



$$c) \pi > \theta_1 > \theta_2 > \theta_3 > \frac{\pi}{2}$$

$$d) 0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$$

5. If R is the radius of a planet, g is the acceleration due to gravity, then the mean density of the planet is given by: [1]

$$a) \frac{3gG}{4\pi R}$$

$$b) \frac{4\pi gR}{3G}$$

$$c) \frac{4\pi GR}{3g}$$

$$d) \frac{3g}{4\pi GR}$$

6. There are 26 tuning forks arranged in the decreasing order of their frequencies. Each tuning fork gives 3 beats with the next. The first one is octave of the last. What is the frequency of 18th tuning fork? [1]

$$a) 103 \text{ Hz}$$

$$b) 96 \text{ Hz}$$

$$c) 100 \text{ Hz}$$

$$d) 99 \text{ Hz}$$

7. The displacement x of a particle varies with time t as  $x = ae^{-\alpha t} + be^{\beta t}$  where a, b,  $\alpha$  and  $\beta$  are positive constants. [1]  
The velocity of the particle will:

a) go on increasing with time

b) be independent of  $\alpha$  and  $\beta$

c) go on decreasing with time

d) drop to zero when  $\alpha = \beta$

8. In a sinusoidal wave, the time required for a particular point to move from maximum displacement to zero displacements is 0.17 sec. The frequency of the wave is: [1]

$$a) 2.94 \text{ Hz}$$

$$b) 1.47 \text{ Hz}$$

$$c) 0.73 \text{ Hz}$$

$$d) 0.36 \text{ Hz}$$

9. The units of pressure in SI system is: [1]

a) Newton

b) Watt

c) Pascal

d) Joule

10. A particle of mass m is at the surface of the earth of radius R. It is lifted to a height h above the surface of the earth. The gain in gravitational potential energy of the particle is [1]

$$a) \text{ Both } \frac{mgh}{\left(1+\frac{h}{R}\right)} \text{ and } \frac{mghR}{(R+h)}$$

$$b) \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

$$c) \frac{mghR}{(R+h)}$$

$$d) \frac{mgh}{\left(1-\frac{h}{R}\right)}$$

11. The front wheel on an ancient bicycle has radius 0.5 m. It moves with angular velocity given by the function  $\omega(t) = 2 + 4t^2$ , where t is in seconds. About how far does the bicycle move between t = 2 and t = 3 seconds? [1]

$$a) 27 \text{ m}$$

$$b) 14 \text{ m}$$

$$c) 36 \text{ m}$$

$$d) 21 \text{ m}$$

12. A cowboy fires a silver bullet with a mass of 2.00 g and with a muzzle speed of 200 m/s into the pine wall of a saloon. Assume that all the internal energy generated by the impact remains with the bullet. What is the temperature change of the bullet? Specific heat of silver is 234J/Kg°C? [1]

$$a) 83.5^\circ\text{C}$$

$$b) 81.5^\circ\text{C}$$

$$c) 85.5^\circ\text{C}$$

$$d) 78.5^\circ\text{C}$$

13. **Assertion:** A body may gain kinetic energy and potential energy simultaneously. [1]

**Reason:** Principle of conservation of mechanical energy may not be valid every time.

a) Assertion and reason both are correct

b) Assertion and reason both are correct

statements and reason is correct explanation for assertion.

statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

14. **Assertion (A):** In isothermal process for ideal gas, change in internal energy is zero. [1]

**Reason (R):** No heat is supplied to system or rejected by system, in an isothermal process.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

15. **Assertion (A):** The ratio of inertial mass to gravitational mass is equal to one. [1]

**Reason (R):** The inertial mass and gravitational mass of a body are equivalent.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

16. **Assertion (A):** The magnitude of resultant of two vectors cannot be less than the magnitude of either vector. [1]

**Reason (R):** The resultant of two vectors  $\vec{P}$  and  $\vec{Q}$  is found out using parallelogram law.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

### Section B

17. Calculate the speed of sound in dry hydrogen at NTP, assuming its density at NTP conditions as  $0.089 \text{ kg m}^{-3}$  and  $\gamma = 1.41$ . [2]

18. A body of mass  $m$  hung at one end of the spring executes SHM. The force constant of a spring is  $k$  while its period of vibration is  $T$ . Prove that the relation  $T = 2\pi m/k$  is incorrect. Also, derive the correct relation. [2]

19. Check the correctness of the relation  $\tau = I\alpha$ , where  $\tau$  is the torque acting on a body,  $I$  is inertia and  $\alpha$  is angular acceleration. [2]

20. One often comes across the following type of statement concerning circular motion: 'A particle moving uniformly along a circle experiences a force directed towards the centre (centripetal force) and an equal and opposite force directed away from the centre (centrifugal force). The two forces together keep the particle in equilibrium'. Explain what is wrong with this statement. [2]

21. Two masses, 800 kg and 600 kg, are at a distance 0.25 m apart. Compute the magnitude of the intensity of the gravitational field at a point distant 0.20 m from the 800 kg mass and 0.15 m from the 600 kg mass. [2]

OR

The radius of a planet is double that of the earth but their average densities are the same. If the escape velocities at the planet and at the earth are  $v_p$  and  $v_E$  respectively, then prove that  $v_p = 2 v_E$ .

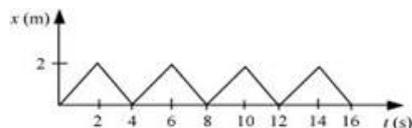
### Section C

22. Find the work done in breaking a water drop of radius 1 mm into 1000 drops. Given the surface tension of water is  $72 \times 10^{-3} \text{ N/m}$ ? [3]

23. Briefly explain, what do you mean by the terms thermal strain and thermal stress? Write expressions for them. [3]

[3]

24. A 100 m sprinter uniformly increases his speed from rest at the rate of  $1 \text{ ms}^{-2}$  up to  $\frac{3}{4}$ th of the total run and then covers the last quarter( $\frac{1}{4}$ th) run with uniform speed. How much time does he take to complete the race?
25. Figure shows the position-time graph of a body of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the body? What is the magnitude of each impulse? [3]



26. A 0.20 Kg aluminum block at  $80^{\circ}\text{C}$  is dropped in a copper calorimeter of mass 0.05 Kg containing  $200 \text{ cm}^3$  of ethyl alcohol at  $20^{\circ}\text{C}$ . What is the final temperature of the mixture? Given Density of ethyl alcohol =  $0.81 \text{ g cm}^3$ ; specific heat of ethyl alcohol =  $0.6 \text{ cal / g } ^{\circ}\text{C}$ ; specific heat of copper =  $0.094 \text{ cal / g } ^{\circ}\text{C}$ , specific heat of Al =  $0.22 \text{ cal / g } ^{\circ}\text{C}$ ? [3]
27. The driver of a three-wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg. [3]
28. What do you understand by capillarity phenomenon? Give two examples to illustrate it. [3]

OR

A ball floats on the surface of water in a container exposed to the atmosphere. Will the ball remain immersed at its initial depth or will it sink or rise somewhat if the container is shifted to the moon?

