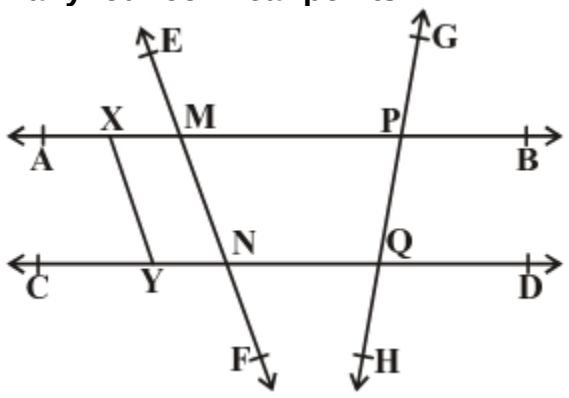
Lines And Angles

Exercise 4.1

Q. 1. In the given figure, name:

i. any six points
ii. any five line segments
iii. any four rays
iv. any four lines
v. any four collinear points



Answer : i) A point specifies the exact location and looks like a small dot.

∴ The points in the given figure are A,B,C,D,E,F,G,H,M,N,P,Q,X,Y.

ii) A part of a line with two end points is a line segment.

∴ The line segment in the given figure are AX,XM,MP,AM,AP,AB,XB,XY…etc

iii) A ray has a starting point but no end point. it may go to infinity.

∴ The ray in the given figure are A,B,C,D,E,F,G,H.

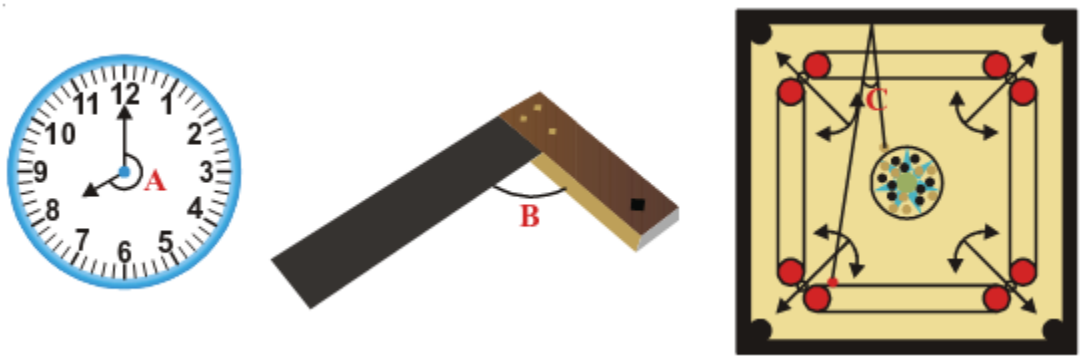
iv) A line goes without end in both direction.

∴ The line segment in the given figure are AB,CD,EF,GH.

v) if three or more points lie on the same line, they are called collinear points

∴ The collinear points in the given figure are AXM,EMN,GPQ,CYN…

Q. 2. Observe the following figures and identify the type of angles in them.



Answer : A) The given figure shows the larger angle area, so it is reflex angle. The angle is more than 180° and less than 360°.

B) The given figure shows the right angle that is B = 90°

C) The given figure shows the smaller angle area, so it is acute angle. The angle is more than 0° and less than 90°.

Q. 3. State whether the following statements are true or false :

i. A ray has no end point.
ii. Line $\overleftrightarrow{AB}$ is the same as line $\overleftrightarrow{BA}$.
iii. A ray $\overrightarrow{AB}$ is same as the ray $\overrightarrow{BA}$.
iv. A line has a define length.
v. A plane has length and breadth but no thickness.
vi. Two distinct points always determine a unique line.
vii. Two lines may intersect in two points.
viii. Two intersecting lines cannot both be parallel to the same line.

Answer : (i) False

A ray has a starting point but no end point but it goes to infinity.

