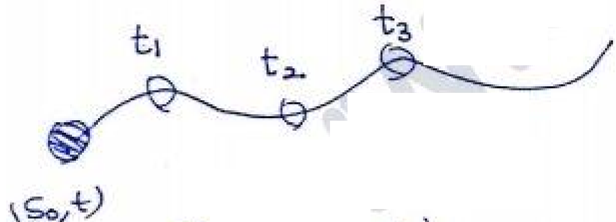
"fluid kinematics"

* Study of motion of fluid without any ref. of force and moment.

The descripition of motion of fluid is defined by

1. lagrangian descripition
2. Eulerian description

<u>lagrangian</u>

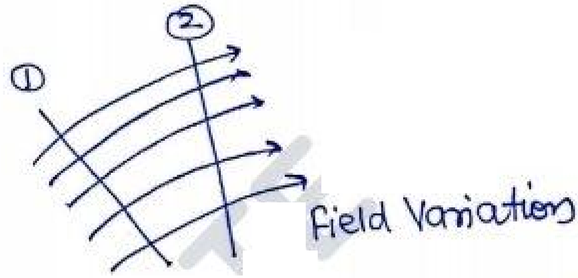


$\vec{S} = (S_o, t)$

$$\vec{V} = \frac{d\vec{S}}{dt}$$

$$\vec{a} = \frac{d^2\vec{S}}{dt^2}$$

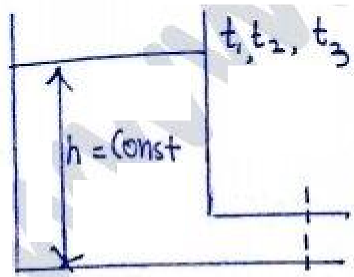
<u>Eulerian</u>



$$\text{Vector}\begin{cases}\vec{V} = f(x,y,z,t)\\ \vec{a} = f(x,y,z,t)\end{cases}$$

$$\text{Scalar}\begin{cases}\rho = f(x,y,z,t)\\ T = f(x,y,z,t)\\ P = f(x,y,z,t)\end{cases}$$

*

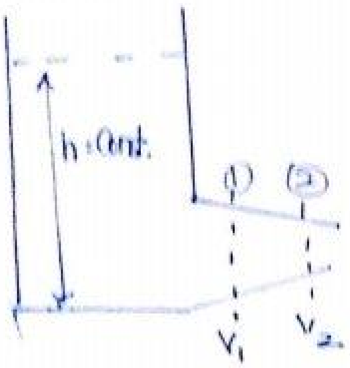


$V = \sqrt{2gh}$

$h \neq$ time

$$\frac{dV}{dt} = 0$$

*



$\frac{ds}{dt} \neq 0$

$$\vec{V} = f(\vec{x}, \vec{y}, \vec{z}, \vec{t})$$

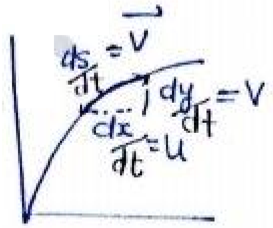
$$d\vec{V} = \frac{\partial \vec{V}}{\partial x} dx + \frac{\partial \vec{V}}{\partial y} dy + \frac{\partial \vec{V}}{\partial z} dz + \frac{\partial \vec{V}}{\partial t} dt$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{D\vec{V}}{Dt}$$

$$\frac{dv}{dt} = \frac{\partial \vec{V}}{\partial x}\left(\frac{dx}{dt}\right) + \frac{\partial \vec{V}}{\partial y}\left(\frac{dy}{dt}\right) + \frac{\partial \vec{V}}{\partial z}\left(\frac{\partial z}{\partial t}\right) + \frac{\partial \vec{V}}{\partial t}$$

$$\vec{V}(x,y,z,t) = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$$

$$\frac{D\vec{V}}{Dt} = u\frac{d\vec{V}}{dx} + v\frac{d\vec{V}}{dy} + w\frac{\partial \vec{V}}{dz} + \frac{d\vec{V}}{dt}$$



$\downarrow$

Total derivative / material derivative

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\Rightarrow \boxed{\vec{V}\cdot\vec{\nabla}} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

↳ time space operator / ~~conve~~ Convective operator.

* $$\boxed{\frac{D}{Dt} = (\vec{V}\cdot\vec{\nabla}) + \frac{\partial}{\partial t}}$$

$$\boxed{\frac{D\rho}{Dt} = (\vec{V}\cdot\vec{\nabla})\rho + \frac{\partial \rho}{\partial t}}$$

$\Rightarrow \frac{\partial}{\partial t} \longrightarrow$ local/temporal operator.

* for incompressible flow $\boxed{\frac{D\rho}{Dt} = 0}$

$(\vec{V}\cdot\vec{\nabla}) = 0$

* $$\vec{a} = (\vec{V}\cdot\vec{\nabla})\vec{V} + \frac{\partial \vec{V}}{\partial t}$$

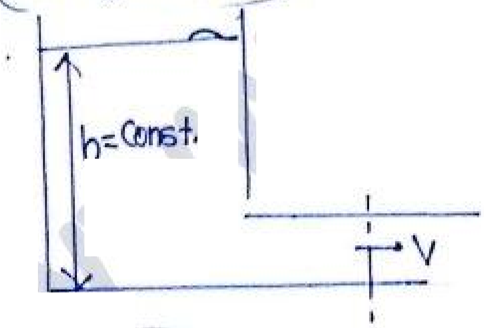
* No acc^n $\boxed{\left(\frac{D\vec{V}}{Dt} = \vec{a} = 0\right)}$

lagrangian Description :→ it is an exact analysis on single fluid particle motion.
In this description single fluid particle motion is described by tracing the space vector with refrence of initial point. Due to large number of variable ~~th~~ in the system, the equation become complex.

Eulerian description:- In this description study is on the bulk flow at the given section by defining the field variable in flow field. It is and approximate analysis

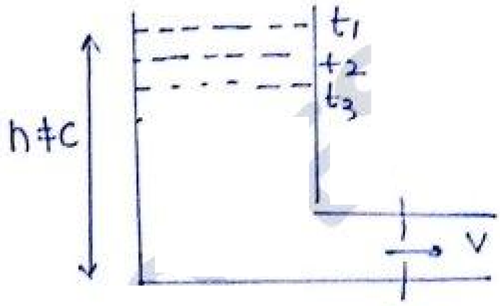
Types of fluid flow:-

* **Steady & Unsteady Flow:-** A flow is said to be steady when properties doesn't change with time (at a given space), otherwise the flow is unsteady.



$h \neq f(\text{time})$ $\quad \frac{\partial V}{\partial t} = 0$

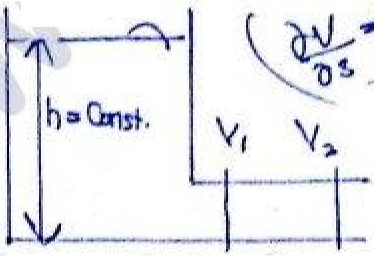
Steady flow



$\frac{\partial V}{\partial t} \neq 0$

Unsteady flow

* **Uniform & Non uniform Flow:-** A flow is said to be uniform if the properties doesn't change with space at a given time otherwise the flow is non-uniform flow.

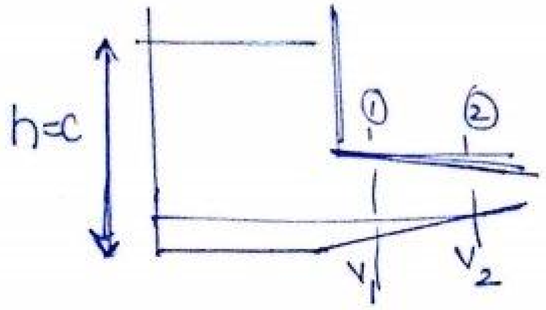


$\left(\frac{\partial V}{\partial s} = 0\right)$

$A_1 V_1 = A_2 V_2 \quad (A_1 = A_2)$

$V_1 = V_2$

(uniform flow)



$\Rightarrow$ Non-Uniform flow

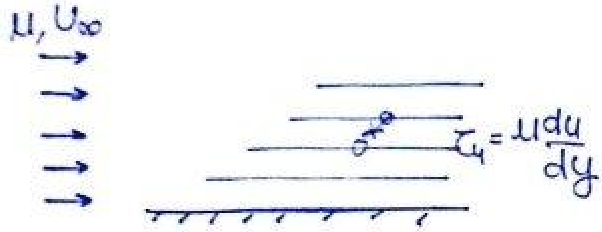
$\left(\frac{\partial u}{\partial s}\right) \neq 0$

$A_1 \neq A_2$, $V_1 \neq V_2$

* laminar flow & turbulant flow:- When fluid flows in the form of laminar with negligible mixing b/w layers is known as laminar flow. &
It Generally occurs at low velocity or high viscosity i.e. low ~~renolds~~ Reynolds. number
Laminar flow is generally know as Viscous flow. there are viscous shear stresses in flow. So laminar flow follow newtons low of viscosity.

