



CONTINUITY AND DIFFERENTIABILITY

BASIC CONCEPTS

- 1. Continuity and Discontinuity of a Function at a Point:** A function $f(x)$ is said to be continuous at a point a of its domain if

$$\lim_{x \rightarrow a^-} f(x), \lim_{x \rightarrow a^+} f(x), f(a) \text{ exist and } \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

A function $f(x)$ is said to be discontinuous at $x = a$ if it is not continuous at $x = a$.

- 2. Properties of Continuous Function:**

- Every constant function is continuous function.
- Every polynomial function is continuous function.
- Identity function is continuous function.
- Every logarithmic and exponential function is a continuous function.

List of Useful Formulae:

- 3.** (i) $\frac{d}{dx}(x^n) = nx^{n-1}$ (ii) $\frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1} \cdot a$
 (iii) $\frac{d}{dx}(e^x) = e^x$ (iv) $\frac{d}{dx}e^{ax} = a \cdot e^{ax}$
 (v) $\frac{d}{dx}a^x = a^x \cdot \log_e a$ (vi) $\frac{d}{dx}a^{bx} = ba^{bx} \log_e a$
 (vii) $\frac{d}{dx} \log_e x = \frac{1}{x}$ and $\frac{d}{dx} \log_e ax = \frac{a}{x}$
 (viii) $\frac{d}{dx} \log_a x = \frac{1}{x \cdot \log_e a}$ and $\frac{d}{dx} \log_a bx = \frac{b}{x \cdot \log_e a}$
- 4.** (i) $\frac{d}{dx} \sin x = \cos x$ and $\frac{d}{dx} \sin ax = a \cos ax$
 (ii) $\frac{d}{dx} \cos x = -\sin x$ and $\frac{d}{dx} \cos ax = -a \sin ax$
 (iii) $\frac{d}{dx} \tan x = \sec^2 x$ and $\frac{d}{dx} \tan ax = a \sec^2 ax$
 (iv) $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$ and $\frac{d}{dx} \cot ax = -a \operatorname{cosec}^2 ax$
 (v) $\frac{d}{dx} \sec x = \sec x \tan x$ and $\frac{d}{dx} \sec ax = a \sec ax \cdot \tan ax$
 (vi) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$ and $\frac{d}{dx} \operatorname{cosec} ax = -a \operatorname{cosec} ax \cdot \cot ax$
- 5.** (i) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ and $\frac{d}{dx} \sin^{-1} ax = \frac{a}{\sqrt{1-a^2x^2}}$

$$\begin{aligned}
(ii) \quad \frac{d}{dx} \cos^{-1} x &= \frac{-1}{\sqrt{1-x^2}} & \text{and} \quad \frac{d}{dx} \cos^{-1} ax &= \frac{-a}{\sqrt{1-a^2x^2}} \\
(iii) \quad \frac{d}{dx} \tan^{-1} x &= \frac{1}{1+x^2} & \text{and} \quad \frac{d}{dx} \tan^{-1} ax &= \frac{a}{1+a^2x^2} \\
(iv) \quad \frac{d}{dx} \cot^{-1} x &= \frac{-1}{1+x^2} & \text{and} \quad \frac{d}{dx} \cot^{-1} ax &= \frac{-a}{1+a^2x^2} \\
(v) \quad \frac{d}{dx} \sec^{-1} x &= \frac{1}{x\sqrt{x^2-1}} & \text{and} \quad \frac{d}{dx} \sec^{-1} ax &= \frac{1}{x\sqrt{a^2x^2-1}} \\
(vi) \quad \frac{d}{dx} \operatorname{cosec}^{-1} x &= \frac{-1}{x\sqrt{x^2-1}} & \text{and} \quad \frac{d}{dx} \operatorname{cosec}^{-1} ax &= \frac{-1}{x\sqrt{a^2x^2-1}}
\end{aligned}$$

5. Chain Rule: Chain rule is applied when the given function is the function of function *i.e.*,

$$\text{if } y \text{ is a function of } x, \text{ then } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{or} \quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

6. Parametric Form: Sometimes we come across the function when both x and y are expressed in terms of another variable say t *i.e.*, $x = \phi(t)$ and $y = \psi(t)$. This form of a function is called parametric form and t is called the parameter.

7. Rolle's Theorem: If $f(x)$ be a real valued function, defined in a closed interval $[a, b]$ such that:

- (i) it is continuous in closed interval $[a, b]$.
- (ii) it is differentiable in open interval (a, b) .
- (iii) $f(a) = f(b)$. Then there exists at least one value $c \in (a, b)$ such that $f'(c) = 0$.

8. Lagrange's Mean Value Theorem:

If $f(x)$ is a real valued function defined in the closed interval $[a, b]$ such that:

- (i) it is continuous in the closed interval $[a, b]$.
- (ii) it is differentiable in the open interval (a, b) .

Then there exists at least one real value $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{(b - a)}$.

9. Some Standard Results:

$$\begin{aligned}
(i) \quad (a) \quad \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} &= na^{n-1}, a > 0, n \in \mathbb{Q} \\
(b) \quad \lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n} &= \frac{m}{n} a^{m-n}, m, n \in \mathbb{Q} \\
(c) \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} &= 1 & (d) \quad \lim_{x \rightarrow 0} \cos x &= 1 \\
(e) \quad \lim_{x \rightarrow 0} \frac{\tan x}{x} &= 1
\end{aligned}$$

(ii) Evaluation of limits of inverse trigonometric functions:

$$\begin{aligned}
(a) \quad \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} &= 1 & (b) \quad \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} &= 1
\end{aligned}$$

(iii) Evaluation of limits of exponential and logarithmic functions:

$$\begin{aligned}
(a) \quad \lim_{x \rightarrow 0} e^x &= 1 & (b) \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} &= 1 \\
(c) \quad \lim_{x \rightarrow 0} \frac{\log |1 + x|}{x} &= 1 & (d) \quad \lim_{x \rightarrow 0} \frac{(1 + x)^n - 1}{x} &= n \\
(e) \quad \lim_{x \rightarrow 0} \frac{a^x - 1}{x} &= \log_e a
\end{aligned}$$

(iv) Limits at infinity: This method is applied when $x \rightarrow \infty$.

Procedure to solve the infinite limits:

- (a) Write the given expression in the form of rational function.
- (b) Divide the numerator and denominator by highest power of x .
- (c) Use the result $\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$, where $n > 0$.
- (d) Simplify and get the required result.

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = -|x - 1|$ is [CBSE 2020 (65/2/1)]
 - (a) continuous as well as differentiable at $x = 1$
 - (b) not continuous but differentiable at $x = 1$
 - (c) continuous but not differentiable at $x = 1$
 - (d) neither continuous nor differentiable at $x = 1$
2. The function $f(x) = e^{|x|}$ is [NCERT Exemplar]
 - (a) continuous everywhere but not differentiable at $x = 0$
 - (b) continuous and differentiable everywhere
 - (c) not continuous at $x = 0$
 - (d) none of these
3. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function, is continuous at
 - (a) 4
 - (b) -2
 - (c) 1
 - (d) 1.5
4. The number of points at which the function $f(x) = \frac{1}{x - [x]}$ is not continuous is [NCERT Exemplar]
 - (a) 1
 - (b) 2
 - (c) 3
 - (d) none of these
5. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is [NCERT Exemplar]
 - (a) 3
 - (b) 2
 - (c) 1
 - (d) 1.5
6. The value of k which makes the function defined by $f(x) = \begin{cases} \sin \frac{1}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$, continuous at $x = 0$ is
 - (a) 8
 - (b) 1
 - (c) -1
 - (d) none of these
7. The function $f(x) = \cot x$ is discontinuous on the set [NCERT Exemplar]
 - (a) $\{x = n\pi : n \in \mathbb{Z}\}$
 - (b) $\{x = 2n\pi : n \in \mathbb{Z}\}$
 - (c) $\left\{x = (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}\right\}$
 - (d) $\left\{x = \frac{n\pi}{2}; n \in \mathbb{Z}\right\}$
8. Let $f(x) = |\sin x|$. Then [NCERT Exemplar]
 - (a) f is everywhere differentiable
 - (b) f is everywhere continuous but not differentiable at $x = n\pi, n \in \mathbb{Z}$
 - (c) f is everywhere continuous but not differentiable at $x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$
 - (d) none of these