#### Section D

29. **Read the text carefully and answer the questions:** [4]

Certain collisions are referred to as elastic collisions. Elastic collisions are collisions in which both momentum and kinetic energy are conserved. The total system kinetic energy before the collision equals the total system kinetic energy after the collision. If total kinetic energy is not conserved, then the collision is referred to as an inelastic collision.

The coefficient of restitution, denoted by ( $e$ ), is the measure of degree elasticity of collision. It is defined as the ratio of the final to initial relative speed between two objects after they collide. It normally ranges from 0 to 1 where 1 would be a perfectly elastic collision. A perfectly inelastic collision has a coefficient of 0. In real life most of the collisions are neither perfectly elastic nor perfectly inelastic and  $0 < e < 1$ .

- (i) The following are the data of a collision between a truck and a car.

Mass of the car = 1000 kg

Mass of the truck = 3000 kg

Mass of the truck Before collision:

Speed of the car = 20 m/s

Momentum of the car = 20000 kg m/s

Speed of the truck = 20 m/s

Momentum of the truck = 60000 kg m/s

After collision:

Speed of the car = 40 m/s in the opposite direction

Momentum of the car = 40000 kg m/s in the opposite direction

Speed of the truck = 0

Momentum of the truck = 0

The collision is

- a) Both elastic since kinetic energy and momentum is conserved
- b) Elastic since momentum is conserved
- c) Inelastic since kinetic energy is conserved
- d) Elastic since kinetic energy is conserved
- (ii) The coefficient of restitution is the measure of
- a) Malleability of a substance
- b) Conductivity of a substance
- c) degree of elasticity of collision
- d) Elasticity of a substance
- (iii) Coefficient of restitution is defined as
- a)  $\frac{\text{Relative velocity before collision}}{\text{Relative velocity after collision}}$
- b) Relative velocity after collision  $\times$  relative velocity before collision
- c) None of these
- d)  $\frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$

**OR**

For perfectly elastic and perfectly inelastic collision, the value of coefficient of restitution are respectively

- a) +1, -1
- b) 0, 1
- c) 0, -1
- d) 1, 0
- (iv) In real life most of the collisions are
- a) Range of coefficient of restitution is  $0 < e < 1$
- b) both neither perfectly nor perfectly inelastic and range of coefficient of restitution is  $0 < e < 1$ .
- c) neither perfectly elastic nor perfectly inelastic
- d) perfectly inelastic

30. **Read the text carefully and answer the questions:**

[4]

The number of independent ways by which a dynamic system can move, without violating any constraint imposed on it, is called the number of **degrees of freedom**. According to the law of equipartition of energy, for any dynamic system in thermal equilibrium, the total energy for the system is equally divided among the degree of freedom.



Define the terms harmonic oscillator, displacement, amplitude, cycle, time period, frequency, angular frequency, phase, and an epoch with reference to an oscillatory system.

32. A particle starts from the origin at  $t = 0$  s with a velocity of  $10.0\hat{j} \text{ m/s}$  and moves in the x-y plane with a constant acceleration of  $(8.0\hat{i} + 2.0\hat{j}) \text{ ms}^{-2}$ . [5]
- At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?
  - What is the speed of the particle at the time?

OR

A projectile is fired horizontally with a velocity of  $98 \text{ ms}^{-1}$  from the hill 490 m high. Find

- time taken to reach the ground
  - the distance of the target from the hill and
  - the velocity with which the projectile strikes the ground.
33. Find position of centre of mass of a semicircular disc of radius  $r$ . [5]

OR

Derive an expression for the moment of inertia of a thin uniform rod about an axis through its centre and perpendicular to its length. Also determine the radius of gyration about the same axis.

# Solution

## Section A

1.

(d)  $[M^0L^0T^{-1}]$

**Explanation:**  $[A] = [y] = L$

$$[B] = [x^{-1}] = L^{-1}$$

$$[C] = [t^{-1}] = T^{-1}$$

$$[D] = 1$$

$$\therefore [ABCD] = [M^0L^0T^{-1}]$$

2. (a) 0.5 second

**Explanation:**  $M = \text{mass string} = 2.5 \text{ kg}, l = 20\text{m}$

$$M = \text{mass per unit length} = \frac{M}{l} = \frac{2.5}{20} = 0.125 \text{ kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/s}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{20\text{m}}{40\text{m/s}} = \frac{1}{2} \text{ sec} = 0.5 \text{ sec}$$

3.

(d) when no external torque acts upon the system

**Explanation:** Angular momentum of a system is conserved only when no external torque acts on it.

4.

(d)  $0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

**Explanation:**  $h = \frac{2\sigma \cos \theta}{\rho r g}$

But  $h, \sigma, r$  and  $g$  are same for the three liquids

$$\therefore \frac{\cos \theta}{\rho} = \text{constant}$$

$$\Rightarrow \frac{\cos \theta_1}{\rho_1} = \frac{\cos \theta_2}{\rho_2} = \frac{\cos \theta_3}{\rho_3}$$

Given:  $\rho_1 > \rho_2 > \rho_3$

$$\Rightarrow \cos \theta_1 > \cos \theta_2 > \cos \theta_3$$

$$\Rightarrow \theta_1 < \theta_2 < \theta_3$$

Also,  $\theta$  is acute for liquid which rise in a capillary

$$\therefore 0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$$

5.