Turbulant flow:- when fluid flow is highly disordered and there is rapid mixing b/w the layer is known as turbulant flow. It generally occurs at high velocity or low viscosity i.e. High Reynolds number.
In turbulant flow due to rapid mixing b/w the layers some additional stress comes into the picture known as ~~edd~~ eddy

eddy shear stress. Their magnitude is high w.r.t. viscous stress in turbulant flow. These stress are also known as Reynold's Stresses

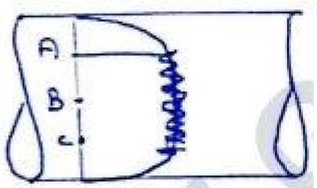


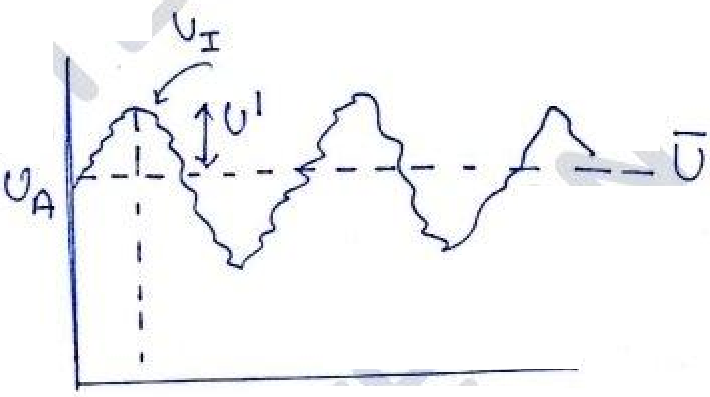
$$\downarrow Re = \frac{F_i \downarrow}{F_v \uparrow}$$

$$\downarrow Re = \frac{\rho V^{\downarrow} L}{\mu \uparrow}$$

In turbulent

⇒





Time avg. velocity

$$\boxed{\bar{U} = \frac{1}{T}\int_0^T U_I \, dt}$$

$U = \bar{U} + U'$

$V = \bar{V} + v'$

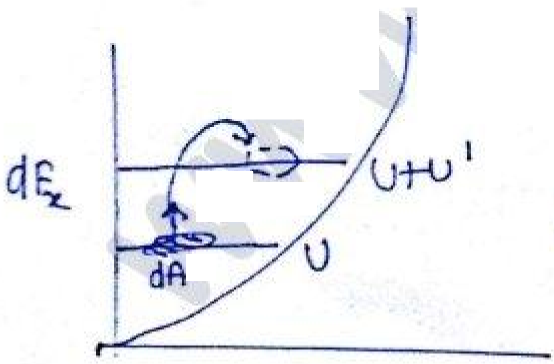
$W = \bar{W} + w'$

Assumption (fluctuation)

$U' = V' = W'$

* $$\boxed{\frac{1}{T}\int_0^T U' dt = 0}$$ time avg. of fluctuation = 0

*



$d\dot{m} = \rho \, dA \, V'$ Net force = momentum

$dF_x = d\dot{m}(U+U') - d\dot{m} U$

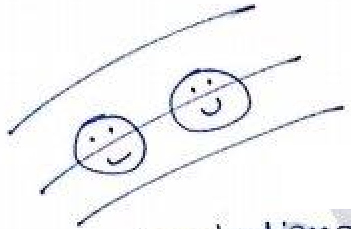
$dF_x = d\dot{m} \, U'$

$$dF_x = (\rho\, dA\, V')\, U'$$

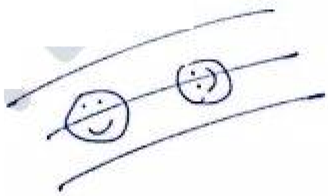
$$\frac{dF_x}{dA} = \rho V' U'$$

* $\boxed{\tau_t = \rho V' U'}$ Reynold's shear stress or eddy stress.

Rotational and irrotational flow:-



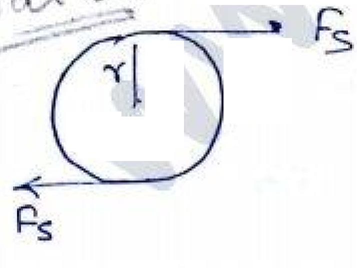
irrotational



Rotational

$u = f(x, y, z)$
$v = f(x, y, z)$
$w = f(x, y, z)$
Rotational flow if change in $\perp^{er}$ dir^n

rotation

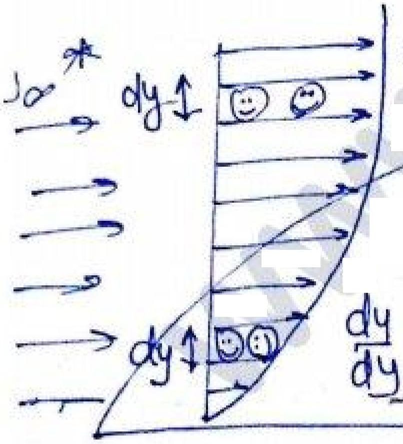


$F_s \times r \neq 0$

$F_s \neq 0$

$\tau \times A \neq 0$, $\underline{\tau \neq 0}$ rotational

$$\mu \frac{du}{dy} \neq 0 \Rightarrow \frac{du}{dy} \neq 0$$



$\frac{du}{dy} = 0$, $\mu \frac{du}{dy} = 0$, $\tau = 0$, $T = 0$ (Non Viscous → irrotational)

No Gradient

$\frac{dy}{dy} \neq 0$, $\tau \neq 0$, $T \neq 0$ (Viscous rotational)

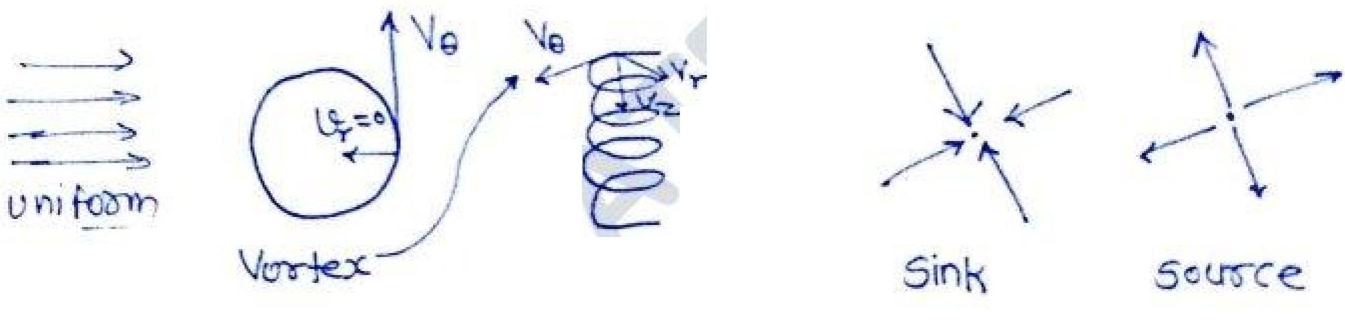
* A flow is said to be irrotational if there is no external torque i.e. No shear force, No Viscous shear stress, ie. the flow is non-viscous flow.

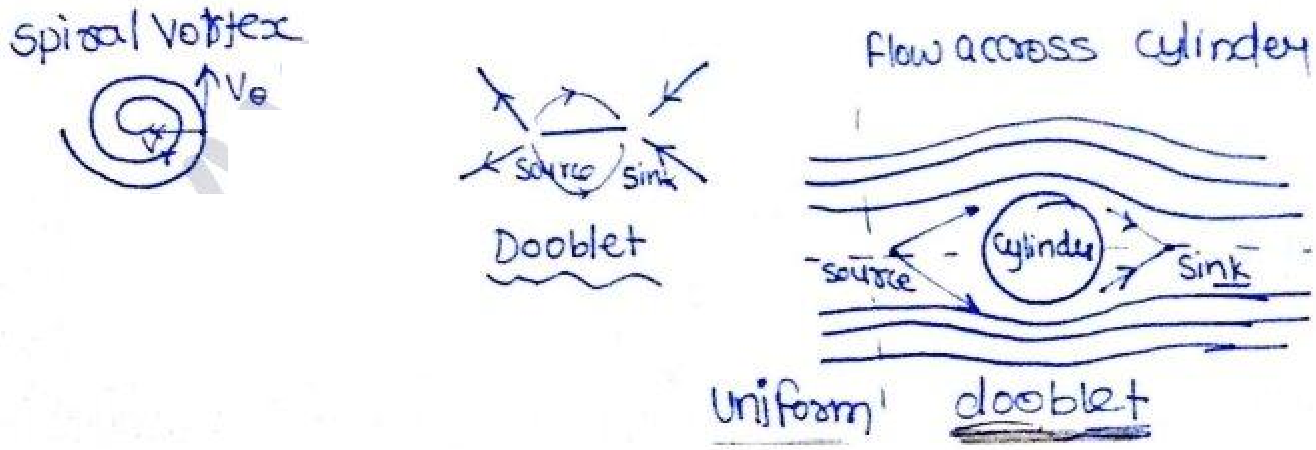
* One/two/three dimension flow → theory book {Read from}

Compressible and incompressible:- A flow is said to incompressible

$$\frac{D\rho}{Dt} = 0 \quad \text{or} \quad \frac{d\rho}{\rho} < 5\% \quad \text{or} \quad Ma < 0.32$$

Vortex Motion:- Motion of fluid along circular path is know as vortex motion.





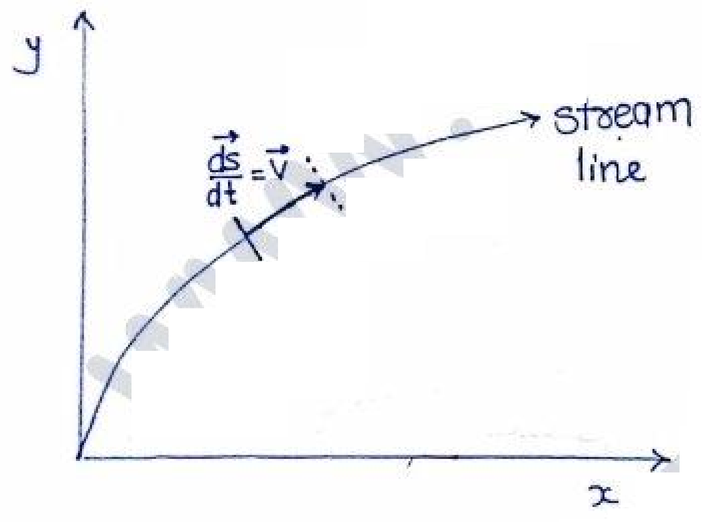
Flow lines:- There are three flow lines to define the fluid flow.

① stream line

② Path line

③ streake line.

