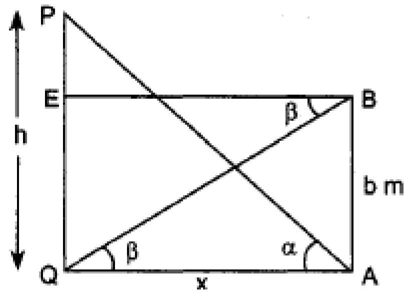
$\therefore$ Distance covered in 30 seconds = 7200 ,

So, Speed = $\frac{7200}{30} = 240\text{m/s}$

= $240 \times \frac{18}{5}$

$= 864\ km/hr.$

OR



Proof: Let AQ = x

$\angle \mathrm{EBQ}=\beta$ [Given]

EB II QA

$\Rightarrow \angle BQA=\beta$ [Alternate angles]

In right angled $\triangle$BAQ,

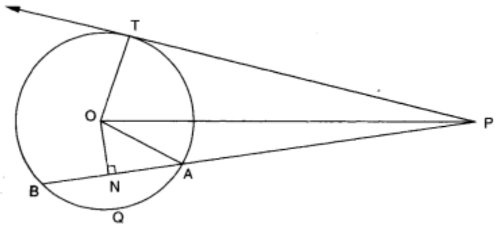
$\frac{\mathrm{AB}}{\mathrm{AQ}}=\frac{b}{x}=\tan \beta$

$\Rightarrow \quad \frac{b}{x}=\tan \beta \Rightarrow x=b \cot \beta$ ....(i)

In right angled $\triangle$PQA,

$\frac{\mathrm{PQ}}{\mathrm{QA}}=\frac{h}{x}=\tan \alpha$

$\Rightarrow \quad h=x \tan \alpha=b \cot \beta \tan \alpha=b \tan \alpha \cot \beta$

9. 

i. PA .PB = (PN - AN)(PN + BN)

= (PN - AN) (PN + AN) $\left[\begin{array}{c}\because ON \perp AB \\ \therefore N \text{ is the mid-point of } AB \\ \Rightarrow AN=BN\end{array}\right]$

= PN^2 - AN^2

ii. Applying Pythagoras theorem in right triangle PNO, we obtain

OP^2 = ON^2 + PN^2

$\Rightarrow PN^2$ = OP^2 - ON^2

$\therefore$ PN^2 - AN^2 = (OP^2 - ON^2) - AN^2

= OP^2 - (ON^2 + AN^2)

= Op^2 - OA ^{2} [Using Pythagoras theorem in ΔONA]

= OP^2 - OT^2 $[\because OA=OT=\text{ radius }]$

iii. From (i) and (ii), we obtain

PA.PB = PN^2 - AN^2 and PN^2 - AN^2 = OP^2 - OT^2

$\Rightarrow$ PA .PB = OP^2 - OT^2

Applying Pythagoras theorem in $\triangle OTP$, we obtain

OP^2 = OT^2 + PT^2

$\Rightarrow$ OP^2 - OT^2 = PT^2

Thus, we obtain

PA.PB = OP^2 - OT^2

and OP^2 - OT^2 = PT^2

Hence, PA.PB = PT^2.

10. The given equation is:

$\frac{3x-4}{7}+\frac{7}{3x-4}=\frac{5}{2}$

put $\frac{3x-4}{7}$ = y, we obtain

$y + \frac{1}{y} = \frac{5}{2}$

$\Rightarrow \frac{y^2+1}{y} = \frac{5}{2}$

$\Rightarrow 2y^2 + 2 = 5y$

$\Rightarrow 2y^2 - 5y + 2 = 0$

By Factorisation we have:

$2y^2 - 4y - y + 2 = 0$

$\Rightarrow 2y(y - 2) - 1(y - 2) = 0$

$\Rightarrow (y - 2)(2y - 1) = 0$

$\Rightarrow y - 2 = 0$ or $2y - 1 = 0$

Therefore, either y = 2 or y = $\frac{1}{2}$

Now, $y = \frac{3x-4}{7}$

$\Rightarrow \frac{3x-4}{7} = 2$ or $\frac{3x-4}{7} = \frac{1}{2}$

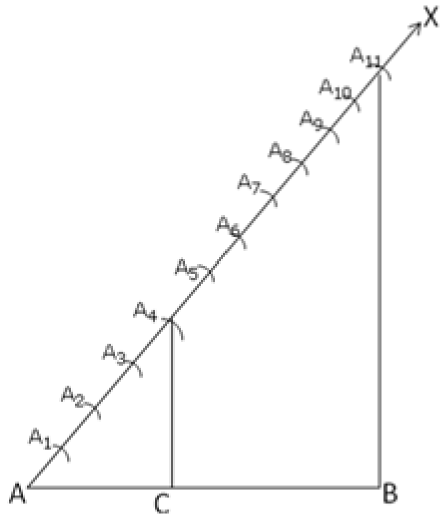
$\Rightarrow 3x - 4 = 14$ or $6x - 8 = 7$

$\Rightarrow 3x = 18$ or $6x = 15$

Therefore, $x = 6\ or\ \frac{5}{2}$

Section C

11.



