

Chapter - Gravitation



Topic-1: Kepler's Laws of Planetary Motion



1 MCQs with One Correct Answer

1. A binary star system consists of two stars A and B which have time period T_A and T_B , radius R_A and R_B and mass M_A and M_B . Then [2006 - 3M, -1]
(a) if $T_A > T_B$ then $R_A > R_B$
(b) if $T_A > T_B$ then $M_A > M_B$
(c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$
(d) $T_A = T_B$
2. If the distance between the earth and the sun were half its present value, the number of days in a year would have been [1996 - 2 Marks]
(a) 64.5 (b) 129
(c) 182.5 (d) 730



6 MCQs with One or More than One Correct Answer

3. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T . If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$ [1989 - 2 Mark]
(a) T^2 is proportional to R^3
(b) T^2 is proportional to $R^{7/2}$
(c) T^2 is proportional to $R^{3/2}$
(d) T^2 is proportional to $R^{3/73}$



10 Subjective Problems

4. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S_2 is closest to S_1 , find
(i) the speed of S_2 relative to S_1 ,
(ii) the angular speed of S_2 as actually observed by an astronaut in S_1 . [1986 - 6 Marks]



Topic-2: Acceleration due to Gravity



1 MCQs with One Correct Answer

1. A planet of radius $R = \frac{1}{10} \times (\text{radius of Earth})$ has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and a linear mass density $10^{-3} \text{ kg m}^{-1}$ into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person

holding it in place is (take the radius of Earth $= 6 \times 10^6 \text{ m}$ and the acceleration due to gravity on Earth is 10 ms^{-2})

[Adv. 2014]

- (a) 96 N (b) 108 N (c) 120 N (d) 150 N



4 Fill in the Blanks

2. The numerical value of the angular velocity of rotation of the earth should berad/s in order to make the effective acceleration due to gravity equal to zero.

[1984 - 2 Marks]



Topic-3: Gravitational Field, Potential and Potential Energy



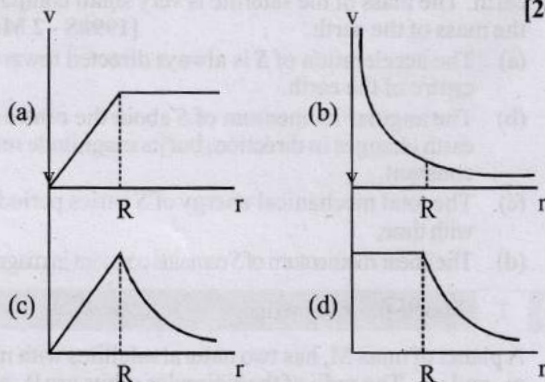
1 MCQs with One Correct Answer

1. A spherically symmetric gravitational system of particles

has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by –

[2008]



6 MCQs with One or More than One Correct Answer

2. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2 respectively. Then: [1994 - 2 Marks]

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

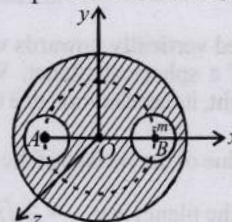
(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

3. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively, are taken out of the solid leaving behind spherical cavities as shown in fig

Then :

[1993-2 Marks]



- (a) The gravitational force due to this object at the origin is zero.
 (b) the gravitational force at the point $B(2, 0, 0)$ is zero.
 (c) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.
 (d) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.



Topic-4: Motion of Satellites, Escape Speed and Orbital Velocity



1 MCQs with One Correct Answer

1. A particle of mass m is under the influence of the gravitational field of a body of mass M ($\gg m$). The particle is moving in a circular orbit of radius r_0 with time period T_0 around the mass M . Then, the particle is subjected to an additional central force, corresponding to the potential

energy $V_c(r) = \frac{m\alpha}{r^3}$, where α is a positive constant of suitable dimensions and r is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius r_0 in the combined gravitational potential due to M and $V_c(r)$, but with a new time period T_1 , then $(T_1^2 - T_0^2)/T_1^2$ is given by [G is the gravitational constant.] [Adv. 2024]

(a) $\frac{3\alpha}{GM r_0^2}$

(b) $\frac{\alpha}{2GM r_0^2}$

(c) $\frac{\alpha}{GM r_0^2}$

(d) $\frac{2\alpha}{GM r_0^2}$

2. Two satellites P and Q are moving in different circular orbits around the Earth (radius R). The heights of P and Q from the Earth surface are h_P and h_Q , respectively, where $h_P = R/3$. The accelerations of P and Q due to Earth's gravity are g_P and g_Q , respectively. If $g_P/g_Q = 36/25$, what is the value of h_Q ? [Adv. 2023]

- (a) $3R/5$ (b) $R/6$ (c) $6R/5$ (d) $5R/6$

3. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? [2012]

(a) $\frac{5GmM}{6R}$

(b) $\frac{2GmM}{3R}$

(c) $\frac{GmM}{2R}$

(d) $\frac{GmM}{2R}$

4. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field

is $v_e = 11.2 \text{ km s}^{-1}$. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet) [Adv. 2017]