Stream line:- It is an imaginary line or curve drawn in space such that the tengent drawn on the curve gives velocity vector i.e. stream line space vector and velocity vector coincides.



$$\vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\boxed{\vec{ds} \times \vec{V} = 0}$$

Stream line eqⁿ.

$$\vec{A} \times \vec{B} = |A||B| \sin\theta$$

$$\theta = 0$$

$$\vec{A} \times \vec{B} = 0$$

$$\vec{ds} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

$$\Rightarrow (w\,dy - v\,dz)\hat{i} + (u\,dz - w\,dx)\hat{j} + (v\,dx - u\,dy)\hat{k} = 0$$

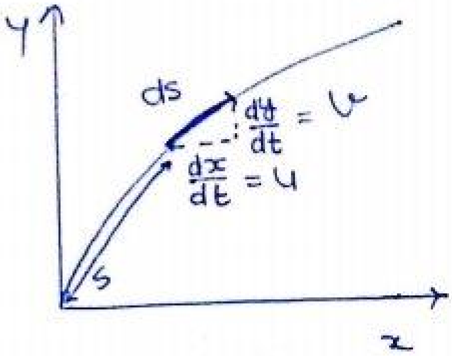
$$w\,dy - v\,dz = 0$$

$$u\,dz - w\,dx = 0$$

$$v\,dx - u\,dy = 0$$

$$* \quad \boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{\omega}}$$

3D eq^n



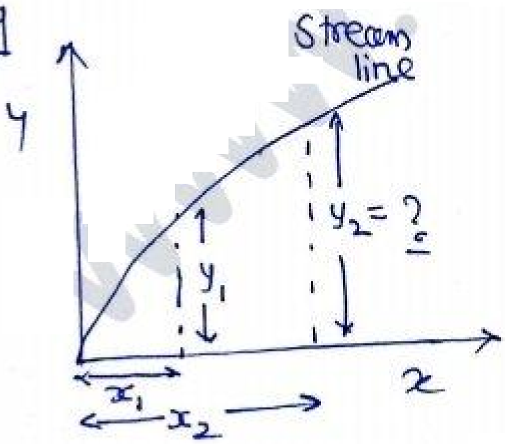
$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}$$

$$dt = \frac{dx}{u} \text{ — (1)}, \quad dt = \frac{dy}{v} \text{ — (2)}$$

(1) & (2)

$$\boxed{\frac{dx}{u} = \frac{dy}{v}}$$

eg



$$u = 2x^2 y$$
$$v = 3xy$$

$$\frac{dx}{2x^2 y} = \frac{dy}{3xy}$$

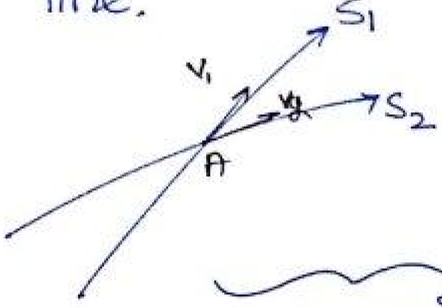
$$\int_{x_1}^{x_2} \frac{dx}{x} = \frac{2}{3} \int_{y_1}^{y_2} dy$$

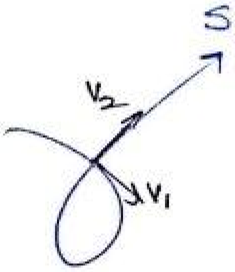
$$\ln x \Big|_{x_1}^{x_2} = \frac{2}{3}(y_2 - y_1)$$

$$\ln\left(\frac{x_2}{x_1}\right) = \frac{2}{3}(y_2 - y_1)$$

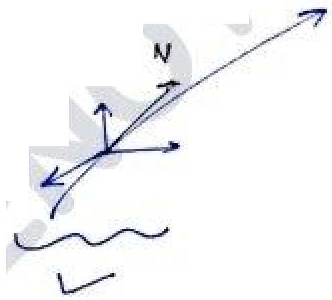
$$y_2 = y_1 + \frac{3}{2}\ln\left(\frac{x_2}{x_1}\right)$$

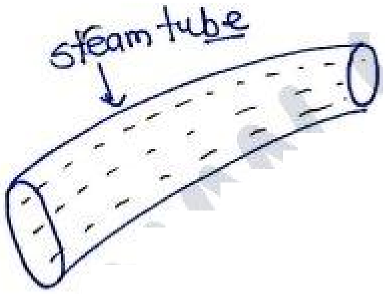
Note:- Two stream lines will never intersect each other or one stream line will never intersect it self because at the point of intersection there will be two velocity field which is impossible so there is no mass flow rate zero across the stream line.

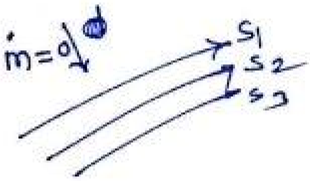




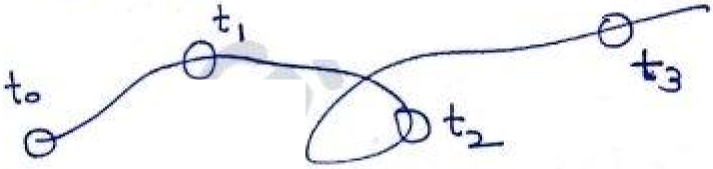
Not possible





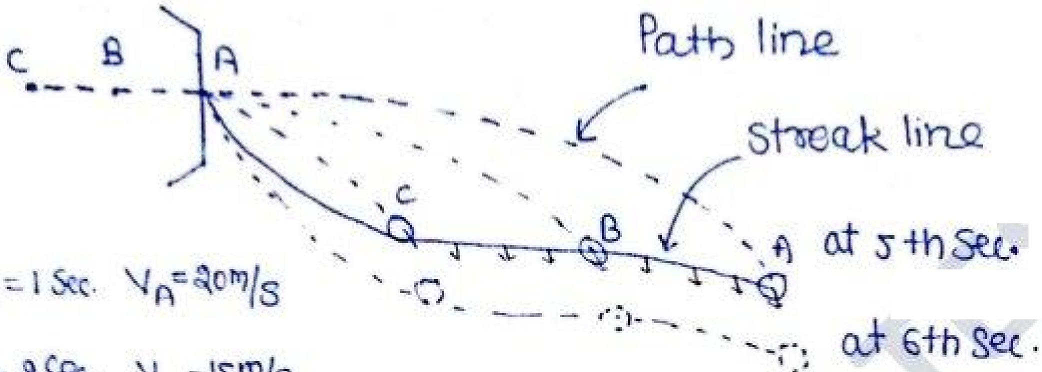


Path line:- It is the line by drawing the path of single fluid particle at diff time interval. It is defined by lagrangian description



Streak line:- It is an instanteneous picture of all the particles passing through single point

* Unsteady flow

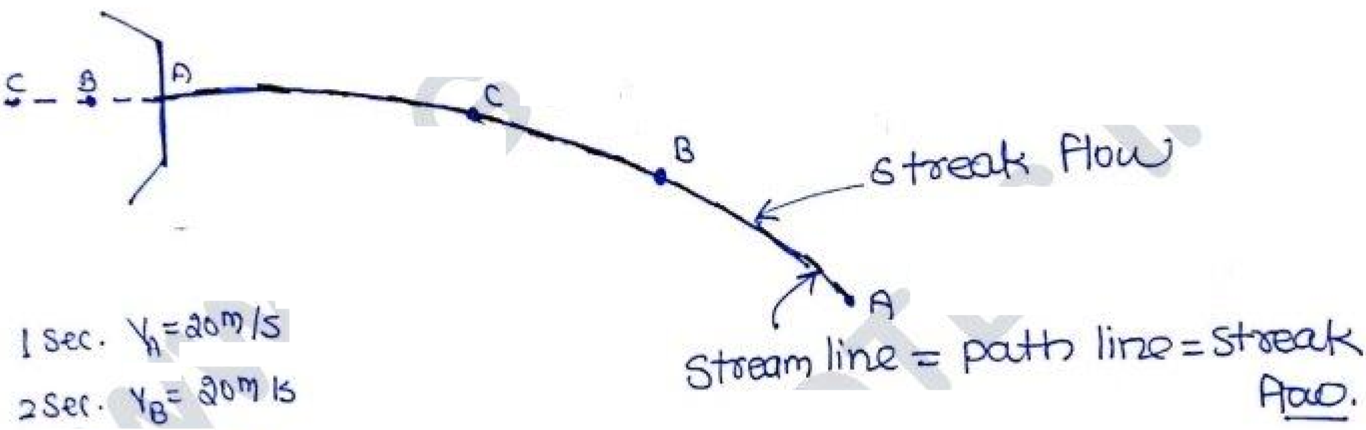


$t = 1$ sec. $V_A = 20 m/s$

$t_2 = 2$ sec. $V_B = 15 m/s$

$t_3 = 3$ sec. $V_C = 10 m/s$

* Steady flow.

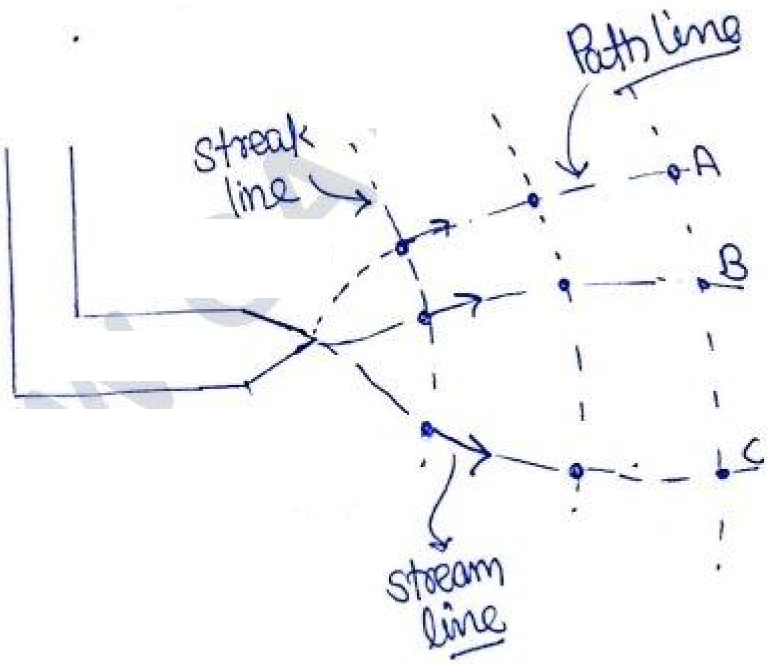


1 sec. $V_A = 20 m/s$

2 sec. $V_B = 20 m/s$

3 sec. $V_C = 20 m/s$

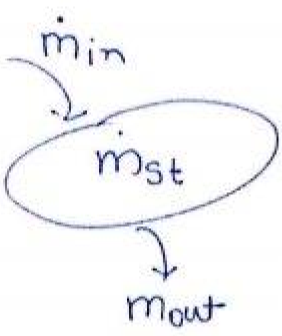
$$\frac{dv}{dt} = 0$$

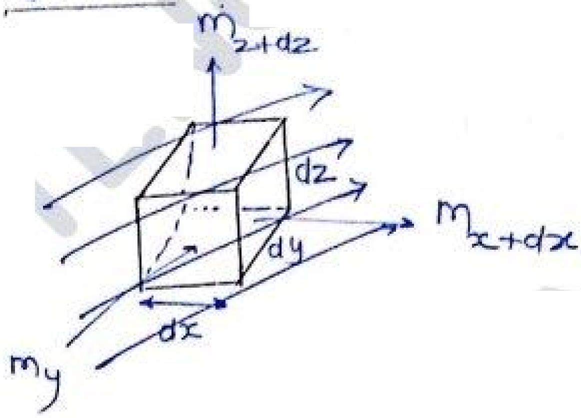


* In steady flow all three lines graphically identical
* In unsteady flow stream line shown intanteneous picture of flow.