Steps of construction:

1. Draw a line segment AB = 6.5 cm
2. Draw a ray AX making an acute ∠BAX with AB
3. Along AX mark (4 + 7) = 11 points
   $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}$
   such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8 = A_8A_9 = A_9A_{10} = A_{10}A_{11}$
4. Join $A_{11}B$.
5. Through the point A_4, draw a line parallel to AB by making an angle equal to $\angle AA_{11}B$ at A_4.
   Suppose this line meets AB at a point C.
   The point C so obtained is the required point, which divides, AB in the ratio 4:7.

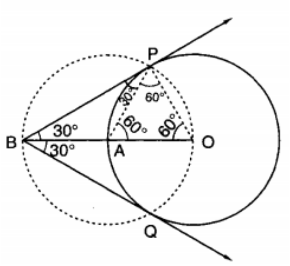
OR

In order to draw the pair of tangents, we follow the following steps.

Steps of construction

STEP I Take a point O on the plane of the paper and draw a circle of radius OA = 5 cm.

STEP II Produce OA to B such that OA = AB = 5 cm.



STEP III Taking A as the centre draw a circle of radius AO = AB = 5 cm. Suppose it cuts the circle drawn in step I at P and Q.

STEP IV Join BP and BQ to get the desired tangents.

Justification: In OAP, we have

OA = OP = 5 cm (= Radius)

Also, AP = 5 cm (= Radius of circle with centre A)

$\therefore \Delta OAP$ is equilateral. $\Rightarrow \angle PAO = 60^\circ \Rightarrow \angle BAP = 120^\circ$

In ΔBAP, we have

BA = AP and $\angle BAP$ = 120°

$\angle ABP = \angle APB = 30^\circ$

$\Rightarrow \quad \angle PBQ = 60^\circ$

12.

C.I.	f	c.f.
0 - 10	2	2
10 - 20	5	7
20 - 30	x	7 + x
30 - 40	12	19 + x
40 - 50	17	36 + x
50 - 60	20	56 + x
60 - 70	y	56 + x + y
70 - 80	9	$65 + x + y$
80 - 90	7	$72 + x + y$
90 - 100	4	$76 + x + y$
	Σf_i = 76 + x + y	

As given, Σf_i = 100

$\Rightarrow 76 + x + y = 100$

$\Rightarrow x + y = 24$

Median = 52.5, n = 100

$\Rightarrow \frac{n}{2} = 50$

Median Class is 50 - 60

Using formula for the median,

52.5 = 50 + $\frac{[50-(36+x)]}{20} \times 10$

$= 50 + \frac{14-x}{2}$

$52.5 - 50 = \frac{14-x}{2}$

$\Rightarrow$ 2.5 $\times$ 2 = 14 - x

$\Rightarrow 5 = 14 - x$

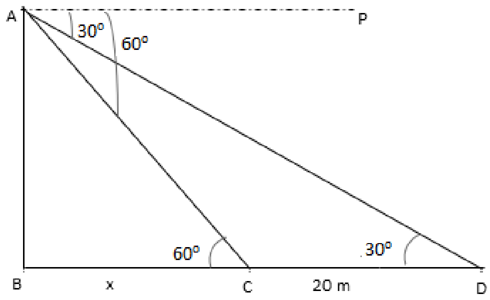
$\Rightarrow x = 14 - 5$

$\Rightarrow x = 9$

Putting in equation(i), we get $9 + y = 24$

$\Rightarrow y = 24 - 9 = 15$

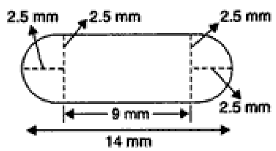
13. The above figure can be redrawn as shown below:



i. From the figure,
let AB = h and BC = x
In ΔABC,
$tan\ 60 = \frac{\mathrm{AB}}{\mathrm{BC}} = \frac{\mathrm{h}}{x}$
$\sqrt{3}$ = $\frac{\mathrm{h}}{x}$
h = $\sqrt{3}$ x ....(i)
In ΔABD,
$tan\ 30 = \frac{\mathrm{AB}}{\mathrm{BD}} = \frac{\mathrm{h}}{x+20}$
$\frac{1}{\sqrt{3}}$ = $\frac{\sqrt{3}x}{x+20}$ [using (i)]
x + 20 = 3x
x = 10m

ii. Height of the building, h = $\sqrt{3}$ x = 10$\sqrt{3}$ = 17.32 m

14. Let r = radius, h = cylindrical height
The radius of the hemisphere or cylinder, r $= \frac{5}{2}$mm
Height of cylinder, h = Total height - 2$\times$ radius of hemisphere
$h = 14 - 2 \times 2.5 = 9$ mm



i. Surface area of cylinder $= 2\pi rh$
$= 2\pi\left(\frac{5}{2}\right)(9) = 45\pi\ \mathrm{mm}^2$

ii. Surface area of the capsule = curved surface area of cylinder + 2 $\times$ surface area of the hemisphere
$= 2\pi rh + 2(2\pi r^2)$
$= 2\pi\left(\frac{5}{2}\right)(9) + 2\left[2 \cdot \pi \cdot \left(\frac{5}{2}\right)^2\right]$
$= 45\pi + 25\pi$
$= 70\pi = 70 \times \frac{22}{7} = 220\ \mathrm{mm}^2$