- (a) $v_s = 22 \text{ km s}^{-1}$ (b) $v_s = 42 \text{ km s}^{-1}$
(c) $v_s = 62 \text{ km s}^{-1}$ (d) $v_s = 72 \text{ km s}^{-1}$



2 Integer Value Answer

5. Two spherical stars A and B have densities ρ_A and ρ_B , respectively. A and B have the same radius, and their masses M_A and M_B are related by $M_B = 2M_A$. Due to an interaction process, star A loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains ρ_A . The entire mass of lost by A is deposited as a thick spherical shell on B with the density of the shell being ρ_A . If v_A and v_B are the escape velocities from A and B after the interaction process, the ratio

$$\frac{v_B}{v_A} = \sqrt{\frac{10n}{15^{1/3}}}$$
 the value of n is _____. [Adv. 2022]

6. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity

is $\frac{1}{4}$ th of its value of the surface of the planet. If the escape

velocity from the planet is $v_{\text{esc}} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) [Adv. 2015]

7. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km s^{-1} , the escape speed on the surface of the planet in km s^{-1} will be [2010]



4 Fill in the Blanks

8. A particle is projected vertically upwards from the surface of earth (radius R_e) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of earth is.... [1997 - 2 Marks]

9. The masses and radii of the Earth and the Moon are M_1, R_1 and M_2, R_2 respectively. Their centres are at a distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is [1988 - 2 Marks]

10. A geostationary satellite is orbiting the earth at a height of $6R$ above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of $2.5R$ from the surface of the earth is hours. [1987 - 2 Marks]



5 True / False

11. It is possible to put an artificial satellite into orbit in such a way that it will always remain directly over New Delhi. [1984 - 2 Marks]



6 MCQs with One or More than One Correct Answer

12. Two spherical planets P and Q have the same uniform density ρ , masses M_P and M_Q and surface areas A and $4A$ respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R , respectively. Then [2012]

- (a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$
(c) $V_R/V_P = 3$ (d) $V_P/V_Q = \frac{1}{2}$

13. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. [1998S - 2 Marks]

- (a) The acceleration of S is always directed towards the centre of the earth.
(b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(c) The total mechanical energy of S varies periodically with time.
(d) The linear momentum of S remains constant in magnitude.



7 Match the Following

14. A planet of mass M , has two natural satellites with masses m_1 and m_2 . The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. [Adv. 2018]

LIST-I

- P. v_1/v_2
Q. L_1/L_2
R. K_1/K_2
S. T_1/T_2

LIST-II

1. $1/8$
2. 1
3. 2
4. 8

- (a) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$
(b) $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 1$
(c) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$
(d) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 4; S \rightarrow 1$



9 Assertion and Reason Type Questions

15. STATEMENT - 1 : An astronaut in an orbiting space station above the earth experiences weightlessness.

because

STATEMENT - 2 : An object moving around the earth under the influence of Earth's gravitational force is in a state of "free-fall". [2008]

- (a) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True



10 Subjective Problems

16. A body is projected vertically upwards from the bottom of a crater of moon of depth $\frac{R}{100}$ where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. [2003 - 4 Marks]
17. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$, respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G , M and a . [1996 - 5 Marks]
18. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. [1990 - 8 Mark]
- Determine the height of the satellite above the earth's surface.
 - If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth.
19. Three particles, each of mass m , are situated at the vertices of an equilateral triangle of side length a . The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual separation a . Find the initial velocity that should be given to each particle and also the time period of the circular motion. [1988 - 5 Marks]



Topic-5: Miscellaneous (Mixed Concepts) Problems



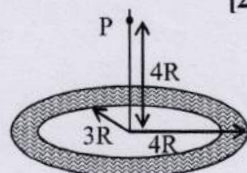
1 MCQs with One Correct Answer

1. Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common centre with the same kinetic energy K . The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time. The particle number density $n(r) = \rho(r)/m$ is [G is universal gravitational constant] [Adv. 2019]

- (a) $\frac{3K}{\pi r^2 m^2 G}$ (b) $\frac{K}{2\pi r^2 m^2 G}$
 (c) $\frac{K}{\pi r^2 m^2 G}$ (d) $\frac{K}{6\pi r^2 m^2 G}$

2. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is [2010]

- (a) $\frac{2GM}{7R}(4\sqrt{2}-5)$
 (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$
 (c) $\frac{GM}{4R}$



- (d) $\frac{2GM}{5R}(\sqrt{2}-1)$



2 Integer/Non-negative Integer Value Answer

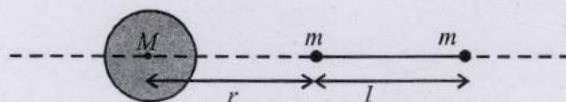
3. The distance between two stars of masses $3M_s$ and $6M_s$ is $9R$. Here R is the mean distance between the centers of the Earth and the Sun, and M_s is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT , where T is the period of Earth's revolution around the Sun. The value of n is ____.

4. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$

from M , the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$.

The value of k is

[Adv. 2015]



6 MCQs with One or More than One Correct Answer

5. Two bodies, each of mass M , are kept fixed with a separation $2L$. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G . The correct statement(s) is (are) [Adv. 2013]

- (a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$
 (b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$
 (c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$
 (d) The energy of the mass m remains constant



Answer Key

Topic-1 : Kepler's Laws of Planetary Motion

1. (d) 2. (b) 3. (b)

Topic-2 : Acceleration due to Gravity

1. (b) 2. $(1.24 \times 10^{-3} \text{ rad/s})$

Topic-3 : Gravitational Field, Potential and Potential Energy

1. (c) 2. (a,b) 3. (a,c,d)

Topic-4 : Motion of Satellites, Escape Speed and Orbital Velocity

1. (a) 2. (a) 3. (a) 4. (b) 5. (2.3) 6. (2) 7. (3) 8. (R)

9. $\left(\sqrt{\frac{4G}{d}} (M_1 + M_2) \right)$ 10. (8.48 h)

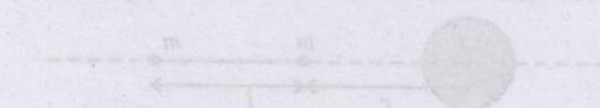
10. False 11. (b,d) 12. (a) 13. (b) 14. (a)

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (b) 2. (a) 3. (9) 4. (7) 5. (b,d)

Two point masses are kept on a horizontal surface. The centre of mass of the system is at a distance of 1 m from the mass of 1 kg. The mass of the other mass is

When the point mass is at a distance of 1 m from the other mass, the centre of mass is at a distance of 1 m from the other mass. The mass of the other mass is



Two bodies, each of mass M , are kept fixed with a separation $2L$. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G . The correct statement(s) is (are)

(a) The minimum initial velocity of the mass m to escape

the gravitational field of the two bodies is $\sqrt{\frac{GM}{L}}$

(b) The minimum initial velocity of the mass m to escape

the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(c) The minimum initial velocity of the mass m to escape

the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

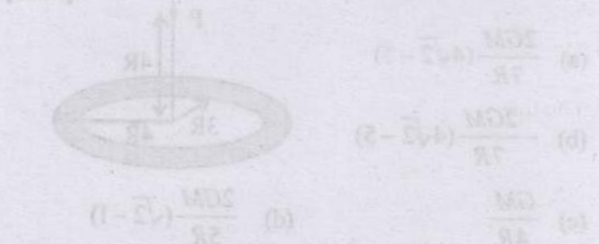
(d) The energy of the mass m remains constant

Consider a spherical mass cloud of mass density $\rho(r)$ in a free space where r is the radial distance from its centre. The cloud is made of particles of equal mass m moving in circular orbits about the common centre with the same kinetic energy E . The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r)$ is

(a) proportional to r^{-1} (b) proportional to r^{-2} (c) proportional to r^{-3} (d) proportional to r^{-4}

A thin uniform circular disc (see figure) of mass M has outer radius $4R$ and inner radius $2R$. The work required to take a unit mass from point P on its axis to infinity is

(a) $\frac{GM}{4R} \left(\sqrt{2} - \frac{1}{2} \right)$ (b) $\frac{GM}{4R} \left(\sqrt{2} - 1 \right)$ (c) $\frac{GM}{4R} \left(\sqrt{2} - \frac{1}{\sqrt{2}} \right)$ (d) $\frac{GM}{4R} \left(\sqrt{2} - \frac{1}{2\sqrt{2}} \right)$



The distance between two stars of masses $3M$ and $4M$ is $2R$. If R is the mean distance between the centres of the Earth and the Sun, and M is the mass of the Sun, the two stars will move around their common centre of mass in circular orbits with period T , where T is the period of Earth's revolution around the Sun. The value of T is

(a) $\frac{2\pi R}{\sqrt{GM}}$ (b) $\frac{2\pi R}{\sqrt{2GM}}$ (c) $\frac{2\pi R}{\sqrt{3GM}}$ (d) $\frac{2\pi R}{\sqrt{4GM}}$

Hints & Solutions



Topic-1: Kepler's Laws of Planetary Motion

1. (d) In case of binary star system, the gravitational force of attraction between the stars will provide the necessary centripetal forces. So angular velocity ω of both stars is the same. Therefore time period T

$$= \frac{2\pi}{\omega} \text{ remains the same.}$$

2. (b) According to Kepler's law $T^2 \propto R^3$

$$\therefore \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$\text{Here } T_1 = 365 \text{ days; } T_2 = ?; R_1 = R \text{ and } R_2 = \frac{R}{2}$$

$$\Rightarrow T_2 = T_1 \left(\frac{R_2}{R_1} \right)^{3/2} = 365 \left[\frac{R/2}{R} \right]^{3/2} = 129 \text{ days}$$

