Chapter - 7

Magnetic Effects of Electric Current

Even before 2000 years, people knew about electricity and magnetism, but as two separate subjects. In 1820 a Danish scientist Orested found a close relation between electricity and magnetism. Ampere and Farady found that a moving charge produced magnetic field and a moving magnet produced electric current. Later the Scotish physicist Maxwell and Lorentz from Holland, showed that both elecricity and magnetism depend on each other. From this, a new field of study as electromagnetism came into existance.

The modern technology is based on science of electricity and magnetism. The important devices for our common use, such as electric power, telecommunication, radio, television, mobile etc, are based on it.

In this chapter, we will study the magnetic field produced by current carrying conductor which is also called as magnetic effect of electric current. We will study, the force on a moving charge in a magnetic field, cyclotron and galvanometer etc.

7.1 Orested's Experiments

To study the magnetic field produced by a current carrying wire, Orested performed an experiment whose arrangement is shown in the diagram (7.1). In it a conducting wire AB is connected to a key and a battery with rehostate. A magnetic needle is placed under the wire and parallel to it, in north-south direction.

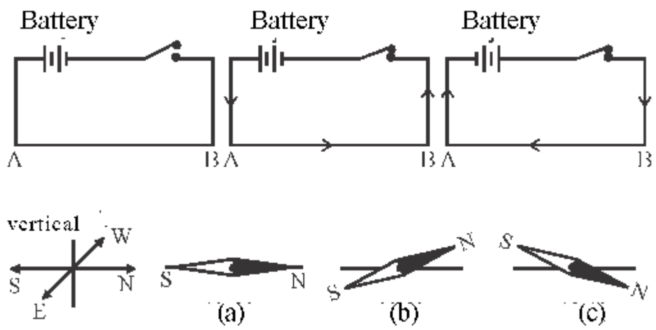


Fig 7.1 Orested's experiment

Orested found from his experiments that -

(i) When there is no current in wire, the magnetic compass needle remain parallel to wire, but as soon as the key is pressed to pass the current, the needle gets deflected.

(ii) The deflection is increased by increasing the current or by bringing the needle close to the wire.

(iii) If the current in the wire is reversed, the deflection is also reversed, to its previous direction.

Similarly, if the compass needle is kept above the wire and the experiment is repeated, the deflection will be opposite to the previous one. Since the magnetic needle is deflected only by external magnetic field, it is clear from Orested's experiment that -

Due to current in a conductor or moving charges, a magnetic field is developed around it, it is called magnetic effect of electric current.

7.1.1 Conculsion from Orested's Experiment

The following conculsions are drawn from Orested experiment -

(i) A magnetic field is developed across a conductor due to electric current in it.

(ii) The magnitude of magnetic field increases with increase in current.

(iii) Magnitude of magnetic field depends on the relative distance from the conductor, it decreases with increase in distance.

(iv) If the current is in S to N direction, the north pole of the magnetic needle placed under the wire deflects towards west directions.

(v) If the current in conductor is in N to S, the deflection will be towards east.

(vi) The direction of magnetic field, above and below the conductor are in opposite direction.

In next discussion we will define magnetic field. It may be a function of space and time.

7.2 Magnetic Field

In chapter 1, we have defined electric field, as the force on a unit positive test charge at that point, $E = F/q$. Had the magnetic mono pole existed, we could have defined magnetic field as simply as above. But since magnetic monopole does not exist, we use another method to define magnetic field. From experiments it is known that a charge at rest in magnetic field does not experience a force. Also if the test charge moves parallel or antiparallel to magnetic field, the force is zero. In the absence of electric field, (neglecting gravitational field), if a moving charge experiences a force in the direction perpendicular to velocity then there must exist a magnetic field B. It is a vector quantity.

The defenition of magnetic field or magnetic induction B, can be given by the force experienced by a moving charge. If a charge q is moving with a velocity $\vec{v}$, at an angle θ with $\vec{B}$, the force on the charge is given by -

$$\vec{F} = q(\vec{v} \times \vec{B}) \qquad ...(7.1)$$

$$\vec{F} = q\,v\,B\sin\theta\hat{n}$$

here θ is the angle between $\vec{B}$ and $\vec{v}$ and $\hat{n}$ is a unit vector in the direction of force $\vec{F}$, which is perpendicular to both $\vec{B}$ and $\vec{v}$.