(d)  $\frac{3g}{4\pi GR}$

**Explanation:**  $g = \frac{GM}{R^2}$

$$= \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi GR \rho$$

$$\therefore \rho = \frac{3g}{4\pi GR}$$

6.

(d) 99 Hz

**Explanation:** Let the frequency of last fork =  $\nu$

Then frequency of first fork =  $2\nu$

$$\Rightarrow \nu = 2\nu - 25 \times 3 \Rightarrow \nu = 75$$

Frequency of first fork =  $2\nu = 150 \text{ Hz}$

Frequency of 18th fork =  $150 - 17 \times 3 = 99 \text{ Hz}$

7. (a) go on increasing with time

**Explanation:**  $x = ae^{-\alpha t} + be^{\beta t}$

Where  $a, b, \alpha, \beta$  are Positive constant

$$V = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$$

$$\therefore \frac{dv}{dt} = a\alpha^2 e^{-\alpha t} + b\beta^2 e^{\beta t} \text{ is always } > 0$$

V is increasing the function of t.

8.

(b) 1.47 Hz

**Explanation:** We know that time required for maximum displacement (t) = 0.17 sec. We know that time period of sinusoidal wave (T) = 4t = 4 × 0.17 = 0.68 sec.

$$\text{Therefore, frequency } \nu = \frac{1}{T} = \frac{1}{0.68} = 1.47 \text{ Hz}$$

9.

(c) Pascal

**Explanation:** Pressure is defined as force per unit area. The SI unit for pressure is the pascal (Pa), equal to one newton per square meter (N/m<sup>2</sup>, or kg m<sup>-1</sup> s<sup>-2</sup>). This name for the unit was added in 1971; before that, pressure in SI was expressed simply in newtons per square meter.

10. (a) Both  $\frac{mgh}{\left(1 + \frac{h}{R}\right)}$  and  $\frac{mghR}{(R+h)}$

**Explanation:** Both  $\frac{mgh}{\left(1 + \frac{h}{R}\right)}$  and  $\frac{mghR}{(R+h)}$

11.

(b) 14 m

**Explanation:**  $\omega t = \frac{d\theta}{dt} = 2 + 4t^2$

$$\int d\theta = \int_2^3 (2 + 4t^2) dt$$

$$\theta = \left[ 2t + \frac{4}{3}t^3 \right]_2^3 = (6 + 36) - \left( 4 + \frac{32}{3} \right)$$

$$= \frac{82}{3} \text{ rad}$$

$$s = \theta r = \frac{82}{3} \times 0.5 = 13.7 \simeq 14 \text{ m}$$

12.

(c) 85.5°C

**Explanation:** Q = K

$$mc\Delta T = \frac{1}{2}mv^2$$

$$\Delta T = \frac{v^2}{2c} = \frac{200 \times 200}{2 \times 234} = 85.5^\circ \text{C}$$

13. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

**Explanation:** Assertion and reason both are correct statements and reason is correct explanation for assertion.

14.

(c) A is true but R is false.

**Explanation:** A is true but R is false.

15. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** Inertial mass and gravitational mass are equivalent. Both are scalar quantities and measured in the same unit. They are quite different in the method of their measurement. Also, the gravitational mass of a body is affected by the presence of other bodies near it whereas internal mass remains unaffected.

16.

(d) A is false but R is true.

**Explanation:** When  $\vec{P}$  and  $\vec{Q}$  are equal, act at angle  $> 90^\circ$ , their resultant

$$\vec{R} = \sqrt{|P|^2 + |Q|^2 + 2|P||Q|\cos\theta}$$

$$= \sqrt{|P|^2 + |P|^2 + 2|P|^2(-\sqrt{3}/2)} = 0.52 P < P$$

Thus magnitude of resultant is smaller than two vector.

### Section B

17. Here it is given that density of hydrogen under NTP conditions  $\rho = 0.089 \text{ kg m}^{-3}$ ,  $\gamma = 1.41$  and normal pressure  $P = 1.013 \times 10^5 \text{ Pa}$

$$\therefore \text{Speed of sound, } v = \sqrt{\frac{\gamma P}{\rho}}$$

$$= \sqrt{\frac{1.41 \times 1.013 \times 10^5}{0.089}}$$

$$= 1267 \text{ ms}^{-1}$$

18. It is given that  $T = \frac{2\pi m}{k}$

LHS,  $T = [T]$

RHS,  $\frac{2\pi m}{k} = \frac{[M]}{[MT^{-2}]} = [T^2]$

As the dimensions of two sides are not equal, hence the equation is incorrect.

To derive the correct relation, suppose  $T = \beta m^a k^b$ ,  $\beta$  is the proportionality constant, then

$$[T]^1 = [M]^a [MT^{-2}]^b = M^{a+b} T^{-2b}$$

Equating dimension on both sides, we get

$$a + b = 0 \dots\dots(i)$$

$$-2b = 1 \dots\dots(ii)$$

On solving the equations. (i) and (ii), we get  $b = \frac{-1}{2}$ ,  $a = \frac{1}{2}$

$$\therefore T = \beta m^{1/2} k^{-1/2}$$

Hence,  $T = \beta \sqrt{\frac{m}{k}}$

This is the correct equation.

19. Given  $\tau = I\alpha$

As torque,  $\tau = \text{Force} \times \text{distance}$

$$\therefore [\tau] = MLT^{-2} \cdot L = ML^2 T^{-2}$$

Moment of inertia

$$I = \text{Mass} \times \text{distance}^2$$

$$\therefore [I] = ML^2$$

Angular acceleration,

$$\alpha = \frac{\text{Angle}}{(\text{Time})^2}$$

$$\therefore [\alpha] = \frac{1}{T^2} = T^{-2}$$

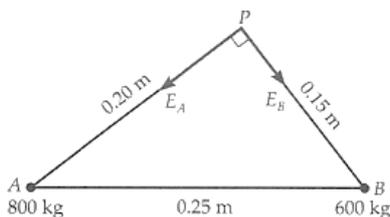
$$[I\alpha] = ML^2 T^{-2}$$

$\therefore$  Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally correct.