3. (b) The centripetal force is provided by the gravitational force of attraction

$$\text{So, } mR\omega^2 = \frac{GMm}{R^2}$$

$$\Rightarrow \frac{mR \times 4\pi^2}{T^2} = \frac{GMm}{R^2} \Rightarrow T^2 \propto R^3$$

4. (i) According to Kepler's third law of planetary motion. $T^2 \propto R^3$

$$\therefore \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow R_2^3 = R_1^3 \times \frac{T_2^2}{T_1^2}$$

$$\Rightarrow R_2^3 = (10^4)^3 \times \frac{8^2}{1^2} = 64 \times 10^{12}$$

$$\Rightarrow R_2 = 4 \times 10^4 \text{ km.}$$

Linear speed of satellite S_1

$$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km/h}$$

Linear speed of satellite S_2 ,

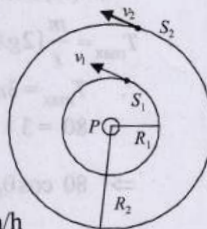
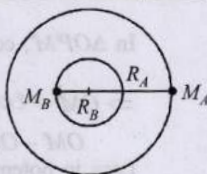
$$v_2 = \frac{2\pi R_2}{T_2} = \frac{(2\pi)(4 \times 10^4)}{8} = \pi \times 10^4 \text{ km/h}$$

Hence the speed of satellite S_2 w.r.t. S_1

$$= v_2 - v_1 = \pi \times 10^4 - 2\pi \times 10^4 = -\pi \times 10^4 \text{ km/h}$$

- (ii) Angular speed of S_2 w.r.t. S_1

$$= \frac{v_2 - v_1}{R_2 - R_1} = \frac{3.14 \times 10^4 \times 5/18}{3 \times 10^4 \times 10^3} = 3 \times 10^{-4} \text{ rad/s}$$



Topic-2: Acceleration due to Gravity

1. (b) The mass of the wire $= 10^{-3} \times 1.2 \times 10^5 = 120 \text{ kg}$
Acceleration due to gravity at the surface of the planet and earth

$$g_p = \frac{4}{3} \pi \rho G R_p;$$

$$\text{and } g_e = \frac{4}{3} \pi \rho G R_e$$

$$\therefore \frac{g_p}{g_e} = \frac{R_p}{R_e} = \frac{1}{10} \Rightarrow g_p = \frac{10}{10} = 1 \text{ ms}^{-2}$$

Let g_{pM} be the acceleration due to gravity at point M which is the mid point of the wire and is at a depth of $\frac{R_p}{10}$.

$$g_{pM} = g_p \left[1 - \frac{R_p/10}{R_p} \right] = 1[1 - 0.1] = 0.9 \text{ ms}^{-2}$$

$$\therefore \text{Force} = \text{mass of wire} \times g_{pM} = 120 \times 0.9 = 108 \text{ N}$$

2. $(1.24 \times 10^{-3} \text{ rad/s})$

$$\text{Using } g' = g - R\omega^2 \cos^2 \phi$$

$$\text{At equator, } \phi = 0, \therefore g' = g - R\omega^2$$

$$\text{Here } g' = 0 \therefore \omega = \sqrt{\frac{g}{R}} = 1.24 \times 10^{-3} \text{ rad/s}$$



Topic-3: Gravitational Field, Potential and Potential Energy

1. (c) For $r \geq R$

$$\frac{mv^2}{r} = m \times \left[\frac{GM}{r^2} \right] \text{ where } M \text{ is the total mass of the spherical system.}$$

$$\therefore v \propto \frac{1}{\sqrt{r}} \quad \left(\because M = \left(\frac{4}{3} \pi R^3 \right) \rho_0 \right)$$

For $r < R$

$$\therefore \frac{mv^2}{r} = m \left[\frac{GMr}{R^3} \right] \Rightarrow v \propto r$$

i.e., $v - r$ graph is a straight line passing through origin.

2. (a,b) For $r > R$, the gravitational field, $F = \frac{GMm}{r^2}$

$$\therefore F_1 = \frac{GMm}{r_1^2} \text{ and } F_2 = \frac{GMm}{r_2^2} \text{ or, } \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$$

$$\text{For } r < R, \text{ the gravitational field, } F = \frac{Gm}{R^3} \times r$$

$$\therefore F_1 = \frac{GMm}{R^3} \times r_1 \text{ and } F_2 = \frac{GMm}{R^3} \times r_2$$

$$\therefore \frac{F_1}{F_2} = \frac{r_1}{r_2}$$

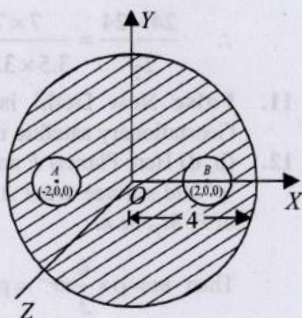
3. (a,c,d) The gravitational field (E) intensity at the point O i.e., centre of a solid sphere is zero. Force acting on a test mass m_0 placed at O

$$F = m_0 E = m_0 \times 0 = 0$$

The gravitational field due to masses at A and B at 'O' is equal and opposite.