The magnitude of force is $|\vec{F}| = qvB\sin\theta$.

If $\theta = 90°$

$$F_{max} = q\,v\,B \text{ or } B = \frac{F_{max}}{qv} \qquad ...(7.2)$$

in equation (7.2), if q = 1C and $v = 1$ m/s

then $B = F_{max}$.

Hence "the magnetic field at any point is equal to the max force experienced by a unit charge moving perpendicular to magnetic field with unit velocity".

Magnetic field is a vector quantity, its S.I. unit is $weber/m^2$ which is also called Tesla T.

$$1\,\text{Tesla} = \frac{1Weber}{m^2} = \frac{1N}{A \times m}$$

In CGS system the unit of B is Maxwell/cm^2 or Gauss. Relation between the two units is 1 T = 10^4 G. The dimensional formula for B is $= M^1L^0T^{-2}A^{-1}$.

The magnetic field B is also known as intensity of magnetic field, magnetic flux density and magnetic induction.

Stationary charge produce only electric field where as a moving charge also produce magnetic field along with electric field.

Just as electric field, the magnetic field also obey law of super position.

7.3 Biot-Savart's Law

The French physicsts Biot and Savart proposed a law about the magnetic field produced by current, on experimental basis, which is known by their name.

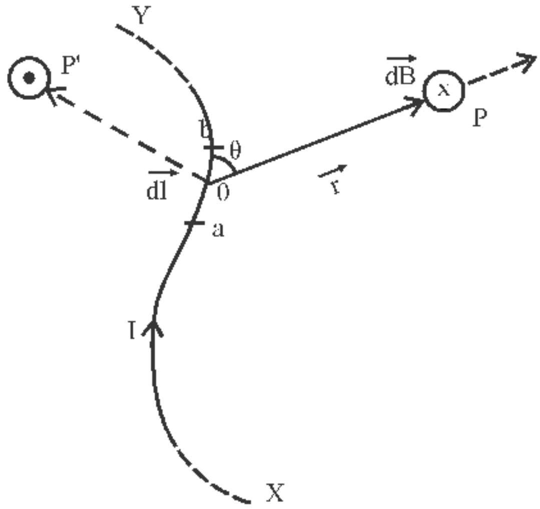


Fig 7.2 Biot-Savart law

The magnetic field $d\vec{B}$ due to a small length element $d\vec{\ell}$ of a conductor XY having a current I at a distance $\vec{r}$ from $d\vec{\ell}$, (shown in fig 7.2) in vacuum as -

(i) $d\vec{B}$ is directly proportional to I, $|d\vec{B}| \propto I$

(ii) Proportional to length elements $d\vec{\ell}$.

$$|d\vec{B}| \propto |d\vec{\ell}|$$

(iii) $d\vec{B}$ is proportional to sine of the angle between $d\vec{\ell}$ and $\vec{r}$.

$$|d\vec{B}| \propto \sin\theta$$

(iv) $d\vec{B}$ is inversely proportional to the square of the distance of the point P from $d\vec{\ell}$.

$$|d\vec{B}|\,\alpha\,\frac{1}{r^2}$$

So combining all above relations we get

$$|d\vec{B}|\,\alpha\,\frac{I\,|d\vec{l}\,|\sin\theta}{r^2} \qquad ...(7.3)$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I\,|d\vec{l}\,|\sin\theta}{r^2} \qquad ...(7.4)$$

Here $\frac{\mu_0}{4\pi}$ = is a proportionality constant. Its value for vacuum is 10^{-7} N/A^2 its unit is $\frac{N}{A^2}$ or

$\frac{Wb}{A \times m}$ or $\frac{T \times m}{A}$.

μ_0 is called magnetic permeability of free space (vacuum).

If the conductor is surrounded by another medium, then

$|d\vec{B}| = \frac{\mu}{4\pi} \frac{I\,|d\vec{l}\,|\sin\theta}{r^2}$ where $\mu = \mu_0\mu_r$, is the magnetic permeability of that medium.

$\mu_r = \frac{\mu}{\mu_0}$ = relative permeability of that medium.