Continuity equation/ Conservation of mass :-

In cartesian coordinate system :-





$\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{st}$

m_x

For x - coordinate

LHS

$$= m_x - \left(m_x + \frac{\partial \dot{m}_x}{\partial x} dx\right) =$$

$$m_x = \rho\,(dydz\,u)$$

$\rho = f(x, y, z, t)$

$u = f(x, y, z, t)$

$$= -\frac{\partial(\rho\, dydz\, u)}{\partial x} dx$$

Similarly

$$\Rightarrow -\frac{\partial}{\partial x}(\rho u)\, dv \qquad -\frac{\partial(\rho v)}{\partial y} dv, \qquad -\frac{\partial(\rho w)}{\partial z} dv$$

RHS

$$\dot{m}_{st} = \frac{d}{dt}(\dot{m}) = \frac{d}{dt}(\rho dv) = dv \frac{d\rho}{dt}$$

So $m_{in} - m_{out} = \dot{m}_{st}$

$$-\frac{\partial}{\partial x}(\rho u)dv - \frac{\partial}{\partial y}(\rho v)\,dv - \frac{\partial}{\partial z}(\rho w) = dv.\frac{\partial \rho}{\partial t}$$

$$\boxed{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0}$$

General Continuity eqn

{Scalar u, v, w निकालने के लिए होगा)

(i) Steady Flow: $\frac{\partial \rho}{\partial t} = 0$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

(ii) $$\frac{\partial(\rho u)}{dx} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

$$\rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial y} + v\frac{\partial \rho}{\partial y} + \rho\frac{\partial w}{\partial z} + w\frac{\partial \rho}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

$$\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

$$\rho(\vec{\nabla}\cdot\vec{V}) + \underbrace{(\vec{V}\cdot\vec{\nabla})\rho + \frac{\partial \rho}{\partial t}} = 0$$

$$\boxed{\rho(\vec{\nabla}\cdot\vec{V}) + \frac{D\rho}{Dt} = 0}$$

General Continuity eqn

(iii) Steady incompressible flow:-

$$\frac{D\rho}{Dt} = 0$$

$$\rho(\vec{\nabla}\cdot\vec{V}) = 0 \qquad \rho \neq 0$$

$$\boxed{(\vec{\nabla}\cdot\vec{V}) = 0}$$

(iv) Unsteady Incompressible flow:

$$\boxed{(\nabla\cdot\vec{V}) = 0} \qquad \frac{D\rho}{Dt} = 0$$

* For incompressible flow field $(\nabla\cdot\vec{V}) = 0$ {Always}

Question:- The velocity component of x & y dirⁿ are given by $u = \lambda xy^3 - x^2y$, $v = xy^2 - \frac{3}{4}y^4$ then the value of λ for incompressible fluid flow.

Sol

$$\nabla\cdot\vec{V} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(\lambda xy^3 - x^2y) + \frac{\partial}{\partial y}(xy^2 - \tfrac{3}{4}y^4) = 0$$

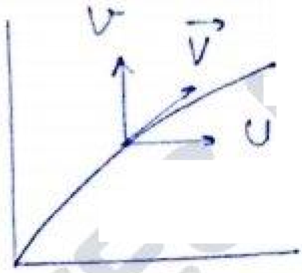
$$\Rightarrow \lambda y^3 - 2xy + 2xy - \tfrac{3}{1}y^3$$

$$\lambda = 3$$

Sol

$$u = \lambda x y^3 - x^2 y$$

$$v = x y^2 - \frac{3}{4} y^4$$



$$\rho(\nabla \cdot \vec{V}) + \cancelto{0}{\frac{D\rho}{Dt}} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancelto{0}{\frac{\partial w}{\partial z}} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$(\lambda y^3 - 2xy) + (2xy - 3y^3) = 0$$

$$(\lambda - 3) y^3 = 0$$

$$\Downarrow$$

$$\boxed{\lambda = 3} \quad 0 = 0$$

Ans

For $\lambda = 3$ we can defined velocity field

Note: each velocity field must satisfield it's continuity eqn otherwise the velocity field is not defined.

Q.6 $$\rho(\nabla \cdot \vec{V}) + \frac{D\rho}{Dt} = 0$$

$$1.2(\nabla \cdot \vec{V}) + 2.4 = 0$$

$$\nabla \cdot \vec{V} = -2 \ s^{-1}$$

Q.7

$$\vec{V} = (5x + 6y + 7z)\hat{i} + (6x + 5y + 9z)\hat{j} + (3x + 2y + \lambda z)\hat{k}$$

$$\rho = \rho_0 \exp(-2t) \qquad \nabla \cdot \vec{V} = (5 + 5 + \lambda)$$

$$\rho(\nabla \cdot \vec{V}) + \frac{D\rho}{Dt} = 0$$

$$\rho_0 e^{-2t}(5 + 5 + \lambda) + \rho_0(-2)e^{-2t} = 0$$

$$\lambda + 10 - 2 = 0$$

$$\lambda = -8$$

or

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

$$\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] + \frac{\partial \rho}{\partial t} = 0$$

$$\rho_0 e^{-2t}(5 + 5 + \lambda) + \rho_0(-2)e^{-2t} = 0$$

$$10 + \lambda - 2 = 0$$

$$\boxed{\lambda = -8}$$

Q.5

$$\cancel{\frac{1}{x}\left(2x + \frac{\partial v}{\partial y}\right) = 0}$$

$$\cancel{2 + \frac{1}{x}\frac{\partial v}{\partial y} = 0} \qquad \cancel{\frac{dv}{dy} = -2x}$$

$$v_y = -2xy + c$$

Q.5

$V_x = x^2$

$\rho = \frac{1}{x}$ $\quad$ Steady Flow

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \cancel{\frac{\partial \rho}{\partial t}} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{x} \times x^2\right) + \frac{\partial}{\partial y}\left(\frac{v}{x}\right) + 0$$

$$1 + \frac{\partial}{\partial y}\left(\frac{v}{x}\right) = 0$$

$$V_y = -xy + f(x)$$

Q.4 incompressible ρ = Const.

$$\nabla \cdot \vec{V} = 0$$

$$\frac{\partial}{\partial x}(2x^2 + z^2 + 6) + \frac{\partial}{\partial y}(y^2 + 2z^2 + 7) + \frac{\partial}{\partial z}(w) = 0$$

$$4x + 2y + \frac{\partial}{\partial z}(w) = 0$$

$$w = -4xz - 2yz + f(x, y)$$

Q.4 $\vec{V} = (x+2y+2)\hat{i} + (4-y)\hat{j}$

$\nabla \cdot V = 1 - 1 = 0$ (incompre

=

Ques Given the velocity field $V = 5x^3\hat{i} - 15x^2y\hat{j}$ obtain the eqn ~~st~~ for stream line

$$V = 5x^3\hat{i} - 15x^2y\hat{j}$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\frac{dx}{5x^3} = \frac{dy}{-15x^2y}$$

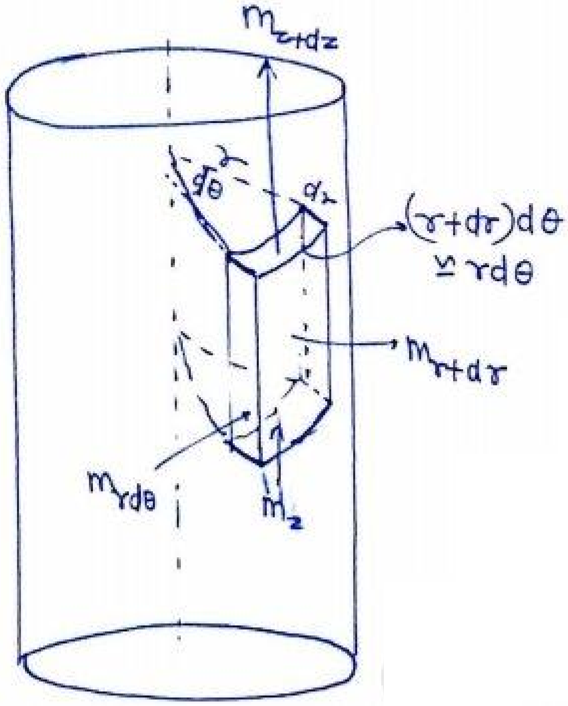
$$\frac{dy}{y} = -3\frac{dx}{x}$$

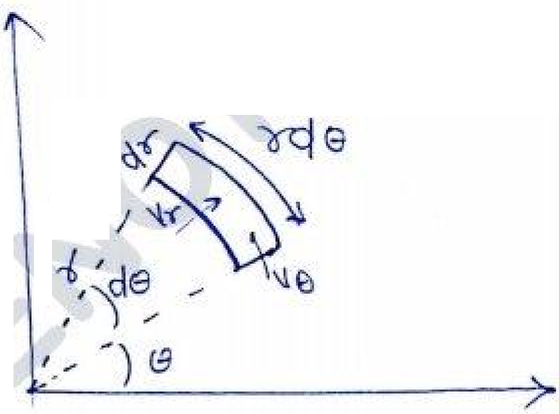
$$\ln y = -3\ln x + \ln c$$

$$\ln(yx^3) = \ln c$$

$$\boxed{yx^3 = c} \text{ or } \boxed{xy^{1/3} = c}$$

Continuity equation in polar Coordinate System:-





$$\dot m_{in} - \dot m_{out} = \dot m_{st}$$

x	y	z
$\downarrow$	$\downarrow$	$\downarrow$
r	$r d\theta$	z

$$u = \frac{dx}{dt} \qquad v_r = \frac{dr}{dt}$$

$$v = \frac{dy}{dt} \qquad v_\theta = \frac{r\,d\theta}{dt}$$

$$w = \frac{dz}{dt} \qquad v_z = \frac{dz}{dt}$$

General Cont. eq^n in polar Coordinate

$$\boxed{\frac{\partial(\rho r v_r)}{r\,\partial r} + \frac{\partial(\rho v_\theta)}{r\,d\theta} + \frac{\partial(\rho v_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