9. The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at [CBSE 2020 (65/2/2)]
 (a) exactly one point (b) exactly two points
 (c) exactly three points (d) no point
10. If $f(x) = x^2 \sin \frac{1}{x}$, where $x \neq 0$, then the value of the function f at $x = 0$, so that the function is continuous at $x = 0$, is [NCERT Exemplar]
 (a) 0 (b) -1 (c) 1 (d) None of these
11. The function $f(x) = |x| + |x-1|$ is
 (a) continuous at $x = 0$ as well as at $x = 1$. (b) continuous at $x = 1$ but not at $x = 0$.
 (c) discontinuous at $x = 0$ as well as at $x = 1$. (d) continuous at $x = 0$ but not at $x = 1$.
12. The function $f(x) = \frac{4-x^2}{4x-x^3}$ is
 (a) discontinuous at only one point (b) discontinuous at exactly two points
 (c) discontinuous at exactly three points (d) none of these
13. The set of points where the functions f given by $f(x) = |x-3| \cos x$ is differentiable is
 (a) \mathbb{R} (b) $\mathbb{R} - \{3\}$ (c) $(0, \infty)$ (d) none of these
14. Differential coefficient of $\sec(\tan^{-1}x)$ w.r.t. x is [NCERT Exemplar]
 (a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\frac{x}{1+x^2}$ (c) $x\sqrt{1+x^2}$ (d) $\frac{1}{\sqrt{1+x^2}}$
15. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is
 (a) $\frac{1}{2}$ (b) x (c) $\frac{1-x^2}{1+x^2} \{4, -4\}, \phi$ (d) 1
16. If $y = \log \sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is
 (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) ∞
17. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to
 (a) $\frac{\cos x}{2y-1}$ (b) $\frac{\cos x}{1-2y}$ (c) $\frac{\sin x}{1-2y}$ (d) $\frac{\sin x}{2y-1}$
18. The function $f(x) = |x| + |x-2|$ is
 (a) differentiable at $x = 0$ and at $x = 2$ (b) differentiable at $x = 0$ but not at $x = 2$.
 (c) not differentiable at $x = 0$ and at $x = 2$. (d) none of these
19. The function given by $f(x) = \tan x$ is discontinuous on the set
 (a) $\{x : x = 2n\pi, n \in \mathbb{Z}\}$ (b) $\{x : x = (n-1)\pi, n \in \mathbb{Z}\}$
 (c) $\{x : x = n\pi, n \in \mathbb{Z}\}$ (d) $\{x : x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\}$
20. The derivative of $\tan x$ w.r.t. $\sin x$ is
 (a) $\tan^2 x$ (b) $\sec x$ (c) $\frac{\tan x}{\sin x}$ (d) $\sec^3 x$
21. The function $f(x) = \frac{x^2 - x - 6}{x - 3}$ is not defined for $x = 3$. In order to make $f(x)$ continuous at $x = 3$, $f(3)$ should be defined as
 (a) 1 (b) 3 (c) 5 (d) none of these

22. If $f(x) = x - 3$ and $g(x) = \frac{x^2}{3} + 1$, then which of the following can be a discontinuous function?

- (a) $f(x) + g(x)$ (b) $f(x) \cdot g(x)$ (c) $f(x) - g(x)$ (d) $\frac{g(x)}{f(x)}$

23. The set of points where the function f given by $|3x - 2| \sin x$ is differentiable is

- (a) \mathbb{R} (b) $(0, \infty)$ (c) $\mathbb{R} - \left\{\frac{2}{3}\right\}$ (d) none of these

24. Let $f(x) = \begin{cases} \cos [x], & x \geq 0 \\ |x| + a, & x < 0 \end{cases}$ where $[x]$ denotes the greatest integer $\leq x$. If $\lim_{x \rightarrow 0} f(x)$ exists then a is equal to

- (a) 1 (b) 3 (c) 0 (d) none of these

25. Let $f(x) = \begin{cases} x[x - 1], & 0 \leq x < 2 \\ [x](x - 1), & 2 \leq x < 3 \end{cases}$ where $[x]$ denotes the greatest integer $\leq x$ then

- (a) $f(x)$ is continuous everywhere.
(b) $f(x)$ is not continuous at $x = 1$ and $x = 2$
(c) $f(x)$ is not differentiable at infinite points
(d) none of these

26. $f : [-2a, 2a] \rightarrow \mathbb{R}$ is an odd function such that the left hand derivative at $x = a$ is zero and $f(x) = f(2a - x) \forall x \in (a, 2a)$. Then its left hand derivative at $x = -a$ is

- (a) 0 (b) a (c) 1 (d) does not exist.

27. The function $f(x) = |x - 3|, x \in \mathbb{R}$ is

- (a) is continuous and differentiable everywhere.
(b) is not continuous but differentiable at $x = 3$
(c) is continuous but not differentiable at $x = 3$
(d) none of these

28. If $f(x) = x^n$, then the value of

$$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \dots + \frac{(-1)^n f^{(n)}(1)}{n!} \text{ is}$$

- (a) 1 (b) 0 (c) 2^n (d) 2

29. If $x = e^{y+e^{y+\dots \text{to } \infty}}$, $x > 0$, then $\frac{dy}{dx}$ is

- (a) $\frac{1}{x}$ (b) $\frac{x}{1+x}$ (c) $\frac{1-x}{x}$ (d) none of these

30. Let $f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

- (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) -1

31. The value of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases} \text{ is continuous for all } x \in \mathbb{R}, \text{ are}$$

- (a) $p = \frac{1}{2}, q = \frac{3}{2}$ (b) $p = \frac{5}{2}, q = \frac{7}{2}$ (c) $p = -\frac{3}{2}, q = \frac{1}{2}$ (d) none of these

32. The function $f(x) = \begin{cases} \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $x = 0$
- (a) is continuous. (b) has removable discontinuity.
(c) has jump discontinuity. (d) has oscillating discontinuity.
33. The function $f(x) = \begin{cases} 2 - x, & \text{if } x < 2 \\ 2 + x, & \text{if } x \geq 2 \end{cases}$ at $x = 2$
- (a) is continuous. (b) has removable discontinuity.
(c) has jump discontinuity. (d) has oscillating discontinuity.
34. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ at $x = 0$
- (a) is continuous. (b) has removable discontinuity.
(c) has jump discontinuity. (d) has oscillating discontinuity.
35. If $f(x) = |x| + |x - 2|$, then
- (a) $f(x)$ is continuous at $x = 0$ but not at $x = 2$.
(b) $f(x)$ is continuous at $x = 0$ and at $x = 2$.
(c) $f(x)$ is continuous at $x = 2$ but not at $x = 0$.
(d) None of these.
36. The function $f(x) = \frac{1}{x - 1}$ at $x = 1$.
- (a) is continuous (b) has removable discontinuity.
(c) has jump discontinuity. (d) has asymptotic discontinuity.
37. The vertical asymptotes to curve $y = \frac{e^x}{x}$ is
- (a) $x = 1$ (b) $x = 0$
(c) $x = 2$ (d) Curve has no any asymptotes.
38. An oblique asymptote to the curve $y = x + e^{-x} \sin x$ is
- (a) $y = x + e$ (b) $y = x$ (c) $y = x + \frac{1}{\pi}$ (d) none of these
39. The function $f(x) = \begin{cases} x^m \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $x = 0$ is continuous if
- (a) $m \geq 0$ (b) $m > 0$ (c) $m < 0$ (d) none of these
40. The function $f(x) = \begin{cases} \frac{1 - \cos 10x}{x^2}, & \text{if } x < 0 \\ m, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{625 + \sqrt{x}} - 25}, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$, if the value of m is
- (a) 25 (b) 50 (c) -25 (d) none of these
41. The set of all points where $f(x) = \sec 2x + \operatorname{cosec} 2x$ is discontinuous is
- (a) $\{n\pi : n = 0, \pm 1, \pm 2, \dots\}$ (b) $\left\{\frac{n\pi}{2} : n = 0, \pm 1, \pm 2, \dots\right\}$
(c) $\left\{\frac{(2n+1)\pi}{4} : n = 0, \pm 1, \pm 2, \dots\right\}$ (d) $\left\{\frac{n\pi}{4} : n = 0, \pm 1, \pm 2, \dots\right\}$

