Control Systems Test 5

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 23: Select the correct alternative from the given choices.

1. The transfer functions of two compensations are given below $G_{c1} = \dfrac{k_1(0.1s+1)}{(0.5s+1)}$ and $G_{c2} = \dfrac{k_2(0.2s+1)}{(0.1s+1)}$

Which one of the following statements is correct?

(A) G_{c1} is lead compensator and G_{c2} is a lag compensator
(B) G_{c1} is lag compensator and G_{c2} is lead compensator
(C) Both G_{c1} and G_{c2} are high pass networks
(D) Both G_{c1} and G_{c2} are low pass networks

2. The following relation involving state transition matrix $\phi(t)$ is True?

(A) $\phi(kt) = \phi^{-k}(t)$
(B) $\phi\,(t_1 - t_2) = \phi\,(t_1 - t_0).\phi(t_0 - t_2)$
(C) $\phi\,(t) = I$
(D) $\phi(t) = e^{-At}$

3. What is represented by state transition matrix of a system?

(A) Impulse response (B) Zero state response
(C) Forced response (D) Step response

4. A first order system with a proportional (p) controller in the negative feed back loop has an off set to a step input. This off set can be eliminated by

(A) adding an integral mode to the controller
(B) adding a derivative mode to the controller
(C) decreasing the magnitude of the gain of the p – controller
(D) increase the stability

5. The state space representation of a system is given by

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \text{ and } y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T x.$$

Then the transfer function of the system is

(A) $\dfrac{1}{s+3}$ (B) $\dfrac{s+1}{s^2+4s+3}$
(C) $\dfrac{s}{s^2+4s+3}$ (D) $\dfrac{1}{s+1}$

6. Let $\dot{X} = \begin{bmatrix} 1 & 2 \\ b & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

Where b is unknown constant. This system is _____

(A) uncontrollable for all values of b
(B) controllable for 'b' = 1
(C) controllable for all values of 'b'
(D) uncontrollable for b = '0'

7. A state variable description of a linear system is $\dot{X}(t) = Ax(t)$, where $x(t)$ is the two dimensional state vector and A is given by $A = \begin{bmatrix} 0 & 2 \\ -1 & -3 \end{bmatrix}$. Then the system is a

(A) under damped system
(B) over damped system
(C) critically damped
(D) undamped system

8. The transfer function G(s) of a PID controller is

(A) $k_p\left[1 + T_i s + \dfrac{1}{k_D s}\right]$ (B) $k\,[1 + T_i S + T_2 S]$

(C) $k_p\left[1 + \dfrac{1}{T_i s} + T_d s\right]$ (D) A and C

9. Which one of the following statements is correct?
A plant is controlled by a proportional controller.
If in that plant a time delay element is introduced in the loop, its

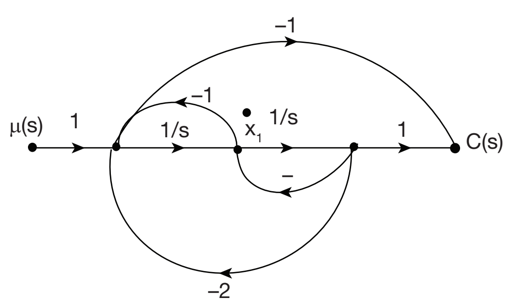
(A) phase margin decrease
(B) gain margin increases
(C) P.M increases
(D) G.M remain same

10. Which of the following are the effects of PI controller?

(i) Steady state error improves
(ii) Reduces high frequency noise
(iii) Rise time decreases
(iv) Damping factor increases
(v) Stability increases

(A) i and ii only (B) ii and iii and v only
(C) i, ii and iv (D) i, ii, iv and v only

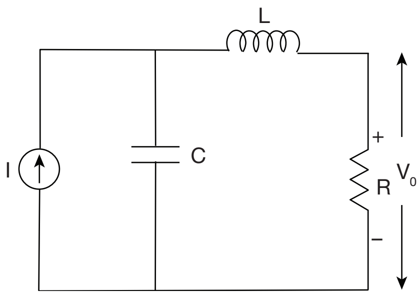
11. A feed back system is represented by a signal flow graph shown in figure