Now, $y^2 + z^2 = 36$ represents the equation of a circle with centre (0, 0, 0) and radius 6 units the plane of the circle is perpendicular to x-axis.

As the plane of these circles is $Y-Z \perp$ to X-axis so potential at any point on these two circles will be constant due to mass M and masses at A and B.



Topic-4: Motion of Satellites, Escape Speed and Orbital Velocity

1. (a) Particle of mass m is moving in a circular orbit of radius r_0 with time period T_0 around the mass M provides centripetal force

$$F_c = \frac{mv^2}{r_0}$$

$$\frac{GMm}{r_0^2} - \frac{3m\alpha}{r_0^4} = \frac{mv^2}{r_0}$$

$$\text{or, } \frac{\omega_1^2}{\omega_0^2} = \frac{\frac{GM}{r_0^2} - \frac{3\alpha}{r_0^4}}{\frac{GM}{r_0^2}}$$

$$\text{or, } \frac{T_0^2}{T_1^2} = 1 - \frac{3\alpha}{GM r_0^2} \Rightarrow \frac{T_1^2 - T_0^2}{T_1^2} = \frac{3\alpha}{GM r_0^2}$$

2. (a) Acceleration due to gravity 'g' varies with height

$$h = \frac{GM}{(R+h)^2}$$

$$\frac{g_P}{g_Q} = \frac{\frac{GM}{(R+h_P)^2}}{\frac{GM}{(R+h_Q)^2}} \therefore \frac{36}{25} = \frac{(R+h_Q)^2}{(R+h_P)^2}$$

$$\therefore R+h_Q = \frac{6}{5} \left(R + \frac{R}{3} \right) \left[\because h_P = \frac{R}{3} \text{ given} \right]$$

$$\therefore h_Q = \frac{24}{15} R - R = \frac{9}{15} R = \frac{3}{5} R$$



3. (a) As we know,

$$\text{Gravitational potential energy} = \frac{-GMm}{r}$$

$$\text{and orbital velocity, } v_0 = \sqrt{GM/R+h}$$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m \frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R} \left(\frac{1}{2} - 1 \right) = \frac{-GMm}{6R} \Rightarrow E_i = \frac{-GMm}{R} + K$$

$$E_i = E_f$$

$$\text{Therefore minimum required energy, } K = \frac{5GMm}{6R}$$

4. (b) Applying energy conservation

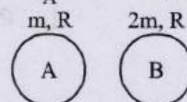
$$\frac{1}{2}mV_s^2 - \frac{GM_e m}{R_e} = \frac{GM_e m \times 3 \times 10^5}{2.5 \times 10^4 R_e}$$

$$\frac{V_s^2}{2} = \frac{GM_e}{R_e} \left[1 + \frac{3 \times 10^5}{2.5 \times 10^4} \right]$$

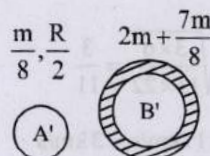
$$V_s = \sqrt{13 \left(\frac{2GM_e}{R_e} \right)} \left(\because V_e = \sqrt{\frac{2GM_e}{R_e}} = 11.2 \text{ km/s} \right)$$

$$\therefore V_s = \sqrt{13} \times 11.2 \approx 42 \text{ m/s}$$

5. (2.3) Initially, let $M_A = m$. Then, $M_B = 2m$



Finally,



$$\text{Now, } V_e = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_B}{V_A} = \sqrt{\frac{M'_B}{M'_A} \times \frac{R'_A}{R'_B}} = \sqrt{\frac{2m + \frac{7m}{8}}{\frac{m}{8}} \times \frac{\frac{R}{2}}{R'_B}}$$

$$\text{Now, } \rho \times \frac{4}{3} \pi \left((R'_B)^3 - R^3 \right) = \frac{7}{8} \times \rho \times \frac{4}{3} \pi R^3$$

$$\Rightarrow (R'_B)^3 - R^3 = \frac{7}{8} R^3 \Rightarrow (R'_B)^3 = \frac{15}{8} R^3$$

$$\Rightarrow R'_B = \frac{15^{1/3}}{2} R$$

$$\text{So, } \frac{V_B}{V_A} = \sqrt{\frac{\frac{23m}{8} \times \frac{R}{2}}{\frac{m}{8} \times \frac{15^{1/3} R}{2}}} = \sqrt{\frac{23m}{8} \times \frac{1}{15^{1/3}}}$$

$$= \sqrt{\frac{23}{15^{1/3}}} = \sqrt{\frac{10 \times 2.3}{15^{1/3}}}$$

Therefore, $n = 2.3$

6. (2) Let h be the height to which the bullet rises with the height acceleration due to gravity varies as

$$g^1 = g \left(1 + \frac{h}{R}\right)^{-2} \Rightarrow \frac{g}{4} = g \left(1 + \frac{h}{R}\right)^{-2} \Rightarrow h = R$$