2-D incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

*
$$\boxed{\frac{\partial (r V_r)}{r\,dr} + \frac{\partial V_\theta}{r\,d\theta} = 0}$$

stream line eqn

$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \boxed{\frac{dr}{V_r} = \frac{r\,d\theta}{V_\theta}}$$ *

if both scalar given use stream line eqn

Que A leaf caught in a whirlpool At a given instant, the leaf at a distance of 120 m from the centre of the whirlpool. The whirlpool can be describe by the velocity distribution

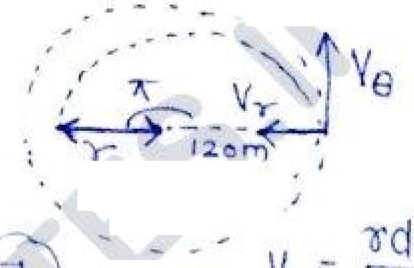
$$V_r = \frac{-60 \times 10^3}{2\pi r} \text{ m/s} \qquad V_\theta = \frac{300 \times 10^3}{2\pi r} \text{ m/s.}$$

where r is the distance in meter from centre what will be the distance of leaf from centre when it has moved through half a revolution.

(a) 48 m (b) 64 m (c) 120 m (d) 142 m

$\underline{Sol^n}$ Stream line eq^n $\dfrac{dr}{V_r} = \dfrac{r\,d\theta}{V_\theta}$

$$\frac{dr \times \cancel{2\pi r}}{-60\times10^3} = \frac{r\,d\theta}{300\times10^3} \times \cancel{2\pi r}$$



$$\frac{dr}{r} = -\frac{1}{5}\,d\theta$$

$$-5\ln r = \theta + c \quad (1)$$

$V_\theta = \dfrac{r\,d\theta}{dt}$

$$-5\ln(120) = 0 + c$$

$V_r = \dfrac{dr}{dt}$

$$c = -5\ln(120)$$

$dt = \dfrac{r\,d\theta}{V_\theta}$

$$-5\ln r = \theta - 5\ln(120)$$

$dt = \dfrac{dr}{V_r}$

$\theta = \pi$

$$\theta = 5\ln\left(\frac{120}{r}\right)$$

$$\pi = 5\ln\left(\frac{120}{r}\right)$$

$$r = \frac{120}{e^{\pi/5}} = \underline{\underline{64.01\,m}}$$

$$\Rightarrow \int_{120}^{r} \frac{dr}{r} = -\frac{1}{5}\int_{0}^{\pi} d\theta$$

$$\ln\left(\frac{r}{120}\right) = -\frac{1}{5}\pi$$

$$\boxed{r = 64.0185\ m}$$

Acceleration of Fluid particle:-

In fluid flow velocity is the function of space and time so the acc^n is the function of space and time. The space component is known as convective acc^n and time component is ~~kno~~ local accelaration.

$$\vec{V} = f(x, y, z, t)$$

$$\boxed{\vec{a} = \frac{D\vec{V}}{Dt} = \underbrace{(\vec{V}\cdot\vec{\nabla})}_{\text{Convective } acc^n}\vec{V} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local } acc^n}}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$a_x = \frac{du}{dt}, \quad a_y = \frac{dv}{dt}, \quad a_z = \frac{dw}{dt}$$

$$u = f(x, y, z, t)$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz + \frac{\partial u}{\partial t}dt$$

$$a_x = \frac{du}{dt}$$

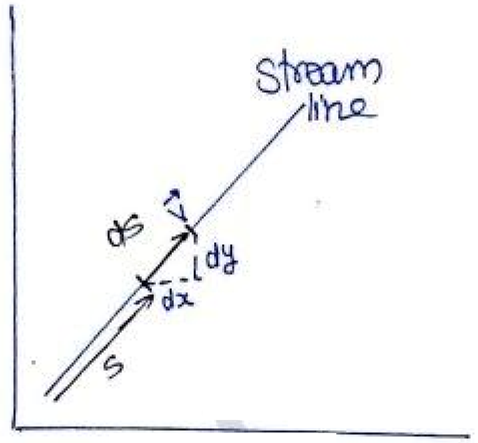
$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_x = (\vec{V}\cdot\vec{\nabla})u + \frac{\partial u}{\partial x}$$

$$a_y = (\vec{V}\cdot\vec{\nabla})v + \frac{\partial v}{\partial y}$$

$$a_z = (\vec{V}\cdot\vec{\nabla})w + \frac{\partial w}{\partial y}$$

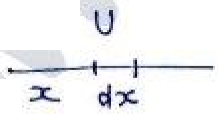
*



$$\vec{V} = f(s, t)$$

$$\vec{a} = \left(\vec{V}\frac{\partial \vec{V}}{\partial s} + \frac{\partial \vec{V}}{\partial t}\right)\hat{s}$$

1. D

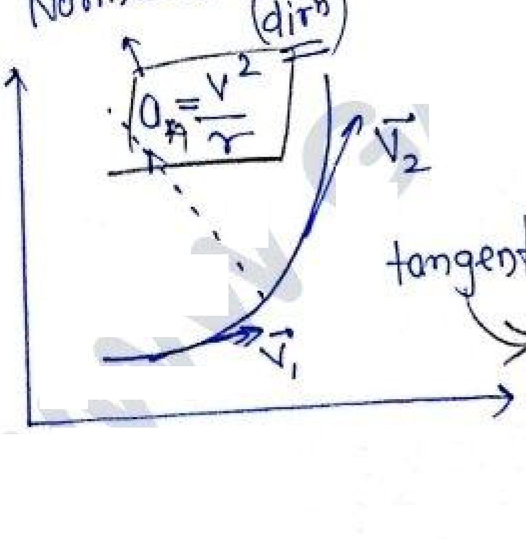


$$\vec{V} = u\hat{i} + 0\hat{j} + 0\hat{k}$$

$$d\vec{s} = dx\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{a} = a_x\hat{i} + 0\hat{j} + 0\hat{k}$$

$$a_x = u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$



tangential acc^n (mag)

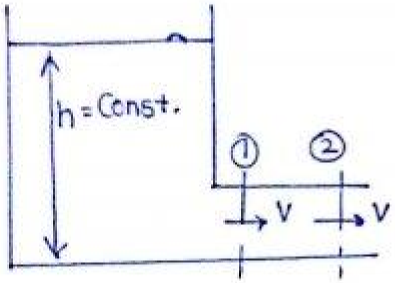
$$\boxed{\vec{a}_{tag} = \left(\vec{V}\frac{dv}{ds} + \frac{dv}{dt}\right)}$$

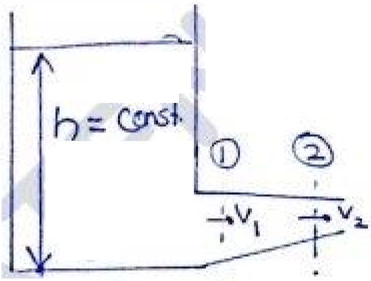
along stream line

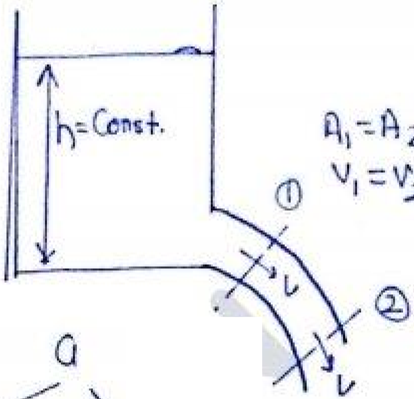
$$\vec{a} = a_{tan}\,\hat{s} + a_x\,\hat{n}$$

*
$$\boxed{\vec{a}_{total} = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)\hat{s} + \left(\frac{V^2}{r}\right)\hat{n}}$$

* **Imp**







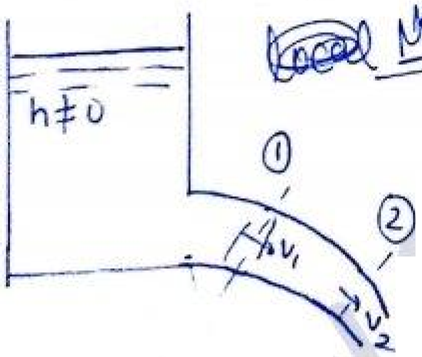
$A_1 = A_2$
$V_1 = V_2$

(mag.) a (dirn)

$a_{tang.}$	a_n	a_t	a_n	a_t	a_n
$\Rightarrow a_{t_c} = 0$	$\Rightarrow a_{n_c} = 0$	$a_{t_c} \neq 0$	$a_{n_c} = 0$	$a_{t_c} = 0$	$a_{n_c} \neq 0$
$\Rightarrow a_{t_L} = 0$	$\Rightarrow a_{n_L} = 0$	$a_{t_L} = 0$	$a_{n_L} = 0$	$a_{t_L} = 0$	$a_{n_L} = 0$

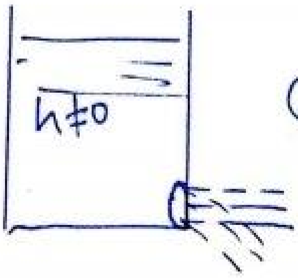
~~Conv.~~ tangential a_{n_c} / a_{t_c} (b/w two points) Convective