20. The statement is correct relative to a non-inertial frame rotating with the particle. For example, consider an observer moving with the same acceleration ( $= v^2/r$ ) as that of the particle. For him, the particle remains at rest. The centripetal force equals the centrifugal force. Centrifugal force is not a real force. It arises only because of the non-inertial nature of the observer himself. The given statement is wrong relative to an inertial frame e.g., the laboratory frame. For an inertial observer (stationary observer), the particle in circular motion is not in equilibrium, it has a centripetal acceleration. There is no force such as centrifugal force.

21. Let A and B be the positions of the two masses and P the point at which the intensity of the gravitational field is to be computed.



Gravitational intensity at P due to mass at A,

$$E_A = \frac{GM}{r^2} = G \frac{800}{(0.20)^2} = 20,000 G, \text{ along PA}$$

Gravitational intensity at P due to mass at B,

$$E_B = G \frac{600}{(0.15)^2} = \frac{80,000}{3} G, \text{ along PB}$$

In  $\triangle APB$ ,

$$PA^2 + PB^2 = AB^2$$

$$\therefore \angle APB = 90^\circ$$

Hence the magnitude of resultant gravitational intensity at P is

$$E = \sqrt{E_A^2 + E_B^2} = G \sqrt{(20,000)^2 + \left(\frac{80,000}{3}\right)^2}$$

$$= 6.66 \times 10^{-11} \times \frac{10,000}{3} = 2.22 \times 10^{-6} \text{N}$$

OR

If  $\rho$  is the average density of the earth, then mass of the earth,

$$M_E = \frac{4}{3} \pi R_E^3 \rho$$

Escape velocity on the earth,

$$v_E = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2G}{R_E} \times \frac{4}{3} \pi R_E^3 \rho}$$

$$= R_E \sqrt{\frac{8}{3} G \pi \rho}$$

Similarly, escape velocity on the planet,

$$v_P = R_P \sqrt{\frac{8}{3} G \pi \rho}$$

$$\therefore \frac{v_P}{v_E} = \frac{R_P}{R_E}$$

But  $R_P = 2 R_E$

$$\therefore v_P = 2v_E$$

### Section C

22. Initial Radius =  $R = 10^{-3}$  m (= 1 mm)

Final Radius =  $r$

Since 1 drop breaks into 1000 small droplets, so

Initial volume = 1000 × Final Volume

$$\frac{4}{3} \pi R^3 = 1000 \times \frac{4}{3} \pi r^3$$

$$R^3 = 10^3 r^3$$

$$r^3 = \frac{R^3}{10^3}$$

On, taking cube root on both sides,  $r = \frac{R}{10} \rightarrow 1)$

Initial Surface Area =  $4\pi R^2$

$$= 4 \times \frac{22}{7} \times (10^{-3})^2$$

$$= 4 \times \frac{22}{7} \times 10^{-5} \text{m}^2 \rightarrow 2)$$

Final Surface Area =  $1000 \times (4\pi r^2)$

$$= 1000 \times 4 \times \frac{22}{7} \times \left(\frac{10^{-3}}{10}\right)^2 \quad r = \frac{R}{10} \text{ form eq } 4 \text{ 1)}$$

$$= 4 \times \frac{22}{7} \times 10^{-3} \times 10^{\frac{1}{3}}$$

$$= 4 \times \frac{22}{7} \times 10^{-5} \text{ -3)}$$

Increase in Surface Area = Final surface Area – Initial surface Area

$$= 4 \times \frac{22}{7} \times 10^{-5} - 4 \times \frac{22}{7} \times 10^{-5} \text{ (} \rightarrow 4)$$

as definition of surface energy says it is the energy associated with the intermolecular forces at the interface between two media.

Now, work Done = Surface Tension × Increase in surface Area

$$= 72 \times 10^{-3} \times \left(4 \times \frac{22}{7} \times 10^{-5} - 4 \times \frac{22}{7} \times 10^{-5}\right) \text{ (from eq 4 4)}$$

$$= 72 \times 4 \times \frac{22}{7} \times 10^{-3} (10^{-5} - 10^{-6})$$

$$= 72 \times 4 \times \frac{22}{7} \times 10^{-3} \times 10^{-5} (1 - 10^{-1})$$

$$\text{Work Done} = 72 \times 4 \times \frac{22}{7} \times 10^{-5} \left(1 - \frac{1}{10}\right)$$

$$= 72 \times 4 \times \frac{22}{7} \times 10^{-3} \times \frac{9}{10}$$

$$\text{Work Done} = 8.14 \times 10^{-6} \text{J}$$

23. When a metal rod, whose ends are rigidly fixed so as to prevent the rod from expansion or contraction, undergoes a change in temperature, thermal strains and thermal stresses are developed in the rod.

The **thermal strain** or deformation for an unrestricted thermal expansion can be defined as the ratio of change in length to the original length.

**Thermal Stress** occurs when restricted expansion is converted to stress.

If a rod of length  $l$  is heated by a temperature  $\Delta T$ , then increase in length of the rod is,  $\Delta l = l \cdot \alpha \cdot \Delta T$

But due to being fixed at ends, the rod does not expand and a compressive thermal strain is developed in it which is given by,

Thermal (compressive) strain =  $\frac{\Delta l}{l} = \alpha \cdot \Delta T$ , here  $\alpha$  = linear expansion coefficient of the material of rod.

Due to this strain, thermal stress is developed in the rod.

Thermal stress =  $Y \times$  thermal strain =  $Y \cdot \alpha \cdot \Delta T$

Thus, the force (tension) exerted by the rod on the supports will be,

$$F = Y\alpha \cdot \Delta T \cdot A$$

24. Given: Total distance covered,  $S = 100$  m,  $u = 0$

For first  $\frac{3}{4}$ th of the run, distance  $S_1 = \frac{3}{4}S = \frac{3}{4} \times 100 = 75$  m,  $a = +1 \text{ ms}^{-2}$ . Let the time for this part of the run be  $t_1$ , then using the second equation of motion,

$$S_1 = ut_1 + \frac{1}{2}at_1^2$$

$$75 = 0 + \frac{1}{2} \times 1 \times t_1^2 \text{ or } t_1^2 = 75 \times 2 = 150$$

$$\Rightarrow t_1 = \sqrt{150} = 12.25 \text{ s}$$

and final velocity  $v = u + at_1 = 0 + 1 \times 12.25 = 12.25 \text{ m s}^{-1}$ .