The state model of signal flow graph is

(A) $\dot{x}_1(t) = -x_1 + x_2$
$\dot{x}_2(t) = -x_1 - x_2 + \mu(s)$
(B) $\dot{x}_1(t) = x_1 - 2x_2$
$\dot{x}_2(t) = -2x_1 - x_2 + \mu(s)$
(C) $\dot{x}_1(t) = -x_1 + x_2$
$\dot{x}_2(t) = x_1 - 2x_2 + \mu(s)$
(D) None of these

12. Consider the electrical network shown in figure



The state equation matrix A by considering V_c and i_L are state variables is

(A) $\begin{bmatrix} 0 & -1/C \\ \frac{1}{L} & -R/L \end{bmatrix}$ (B) $\begin{bmatrix} 0 & C \\ L & -R/L \end{bmatrix}$

(C) $\begin{bmatrix} 0 & C \\ L & -L/R \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1/L \\ 1/C & -R/L \end{bmatrix}$

13. A system is described by the following equations $\dot{x}_1(t) = x_2(t)$ $\dot{x}_2(t) = -x_1(t) - 2x_2(t) + u(t)$ and output $y(t) = x_1(t) + x_2(t)$. The steady state error due to step input is

(A) 0 (B) ∞

(C) $\frac{1}{2}$ (D) $\frac{-1}{3}$

14. A state space representation of a system is given by $\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}x$, $y = \begin{bmatrix} -1 & 2 \end{bmatrix}x$, and $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The output of this system is

(A) $\cos\sqrt{2}t + \sin\sqrt{2}t$

(B) $-\cos\sqrt{2}t - 2\sqrt{2}\sin\sqrt{2}\,t$

(C) $\cos\sqrt{2}\,t + 2\sqrt{2}\,\sin\sqrt{2}\,t$

(D) $\cos t + 2\sqrt{2}\,t$

15. In the following state equation u is the unit step input $\dot{X} = \begin{bmatrix} -2 & 0 \\ -3 & -1 \end{bmatrix}x + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u$, $y = \begin{bmatrix} 0 & 1 \end{bmatrix}x$, the system is

(A) Controllable but not observable
(B) not controllable but observable
(C) controllable and observable
(D) not controllable and not observable

16. The open loop transfer function of a plant is $G(s) = \frac{(s+1)^2}{s^2-4}$.

If the plant is operated in a unity feed back configuration then the lead compensator that can stabilize this control system is

(A) $4\frac{(s+2)}{3}$ (B) $\frac{4(s-2)}{(s+1)}$

(C) $\frac{2(s-2)}{(s-1)}$ (D) $\frac{3(s-4)}{s+1}$

17. A particular control system is described by the following state equations

$\dot{X} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}X + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mu$ and $y = \begin{bmatrix} k & 0 \end{bmatrix}x$

The transfer function of this system is

(A) $\frac{y(s)}{\mu(s)} = \frac{k}{s^2+2s+3}$ (B) $\frac{y(s)}{\mu(s)} = \frac{k}{s^2+3s+2}$

(C) $\frac{y(s)}{\mu(s)} = \frac{1}{s^2+s+5}$ (D) None of these

18. The closed loop transfer function of a system is $\frac{1}{(s+1)(s+2)}$, the state model matrices are

(A) $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 1 \end{bmatrix}, D = [1]$

(B) $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \end{bmatrix}, D = [0]$

(C) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}, D = [1]$

(D) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, D = [0]$

19. The maximum phase shift that can be provided by a compensator with transter function $G_c(s) = \frac{k(1+2s)}{(1+6s)}$ is

(A) 60° (B) –60°
(C) 45° (D) –30°

20. A system has the transfer function

$G_c(s) = \frac{0.2(0.2s+1)}{(0.5s+1)}$

Its gain at frequency of maximum phase

(A) 0.126 (B) 0.632
(C) 1.2 (D) 0.3

21. For the given network, the maximum phase lead ϕ_m of V_0 with respect to V_i is

(A) $\sin^{-1}\left[\frac{R_1}{R_1+2R_2}\right]$ (B) $\cos^{-1}\left[\frac{R_2}{R_1+R_2}\right]$