42. If $f(x) = x^n$, then the value of $f(1) + \frac{f(1)}{1!} + \frac{f^2(1)}{2!} + \frac{f^3(1)}{3!} + \dots + \frac{f^n(1)}{n!}$, where $f^r(1)$ is the r^{th} derivative of $f(x)$ w.r.t. x .
- (a) 1 (b) n (c) 2^n (d) none of these
43. If $f(x), g(x), h(x)$ are polynomials in x of degree 2 and $F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$, then $F'(x)$ is equal to
- (a) -1 (b) 2 (c) 0 (d) none of these
44. The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x = \frac{\pi}{4}$, where $f'(1) = g'(\sqrt{2}) = 4$ is
- (a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) none of these
45. If g is inverse function of f and $f'(x) = \sin x$, then $g'(x)$ is
- (a) $\sin(g(x))$ (b) $\sin^{-1} x$ (c) $\frac{1}{\sqrt{1-x^2}}$ (d) $\operatorname{cosec}(g(x))$
46. If $y = \left(\frac{x}{n}\right)^{nx} \left(1 + \log \frac{x}{n}\right)$, $y'(n)$ is given by
- (a) $\frac{n^2+1}{n}$ (b) $\frac{1}{n}$ (c) $\left(\frac{1}{n}\right)^n$ (d) $\left(\frac{1}{n}\right)^n \left(\frac{n^2+1}{n}\right)$
47. If $f(x) = \log \sqrt{\tan x}$, then the value of $f'(x)$ at $x = \frac{\pi}{4}$ is
- (a) ∞ (b) 1 (c) 0 (d) $\frac{1}{2}$
48. Let y be a function of x such that $\log(x+y) - 2xy = 0$, then $y'(0)$ is
- (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$
49. If $x \cos y + y \cos x = \pi$ then $y''(0)$ is
- (a) π (b) $-\pi$ (c) 0 (d) 1
50. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is constant, then find $f''(0)$.
- (a) p (b) 0 (c) $p + p^3$ (d) $p + p^2$
51. If $y = f\left(\frac{3x+4}{5x+6}\right)$ and $f'(x) = \tan x^2$ then $\frac{dy}{dx}$ is equal to
- (a) $-2 \tan\left(\frac{3x+4}{5x+6}\right)^2 \times \frac{1}{(5x+6)^2}$ (b) $\tan x^2$
- (c) $f\left(\frac{3 \tan x^2 + 4}{5 \tan x^2 + 6}\right)$ (d) none of these
52. The set of all points where the function $f(x) = x|x|$ is differentiable is
- (a) $(-\infty, \infty)$ (b) $(-\infty, 0) \cup (0, \infty)$ (c) $(0, \infty)$ (d) $[0, \infty)$
53. Let $f(x) = \cos^{-1}(\cos x)$ then $f(x)$ is
- (a) continuous at $x = \pi$ and not differentiable at $x = \pi$.
- (b) continuous at $x = -\pi$

- (c) differentiable at $x = 0$
 (d) differentiable at $x = \pi$
54. Let $f(x) = x - |x|$ then $f(x)$ is
 (a) differentiable $\forall x \in \mathbb{R}$
 (b) continuous $\forall x \in \mathbb{R}$ and not differentiable at $x = 0$
 (c) neither continuous nor differentiable at $x = 0$
 (d) discontinuous at $x = 0$
55. Let $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 < x \leq 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$
 then the number of points at which $g(x)$ is non-differentiable is
 (a) at most one point (b) two (c) exactly one point (d) infinite
56. Let $f(x)$ be differentiable function such that

$$f\left(\frac{x+y}{1-xy}\right) = f(x) + f(y) \quad \forall x \text{ and } y.$$

 If $\lim_{x \rightarrow 0} \frac{f(x)}{x} = \frac{1}{3}$ then $f(1)$ is equal to
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{6}$ (d) None of these
57. Let $f(x) = \begin{cases} 1-3x, & x < 0 \\ 3, & x = 0 \\ x^2 + 3, & x > 0 \end{cases}$ then at $x = 0$
 (a) $f(x)$ is continuous from left (b) $f(x)$ is continuous
 (c) $f(x)$ is right continuous (d) $f(x)$ has removable discontinuity
58. Let $f(x) = |\sin x|$; $0 \leq x \leq 2\pi$ then
 (a) $f(x)$ is differentiable function at infinite number of points
 (b) $f(x)$ is non-differentiable at 3 points and continuous everywhere.
 (c) $f(x)$ is discontinuous everywhere
 (d) $f(x)$ is discontinuous at 3 points
59. Let $f(x) = \begin{cases} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$
 then the value of k such that $f(x)$ holds continuity at $x = 0$ is
 (a) e (b) $\frac{1}{e^2}$ (c) e^2 (d) None of these
60. Let $f(x) = x^2|x|$ then the set of values, where $f(x)$ is three times differentiable, is
 (a) Infinite (b) 2 (c) 3 (d) None of these
61. Let $f(x) = \begin{cases} -3 & -3 \leq x < 0 \\ x^2 - 3 & 0 \leq x \leq 3 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$ then which of the following is true?
 (a) At $x = 0$, $g(x)$ is continuous as well as differentiable and at $x = \sqrt{3}$, $g(x)$ is continuous but not differentiable.
 (b) At $x = \sqrt{3}$, $g(x)$ is neither continuous nor differentiable
 (c) At $x = 2$, $g(x)$ is neither continuous nor differentiable
 (d) None of these

62. Let $f(x) = x^{3/2} - \sqrt{x^3 + x^2}$ then
- LHD at $x = 0$ exists but RHD at $x = 0$ does not exist
 - $f(x)$ is differentiable at $x = 0$
 - RHD at $x = 0$ exists but LHD at $x = 0$ does not exist
 - None of these
63. Number of points at which $f(x) = \frac{1}{\log |x|}$ is discontinuous is
- 2
 - 3
 - 1
 - 4
64. If $f(x) = \frac{\sin 4\pi [\pi^2 x]}{7 + [x]^2}$, $[\cdot]$ denotes the greatest integer function, then $f(x)$ is
- continuous for all x but $f'(x)$ does not exist
 - discontinuous at some x
 - $f''(x)$ exists for all x
 - $f'(x)$ exists but $f''(x)$ does not exist for some value of x .
65. The function $f(x) = \begin{cases} \frac{\sin^3 x^2}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ is
- continuous but not derivable at $x = 0$
 - neither continuous nor differentiable at $x = 0$
 - continuous and differentiable at $x = 0$
 - none of these
66. Let $f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ then $f(x)$ is
- differentiable at $x = 1$
 - continuous $\forall x \in \mathbb{R}$
 - neither continuous nor differentiable at $x = 1$
 - continuous but not differentiable at $x = 1$
67. Let $f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \leq x \leq 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$ then $f(x)$ is
- differentiable at $x = 1$
 - differentiable at $x = 2$
 - differentiable at $x = 1$ and $x = 2$
 - none of these
68. Let $f(x) = |\log |x||$ then
- $f(x)$ is continuous in its domain but not differentiable at $x = \pm 1$
 - $f(x)$ is continuous and differentiable for all x in its domain
 - $f(x)$ is neither continuous nor differentiable at $x = \pm 1$
 - all of these
69. Let $g(x) = \begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$ then $g(x)$ does not satisfy the condition
- continuous $\forall x \in \mathbb{R}$
 - not differentiable at $x = 0$
 - continuous $\forall x \in \mathbb{R}$ and non differentiable at $x = \pm 1$
 - none of these

70. Let $f(x) = [x]^2 + \sqrt{\{x\}}$, where $[\cdot]$ and $\{ \cdot \}$ respectively denotes the greatest integer and fractional part functions, then

- (a) $f(x)$ is continuous at all integral points
- (b) $f(x)$ is non differentiable $\forall x \in \mathbb{Z}$
- (c) $f(x)$ is discontinuous $\forall x \in \mathbb{Z} - \{1\}$
- (d) $f(x)$ is continuous and differentiable at $x = 0$

71. Find the value of a if the function $f(x)$ defined by

$$f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases} \text{ is continuous at } x = 2.$$

- (a) 3
- (b) -3
- (c) 0
- (d) 4

72. If the function $f(x)$ defined by

$$f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \text{ is continuous at } x = 0, \text{ find } k.$$

- (a) a
- (b) b
- (c) $a + b$
- (d) 0

73. If function $f(x) = e^{-|x|}$ is

- (a) continuous everywhere but not differentiable at $x = 0$
- (b) continuous and differentiable everywhere
- (c) not continuous at $x = 0$
- (d) None of these.

74. If $x^y = e^{x-y}$ find $\frac{dy}{dx}$.

- (a) $\frac{\log x}{(1 + \log x)^2}$
- (b) $\frac{x}{\log x}$
- (c) $\frac{\log x}{(1 - \log x)^2}$
- (d) None of these

75. If $x^y = y^x$, find $\frac{dy}{dx}$.

- (a) $x \log x$
- (b) $\frac{y}{x} \cdot \left(\frac{x \log y - y}{y \log x - x} \right)$
- (c) 0
- (d) None of these

76. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$, find $\frac{dy}{dx}$.