We know escape speed, $v_e = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$ (given) ... (i)

Now applying conservation of energy principle
Loss of kinetic energy = gain in gravitational potential energy

$$\therefore \frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$\therefore v = \sqrt{\frac{GM}{R}} \quad \dots (ii)$$

Comparing eq. (i) & (ii) we get $N = 2$

7. (3) We know that escape speed $v = \sqrt{2gR}$

$$\therefore \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g} \times \frac{R_p}{R}} \quad \dots (i)$$

$$\text{Given } \frac{g_p}{g_e} = \frac{\sqrt{6}}{11} \quad \dots (ii)$$

$$\text{Also } g = \frac{4}{3}\pi G \rho R \quad \therefore \frac{g_p}{g_e} = \frac{\rho_p}{\rho} \times \frac{R_p}{R} \quad \left[\because \frac{\rho_p}{\rho} = \frac{2}{3} \text{ (given)} \right]$$

$$\therefore \frac{R_p}{R} = \frac{3\sqrt{6}}{22} \quad \dots (iii)$$

From (i), (ii) & (iii)

$$\frac{v_p}{v_e} = \sqrt{\frac{\sqrt{6}}{11} \times \frac{3\sqrt{6}}{22}} = \sqrt{\frac{3 \times 6}{11 \times 22}} = \frac{3}{11}$$

$$\therefore v_p = \frac{3}{11} \times v_e = \frac{3}{11} \times 11 \text{ km/s} = 3 \text{ km/s}$$

8. (R) From conservation of mechanical energy

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{(R+h)}$$

$$\text{KE provided} = \frac{1}{2} \times \text{KE of escape}$$

$$\text{or } \frac{1}{2}mv^2 = \frac{1}{2} \times \frac{GMm}{R} \quad \left[\because V_e = \sqrt{\frac{GM}{R} \times 2} \right]$$

$$\therefore \frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\Rightarrow -\frac{1}{2R} = -\frac{1}{R+h} \Rightarrow R+h = 2R \quad \text{or, } h = R$$

9. $\left(\sqrt{\frac{4G}{d}}(M_1 + M_2)\right)$ Total mechanical energy mass m at a

midway point between the centres of earth and moon

$$= \frac{GM_1m}{d/2} - \frac{GM_2m}{d/2} + \frac{1}{2}mV_e^2$$

$$= \frac{Gm}{d/2}(M_1 + M_2) + \frac{1}{2}mV_e^2 \therefore \frac{1}{2}mV_e^2 = \frac{Gm}{d/2}(M_1 + M_2)$$

[\because final mechanical energy is zero]
where V_e is the velocity with which mass m is projected.

$$\Rightarrow V_e = \sqrt{\frac{4G}{d}(M_1 + M_2)}$$

10. (8.48 h) According to Kepler's law $T^2 \propto R^3$

$$\therefore \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Here $R_1 = R + 6R = 7R$ and $R_2 = 2.5R + R = 3.5R$
 $T_1 = 24$ hours

$$\therefore \frac{24 \times 24}{T_2^2} = \frac{7 \times 7 \times 7 \times R^3}{3.5 \times 3.5 \times 3.5 \times R^3} \Rightarrow T_2 = 8.48 \text{ h}$$

11. False New Delhi is not on the equatorial plane.

Geostationary satellite is launched on the equatorial plane.

12. (b, d) Here Planets P and Q have the same uniform density ' ρ ' and surface areas A and $4A$ respectively. Let the mass of P , M_P be m .

$$\text{Then } m = \rho \times \frac{4}{3}\pi r^3 = \rho \times \frac{4}{3}\pi \left[\frac{A}{4\pi}\right]^{3/2}$$

$$\text{The mass of } M_Q = \rho \times \frac{4}{3}\pi \left[\frac{4A}{4\pi}\right]^{3/2} = 8m$$

\therefore The mass of Planet $R = 8m + m = 9m$

If the radius of $P = r$

Then the radius of $Q = 2r$

$$\therefore r_Q = \left(\frac{4A}{4\pi}\right)^{3/2} = 2\left(\frac{A}{4\pi}\right)^{3/2}$$

$$\text{and radius of } R = 9^{1/3}r \quad \left[\because M_R = M_P + M_Q \right]$$

As we know, escape velocity from the planet

$$V_e = \sqrt{\frac{2GM}{R}} \therefore v_P = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2Gm}{r}}$$

$$v_Q = \sqrt{\frac{2GM_Q}{R_Q}} = \sqrt{\frac{2G(8m)}{2r}} = 2v_P$$

$$v_R = \sqrt{\frac{2G(9m)}{9^{1/3}r}} = 9^{1/3}v_P$$

13. (a) Force on satellite is always towards earth which attracts the satellite with the gravitational force F , therefore, acceleration of satellite S is always directed towards centre of the earth.