~~Local~~ Normal a_{n_L} / a_{t_L} (at a point) local

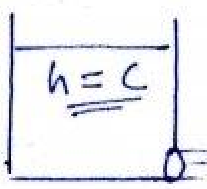


a

a_t	a_n
$a_{t_c} \neq 0$	$a_{n_c} \neq 0$
$a_{t_L} \neq 0$	$a_{n_L} = 0$



$a_{n_L} \neq 0$

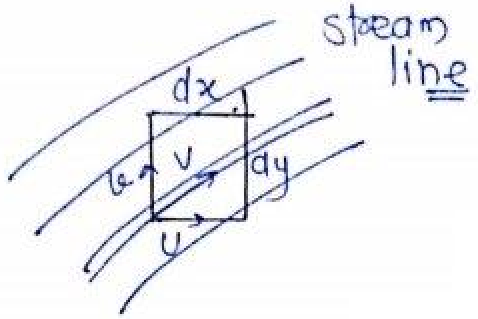


$a_{n_L} = 0$

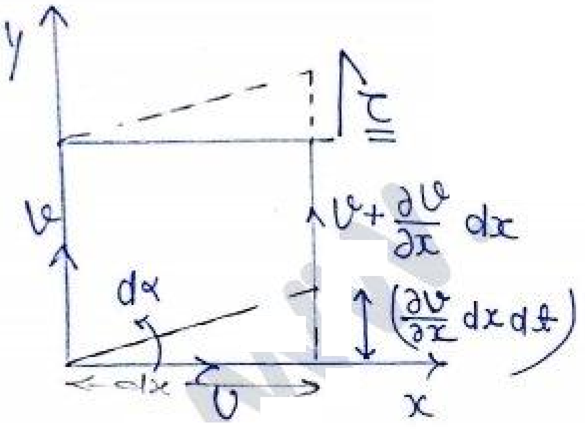
Rotation of Fluid Motion:-

$u = F(x, y)$
$v = F(x, y)$

(variation in $\perp$ dirn)
Rotation



$$\omega_{mean} = \frac{\omega_1 + (-\omega_2)}{2} = \frac{\omega_1 - \omega_2}{2}$$



$$\tan(d\alpha) = \frac{\frac{\partial v}{\partial x} dx\, dt}{dx}$$

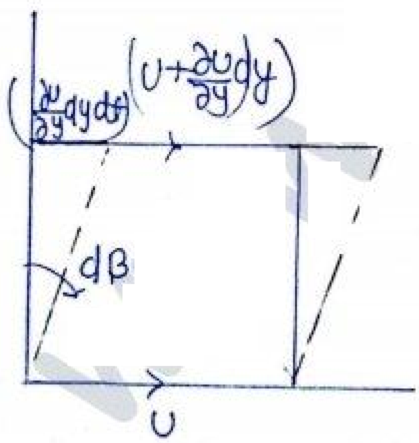
Small angle

$$d\alpha = \frac{\frac{\partial v}{\partial x} dx\, dt}{dx}$$

$\left(\omega = \frac{d\theta}{dt}\right)$

Rate of Shear strain

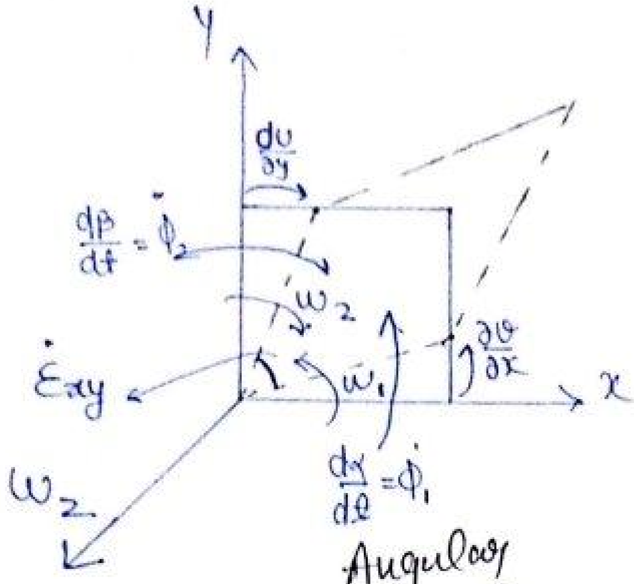
$$\omega_1 = \frac{d\alpha}{dt} = \frac{\partial v}{\partial x} \quad \text{(ACW)} \quad +ve$$



Similarly

$$\frac{d\beta}{dt} = \frac{\partial u}{\partial x}$$

$$\omega_2 = \frac{\partial u}{\partial x} \quad -ve$$



Angular Velocity

$$\omega_{mean} = \frac{\omega_1 + (-\omega_2)}{2} = \omega_z$$

$$\omega_z = \frac{\omega_1 - \omega_2}{2}$$

$$\boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$

Angular Velocity

$$\dot{\varepsilon}_{xy} = \frac{|\phi_1| + |\phi_2|}{2}$$

Rate of shear strain

$$\boxed{\dot{\varepsilon}_{xy} = \frac{1}{2}\left[\left|\frac{dv}{dx}\right| + \left|\frac{du}{dy}\right|\right]}$$

Rate of shear strain/deformation

Q.22 $u = 6y, \quad v = 0$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}(0-6) = -3$$

$$\ddot{\varepsilon}_x = \frac{1}{2}\left[0 + |-6|\right) = 6$$

Note:-

Rotationality

$$\boxed{\omega = \frac{1}{2}(\text{curl } \vec{V})}$$

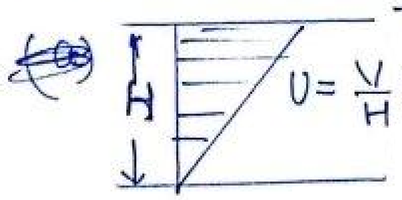
$$\boxed{\omega = \frac{1}{2}\left(\vec{\nabla} \times \vec{V}\right)}$$

$$\omega = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\omega = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

Ques The laminar flow takes place b/w closely spaced parallel plate as shown in fig Velocity profile is given by $u = \frac{V}{H}y$. The rate of rotation of fluid particles is given.



$$\vec{V} = u\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{V} = \frac{V}{H}y\,\hat{i}$$

$$\omega = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{V}{h}y & 0 & 0 \end{vmatrix}$$

$$\omega = \frac{1}{2}(0-0)\hat{i} - \frac{1}{2}(0-0)\hat{j} + \frac{1}{2}\left(0 - \frac{V}{h}\right) = 0\hat{i} + 0\hat{j} - \frac{V}{2h}\hat{k}$$

irrotational flow :-

$$\omega = 0 \quad , \omega_x = \omega_y = \omega_z = 0$$

$$\frac{1}{2}(\vec{\nabla} \times \vec{V}) = 0$$

$$\text{curl } \vec{V} = 0$$

Q.30

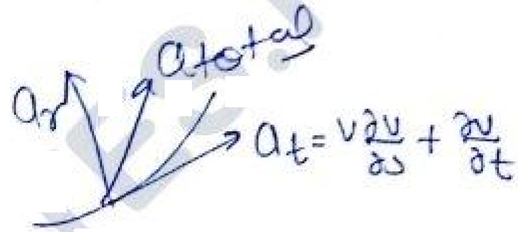
$$V = 2t\left(1 - \frac{x}{2L}\right)^2$$

a_n = Normal acc^n $= 0$

$$V = f(x, t)$$

$$a_{total} = V\left(\frac{\partial V}{\partial s}\right) + \left(\frac{\partial V}{\partial t}\right)$$

$$= \underbrace{V\left(\frac{\partial V}{\partial x}\right)}_{\text{Conv.}} + \underbrace{\frac{\partial V}{\partial t}}_{\text{local}}$$



(i) $a_c = V\frac{\partial V}{\partial x} = 2t\left(1-\frac{x}{2L}\right)^2\left[2t \times 2\left(1-\frac{x}{2L}\right)\left(-\frac{1}{2L}\right)\right]$

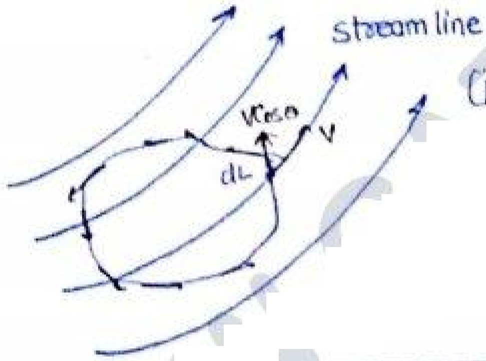
$$a_c = -14.623 \text{ m/s}^2$$

(ii) $a_{local} = \frac{\partial V}{\partial t} = 2\left[1 - \frac{x}{2L}\right) =$

(iii) $a_{to} = -13.62 \text{ m/s}^2$

Circulation:-

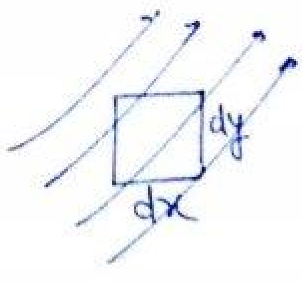
- It is the line integration of tangential component of velocities along closed loop.
- It is measure of rotationality in the flow.
- It does not depend on shape of the loop.
- Counter cw movement should be considered +ve.