- (a) $\frac{\cos x}{2y + 1}$
- (b) $\frac{\cos x}{2y - 1}$
- (c) 0
- (d) None of these

77. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, $a > 0$ and $-1 < t < 1$, then $\frac{dy}{dx}$ is:

- (a) $\frac{y}{x}$
- (b) $\frac{x}{y}$
- (c) $\frac{-y}{x}$
- (d) None of these

78. Derivative of $\tan^{-1} \left(\frac{1+2x}{1-2x} \right)$ w.r.t $\sqrt{1+4x^2}$:

- (a) $\frac{1}{2x\sqrt{1+4x^2}}$
- (b) $\frac{1}{x\sqrt{1+x^2}}$
- (c) $\frac{1}{4x\sqrt{1+2x^2}}$
- (d) $\frac{1}{2x\sqrt{1-4x^2}}$

79. If $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, find $f'(x)$.

- (a) $3x^2 - 2x(a^2 + b^2 + c^2)$
- (b) $3x^2 + 2x(a^2 + b^2 + c^2)$

(c) 0

(d) None of these

80. If $y = x^x$ find $\frac{d^2 y}{dx^2}$.

(a) $x^x \left\{ (1 + \log x)^2 - \frac{1}{x} \right\}$

(b) $x^x \left\{ (1 + \log x)^2 + \frac{1}{x} \right\}$

(c) 0

(d) $x^x \left\{ (1 - \log x)^2 + \frac{1}{x} \right\}$

81. Find $\frac{d^2 y}{dx^2}$, if $x = at^2$, $y = 2at$.

(a) $\frac{-1}{2at^3}$

(b) $\frac{1}{2at^2}$

(c) $\frac{-1}{2at^2}$

(d) 0

82. Determine the value of the constant k so that the function

(a) $\frac{3}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{2}$

(d) 0

Answers

- | | | | | | |
|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (a) | 3. (d) | 4. (d) | 5. (b) | 6. (d) |
| 7. (a) | 8. (b) | 9. (c) | 10. (a) | 11. (a) | 12. (c) |
| 13. (b) | 14. (a) | 15. (d) | 16. (b) | 17. (a) | 18. (c) |
| 19. (d) | 20. (d) | 21. (c) | 22. (d) | 23. (c) | 24. (a) |
| 25. (b) | 26. (a) | 27. (c) | 28. (b) | 29. (c) | 30. (b) |
| 31. (c) | 32. (d) | 33. (c) | 34. (b) | 35. (b) | 36. (d) |
| 37. (b) | 38. (b) | 39. (b) | 40. (b) | 41. (d) | 42. (c) |
| 43. (c) | 44. (a) | 45. (d) | 46. (a) | 47. (b) | 48. (b) |
| 49. (a) | 50. (b) | 51. (a) | 52. (a) | 53. (a) | 54. (b) |
| 55. (c) | 56. (b) | 57. (c) | 58. (b) | 59. (c) | 60. (a) |
| 61. (a) | 62. (c) | 63. (b) | 64. (c) | 65. (c) | 66. (c) |
| 67. (b) | 68. (a) | 69. (c) | 70. (c) | 71. (a) | 72. (c) |
| 73. (a) | 74. (a) | 75. (b) | 76. (b) | 77. (c) | 78. (a) |
| 79. (b) | 80. (b) | 81. (a) | 82. (b) | | |

CASE-BASED QUESTIONS

Choose the correct option in the following questions.

1. Read the following and answer any four questions from (i) to (v).

A potter made a mud vessel, where the shape of the pot is based on $f(x) = |x - 3| + |x - 2|$, where $f(x)$ represents the height of the pot.



[CBSE Question Bank]

Answer the questions given below.

(i) When $x > 4$ What will be the height in terms of x ?

(a) $x - 2$

(b) $x - 3$

(c) $2x - 5$

(d) $5 - 2x$

(ii) Will the slope vary with x value?

- (a) Yes (b) No
(c) may or may not vary (d) none of these

(iii) What is $\frac{dy}{dx}$ at $x = 3$

- (a) 2 (b) -2
(c) Function is not differentiable (d) 1

(iv) When the x value lies between (2, 3) then the function is

- (a) $2x - 5$ (b) $5 - 2x$ (c) 1 (d) 5

(v) If the potter is trying to make a pot using the function $f(x) = [x]$, will he get a pot or not? Why?

- (a) Yes, because it is a continuous function
(b) Yes, because it is not continuous
(c) No, because it is a continuous function
(d) No, because it is not continuous

Sol. (i) We have, $f(x) = |x - 3| + |x - 2|$

When $x > 4$

$$\therefore f(x) = (x - 3) + (x - 2) = 2x - 5$$

\therefore Option (c) is correct.

(ii) Yes, because when $2 < x < 3$, we have

$$f(x) = -(x - 3) + (x - 2) = 1$$

$$\Rightarrow \text{Slope} = f'(x) = 0$$

but when $x > 3$, we have

$$f(x) = x - 3 + x - 2 = 2x - 5$$

$$\text{then slope} = f'(x) = 2$$

\therefore Option (a) is correct.

(iii) At $x = 3$

$$\text{L.H.D} = \lim_{\lambda \rightarrow 0} \frac{f(3 - \lambda) - f(3)}{-\lambda} = \lim_{\lambda \rightarrow 0} \frac{-(3 - \lambda - 3) + (3 - \lambda - 2) - 1}{-\lambda}$$

$$= \lim_{\lambda \rightarrow 0} \frac{\lambda + 1 - \lambda - 1}{-\lambda} = \lim_{\lambda \rightarrow 0} \frac{0}{-\lambda} = 0$$

$$\text{R.H.D} = \lim_{\lambda \rightarrow 0} \frac{f(3 + \lambda) - f(3)}{\lambda}$$

$$= \lim_{\lambda \rightarrow 0} \frac{(3 + \lambda - 3) + (3 + \lambda - 2) - 1}{\lambda}$$

$$= \lim_{\lambda \rightarrow 0} \frac{\lambda + 1 + \lambda - 1}{\lambda} = 2$$

$$\text{L.H.D} \neq \text{R.H.D at } x = 3$$

$\therefore f(x)$ is not differentiable at $x = 3$

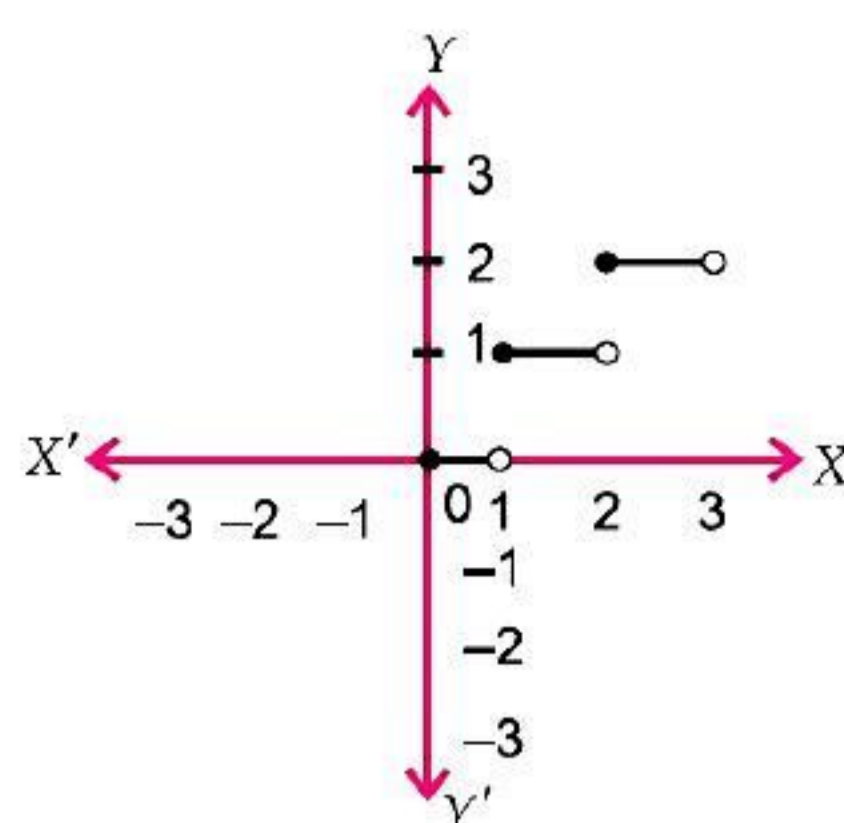
\therefore Option (c) is correct.

(iv) When $2 < x < 3$, we have

$$f(x) = -(x - 3) + (x - 2) = 1$$

∴ Option (c) is correct.

(v) We have the function $f(x) = [x] \leq x$, where x is an integer.



It is not a continuous function, so the potter can not make a pot using the function $f(x) = [x]$.

∴ Option (d) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false and R is also false.

1. **Assertion (A) :** If $f(x).g(x)$ is continuous at $x = a$, then $f(x)$ and $g(x)$ are separately continuous at $x = a$.

Reason (R) : Any function $f(x)$ is said to be continuous at $x = a$, if $\lim_{h \rightarrow 0} f(a + h) = f(a)$.

2. **Assertion (A) :** If $f(x)$ and $g(x)$ are two continuous functions such that $f(0) = 3$, $g(0) = 2$, then $\lim_{x \rightarrow 0} \{f(x) + g(x)\} = 5$.

Reason (R) : If $f(x)$ and $g(x)$ are two continuous functions at $x = a$ then $\lim_{x \rightarrow a} \{f(x) + g(x)\} = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$.

3. **Assertion (A) :** $|\sin x|$ is a continuous function.

Reason (R) : If $f(x)$ and $g(x)$ both are continuous functions, then $gof(x)$ is also a continuous function.