(ii) True

A line goes without end in both direction. so, $\overleftrightarrow{AB}$ is same as $\overleftrightarrow{BA}$

(iii) False

A ray has a starting point but no end point but it goes to infinity. so

a ray $\overline{AB}$ is not same as $\overline{BA}$

(iv) False

A line goes without end in both direction. So, it does not have a define length.

(v) True

A line goes without end in both direction. so, $\overline{AB}$ is same as $\overline{BA}$

(vi) True

If two points are joined together then forms a line.

(vi) False

Two lines intersected at one point.

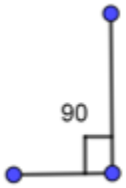
(vii) True

A parallel line does not have any intersecting point.

Q. 4. What is the angle between two hands of a clock when the time in the clock is

(a) 9'O clock
(b) 6'O clock
(c) 7:00 PM

Answer : (a) Let draw the 9'O clock and find the angle between the lines

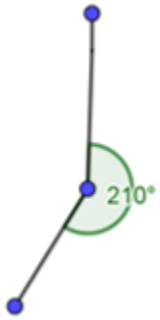


∴ The angle is 90°

(b) Let draw the 6'O clock and find the angle between the lines

∴ The angle is 180°

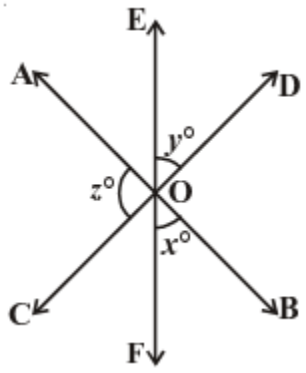
(c) Let draw the 7.00 PM and find the angle between the lines



∴ The angle is 210°

Exercise 4.2

Q. 1. In the given figure three lines $\overline{AB}, \overline{CD}$ and $\overleftrightarrow{EF}$ intersecting at O. Find the values of x, y and z it is being given that x : y : z = 2 : 3 : 5



Answer : From the given, the three angles are x,y,z

If the two lines intersect at a point then its vertically opposite angles are equal.

∴ A = x, B = z and C = y

We know that the sum of all the angles around at a point is equal to 360°

∴ A + B + C + x + y + z = 360°

⇒ x + y + z + x + y + z = 360°

⇒ 2x + 2y + 2z = 360°

⇒ 2(x + y + z) = 360°

⇒ $(x + y + z) = \frac{360^\circ}{2}$

⇒ x + y + z = 180° ------(1)

Given that, x : y : z = 2 : 3 : 5

Let x = 2m,y = 3m,z = 5m (∵ m = constant)