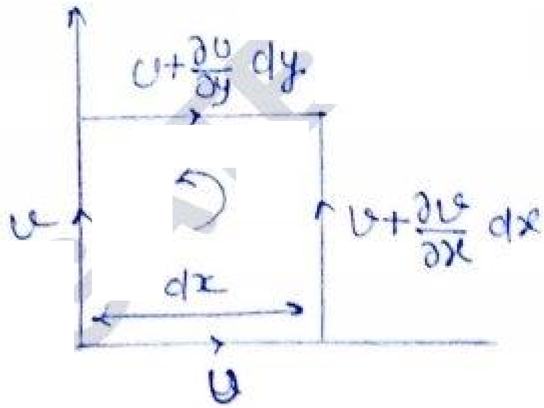


Circulation $\Gamma = \oint V\cos\theta \cdot dL$

$$\Gamma = \oint \vec{V}\cdot d\vec{L}$$

$$\boxed{\oint_L \vec{V}\cdot d\vec{L} = \iint_A (\vec{\nabla}\times\vec{V})\cdot\hat{n}\, dA}$$





$$\Gamma_{ABCD} = \Gamma_{AB} + \Gamma_{BC} + \Gamma_{CD} + \Gamma_{DA}$$

$$= u\,dx\cos 0^\circ + \left(v+\frac{\partial v}{\partial x}dx\right)\cos 0^\circ + \left(u+\frac{\partial u}{\partial y}dy\right)dx\cos 180^\circ$$

$$+ v\,dy\cos 180^\circ$$

$$\Gamma_{ABCD} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dx\,dy$$

Circulation $$\boxed{\Gamma_{ABCD} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dA}$$

$$\frac{\Gamma_{ABCD}}{dA} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$\boxed{\frac{\Gamma_{ABCD}}{dA} = \Omega} \quad \text{(Vorticity)}$$

$$\boxed{\Omega = 2\omega}$$

$$\boxed{\Omega = \text{Curl}\,\vec{V} = \vec{\nabla} \times \vec{V}}$$

$$\Omega = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \Omega_x \hat{i} + \Omega_y \hat{j} + \Omega_z \hat{k}$$

Vorticity :- Vorticity is the mathematical measure of rotationality. It is defined as circulation per Area. It is twice of Rotation.

The dir^n of vorticity is same as dir^n of rotation

$$\Omega = 2\omega$$

Q.24 $\Gamma = \iint (\nabla \times \vec{V}) \cdot \hat{n} \, dA$

$\nabla \times \vec{V}$

$\Gamma = \Omega \times A$

$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) A$

$\Gamma = [0-3] \pi \, 2^2$

$\Gamma = -12\pi$ unit

Q.25 $\vec{V} = K(y\hat{i} + x\hat{k})$

$$\Omega = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ky & 0 & kx \end{vmatrix}$$

$\Omega = (0-0)\hat{i} - \hat{j}(k-0) + (0-k)\hat{k}$

$\Omega = -k\hat{j} - k\hat{k}$

$\boxed{\Omega_z = -k}$ Ans

Velocity potential function

$$\phi = f(x, y, z, t)$$

* Scalar function
* function of space & time
* it is defined for irrotational flow
* it is defined in such a way the $-ve$ derivative of potential function in any direction gives velocity component in that dirn

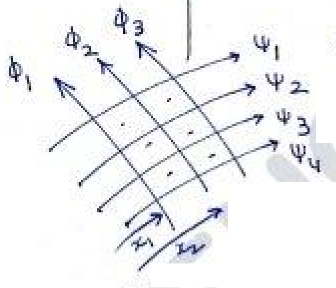
* $dx > 0$, $d\phi < 0$

$$\left(-\frac{d\phi}{dx}\right) > 0$$

* $-\frac{\partial \phi}{\partial x} = u$, $-\frac{\partial \phi}{\partial y} = v$, $\frac{-\partial \phi}{\partial z} = w$

steady, incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$



$dx > 0$

$\phi_1 > \phi_2 > \phi_3$

$d\phi < 0$

Stream function

$$\psi = f(x, y, z, t)$$

* It is the scalar function
* function of space & time
* it is defined for continuity equation i.e. for conservation of mass.
* The derivative of stream function in any direction gives velocity scalar in $\perp$er dirn (ccw and ~~cw~~)

*

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$

2D steady incompressible.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$-\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial y^2} - \frac{\partial^2\psi}{\partial z^2} = 0$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad , \nabla^2\phi = 0$$

$\nabla^2 f(x)$ — poissoin $\neq 0$; laplace $= 0$

$\nabla^2\phi$

laplace: $\nabla^2\phi = 0$, $0 = 0$

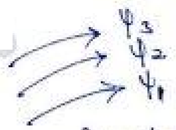
Poissions: $\nabla^2\phi \neq 0$, $\cancel{\phi} \neq 0$

$$\boxed{\nabla^2\phi = 0}$$ (only defined for irrotational)

$\nabla^2\phi = 0$

$\nabla^2\psi = 0$

irrotation



$\nabla^2\psi \neq 0$

Rotation

$$\frac{\partial}{\partial x}\left(\frac{-\partial\psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\right) = 0$$

$$-\frac{\partial^2\psi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial x\partial y} = 0$$

$$0 = 0$$

Rotationality $\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial\psi}{\partial y}\right)\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right]$$

$$\omega_z = \frac{1}{2}\left(\nabla^2\psi\right)$$

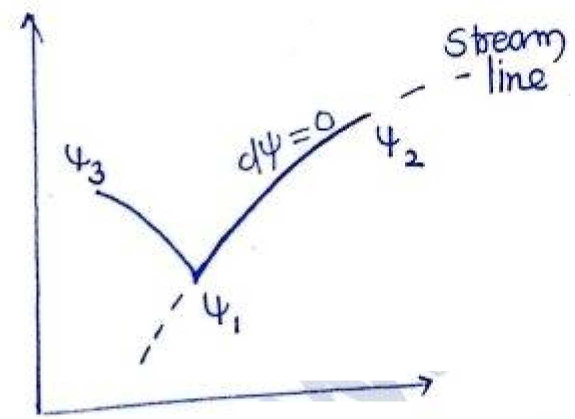
laplace: $\nabla^2\psi = 0$, $\omega_z = 0$ — irrotational

poission: $\nabla^2\psi \neq 0$, $\omega_z \neq 0$ — Rotational

* Potential function (ϕ) defined for irrotational flow so irrotational flow is also known as potential flow.
* Potential flow is defined for 3-D flow.
* If potential function satisfied laplace eq^n then potential function is valid

* Stream function (ψ) defined for both rotational & irrotational flow
* It is defined for 2-D flow
* If ψ satisfied laplace eq^n then flow is irrotational.

Physical significance of stream function (ψ):-



$$\psi = F(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$d\psi = v\,dx - u\,dy \qquad \because \frac{dx}{u} = \frac{dy}{v}$$

$\boxed{d\psi = 0}$ along stream line.

Stream function is the point function. The locus of constant stream functions is the stream line i.e. for the given stream line there is a given stream function.

* The discharge per unit depth in flow field is defined by

$$Q_{12} = |\psi_1 - \psi_2|$$

$\Rightarrow$ $d\psi = v\,dx - u\,dy$

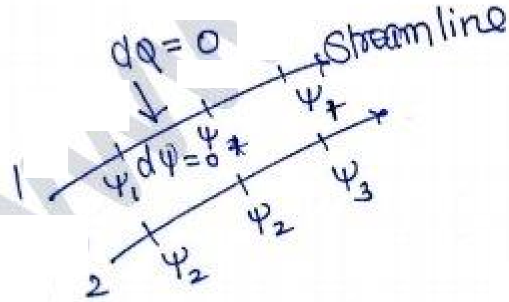
$$\frac{dx}{u} = \frac{dy}{v} \quad \text{stream line}$$

$$v\,dx - u\,dy = 0$$

$d\psi = 0$ along stream line

$\boxed{\int d\psi = 0}$ $\boxed{\psi = \text{Const.}}$

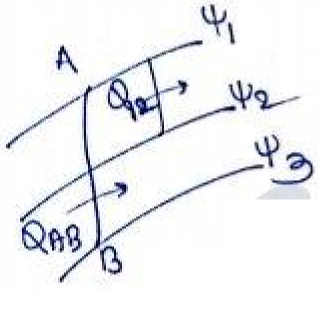
*



$$|dQ| = |d\psi|$$

$\downarrow$

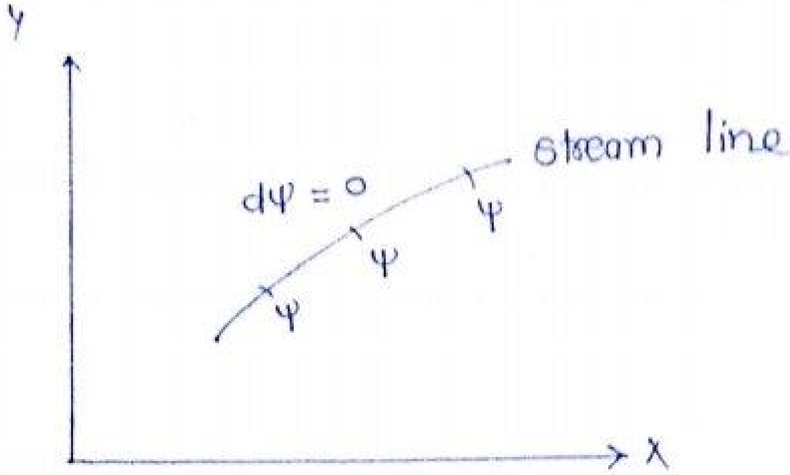
discharge per unit depth



$$Q_{12} = |\psi_1 - \psi_2|$$

$$\boxed{Q_{AB} = |\psi_A - \psi_B|}$$

Note:-

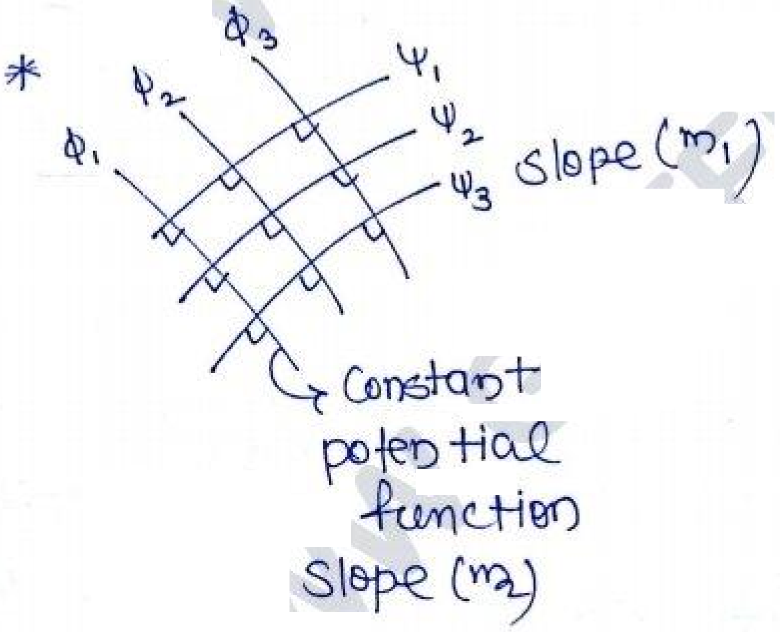


$$\frac{dx}{u} = \frac{dy}{v}$$

Slope of stream line $\Rightarrow \dfrac{dy}{dx} = \dfrac{v}{u}$, $m_1 = \dfrac{v}{u}$

$$* \boxed{\frac{dy}{dx} = \frac{\left(\frac{\partial\psi}{\partial x}\right)}{\left(\frac{-\partial\psi}{\partial y}\right)}}$$