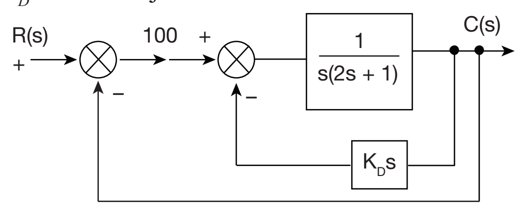
(C) $\text{Tan}^{-1}\left[\frac{R_1}{2}\sqrt{1+\frac{R_1}{R_2}}\right]$ (D) A and C only

22. Which of the following are NOT effects of Derivative controller?

(i) Steady state error increase
(ii) Type of system increases
(iii) Stability increases
(iv) Both steady state error and stability decreases

(A) only ii (B) I, ii and iii only
(C) ii, iii and iv only (D) ii and iv only

23. Derivative feedback is employed in the control system shown in the figure, to improve its damping. If the required damping factor of the system is 0.5, the value of K_D must be adjusted to



(A) 7 (B) 13.14
(C) 6 (D) None of these

Direction for questions 24 and 25: Consider a control system with state model

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} u;\ x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

24. The state transition matrix is

(A) $\begin{bmatrix} 2e^{-t} + e^{-2t} & e^{-t} + e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$

(B) $\begin{bmatrix} 2e^{-t} - e^{2t} & e^{-t} - e^{2t} \\ -2e^{-t} + 2e^{2t} & -e^{t} + 2e^{2t} \end{bmatrix}$

(C) $\begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$

(D) $\begin{bmatrix} 2e^{-t} + e^{-2t} & e^{-t} - e^{-2t} \\ 2e^{-t} - 2e^{-2t} & e^{-t} - 2e^{-2t} \end{bmatrix}$

25. Find the zero state response.

(A) $\begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$ (B) $\begin{bmatrix} e^{-t} - e^{-2t} \\ e^{-t} - 2e^{-2t} \end{bmatrix}$

(C) $\begin{bmatrix} e^{-t} + e^{-2t} \\ e^{-t} + 2e^{-2t} \end{bmatrix}$ (D) $\begin{bmatrix} e^{-t} + e^{-2t} \\ e^{-t} - 2e^{-2t} \end{bmatrix}$

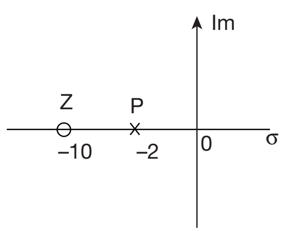
Answer Keys

1. B	**2.** B	**3.** B	**4.** A	**5.** D	**6.** C	**7.** B	**8.** C	**9.** A	**10.** C
11. A	**12.** A	**13.** A	**14.** B	**15.** C	**16.** B	**17.** A	**18.** B	**19.** D	**20.** A
21. D	**22.** D	**23.** B	**24.** C	**25.** A					

Hints and Explanations

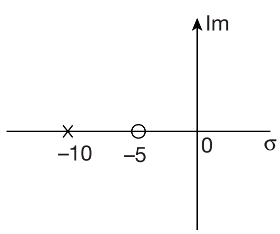
1. Pole – zero

G_{c1}



Pole near by origin so it is a lag n/w (or) LPF

G_{c2}



Zero near by origin it indicates lead n/w or HPF.

Choice (B)

2. We know

State transition matrix $\phi(t) = e^{At}$

$\therefore$ $\phi(0) = I$

And $\phi(kt) = \phi\, k(t)$

So only option B is correct Choice (B)

3. We know for zero input

$x(t) = \phi(t)\, x(0)$

$\therefore$ $\phi(t)$ is a zero state or free response.