For remaining  $\frac{1}{4}$ th run, distance  $S_2 = S - S_1 = 100 - 75 = 25$  m, uniform velocity  $v = 12.25 \text{ m s}^{-1}$

$$\therefore \text{Time for this run } t_2 = \frac{S_2}{v} = \frac{25}{12.25} = 2.04 \text{ s}$$

Thus, total time taken by the sprinter to complete the race is,  $t = t_1 + t_2 = 12.25 \text{ s} + 2.04 \text{ s} = 14.29 \text{ s} = 14.3 \text{ s}$ .

25. A ball rebounding between two walls located between at  $x = 0$  cm and  $x = 2$  cm; after every 2 s, the ball receives an impulse of magnitude  $0.08 \times 10^{-2} \text{ kgm/s}$  from the walls.

If we take any one of the triangular portion of the graph, we can see that the position of the ball is increasing uniformly in first 2s and then decreasing at the same rate in the next 2s. i.e. The ball is coming back to the same position after every 4s. The given graph shows that a body changes its direction of motion after every 2 s. Physically, this situation can be visualized as a ball rebounding to and fro between two stationary walls situated between positions  $x = 0$  cm and  $x = 2$  cm. Since the slope of the  $x$ - $t$  graph reverses after every 2 s, the ball collides with a wall after every 2 s. Therefore, ball receives an impulse after every 2 s.

Mass of the ball is given by,  $m = 0.04$  kg

The slope of the graph gives the velocity of the ball. Using the graph (in first 2s), we can calculate initial velocity ( $u$ ) as:

$$u = \frac{(2-0) \times 10^{-2}}{(2-0)} = 10^{-2} \text{ m/s}$$

Velocity of the ball before collision (taking any one of the triangle of the graph and time for the first 2s) is given by,  $u = 10^{-2} \text{ m/s}$

Velocity of the ball after collision (taking the same triangle and time for next 2s) is given by,  $v = -10^{-2} \text{ m/s}$

(Here, the negative sign arises as the ball reverses its direction of motion i.e. the decrease of position of the ball for next 2s.)

Now from the mathematical explanation of Newton's 2nd law of motion, Magnitude of impulse = Change in momentum, Hence

$$= |mv - mu|$$

$$= |0.04(v - u)|$$

$$= |0.04(-10^{-2} - 10^{-2})|$$

$$= 0.08 \times 10^{-2} \text{ kgm/s}$$

26. Let final temperature of the mixture (aluminium block and ethyl alcohol) =  $\theta^\circ\text{C}$

Mass of ethyl alcohol = volume  $\times$  Density

$$m = 200 \times 0.81 = 162 \text{ g}$$

Heat lost by Aluminium block =  $m \times c \times \Delta T$

$c$  - specific heat of aluminium

$m$  - mass of piece of aluminium

$\Delta T$  - fall in temperature of aluminium block

$$\Delta T = 80 - \theta$$

$$\text{heat lost by aluminium block} = (0.20 \times 10^3) \times 0.22 \times (80 - \theta)$$

$$= 20 \times 22 \times 10^{-4} \times 10^3 (80 - \theta)$$

$$= 440 \times 10^{-1} (80 - \theta)$$

$$= 44(80 - \theta) \dots(i)$$

Heat gained by the mixture (ethyl alcohol and calorimeter) = (Mass of ethyl alcohol  $\times$  specific heat  $\times$  gain in Temperature) + Mass of copper calorimeter  $\times$  specific heat  $\times$  gain in Temperature

$$= [162 \times 0.6 \times (\theta - 20)] + [0.05 \times 10^3 \times 0.094 \times (\theta - 20)]$$

$$= 101.9 \times (\theta - 20) \dots(ii)$$

According to principle of calorimetry, Heat gained = Heat Lost

Using, (i) & (ii)

$$44(80 - \theta) = 101.9(\theta - 20)$$

$$\Rightarrow 3520 - 44\theta = 101.9\theta - 2038$$

$$3520 + 2038 = 101.9\theta + 44\theta$$

$$5558 = 145.9\theta$$

$$\theta = \frac{5558}{145.9}$$

$$\theta = 38.1^\circ C$$

27. Initial speed of the three-wheeler,  $u = 36 \text{ km/h} = (36 \times 5) \div 18 \text{ m/s} = 10 \text{ m/s}$

Final speed of the three-wheeler,  $v = 0 \text{ m/s}$

Time,  $t = 4 \text{ s}$

Mass of the three-wheeler,  $m = 400 \text{ kg}$

Mass of the driver,  $m' = 65 \text{ kg}$

Total mass of the system,  $M = m + m' = (400 + 65) = 465 \text{ kg}$

Using Newton's first law of motion, the acceleration ( $a$ ) of the three-wheeler can be calculated as:

$$v = u + at$$

$$\therefore a = \frac{v-u}{t} = \frac{0-10}{4} = -2.5 \text{ m/s}^2$$

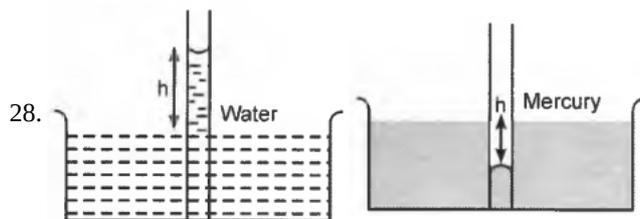
The negative sign indicates that the velocity of the three-wheeler is decreasing with time.

Using Newton's second law of motion, the net force acting by the three-wheeler can be calculated as:

$$F = Ma$$

$$= 465 \times (-2.5) = -1162.5 \text{ N}$$

[The negative sign indicates that the force is acting against the direction of motion of the three-wheeler.]



It is the phenomenon of rise or fall of a liquid in a capillary tube as compared to the surroundings. All those liquids which wet the walls of container e.g., water, alcohol etc., show the phenomenon of rise in a capillary tube. On the other hand, for liquids which do not wet the walls of container, the liquid level falls in a capillary tube as compared to the surroundings. Thus, mercury level falls in a capillary tube. The phenomenon of capillarity is due to the property of surface tension of the liquid.