4. **Assertion (A) :** If $y = \sin x$, then $\frac{d^3 y}{dx^3} = -1$ at $x = 0$.

Reason (R) : If $y = f(x) \cdot g(x)$, then $\frac{dy}{dx} = f(x) \cdot \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$.

Answers

1. (d) 2. (a) 3. (a) 4. (b)

HINTS/SOLUTIONS OF SELECTED MCQS

1. We have,

$$f(x) = -|x-1| = \begin{cases} x-1, & \text{if } x \leq 1 \\ -(x-1), & \text{if } x > 1 \end{cases}$$

At $x = 1$

$$\text{LHL} = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \{(1-h)-1\} = 0$$

$$\text{RHL} = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} -(1+h-1) = 0$$

$$f(1) = 1-1 = 0$$

$\therefore \text{LHL} = \text{RHL} = f(1) \Rightarrow f(x)$ is continuous every where.

Now, at $x = 1$

$$\text{LHD} = \frac{d}{dx}(x-1) = 1; \quad \text{RHD} = \frac{d}{dx}\{-(x-1)\} = -1$$

$$\text{LHD} \neq \text{RHD}$$

$\therefore f(x)$ is not differentiable of $x = 1$.

\therefore Option (c) is correct.

3. $f(x) = [x]$

Let $c \in \mathbb{Z}$

$$\text{LHL} = \lim_{x \rightarrow c^-} f(x) = \lim_{h \rightarrow 0} f(c-h) = \lim_{h \rightarrow 0} [c-h]$$

$$= c-1 \quad [\because h > 0 \Rightarrow -h < 0 \Rightarrow c-h < c \text{ and } c \in \mathbb{Z}]$$

$$\text{RHL} = \lim_{x \rightarrow c^+} f(x) = \lim_{h \rightarrow 0} f(c+h) = \lim_{h \rightarrow 0} [c+h]$$

$$= c \quad [\because h > 0 \Rightarrow c+h > c \text{ and } c \in \mathbb{Z}]$$

As $\text{LHL} \neq \text{RHL}$ for $c \in \mathbb{Z}$

$\therefore f(x)$ is not continuous at and $c \in \mathbb{Z}$ and $c \in \mathbb{Z}$ is an arbitrary so $f(x)$ is not continuous at all the integers.

\therefore In the given option (d) is the correct option

4. $f(x) = \frac{1}{x-[x]}$

$$\because x - [x] = 0 \quad \forall x \in \mathbb{Z}$$

$f(x)$ is not defined $\forall x \in \mathbb{Z}$ number of points of discontinuity is infinite.

\therefore Option (d) is correct.

6. Indeed $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist.

\therefore Option (d) is correct.

7. We know that, $f(x) = \cot x$ is continuous in $\mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$.

$$\text{Since,} \quad f(x) = \cot x = \frac{\cos x}{\sin x} \quad [\text{since, } \sin x = 0 \text{ at } \{n\pi, n \in \mathbb{Z}\}]$$

Hence, $f(x) = \cot x$ is discontinuous on the set $\{x = n\pi : n \in \mathbb{Z}\}$.

\therefore Option (a) is correct.

8. $f(x) = |\sin x|$

As $\sin x$ is continuous everywhere so $f(x) = |\sin x|$ is continuous everywhere.

And $\sin x = 0$ for $x = n\pi, n \in \mathbb{Z}$

$\therefore f(x) = |\sin x|$ is not differentiable at $x = n\pi : n \in \mathbb{Z}$

\therefore Option (b) is correct.

10. $\because f(x) = x^2 \sin\left(\frac{1}{x}\right)$, where $x \neq 0$; $\therefore \lim_{x \rightarrow 0} f(x) = 0$

Hence, value of the function f at $x = 0$, so that it is continuous at $x = 0$ is 0.

\therefore Option (a) is correct.

12. $f(x) = \frac{4 - x^2}{4x - x^3}$

$f(x)$ is discontinuous where $4x - x^3 = 0$

i.e., $x(4 - x^2) = 0$ i.e., $x(2 + x)(2 - x) = 0$ i.e., At $x = 0, -2, 2$

Hence $f(x)$ is discontinuous at exactly three points.

\therefore Option (c) is correct.

13. $f(x) = |x - 3| \cos x = g(x)h(x)$ where $g(x) = |x - 3|$ and $h(x) = \cos x$

$h(x) = \cos x$ is differentiable everywhere

But $g(x) = |x - 3|$ is differentiable everywhere except at $x = 3$.

$\therefore f(x) = g(x)h(x)$ is differentiable everywhere except at $x = 3$.

$\therefore f(x) = g(x)h(x)$ is differentiable at $x \in \mathbb{R} - \{3\}$.

\therefore Option (b) is correct.

15. $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2 \tan^{-1} x \Rightarrow \frac{du}{dx} = \frac{2}{1+x^2}$

$v = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2 \tan^{-1} x \Rightarrow \frac{dv}{dx} = \frac{2}{1+x^2}$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{2/1+x^2}{2/1+x^2} = 1$

\therefore Option (d) is correct.

16. We have, $y = \log \sqrt{\tan x}$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{\tan x}} \times \frac{1}{2\sqrt{\tan x}} \times \sec^2 x$

$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x}{2 \tan x} \Rightarrow \frac{dy}{dx} \text{ at } x = \frac{\pi}{4} = \frac{(\sqrt{2})^2}{2 \times 1} = \frac{2}{2} = 1$

\therefore Option (b) is correct.

18. LHD (at $x = 0$) \neq RHD (at $x = 0$)

Also, LHD (at $x = 2$) \neq RHD (at $x = 2$)

Therefore, $f(x) = |x| + |x - 2|$ is not differentiable at $x = 0$ and $x = 2$.

\therefore Option (c) is correct.

19. Since, the value of function $f(x) = \tan x$ is $\pm \infty \quad \forall x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$.

Hence $f(x) = \tan x$ is discontinuous on the set $\left\{x : x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\right\}$.

\therefore Option (d) is correct.

20. Let $y = \tan x$ and $t = \sin x$

$$\Rightarrow \frac{dy}{dx} = \sec^2 x \text{ and } \frac{dt}{dx} = \cos x$$

$$\text{Now, derivative of } \tan x \text{ w.r.t. } \sin x = \frac{dy}{dt} = \frac{dy/dx}{dt/dx} = \frac{\sec^2 x}{\cos x} = \sec^3 x$$

\therefore Option (d) is correct.

$$22. \because \frac{g(x)}{f(x)} = \frac{\frac{x^2}{3} + 1}{x - 3} = \frac{x^2 + 3}{3(x - 3)} \Rightarrow \lim_{x \rightarrow 3} \frac{g(x)}{f(x)} = \infty$$

\Rightarrow Function $\frac{g(x)}{f(x)}$ is discontinuous at $x = 3$.

\therefore Option (d) is correct.

$$24. \because \lim_{x \rightarrow 0} f(x) \text{ exists} \Rightarrow \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} f(0 + h)$$

$$\Rightarrow \lim_{h \rightarrow 0} f(-h) = \lim_{h \rightarrow 0} f(h)$$

$$\Rightarrow \lim_{h \rightarrow 0} \{|-h| + a\} = \lim_{h \rightarrow 0} \cos[h]$$

$$\Rightarrow a = \cos 0 \quad \Rightarrow a = 1$$

\therefore Option (a) is correct.

$$25. \text{ Since, } 0 \leq x < 1 \Rightarrow -1 \leq x - 1 < 0 \Rightarrow [x - 1] = -1$$