Choice (B)

4. Choice (A)

5. $T/F = C[sI - A]^{-1}B + D$

$D = 0$

$$(SI - A) = \begin{bmatrix} s+1 & 0 \\ 0 & s+3 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{adj(sI - A)}{|sI - A|} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$T/F = [1\ 1]\begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix} = \frac{1}{s+1}.$$

Choice (D)

6. Controllability matrix

$Q_c = [B\ AB\ A^2B \ldots\ldots A^{n-1}B]$

Where $n \Rightarrow$ order of matrix A

$\therefore \quad n = 2$

$Q_c = \begin{bmatrix} B & AB \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 \\ b & 0 \end{bmatrix}_{2\times 2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}_{2\times 1}$

$Q_c = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

$|Q_c| = -2$

$\therefore$ $|Q_c| \neq 0$ so the system is controllable for all values of 'b'. Choice (C)

7. Characteristic equation of the system is

$|SI - A| = 0$

$(SI - A) = \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}$

$|SI - A| = \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}$

$s(s + 3) + 2 = 0$

$s^2 + 3s + 2 = 0$

$\therefore \quad \omega_n = \sqrt{2}$ rad/sec

$2\zeta\omega_n = 3$

$\zeta = \dfrac{3}{2\sqrt{2}} \quad \therefore > 1.$ Choice (B)

8. PID – controller $\Rightarrow P + I + D$

$G(s) = K_p + \dfrac{k_I}{S} + k_D\, s.$ Choice (C)

9. a time delay element reduces the both $G.M$ and $P.M.$ Choice (A)

10. Effects of PI controller

(i) Type of the system increases, hence e_{ss} decreases

(ii) It acting as a LPF so $BW \downarrow$and Rise time

(iii) $\xi \Rightarrow MP\,\% \downarrow$

(iv) LPF so it reduces high frequency noise. Choice (C)

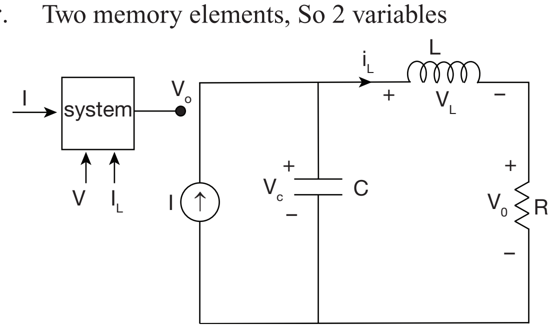
11. $\dot{x}_1 = -x_1 + x_2$

$\dot{x}_2 = \mu(s) - 2x_1 - \dot{x}_1$

$\dot{x}_2 = \mu(s) - x_1 - x_2$ Choice (A)

12. Initial conditions are state variables

$\therefore$ Two memory elements, So 2 variables



$I = I_c + I_L$

$I_c = \dfrac{c.dV_c}{dt}$

$I = c\dfrac{dV_c}{dt} + I_L$

$\dfrac{dV_c}{dt} = \dfrac{1}{c}\,[I - I_L]$ ----------- (i)

$V_c - V_L - i_L R = 0$

$L.\dfrac{di_L}{dt} = V_c - i_L R$

$\dfrac{di_L}{dt} = \dfrac{1}{L}\,\{V_c - i_L R\}$ ---------- (ii)

$$\begin{bmatrix} \dfrac{dV_c}{dt} \\ \dfrac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{-1}{C} \\ \dfrac{1}{L} & \dfrac{-R}{L} \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} \dfrac{1}{c} \\ 0 \end{bmatrix} I$$

Choice (A)

13. $T/F = C(SI - A)^{-1} B + D$

Where $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = [1\ 1]$

$(sI - A)^{-1} = \dfrac{adj(sI - A)}{|sI - A|}$

$(sI - A) = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}$

$|sI - A| = s(s + 2) + 1 = s^2 + 2s + 1 = (s + 1)^2$

Adj $(sI - A) = \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}$

$T(s) = [1\ 1] \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(s) = \dfrac{1}{(s+1)^2} \begin{bmatrix} s+1 & s+1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \dfrac{s+1}{(s+1)^2} = \dfrac{1}{s+1}$

$\dfrac{y(s)}{u(s)} = \dfrac{1}{s+1}$

$y(s) = \dfrac{u(s)}{s+1}$

But $u(s) = \dfrac{1}{s}$

$y(s) = \dfrac{1}{s(s+1)}$

$y(s) = \dfrac{1}{s(s+1)}$

$y = \underset{s\to 0}{Lt}\; s\, y(s) = 1$

$\therefore \quad e_{ss} = 1 - y_{ss} = 0.$ Choice (A)