Following examples can illustrate it:

- i. Process of perspiration takes place in human beings because fine pores are present in skin which behave as capillaries. Due to capillary action water from under the skin rises in fine pores, comes out and then evaporates
- ii. Sap rises from the roots of a plant due to capillarity phenomenon, as fine capillaries are present in stem, branches and every leaf of the plant.

OR

The gravity on moon is about one-sixth of that on the earth. But gravity has equal effect both on weight of the body and the upthrust. So equilibrium of the floating body is not affected. On the earth, weight of the floating body is balanced by upthrust due to both water and air.

$$\therefore W = mg = V_w \rho_w g + V_a \rho_a g$$

$$\text{or } m = V_w \rho_w + V_a \rho_a \dots (i)$$

But the moon has no atmosphere. So

$$W = mg = V'_w \rho_w g$$

$$\text{or } m = V'_w \rho_w$$

From (i) and (ii), we note that

$$V'_w = V_w + \frac{V_a \rho_a}{\rho_w}$$

Clearly,  $V'_w > V_w$

That is, the volume of ball immersed in water on the moon is greater than that on earth. Hence ball will sink slightly more in water when taken to the moon.

Section D

29. Read the text carefully and answer the questions:

Certain collisions are referred to as elastic collisions. Elastic collisions are collisions in which both momentum and kinetic energy are conserved. The total system kinetic energy before the collision equals the total system kinetic energy after the collision. If total kinetic energy is not conserved, then the collision is referred to as an inelastic collision.

The coefficient of restitution, denoted by ( $e$ ), is the measure of degree elasticity of collision. It is defined as the ratio of the final to initial relative speed between two objects after they collide. It normally ranges from 0 to 1 where 1 would be a perfectly elastic collision. A perfectly inelastic collision has a coefficient of 0. In real life most of the collisions are neither perfectly elastic nor perfectly inelastic and  $0 < e < 1$ .

- (i) (b) Elastic since momentum is conserved

**Explanation:** From the given data kinetic energy is 800000 Joules, before and after collision and momentum is 40000 kg m/s before and after the collision. So the collision is elastic.

- (ii) (c) degree of elasticity of collision

**Explanation:** degree of elasticity of collision

- (iii) (d)  $\frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$

**Explanation:**  $\frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$

OR

- (d) 1, 0

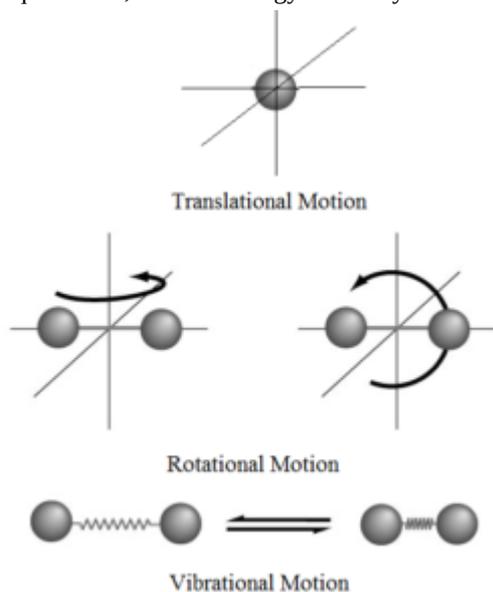
**Explanation:** 1, 0

- (iv) (b) both neither perfectly nor perfectly inelastic and range of coefficient of restitution is  $0 < e < 1$ .

**Explanation:** both neither perfectly nor perfectly inelastic and range of coefficient of restitution is  $0 < e < 1$ .

30. Read the text carefully and answer the questions:

The number of independent ways by which a dynamic system can move, without violating any constraint imposed on it, is called the number of **degrees of freedom**. According to the law of equipartition of energy, for any dynamic system in thermal equilibrium, the total energy for the system is equally divided among the degree of freedom.



- (i) (c)  $1 + 2/n$

**Explanation:**  $1 + 2/n$

- (ii) (c)  $kT$

**Explanation:**  $kT$

- (iii) (c) the average distance covered by a molecule between two successive collisions

**Explanation:** the average distance covered by a molecule between two successive collisions

- (iv) (b) in random motion

**Explanation:** in random motion

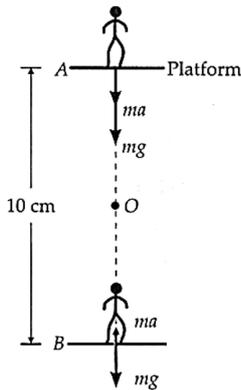
OR

(b) 4.148 joule

Explanation: 4.148 joule

### Section E

31. The platform vibrates between the positions A and B about the mean position O, as shown in figure.



Given  $A = 5.0 \text{ cm}$ ,  $m = 60 \text{ kg}$ ,  $v = 2 \text{ Hz}$

At A and B, the acceleration is maximum and is directed towards the mean position.

It is given by

$$a_{\max} = \omega^2 A$$

$$= 4\pi^2 v^2 A$$

$$= 4 \times 9.87 \times (2)^2 \times 0.05 = 7.9 \text{ ms}^{-2}$$

At A, both the weight  $mg$  and the restoring force  $F$  are directed towards O. Therefore, the weight at A is maximum and is given by

$$W_1 = (mg + F) = (mg + ma_{\max}) = m(g + a_{\max})$$

$$= 60(10 + 7.9) = 60 \times 17.9 = 1074 \text{ N}$$

$$= \frac{1074}{g} = \frac{1074}{10} = 107.4 \text{ kg f}$$

At B,  $mg$  and  $F$  are opposed to each other so that the weight is minimum. It is given by

$$W_2 = (mg - F) = (mg - ma_{\max}) = m(g - a_{\max})$$

$$= 60(10 - 7.9) = (60 \times 2.1) \text{ N} = 126 \text{ N}$$

$$= \frac{126}{10} = 12.6 \text{ kg f}$$

OR

i. **Harmonic oscillator:** A particle executing simple harmonic motion is called harmonic oscillator.

ii. **Displacement:** The distance of the oscillating particle from its mean position at any instant is called its displacement. It is denoted by  $x$ .