$$1 \leq x < 2 \Rightarrow 0 \leq x - 1 < 1 \Rightarrow [x - 1] = 0$$

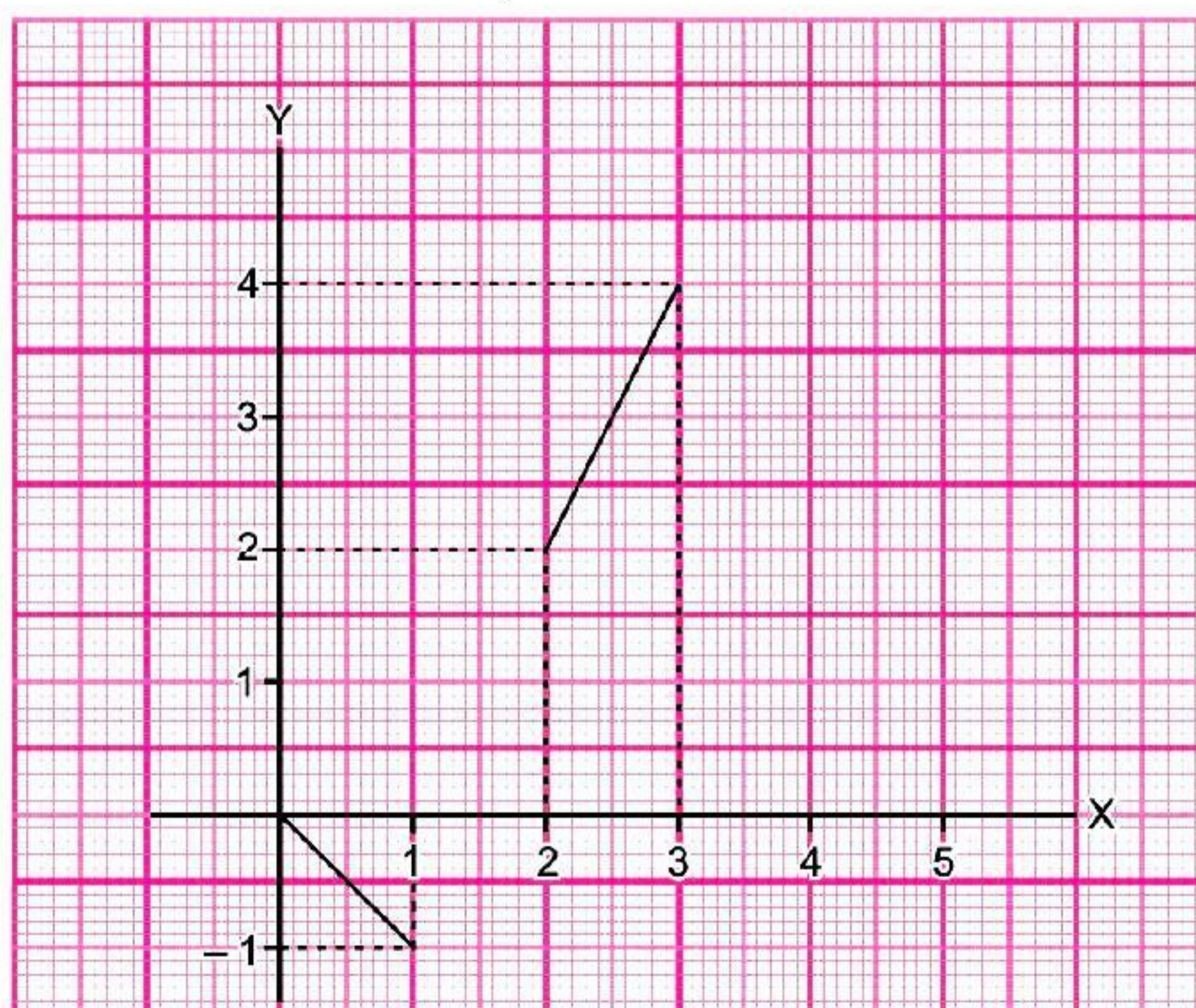
$$\text{And } 2 \leq x < 3 \Rightarrow [x] = 2$$

Therefore, given function may be written as

$$f(x) = \begin{cases} -x, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2(x-1), & 2 \leq x < 3 \end{cases}$$

From the graph it is obvious that

$f(x)$ is not continuous at $x = 1$ and 2 , and thus not differentiable



\therefore Option (b) is correct.

29. Given expression is

$$x = e^{y+e^{y+\dots\text{to } \infty}} \Rightarrow x = e^{y+x}$$

Taking log on both sides, we have

$$\log x = \log e^{y+x} \Rightarrow \log x = y + x$$

Differentiating, w.r.t. x , we get

$$\frac{1}{x} = \frac{dy}{dx} + 1 \Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$$

\therefore Option (c) is correct.

32. $f(x) = \begin{cases} \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \sin \left(\frac{1}{x} \right) \text{ does not exist.}$$

But value of $\sin \left(\frac{1}{x} \right)$ oscillates between -1 and 1 .

\therefore Option (d) is correct.

36. $f(x) = \frac{1}{x-1}$

$$\text{As } \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{1}{x-1} \text{ does not exist.}$$

$\therefore f(x)$ has asymptotic discontinuity.

\therefore Option (d) is correct.

37. $y = \frac{e^x}{x}$ is the curve which has vertical asymptotes at $x = 0$.

\therefore Option (b) is correct.

38. $y = x + e^{-x} \sin x = x + \frac{\sin x}{e^x}$

$$\therefore \lim_{x \rightarrow \infty} \frac{\sin x}{e^x} = 0$$

$$\text{As } -1 \leq \sin x \leq 1$$

$$\therefore -e^{-x} \leq e^{-x} \sin x \leq e^{-x}$$

$$\lim_{x \rightarrow \infty} \pm e^{-x} = 0$$

$$\therefore \lim_{x \rightarrow \infty} e^{-x} \sin x = 0$$

\therefore Line $y = x$ is oblique asymptote to the given curve.

\therefore Option (b) is correct.

41. $f(x) = \sec 2x + \operatorname{cosec} 2x = \frac{1}{\cos 2x} + \frac{1}{\sin 2x}$

$$\cos 2x = 0 \Rightarrow 2x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{4} \text{ where } n = 0, \pm 1, \pm 2, \dots$$

$$\text{And } \sin 2x = 0 \Rightarrow 2x = n\pi \Rightarrow x = \frac{n\pi}{2} \text{ where } n = 0, \pm 1, \pm 2, \pm 3, \dots$$

None of the options (a), (b) and (c) are correct.

∴ Option (d) is correct.

42. $f(x) = x^n$

$$\begin{aligned} f(1) + \frac{f^1(1)}{1} + \frac{f^2(1)}{2} + \frac{f^3(1)}{3} + \dots + \frac{f^n(1)}{n} \\ = 1 + \frac{n}{1} + \frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{3} + \dots + \frac{n(n-1)\dots 1}{n} \\ = (1+1)^n = 2^n \text{ [By using Binomial expansion]} \end{aligned}$$

Hence option (c) is correct.

43. $F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

where $f(x), g(x), h(x)$ are polynomial of degree 2.

$$\therefore f'''(x) = 0 = g'''(x) = h'''(x)$$

$$\begin{aligned} F'(x) &= \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ f''(x) & g''(x) & h''(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix} \\ &= 0 + 0 + 0 = 0 \end{aligned}$$

∴ Option (c) is correct.

45. Given that $g(x)$ is the inverse of $f(x)$

$$\therefore (fog)(x) = x \Rightarrow f(g(x)) = x$$

$$\therefore f'(g(x))g'(x) = 1$$

$$\Rightarrow g'(x) = \frac{1}{f'(g(x))}$$

We have $f'(x) = \sin x$ so $f'(g(x)) = \sin(g(x))$

$$\therefore g'(x) = \frac{1}{\sin(g(x))} = \operatorname{cosec}(g(x))$$

∴ Option (d) is correct.

47. $f(x) = \log \sqrt{\tan x}$

$$f'(x) = \frac{1}{\sqrt{\tan x}} \times \frac{1}{2\sqrt{\tan x}} \times \sec^2 x = \frac{1}{2 \tan x} \times \sec^2 x$$

$$f'\left(\frac{\pi}{4}\right) = \frac{1}{2 \times 1} \times (\sqrt{2})^2 = \frac{2}{2} = 1$$

Option (b) is correct.

50. $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$$f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} x^3 & \sin x & \cos x \\ 0 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\begin{aligned}
&= \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + 0 + 0 \\
\Rightarrow f'(x) &= \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} \\
\Rightarrow f''(x) &= \begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 0 & 0 & 0 \\ p & p^2 & p^3 \end{vmatrix} + \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \\
&= \begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} \\
\text{Similarly, } f'''(x) &= \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} \\
\Rightarrow f'''(0) &= \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = 0
\end{aligned}$$

∴ Option (b) is correct.

52. $f(x) = x|x| = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

At $x = 0$

$$\begin{aligned}
\text{LHD} &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{-x^2 - 0}{x} \\
&= \lim_{x \rightarrow 0^-} (-x) = 0
\end{aligned}$$

$$\text{RHD} = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{x^2 - 0}{x} = \lim_{x \rightarrow 0^+} (x) = 0$$

LHD = RHD so f is differentiable at $x = 0$.

And $f(x)$ is also differentiable at $x \neq 0$.

∴ Set of all points where given function $f(x)$ is differentiable is $R = (-\infty, \infty)$.

Option (a) is correct.

54. $f(x) = x - |x| = g(x) + h(x)$

where $g(x) = x$, $h(x) = -|x|$

As $g(x)$ and $h(x)$ are both continuous $\forall x \in \mathbb{R}$

∴ $f(x)$ is continuous $\forall x \in \mathbb{R}$

And $g(x)$ is differentiable $\forall x \in \mathbb{R}$

but $h(x)$ is not differentiable at $x = 0$

∴ $f(x) = g(x) + h(x)$ is not differentiable at $x = 0$

∴ Option (b) is correct.