Substitute these values in equation (1) we get

2m + 3m + 5m = 180

10m = 180

$m = \frac{180}{10}$

∴ m = 18

Substituting m = 18 in x,y,z

x = 2m,x = 2(18) = 36°

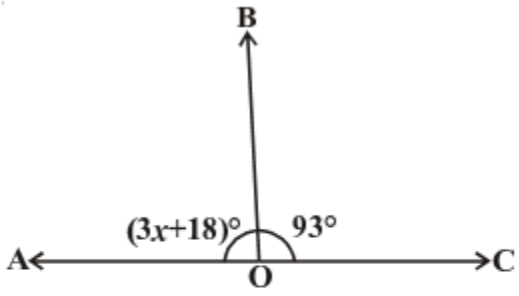
y = 3m,y = 3(18) = 54°

z = 5m,z = 5(18) = 90°

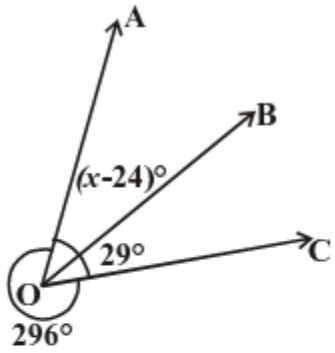
∴ x = 36°,y = 54°,z = 90°

Q. 2. Find the value of x in the following figures.

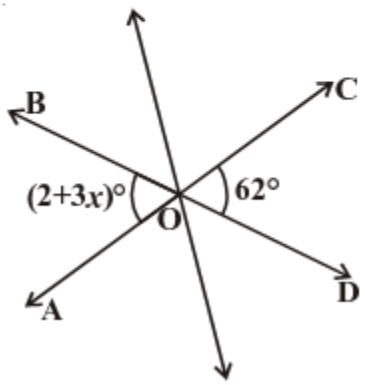
i.



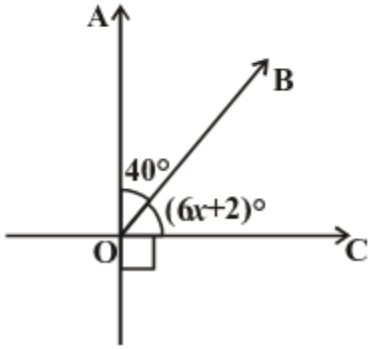
ii.



iii.



iv.



Answer : (i) From the given figure,

$3x + 18° + 93° = 180°$

$\Rightarrow 3x + 111° = 180°$

$\Rightarrow 3x = 180° - 111°$

$\Rightarrow 3x = 69°$

$\Rightarrow x = \frac{69°}{3}$

$\therefore x = 24°$

(ii) From the given figure,

$(x-24)° + 29° + 296° = 360°$

$\Rightarrow (x-24)° = 360° - 325°$

$\Rightarrow (x-24)° = 35°$

⇒ x = 35° + 24°

∴ x = 59°

(iii) From the given figure,

(2 + 3x)° = 62°

⇒ 3x = 62°-2° = 60°

⇒ $x = \frac{60^\circ}{3}$

∴ x = 20°

(iv) From the given figure,

40° + (6x + 2)° = 90°

⇒ (6x + 2)° = 90°-40°

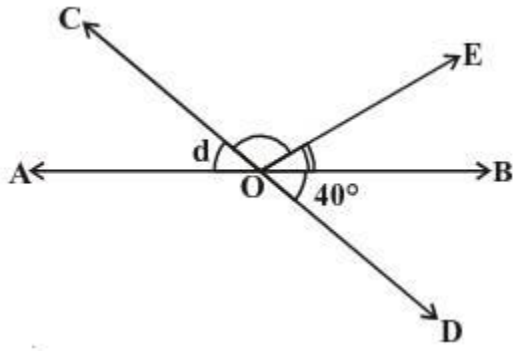
⇒ (6x + 2)° = 50°

⇒ 6x = 50°-2° = 48°

⇒ $x = \frac{48^\circ}{6}$

∴ x = 8°

Q. 3. In the given figure lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Answer : Given that,

The lines $\overline{AB}$ and $\overline{BC}$ intersect at O.

∠AOC + ∠BOE = 70° ----(1)

∠BOD = 40° ----(2)

If the two lines intersect at a point then its vertically opposite angles are equal.

∴ ∠AOC = ∠BOD

Substitute (2) in (1)

⇒ 40° + ∠BOE = 70°

⇒∠BOE = 70°-40°

∴∠BOE = 30°

From the figure, AOB is a straight line and its angle is 180°

So, ∠AOC + ∠BOE + ∠COE = 180°

From equation (1)