*



$$m_1 \times m_2 = -1$$

$$m_2 = -\frac{1}{m_1} = -\frac{u}{v}$$

$$\boxed{m_2 = -\frac{u}{v}}$$

Cauchy - Riemann eqn:- (C·R eqn)

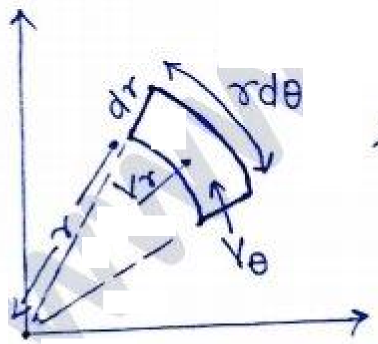
$$-\frac{\partial \phi}{\partial x} = u \qquad \frac{\partial \psi}{\partial x} = v$$

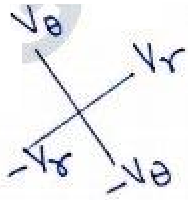
$$-\frac{\partial \phi}{\partial y} = v \qquad \frac{\partial \psi}{\partial y} = -u$$

$$\boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}} \qquad \boxed{\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}}$$

Valid for irrotational flow

In polar Coordinates:-





$$-\frac{\partial \phi}{\partial r} = V_r, \quad \frac{\partial \psi}{\partial r} = V_\theta$$

$$-\frac{\partial \phi}{r\partial \theta} = V_\theta, \quad \frac{\partial \psi}{r\partial \theta} = -V_r.$$

$$\boxed{\frac{\partial \phi}{\partial r} = \frac{\partial \psi}{r\partial \theta}}, \quad \boxed{-\frac{\partial \phi}{r\partial \theta} = \frac{\partial \psi}{\partial r}}$$

Q 34
P.928
WB

$u = \frac{75}{1} y \qquad u = 75y$

$v = 0$ (assume)

$$\frac{\partial \psi}{\partial y} = -u = -75y$$

$$\partial \psi = -75y\,dy$$

$$\psi = -37.5y^2 + f(x)$$

$$\frac{\partial \psi}{\partial x} = v = 0$$

$$0 + f'(x) = 0$$

$$f(x) = C$$

$$\psi = -37.5y^2 + c$$

Now find $\phi = ?$

$$-\frac{\partial \phi}{\partial x} = u$$

$$\frac{\partial \phi}{\partial x} = -75y$$

$$\phi = -75xy + f(y)$$

$$-\frac{\partial \phi}{\partial y} = v = 0$$

$$\frac{\partial \phi}{\partial y} = -75x + f'(y) = 0$$

$$f'(y) = 75x$$

or by C-R eqn

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y}$$

$$\frac{\partial \phi}{\partial x} = -75y$$

$$\phi = -75xy + c$$

wrong because $u = 75y$

ϕ can't be determined

Flow is rotational

Ques $\psi = m \ln r$

$$-\frac{\partial \phi}{r\partial \theta} = \frac{\partial \psi}{\partial r}$$

$$-\frac{\partial \phi}{r\partial \theta} = \frac{m}{r} \Rightarrow \phi = m\theta + c$$

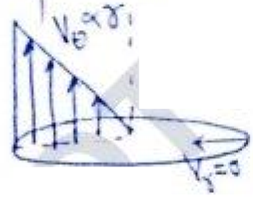
Vortex Motion:-

Forced vortex:-

$\tau \neq 0$

rotational

$\left\{ \begin{array}{l} \psi \checkmark \\ \phi \times \end{array} \right.$



$$\boxed{V_\theta = r\omega}$$

$$V_r = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial(rV_\theta)}{r\partial r} - \frac{\partial(V_r)}{r\partial\theta}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial(r^2\omega)}{r\partial r} - 0\right)$$

$$\omega_z = \frac{1}{2}\cdot\frac{\omega}{r}\cdot 2r$$

$$\boxed{\omega_z = \omega}$$ Rotational

Trick

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

$$\frac{\partial(rV_r)}{r\partial r} + \frac{\partial(V_\theta)}{r\partial\theta} = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial(rV_\theta)}{r\partial r} - \frac{\partial(V_r)}{r\partial\theta}\right)$$

Free Vortex

$T = 0$

$F \times r = 0$

$\left\{ \begin{array}{l} \psi \checkmark \\ \phi \checkmark \end{array} \right.$

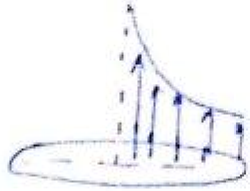
$$\frac{d\,mv}{dt} \times r = 0$$

$$\frac{d}{dt}(mVr) = 0$$

$$mV_\theta r = C$$

$$\boxed{V_\theta r = C}$$

$$\boxed{V_\theta = \frac{c}{r}}$$



$r \to 0 , \quad V_\theta \to \infty$

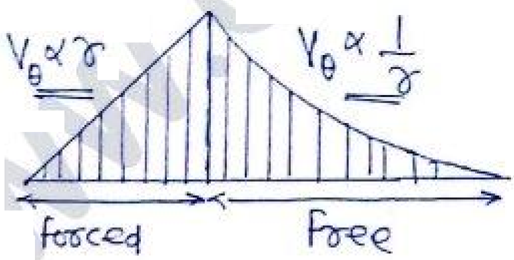
$$\omega_z = \frac{1}{2}\left(\frac{\partial (r V_\theta)}{r \partial \theta} - \cancel{\frac{\partial V_r}{r \partial \theta}}^{\,0} \right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial c}{r \partial \theta} \right)$$

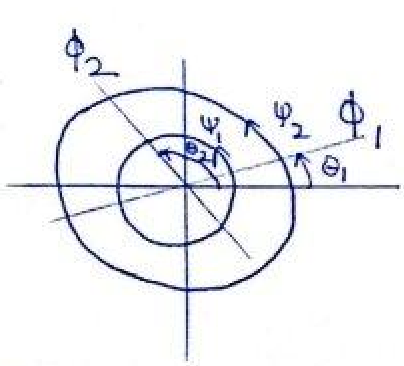
$$\boxed{\omega_z = 0}$$

irrotation flow

*



free vortex → ϕ, ψ



$V_\theta = \frac{c}{r} \qquad V_r = 0$

$$-\frac{\partial \phi}{r \partial \theta} = V_\theta \qquad \frac{\partial \psi}{\partial r} = V_\theta$$

$$-\frac{\partial \phi}{r \partial \theta} = \frac{c}{r} \qquad \frac{\partial \psi}{\partial r} = \frac{c}{r}$$

$$\boxed{\phi = -c\theta + c_1} \qquad \boxed{\psi = c \ln r + c_2}$$

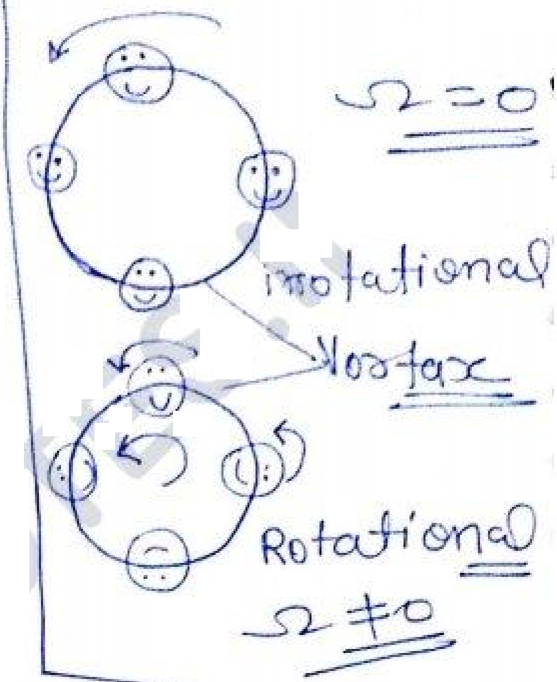
with θ with radius

Sink/Source $V_r \neq 0$

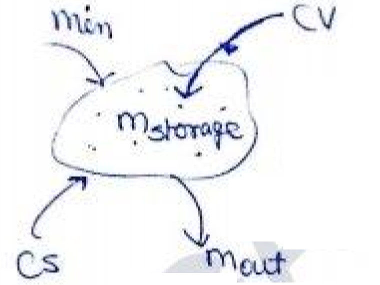
$$V_\theta = 0$$

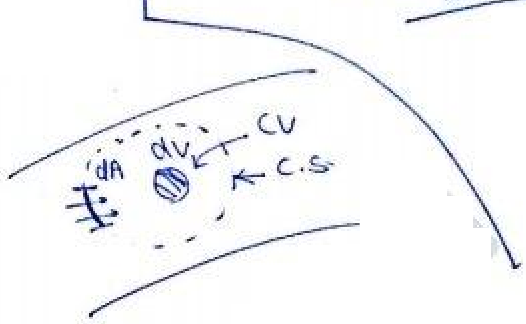
$$\phi = C \ln r + C_2$$

$$\psi = -C\theta + C_1$$



Continuity eqn





$$\cancel{\frac{dM}{dt}}^{\,0} = \int_{C.S} \underbrace{\rho V dA}_{kg/s} + \underbrace{\frac{d}{dt}\int \rho dV}_{1/s} \quad m = kg$$

$$* \quad \boxed{\int_{C.S.} \rho V dA + \frac{d}{dt}\int_{C.V.} \rho dV = 0}$$ Continuity eqn

Now $\dot{P} = \dot{m}.V$ momentum

$$* \quad \boxed{\dot{P} = \int_{C.S.} \rho V^2 dA + \frac{d}{dt}\int_{C.V.} \rho V . dV}$$