2^{nd} method:–

$E_{step}(\infty) = (1 + CA^{-1}B)$

14. $A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} : x(t) = \phi(t).x(o)$

$\phi(t) = L^{-1}\{(sI - A)^{-1}\}$

$\therefore \quad y = \begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -x_1 + 2x_2$

$\phi(s) = (sI - A)^{-1}$

$(sI - A) = \begin{bmatrix} s & -1 \\ 2 & s \end{bmatrix}$

$|sI - A| = s^2 + 2$

$$\phi(s) = \frac{adj(sI - A)}{|sI - A|} = \begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix} \times \frac{1}{s^2 + 2}$$

$X(s) = \phi(s)\, x(0)$

$$= \frac{1}{s^2+2}\begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{s^2+2}\begin{bmatrix} s \\ -2 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} \cos\sqrt{2}t \\ -\sqrt{2}.\sin\sqrt{2}t \end{bmatrix}$$

$$y(t) = \begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} \cos\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t \end{bmatrix}$$

$$y(t) = \left(-\cos\sqrt{2}t - 2\sqrt{2}\sin\sqrt{2}t\right)$$

Choice (B)

15. Controllability matrix Q_c:

$Q_c = \begin{bmatrix} B & AB \end{bmatrix}$

$$AB = \begin{bmatrix} -2 & 0 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$$

$|Q_c| \neq 0$

$\therefore$ Controllable observability matrix Q:

$Q_0 = [C^T A^T C^T \ldots\ldots (A^T)^{n-1} C^T]$

$Q_0 = [C^T A^T C^T]$

$$A^T C^T = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

$$Q_0 c = \begin{bmatrix} 0 & -3 \\ 1 & -1 \end{bmatrix}$$

$|Q_0| \neq 0$ Observable

Choice (C)

16. Overall T/F is $G_T(s) = G(s).G_c(s)$

$G_c(s)$ is a lead network

$$G_T(s) = \frac{(s+1)^2}{(s+2)(s-2)} \times \frac{k(s-2)}{(s+1)} = \frac{k(s+1)}{s+2}$$

$\Rightarrow$ stable

From the given options. Option B is satisfies

Choice (B)

17. From the given state equations

$Y(s) = x_1(s)$

$\dot{X}_1(t) = x_2(t)$

$\dot{X}_2(t) = -3x_1 - 2x_2(t) + \mu(t)$

Apply both sides laplace transform

$SX_1(s) = X_2(s)$

$sX_2(s) = -3X_1(s) - 2X_2(s) + \mu(s)$

$s^2x_1(s) = -3x_1(s) - 2x_2(s) + \mu(s)$

$X_1(s)\,[s^2 + 2s + 3] = \mu(s)$

$$\therefore \quad Y(s) = \frac{k.\mu(s)}{s^2 + 2s + 3}$$

$$\frac{Y(s)}{\mu(s)} = \frac{k}{s^2 + 2s + 3}.$$

Choice (A)

18. Given $H(s) = CLTF = \dfrac{1}{(s+1)(s+2)}$

$$H(s) = \frac{k_1}{s+1} + \frac{k_2}{s+2}$$

$K_1 = H(s) \times (s + 1)$ at $s = -1$

$K_1 = 1$

$K_2 = H(s)\,(s + 2)$ at $s = -2 = -1$

$$\therefore \quad H(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

A = [diagonal matrix of poles]

$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $p \neq z$ and no repeated poles

C = [residues of poles]

$= [k_1\ k_2] = [1\ -1]$

$D[0]$; $p \neq z$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

Any T/F having 'n' multiple poles then 'B' have $(n - 1)$ zero's in B matrix

No of poles (p) = no of zeros (z) then $D = [1]$

Choice (B)

19. $P < z$

$\therefore$ Lag compensator

$$G_c(s) = \frac{k(1 + s\tau)}{(1 + \beta\tau s)} \quad \tau = 2 \text{ and } \beta\tau = 6.$$