The net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout.

As the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

14. (b) \therefore Orbital velocity,

$$V = \sqrt{\frac{GM}{R}}, \text{ or, } V \propto \frac{1}{\sqrt{R}} \therefore \frac{V_1}{V_2} = \sqrt{\frac{R_2}{R_1}} = \frac{2}{1}$$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_1 v_2 R_2} = \frac{2 \times 2 \times 1}{1 \times 1 \times 4} = 1$$

$$\text{Kinetic energy, } K = \frac{GMm}{2R}$$

$$\therefore \frac{k_1}{k_2} = \frac{m_1}{m_2} \times \frac{R_1}{R_2} = \frac{2 \times 4}{1 \times 1} = 8$$

From Kepler's law of planetary motion.

$$T^2 \propto R^3 \therefore \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \frac{1}{8}$$

15. (a) An astronaut in an orbiting space station above the earth experiences weight less as he is in a state of free fall. The force acting on astronaut is utilised in providing necessary centripetal force.

16. Applying mechanical energy conservation,

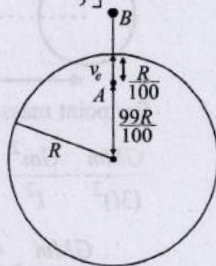
Total energy at A = Total energy at B

$$(K.E.)_A + (P.E.)_A = (P.E.)_B$$

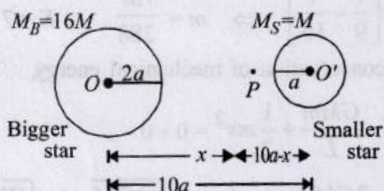
$$\Rightarrow \frac{1}{2} m \times \frac{2GM}{R} + \left[\frac{-GMm}{2R^3} \left\{ 3R^2 - \left(\frac{99R}{100} \right)^2 \right\} \right] = -\frac{GMm}{R+h}$$

Solving above equation, we get $h = 99.5 R$.

(Maximum height attained by the body from the surface of the earth)



17. Let the force of attraction is zero at a distance x from the bigger star.



Then force on mass m due to bigger star = force on mass m due to small star

$$\frac{GM_B m}{x^2} = \frac{GM_S m}{(10a-x)^2} \Rightarrow \frac{16M}{x^2} = \frac{M}{(10a-x)^2} \Rightarrow x = 8a$$

Let v denote velocity with which the body of mass m is fired so that it crosses P along OP . The energy is conserved.

The energy of the system (of these masses) initially = final energy when m is at P

$$-\frac{GM_B M_S}{10a} - \frac{GM_B m}{2a} - \frac{GM_S m}{8a} + \frac{1}{2} m v^2 = -\frac{GM_B M_S}{10a} - \frac{GM_B m}{8a} - \frac{GM_S m}{2a}$$

$$[\because M_B = 16M; M_S = M]$$

$$\therefore v = \frac{3}{2} \sqrt{\frac{5GM}{a}}$$

18. (i) Here centripetal force is provided by the gravitational pull

$$\therefore \frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$

$$\Rightarrow v^2 = \frac{GM}{R+h}$$

$$\text{Given } v = \frac{1}{2} v_e = \frac{1}{2} \sqrt{\frac{2GM}{R}}$$

$$\therefore \frac{1}{4} \left(\frac{2GM}{R} \right) = \frac{GM}{R+h} \Rightarrow 2R+2h=4R \therefore h=R=6400 \text{ km.}$$

- (ii) Let V be the speed with which the satellite hits the surface of the earth. If the satellite is stopped, its kinetic energy is zero. When it falls freely on the Earth, its potential energy decreases and converts into kinetic energy.

$$\therefore (P.E.)_A - (P.E.)_B = K.E.$$

$$\Rightarrow \frac{-GMm}{2R} - \left(\frac{-GMm}{R} \right) = \frac{1}{2} m v^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7920 \text{ m/s} = 7.92 \text{ km/s}$$

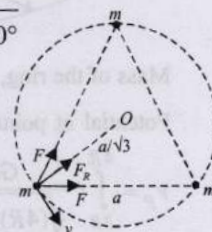
19. Let the initial velocity that should be given to each particle be ' V '. The centripetal force is provided by the resultant gravitational attraction of the two masses.

$$F_R = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ}$$

$$= \sqrt{3} F = \sqrt{3} G \frac{m \times m}{a^2}$$

$$\therefore \sqrt{3} G \frac{m^2}{a^2} = \frac{mv^2}{r}$$

$$\left(\frac{mv^2}{r} = \text{centripetal force} \right)$$



$$\text{Radius of the circular path } r = \frac{2}{3} \sqrt{a^2 - \frac{a^2}{4}} = \frac{a}{\sqrt{3}}$$

$$v^2 = \frac{\sqrt{3} G m r}{a^2} = \frac{\sqrt{3} G m a}{a^2 \times \sqrt{3}} \Rightarrow v = \sqrt{\frac{Gm}{a}}$$

Time period of circular motion

$$T = \frac{2\pi r}{v} = \frac{2\pi a / \sqrt{3}}{\sqrt{\frac{Gm}{a}}} = 2\pi \sqrt{\frac{a^3}{3Gm}}$$



Topic-5: Miscellaneous (Mixed Concepts) Problems

1. (b) Gravitational pull of the mass ' M ' present in the sphere of radius ' r '. Provide the required centripetal force of particle of mass ' m ' to revolve in a circular path.