56. We have, $f\left(\frac{x+y}{1-xy}\right) = f(x) + f(y) \quad \forall x, y \in \mathbb{R}$

Since it is of the form $\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}x + \tan^{-1}y$

Let $f(x) = A \tan^{-1}x$

$$\lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{A \tan^{-1}x}{x} = \frac{1}{3}$$

$$\Rightarrow A \lim_{x \rightarrow 0} \frac{\tan^{-1}x}{x} = \frac{1}{3} \quad \Rightarrow A \times 1 = \frac{1}{3} \quad \Rightarrow A = \frac{1}{3}$$

$$\therefore f(x) = \frac{1}{3} \tan^{-1}x$$

$$\Rightarrow f(1) = \frac{1}{3} \tan^{-1}(1) = \frac{1}{3} \times \frac{\pi}{4} = \frac{\pi}{12}$$

\therefore Option (b) is correct.

57. LHL = $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (1 - 3x) = 1$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (x^2 + 3) = 3$$

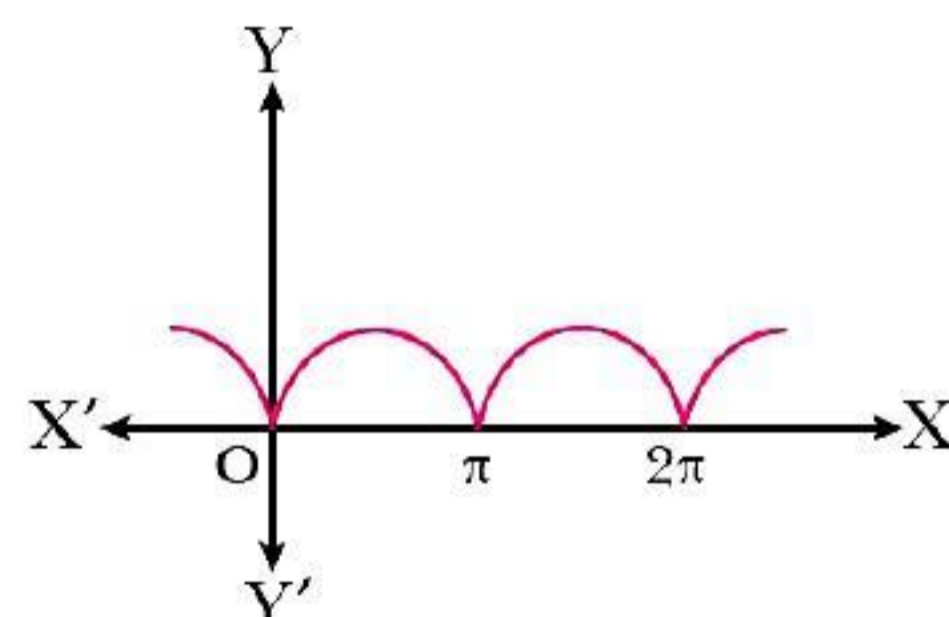
$$f(0) = 3, \text{LHL} \neq \text{RHL} = f(0)$$

Here $\lim_{x \rightarrow 0^+} f(x) = f(0)$

$\Rightarrow f(x)$ is right continuous but discontinuous from left.

\therefore Option (c) is correct.

58.



It is clear from graph that $f(x)$ is continuous everywhere in $0 \leq x \leq 2\pi$. And has sharp edge at $x = 0, \pi, 2\pi$ so it is not differentiable at $x = 0, \pi, 2\pi$.

Because it has no unique tangents.

\therefore Option (b) is correct.

$$\begin{aligned} 59. \quad \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \left[\tan\left(\frac{\pi}{4} + x\right) \right]^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left[\frac{1 + \tan x}{1 - \tan x} \right]^{\frac{1}{x}} \\ &= \lim_{x \rightarrow 0} \left[(1 + \tan x)^{\frac{1}{\tan x}} \right]^{\frac{\tan x}{x}} \times \lim_{x \rightarrow 0} \left[(1 - \tan x)^{-\frac{1}{\tan x}} \right]^{\frac{\tan x}{x}} \\ &= e \times e = e^2 \quad \left[\because \lim_{x \rightarrow 0} [1 + x]^{\frac{1}{x}} = e \right] \end{aligned}$$

$\therefore f(x)$ is continuous at $x = 0$.

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$$\Rightarrow e^2 = k \quad \Rightarrow k = e^2$$

∴ Option (c) is correct.

62. $f(x) = x^{3/2} - \sqrt{x^3 + x^2}$

Domain of $f = [0, \infty)$

So, LHD at $x = 0$ does not exist.

∴ Option (c) is correct.

63. $f(x) = \frac{1}{\log |x|}$

$f(x)$ is not defined for $x = 0, -1, 1$

∴ $f(x)$ is not continuous at $x = 0, -1, 1$

∴ Option (b) is correct.

64. We have $f(x) = \frac{\sin 4\pi [\pi^2 x]}{7 + [x]^2}$

We know that $[\pi^2 x]$ is an integer for every x .

∴ $4\pi[\pi^2 x]$ is an integral multiple of π .

∴ $\sin 4\pi[\pi^2 x] = 0$ and $7 + [x]^2 \neq 0 \forall x$

∴ $f(x) = 0 \forall x$

⇒ $f(x)$ is a constant function so $f(x), f'(x), f''(x), \dots, f^{(n)}(x)$ exists $\forall x$.

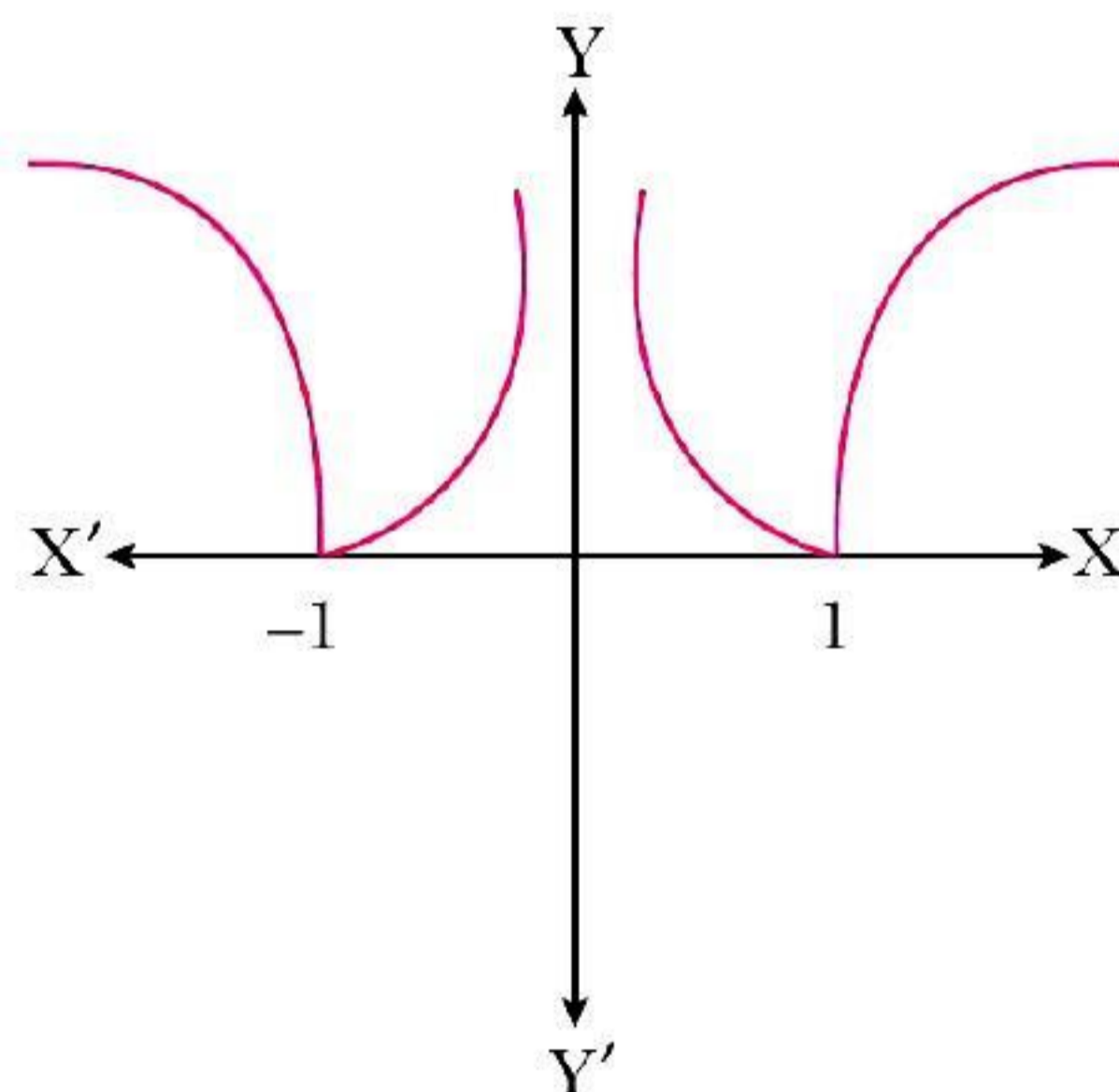
∴ Option (c) is correct.