$\therefore \quad \beta = 3$

$$\phi_m = \tan^{-1}\left\{\frac{1-\alpha}{2\sqrt{\alpha}}\right\}$$

$\alpha = \beta = 3$

$$\phi_m = \tan^{-1}\left\{\frac{1-3}{2\sqrt{3}}\right\}$$

$$= \mathrm{Tan}^{-1}\left\{\frac{-1}{\sqrt{3}}\right\} = -30°$$

Choice (D)

20. $|G_c(j\omega)|$ at $\omega = \omega_m$

$$\omega_m = \frac{1}{T\sqrt{a}} = \frac{1}{\sqrt{0.2\times 0.5}} = \sqrt{10}\text{ rad/sec} = 3.162$$

$$|G_c| = 0.2\frac{\sqrt{1+(0.2\omega_m)^2}}{\sqrt{1+(0.5\omega_m)^2}} = \frac{0.2366}{1.87} = 0.126.$$

Choice (A)

21. For lead n/w

$$\alpha = \frac{R_2}{R_1+R_2} < 1$$

We know $\phi_m = \tan^{-1}\left\{\frac{1-a}{2\sqrt{a}}\right\}$

$$= \tan^{-1}\left\{\frac{\left(R_1 / R_1+R_2\right)}{2\sqrt{\frac{R_2}{R_1+R_2}}}\right\} = \tan^{-1}\left\{\frac{R_1\left(\sqrt{R_1+R_2}\right)}{2\sqrt{R_2}}\right\}$$

$$= \tan^{-1}\left\{\frac{R_1}{2}.\sqrt{1+\frac{R_1}{R_2}}\right\}$$

$$\phi_m = \sin^{-1}\left[\frac{1-a}{1+a}\right];\ \phi_m = \sin^{-1}\left\{\frac{R_1}{R_1+2R_2}\right\}$$

$$\phi_m = \cos^{-1}\left\{\frac{2\sqrt{a}}{1+a}\right\}$$

$$\phi_m = \cos^{-1}\left\{\frac{2.\sqrt{\frac{R_2}{R_1+R_2}}}{1+\frac{R_2}{R_1+R_2}}\right\}$$

$$\phi_m = \cos^{-1}\left\{\frac{2.\sqrt{R_2\left(R_1+R_2\right)}}{R_1+2R_2}\right\}.$$

Choice (D)

22. Derivative controller $G_c(s)$

$G_c(s) = k_D\, S$

(i) Zero adding at origin so type of system decrease
(ii) stability increases
(iii) Steady state error increases

Choice (D)

23. The closed loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{\frac{100}{s(2s+1)}}{1+\frac{k_D s}{s(2s+1)}+\frac{100}{s(2s+1)}}$$

$$= \frac{100}{2s^2+s+k_D s+100} = \frac{50}{s^2+0.5(k_D+1)s+50}$$

$$\frac{C(s)}{R(s)} = \frac{50}{s^2+0.5(k_D+1)s+50}$$

Given $\zeta = 0.5$

$\omega_n^2 = 50$

$\omega_n = 7.07$ rad/sec

$2\,\zeta\omega_n = 0.5\,(k_D + 1)$

$2 \times 7.07 = k_D + 1$

$K_D = 13.14.$

Choice (B)

24. We know

$\phi(t) = L^{-1}\,[(SI - A)^{-1}]$

$$\phi(s) = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$|(SI - A)| = s\,(s + 3) + 2$
$= s^2 + 3s + 2$

$|SI - A| = (s + 1)\,(s + 2)$

$$\phi(s) = \frac{adj(SI-A)}{|SI-A|}$$

$$\phi(s) = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$\phi(s) = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$\phi(t) = L^{-1}\{\phi(s)\}$

$$= \begin{bmatrix} 2.e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2.e^{-t}+2.e^{-2t} & -e^{-t}+2.e^{-2t} \end{bmatrix}.$$

Choice (C)

25. State response: Input u(s) = 0

$X(s) = [(sI - A)^{-1}]\,x(0) + [(sI - A)^{-1}.B.u(s)]$

$\therefore\quad x(t) = \phi(t).x(0)$

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$X(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^{-t}-e^{-2t} \\ -e^{-t}+2e^{-2t} \end{bmatrix}$$

Choice (A)