There can be other kind of displacement variables. These can be voltage variations in time across a capacitor in an a.c. circuit, pressure variations in time in the propagation of a sound wave, the changing electric and magnetic fields in the propagation of a light wave, etc.

iii. **Amplitude:** The maximum displacement of the oscillating particle on either side of its mean position is called its amplitude. It is denoted by  $A$ . Thus  $x_{\max} = \pm A$

iv. **Oscillation or cycle:** One complete back-and-forth motion of a particle starting and ending at the same point is called a cycle or oscillation or vibration.

v. **Time period:** The time taken by a particle to complete one oscillation is called its time period. Or, it is the smallest time interval after which the oscillatory motion repeats. It is denoted by  $T$ .

vi. **Frequency:** It is defined as the number of oscillations completed per unit time by a particle. It is denoted by  $\nu$  (nu). Frequency is equal to the reciprocal of the time period. That is,

$$\nu = \frac{1}{T}$$

Clearly, the unit of frequency is  $(\text{second})^{-1}$  or  $\text{s}^{-1}$ . It is also expressed as cycles per second (cps) or hertz (Hz). SI unit of frequency =  $\text{s}^{-1} = \text{cps} = \text{Hz}$

vii. **Angular frequency:** It is the quantity obtained by multiplying frequency  $\nu$  by a factor of  $2\pi$ . It is denoted by  $\omega$ .

$$\text{Thus, } \omega = 2\pi\nu = \frac{2\pi}{T}$$

SI unit of angular frequency =  $\text{rad s}^{-1}$ .

viii. **Phase:** The phase of a vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant. It is equal to the argument of sine or cosine function occurring in the displacement equation of the S.H.M. Suppose a simple harmonic equation is represented by

$$x = A \cos(\omega t + \phi_0)$$

Then phase of the particle is:  $\phi = \omega t + \phi_0$

Clearly, the phase  $\phi$  is a function of time  $t$ . It is usually expressed either as the fraction of the time period  $T$  or fraction of angle  $2\pi$  that has elapsed since the vibrating particle last passed its mean position in the positive direction.

$\phi = \omega t + \phi_0$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$x = A \cos(\omega t + \phi_0)$	+A	0	-A	0	+A

Thus the phase  $\phi$  gives an idea about the position and the direction of motion of the oscillating particle.

ix. **Initial phase or epoch:** The phase of a vibrating particle corresponding to time  $t = 0$  is called the initial phase or epoch.

At  $t = 0$ ,  $\phi = \phi_0$

The constant  $\phi_0$  is called the initial phase or epoch. It tells about the initial state of motion of the vibrating particle.

32. Given: Velocity at time  $t = 0$  is given as

$$\vec{u} = 0\hat{i} + 10\hat{j} \text{ m/s}$$

$$\Rightarrow u_x = 0 \text{ m/s}, u_y = 10 \text{ m/s}$$

$$\text{Acceleration, } \vec{a} = 8.0\hat{i} + 2.0\hat{j} \text{ m/s}^2 \Rightarrow a_x = 8.0 \text{ m/s}^2, a_y = 2.0 \text{ m/s}^2$$

a. time taken by particle for  $x = 16$  m

Using equation,  $S = ut + \frac{1}{2}at^2$  along x axis

$$x = u_x t + \frac{1}{2}a_x t^2 \text{ we get}$$

$$16 = (0 \times t) + \frac{1}{2}(8)(t)^2$$

$$t = 2 \text{ s}$$

y-coordinate at this time will be:

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$y = (10 \times 2) + \frac{1}{2}(2)(2)^2$$

$$y = 24 \text{ m}$$

b. Velocity along x and y-axis after time,  $t = 2$  s

$$v_x = u_x + a_x t \Rightarrow v_x = 0 + (8 \times 2)$$

$$v_x = 16 \text{ m/s}$$

$$v_y = 10 + (2 \times 2)$$

$$v_y = 14 \text{ m/s}$$

Net speed of the particle is:

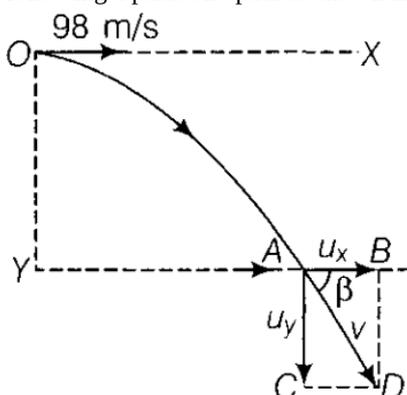
$$v = \sqrt{(v_x)^2 + (v_y)^2}$$

$$v = \sqrt{(16)^2 + (14)^2}$$

$$v = 21.26 \text{ m/s}$$

OR

From the given figure,  $YO = 490$  m. A body projected horizontally from  $O$  with velocity  $u = 98 \text{ ms}^{-1}$  hits the ground at position  $A$  following a parabolic path as shown in the figure.



i. Let  $T$  be the time of flight of the projectile.

Taking vertical downward motion of projectile from  $O$  to  $A$ , we have

$$y_0 = 0, y = 490 \text{ m}, u_y = 0, a_y = 9.8 \text{ m/s}^2, t = T$$

$$\text{From equation of kinematics, } y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow 490 = 0 + 0 \times T + \frac{1}{2} \times 9.8 \times T^2 = 4.9T^2$$

$$\text{or } T = \sqrt{\frac{490}{4.9}} = 10 \text{ s}$$

ii. Taking horizontal motion (i.e. motion along OX axis) of projectile from O to A, we have

$$x_0 = 0, x = R \text{ (say)}, u_x = 98 \text{ m/s}, t = T = 10 \text{ s}, a_x = 0 \text{ (as there is no acceleration along horizontal)}$$

$$\text{As, } x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

$$\therefore R = 0 + 98 \times 10 + \frac{1}{2} \times 0 \times 10^2 = 980 \text{ m}$$

iii. Let  $v_x, v_y$  be the horizontal and vertical component velocity of the projectile at point A.