⇒ 70° + ∠COE = 180°

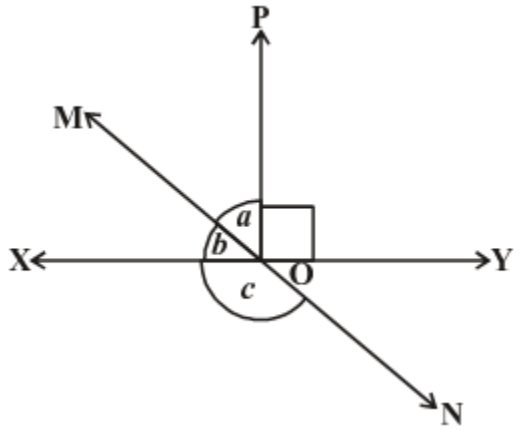
⇒∠COE = 180°-70°

∴∠COE = 110°

Reflex ∠COE = 360° - 110° = 250°

∴∠BOE = 30° and Reflex ∠COE = 250°

Q. 4. In the given figure lines $\overline{XY}$ and $\overline{MN}$ intersect at O. If ∠POY = 90° and a: b = 2 : 3, find c.



Answer : Given that,

The lines $\overline{XY}$ and $\overline{MN}$ intersect at O.

From the figure, XOY is a straight line and its angle is 180°

So, ∠XOM + ∠MOP + ∠POY = 180°

From the given, Let ∠a = 2x and ∠b = 3x

⇒∠b + ∠a + ∠POY = 180°----(1)

Given that ∠POY = 90°

Substitute the values in equation (1),

⇒ 2x + 3x + 90° = 180°

⇒ 5x + 90° = 180°

⇒ 5x = 180°-90° = 90°

⇒ $x = \frac{90^\circ}{5}$

∴ x = 18°

⇒ ∠a = 2x = 2×18° = 36°

⇒ ∠b = 3x = 3×18° = 54°

From the figure, MON is a straight line and its angle is 180°

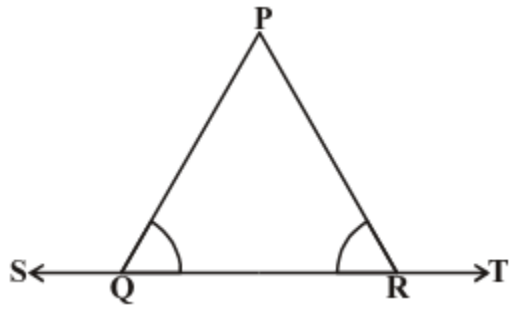
⇒ ∠b + ∠c = 180°

⇒ 54° + ∠c = 180°

⇒ ∠c = 180°-54°

∴ ∠c = 126°

Q. 5. In the given figure ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.



Answer : In the figure, ST is a straight line and its angle is 180°

So, ∠PQS + ∠PQR = 180° ----(1)

And ∠PRT + ∠PRQ = 180°-----(2)

From the two equations, we get

∠PQS + ∠PQR = ∠PRT + ∠PRQ

Given that,

∠PQR = ∠PRQ

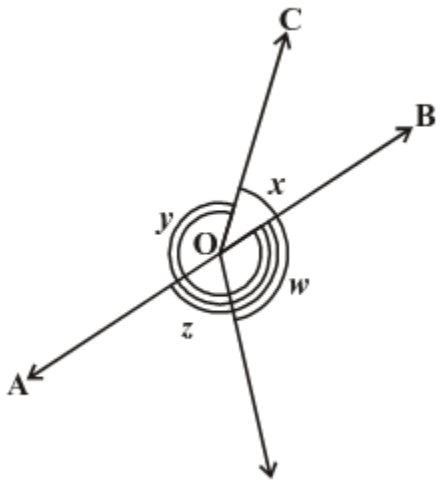
⇒ ∠PQS + ∠PRQ = ∠PRT + ∠PRQ

⇒ ∠PQS = ∠PRT + ∠PRQ -∠PRQ

⇒ ∠PQS = ∠PRT

So, ∠PQS = ∠PRT is proved.