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2r} \Rightarrow K = \frac{GMm}{2r}$$

$$\therefore M = \frac{2Kr}{Gm}$$

Differentiating the above equation w.r.t 'r' we get

$$\frac{dM}{dr} = \frac{2K}{Gm}$$

$$\text{or } dM = \frac{2K}{Gm} dr$$

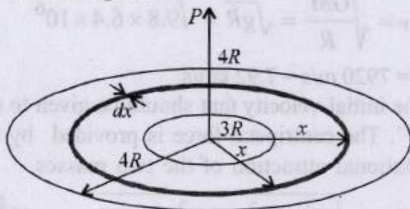
$$\therefore 4\pi r^2 dr \rho = \frac{2K}{Gm} dr \Rightarrow \rho = \frac{K}{2\pi r^2 m G}$$

$$\therefore \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G} \text{ or } \frac{\rho(r)}{m} = \frac{K}{2\pi r^2 m^2 G}$$

2. (a) Mass per unit area of the shaded part.

$$\sigma = \frac{\text{mass}}{\text{area}} = \frac{M}{\pi((4R)^2 - (3R)^2)} = \frac{M}{7\pi R^2}$$

Let us consider a ring of radius x and thickness dx as shown in the figure.



$$\text{Mass of the ring, } dM = \sigma 2\pi x dx = \frac{2\pi M x dx}{7\pi R^2}$$

Potential at point P due to shaded part

$$V_P = \int_{3R}^{4R} -\frac{GdM}{\sqrt{(4R)^2 + (x)^2}} = -\frac{GM2\pi}{7\pi R^2} \int_{3R}^{4R} \frac{xdx}{\sqrt{16R^2 + x^2}}$$

Solving, we get

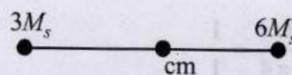
$$V_P = -\frac{GM2\pi}{7\pi R^2} \left[\sqrt{16R^2 + x^2} \right]_{3R}^{4R} = -\frac{2GM}{7R} (4\sqrt{2} - 5)$$

Workdone in moving a unit mass from P to $\infty = V_\infty - V_P$

$$\text{or } W_{P\infty} = 0 - \left(-\frac{2GM}{7R} (4\sqrt{2} - 5) \right) = \frac{2GM}{7R} (4\sqrt{2} - 5)$$

3. (9) The centre of mass lies at a distance $6R$ from lighter mass
In circular orbit,

$$\text{Time period, } T = 2\pi \sqrt{\frac{R^3}{GM_S}}$$



Binary stars system

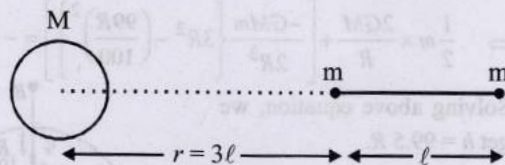
$$nT = 2\pi \sqrt{\frac{(9R)^3}{G(3M_S + 6M_S)}}$$

$$\text{or, } n \times 2\pi \sqrt{\frac{R^3}{GM_S}} = 9 \times 2\pi \sqrt{\frac{R^3}{GM_S}}$$

$$\therefore T = 2\pi \sqrt{\frac{R^3}{GM_S}} \therefore n = 9$$

4. (7) For point mass at distance $r = 4l$

$$\frac{GMm}{(4l)^2} + \frac{Gm^2}{l^2} = ma$$



For point mass at distance $r = 3l$

$$\frac{GMm}{(3l)^2} - \frac{Gm^2}{l^2} = ma$$

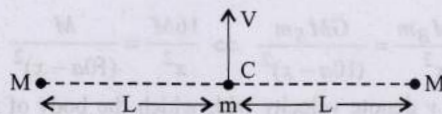
$$\therefore \frac{GMm}{(4l)^2} + \frac{Gmm}{l^2} = \frac{GMm}{(3l)^2} - \frac{Gmm}{l^2}$$

$$\therefore 2m = M \left[\frac{1}{9} - \frac{1}{16} \right] \Rightarrow m = \frac{7M}{288} \therefore K = 7$$

5. (b, d) From conservation of mechanical energy,

$$\frac{-GMm}{L} - \frac{GMm}{L} + \frac{1}{2}mv^2 = 0 + 0$$

$$\text{or, } \frac{1}{2}mv^2 = \frac{2GMm}{L} \therefore V = \sqrt{\frac{4GM}{L}} = 2\sqrt{\frac{GM}{L}}$$



Total energy of mass 'm' is conserved as there is no external force involved.