$$\text{Using the relation, } v_x = u_x + a_x t = 98 + 0 \times 10 = 98 \text{ m/s, which is represented by AB.}$$

$$\text{Similarly, } v_y = u_y + a_y t = 0 + 9.8 \times 10 = 98 \text{ m/s as represented by AC}$$

$\therefore$  The magnitude of the resultant velocity is given by

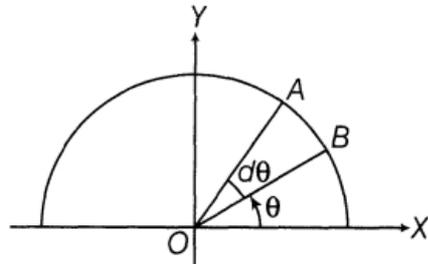
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2} \text{ m/s}$$

And the direction of the resultant velocity is given by

$$\tan \beta = \frac{v_y}{v_x} = \frac{98}{98} = 1 \text{ or } \beta = 45^\circ \text{ with the horizontal.}$$

33. As semicircular disc is symmetrical about its one of diameter, we take axes as shown. So, now we only have to calculate  $Y_{CM}$

(As,  $X_{CM}$  is zero by symmetry and choice of origin).



Axis of symmetry  $CM$  lies on this

$$\text{Now, } Y_{CM} = \frac{\int y dm}{\int dm}$$

$$\text{From our text, } M \cdot Y_{CM} = \int y dm$$

$$Y_{CM} = \frac{1}{M} \int y dm = \frac{\int y dm}{\int dm}$$

Now, for a small element OAB, as element is small and it can be treated as a triangle so,

$$\text{Area of sector OAB} = \frac{1}{2} \times r \times r d\theta$$

$$\text{Height of triangle} = r$$

$$\text{Base of triangle} = AB = r d\theta$$

$$\text{So, its mass } dm = \frac{1}{2} r^2 d\theta \cdot \rho \left[ \because \rho = \frac{\text{mass}}{\text{area}} \right]$$

As centre of mass of a triangle is at a distance of  $\frac{2}{3}$  from its vertex (at centroid, intersection of medians).

$$\text{So, } y = \frac{2}{3} r \sin \theta \text{ (location of CM of small sector AOB).}$$

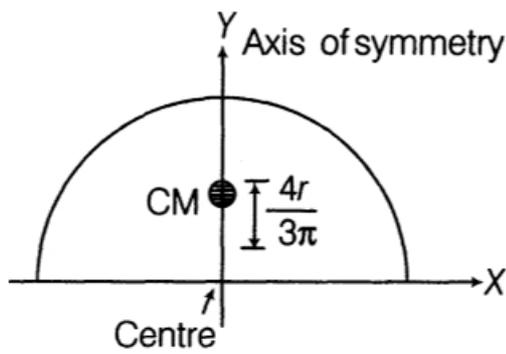
$$\text{So, } Y_{CM} = \frac{\int_0^\pi y dm}{\int_0^\pi \frac{1}{2} r^2 \rho d\theta}$$

$$= \frac{\int_0^\pi \frac{2}{3} r \sin \theta \times \frac{1}{2} r^2 d\theta \cdot \rho}{\int_0^\pi \frac{1}{2} r^2 \rho d\theta}$$

$$= \frac{\frac{1}{2} \times \frac{2}{3} r^3 \cdot \rho \int_0^\pi \sin \theta d\theta}{\frac{1}{2} r^2 \rho \cdot \int_0^\pi d\theta} = \frac{\frac{2r}{3} \int_0^\pi \sin \theta d\theta}{\int_0^\pi d\theta}$$

$$= \frac{\frac{2r}{3} [-\cos \theta]_0^\pi}{[\theta]_0^\pi} = \frac{\frac{-2r}{3} (\cos \pi - \cos 0^\circ)}{(\pi - 0)}$$

$$= \frac{4r}{3\pi}$$



So, CM of disc is at a distance of  $\frac{4r}{3\pi}$  from its centre on its axis of symmetry.

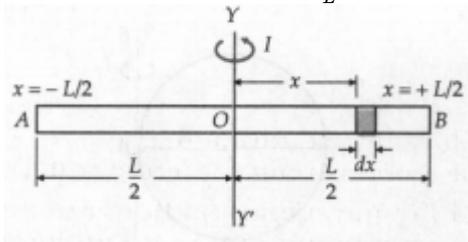
OR

M.I. of a thin uniform rod about a perpendicular axis through its centre. Consider a thin uniform rod AB of length L and mass M, free to rotate about an axis YY' through its centre O and perpendicular to its length

$\therefore$  Mass per unit length of rod =  $\frac{M}{L}$

Consider a small mass element of length dx at a distance x from O.

Mass of the small element =  $\frac{M}{L} dx$



Moment of inertia of the small element about YY'

$$dI = \text{Mass} \times (\text{distance})^2 = \frac{M}{L} dx \times x^2$$

The moment of inertia of the whole rod about the axis YY' can be obtained by integrating the above expression between the limits  $x = -\frac{L}{2}$  and  $x = +\frac{L}{2}$ .

$$\therefore I = \int dI = \int_{-L/2}^{+L/2} \frac{M}{L} x^2 dx$$

$$= \frac{M}{L} \int_{-L/2}^{+L/2} x^2 dx = \frac{M}{L} \left[ \frac{x^3}{3} \right]_{-L/2}^{+L/2}$$

$$= \frac{M}{3L} \left[ \left( \frac{L}{2} \right)^3 - \left( -\frac{L}{2} \right)^3 \right]$$

$$= \frac{M}{3L} \left[ \frac{L^3}{8} + \frac{L^3}{8} \right] = \frac{M}{3L} \times \frac{L^3}{4}$$

$$\text{or } I = \frac{ML^2}{12}$$

The radius of gyration. Let k be the radius of gyration of the rod about the axis YY'. Then

$$I = Mk^2$$

$$\therefore Mk^2 = \frac{ML^2}{12} \text{ or } k^2 = \frac{L^2}{12}$$

$$\text{or } k = \frac{L}{2\sqrt{3}}$$

Thus, the radius of gyration of a uniform thin rod rotating about an axis passing through its centre and perpendicular to its length is  $\frac{L}{2\sqrt{3}}$ .