Q. 6. In the given figure, if x + y = w + z, then prove that AOB is a line.



Answer : In a circle, the sum of all angles is 360°

∴ ∠AOC + ∠BOC + ∠DOB + ∠AOD = 360°

⇒ x + y + w + z = 360°

Given that, x + y = w + z

⇒ w + z + w + z = 360°

⇒ 2w + 2z = 360°

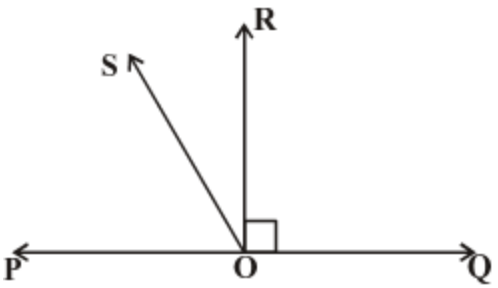
⇒ 2(w + z) = 360°

⇒ w + z = 180° or ∠DOB + ∠AOD = 180°

If the sum of two adjacent angles is 180° then it forms a line.

So AOB is a line.

Q. 7. In the given figure $\overleftrightarrow{PQ}$ is a line. Ray $\overrightarrow{OR}$ is perpendicular to line $\overleftrightarrow{PQ}$. $\overrightarrow{OS}$ is another ray lying between rays $\overrightarrow{OR}$ and $\overrightarrow{OR}$.



Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Answer : Given that, $\overline{PQ}$ is a line and $\overline{OR}$ is perpendicular to line $\overline{PQ}$.

⇒ ∠POR = 90°

The sum of linear pair is always equal to 180°

∴ ∠POS + ∠ROS + ∠POR = 180°

Substitute ∠POR = 90°

⇒ 90° + ∠POS + ∠ROS = 180°

⇒ ∠POS + ∠ROS = 90°

∴ ∠ROS = 90°-∠POS----(1)

⇒ ∠QOR = 90°

Given that OS is another ray lying between OP and OR

⇒ ∠QOS-∠ROS = 90°

∴∠ROS = ∠QOS-90°----(2)

On adding two equations (1) and (2) we get

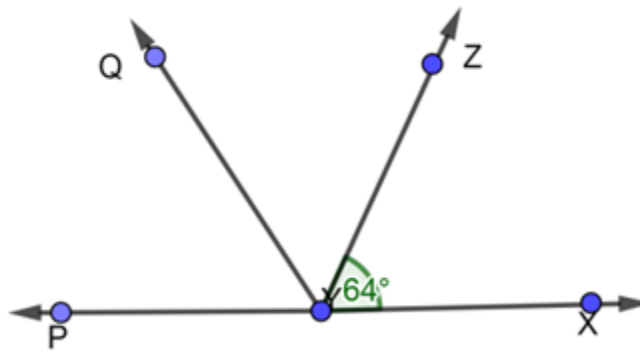
2∠ROS = ∠QOS-∠POS

⇒ ∠ROS = $\frac{1}{2}$(∠QOS-∠POS)

So, ∠ROS = $\frac{1}{2}$(∠QOS-∠POS) is proved.

Q. 8. It is given that ∠XYZ = 64° and XY is produced to point P. A ray YQ bisects ∠ZYP. Draw a figure from the given information. Find ∠XYQ and reflex ∠QYP.

Answer : Let us draw a figure from the given,



Given that, a ray YQ bisects ∠ZYP

So, ∠QYP = ∠ZYQ

Here, PX is a straight line, so the sum of the angles is equal to 180°

∠XYZ + ∠ZYQ + ∠QYP = 180°

Given that, ∠XYZ = 64° and ∠QYP = ∠ZYQ

⇒ 64° + 2∠QYP = 180°

⇒ 2∠QYP = 180°-64° = 116°

∴ $\angle QYP = \frac{116^\circ}{2} = 58^\circ$

Also, ∠QYP = ∠ZYQ = 58°

Using the angle of reflection,

∠QYP = 360°-58° = 302°

∠XYQ = ∠XYZ + ∠ZYQ

⇒∠XYQ = 64° + 58° = 122°

∴ Reflex ∠QYP = 302° and ∠XYQ = 122°