66. We know that domain of $\sin^{-1} x$ is $|x| < 1$ and so $\sin^{-1} \left(\frac{1+x^2}{2x} \right)$ is defined only for $|x| < 1$.

Hence, $f(x)$ is neither continuous nor differentiable at $x = 1$

∴ Option (c) is correct.

68. $y = |\log |x||$



From the graph, $f(x)$ is continuous in its domain but LHD at $x = 1$ is negative.

RHD at $x = 1$ is positive (as shown from the graph)

as LHD at $x = 1 \neq$ RHD at $x = 1$

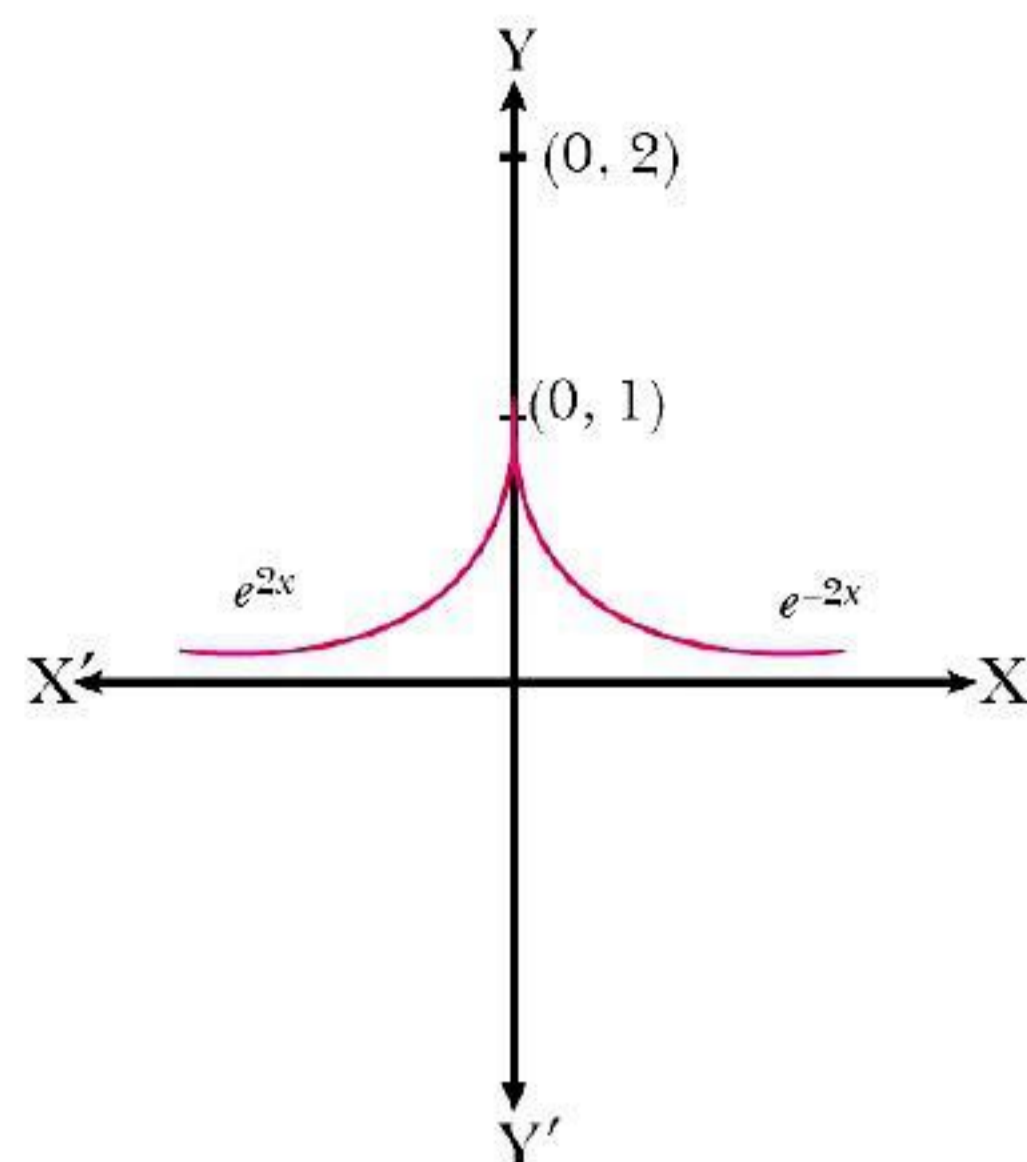
∴ $f(x)$ is not differentiable at $x = 1$.

Also LHD at $x = -1$ is negative (as shown from the graph) and RHD at $x = -1$ is positive.

∴ $f(x)$ is not differentiable at $x = -1$.

∴ Option (a) is correct.

69.



Given $g(x) = \begin{cases} e^{2x}, & x < 0 \\ e^{-2x}, & x \geq 0 \end{cases}$

$$g'(x) = \begin{cases} 2e^{2x}, & x < 0 \\ -2e^{-2x}, & x \geq 0 \end{cases}$$

\therefore LHD at $x = 0$, $g'(0) = 2e^{2 \times 0} = 2e^0 = 2$

RHD at $x = 0$, $g'(0) = -2e^0 = -2 \times 1 = -2$

As LHD \neq RHD at $x = 0$

$\therefore g(x)$ is not differentiable at $x = 0$.

Again RHL = $\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} e^{-2x} = e^0 = 1$

LHL = $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} e^{2x} = e^0 = 1$

$g(0) = e^0 = 1$

As LHL = RHL = $f(0)$

$\therefore g(x)$ is continuous $\forall x \in \mathbb{R}$

\therefore Option (c) is correct.

71. $f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$

RHL = $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x + 1) = 3$

LHL = $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (2x - 1) = 4 - 1 = 3$

\therefore Since f is continuous at $x = 2$ so LHL = RHL = $f(2)$

$\Rightarrow 3 = a \Rightarrow a = 3$

\therefore Option (a) is correct.

73. $f(x) = e^{-|x|} = \frac{1}{e^{|x|}}$ is continuous everywhere but not differentiable at $x = 0$

\therefore Option (a) is correct.

74. $x^y = e^{x-y}$

Taking logarithm on both sides, we get

$y \log x = (x - y) \log e = x - y$

$\Rightarrow (1 + \log x)y = x \Rightarrow y = \frac{x}{1 + \log x}$

$$\frac{dy}{dx} = \frac{(1 + \log x) \times 1 - x \left(0 + \frac{1}{x}\right)}{(1 + \log x)^2} = \frac{1 + \log x - 1}{(1 + \log x)^2} = \frac{\log x}{(1 + \log x)^2}$$

∴ Option (a) is correct.

75. $x^y = y^x \Rightarrow y \log x = x \log y$

$$\frac{y}{x} + \log x \frac{dy}{dx} = \frac{x}{y} \frac{dy}{dx} + \log y$$

$$\Rightarrow \left(\log x - \frac{x}{y} \right) \frac{dy}{dx} = \left(\log y - \frac{y}{x} \right) = \frac{x \log y - y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x \log y - y}{y \log x - x} \times \frac{y}{x} = \frac{y}{x} \times \frac{x \log y - y}{y \log x - x}$$

∴ Option (b) is correct.

76. Let $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}$

$$\Rightarrow y = \sqrt{\sin x + y} \text{ should be out of square root}$$

$$\Rightarrow y^2 = \sin x + y \Rightarrow y^2 - y = \sin x$$

Differentiate with respect to x , we get

$$\therefore 2y \frac{dy}{dx} - \frac{dy}{dx} = \cos x$$

$$\Rightarrow (2y - 1) \frac{dy}{dx} = \cos x \Rightarrow \frac{dy}{dx} = \frac{\cos x}{\sqrt{2y - 1}}$$

∴ Option (b) is correct.

80. $y = x^x = e^{x \log x}$

$$\therefore \frac{dy}{dx} = e^{x \log x} \left(x \times \frac{1}{x} + \log x \right)$$

$$= e^{x \log x} (1 + \log x)$$

$$\therefore \frac{d^2 y}{dx^2} = e^{x \log x} \left(\frac{1}{x} \right) + (1 + \log x) e^{x \log x} (1 + \log x)$$

$$= \frac{x^x}{x} + (1 + \log x)^2 e^{x \log x}$$

$$= x^{x-1} + (1 + \log x)^2 x^x = x^x \left\{ (1 + \log x)^2 + \frac{1}{x} \right\}$$

∴ Option (b) is correct.

81. $x = at^2 \Rightarrow \frac{dx}{dt} = 2at$

$$y = 2at \Rightarrow \frac{dy}{dt} = 2a$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

$$\therefore \frac{d^2 y}{dx^2} = -\frac{1}{t^2} \times \frac{dt}{dx} = -\frac{1}{t^2} \times \frac{1}{2at} = -\frac{1}{2at^3}$$

∴ Option (a) is correct.