The Biot-Savart law in vector notation is

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2} \qquad \ldots (7.6)$$

or

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3} \qquad \ldots (7.7)$$

$$\left[\because \hat{r} = \frac{\vec{r}}{r}\right]$$

From equation (7.6) it is clear that the direction of $d\vec{B}$ is always perpendicular to the plane of $d\vec{\ell}$ and $\hat{r}$ according to right hand screw rule. In the fig 7.2 the direction of $d\vec{B}$ at P, is perpendicular to the page and downwards shown by $\otimes$. At P' it is perpendicular to the page but upwards as shown by $\odot$.

Different Positions

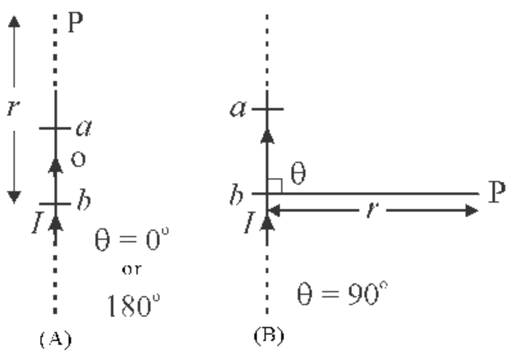


Fig 7.3 $d\vec{B}$ for (A) $\theta = 0°$, 180° (B) $\theta = 90°$

(i) If the P and P' are situated on the line of the current, then $\theta = 0°$ and 180° respectively. $\sin\theta = \sin 0 = \sin 180° = 0$

hence $|d\vec{B}| = 0$ $\qquad \ldots (7.8)$

(ii) If the required point P is normal to $d\vec{\ell}$, as in [fig 7.3(B)], then $\theta = 90°$, and $\sin 90° = 1$.

$$|d\vec{B}| = \frac{\mu_0}{4\pi} I \frac{|d\vec{\ell}\,|\sin 90°}{r^2}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi} I \frac{|d\vec{\ell}\,|}{r^2} \qquad \ldots (7.9)$$

This is the max. value.

(iii) The resultant magnetic field due to the whole of the conductor at a point P for is

$$\vec{B} = \frac{\mu_0}{4\pi} \sum \frac{I\, d\vec{l} \times \vec{r}}{r^3} \qquad \ldots (7.10)$$

Comparision of $d\vec{B}$ due to small current element $Id\vec{\ell}$, from Biot-Savart law equation (7.7), and the $d\vec{E}$ due to a small charge dq by coulomb's law $\left(d\vec{E} = \frac{kdq}{r^2}\hat{r}\right)$.

In both the cases there are two similarlities and two important differences. The current $Id\vec{\ell}$ produces magnetic field where as dq produces electric field. Both obey inverse square law. But there is difference in the direction of the field, due to dq the field $\vec{E}$ is radial, where as $d\vec{B}$ is normal to the plane of $\vec{r}$ and $d\vec{\ell}$. The second difference is that $d\vec{E}$ can be due to single charge or a charge distribution whereas the magnetic field is due to only current.

7.3.1 Direction of Magnetic Field

The direction of $\vec{B}$ can be given by following rules -

(i) Snow Rule - The direction of $d\vec{B}$ near a conductor can be given by the deflection of north pole of a magnetic needle placed near it. According to this law- "If the current in a conductor is form south to north and wire is situated over the compass needle, then the deflection of its north pole is towards west" (fig 7.1 B).

(ii) Right Hand Rule - According to this rule, if we hold a current carrying conductor by our right hand as shown in fig 7.4 and the direction of thumb indicates the direction of I, then the currled fingers will give the direction of magnetic field arround the conductor.

Fig 7.4 : Right hand thumb rule

(iii) Right hand palm rule for circular current- According to this rule, if the direction of currled fingers of right hand gives the direction of current, then the direction of the thumb gives the direction of magnettic field (fig 7.5).

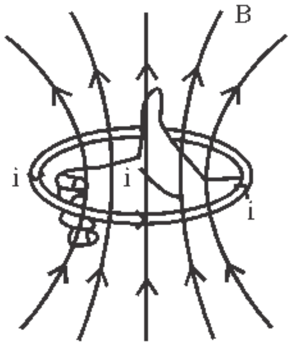


Fig 7.5 Right hand palm rule

(iv) Maxwell's Cork Screw Rule-Right handed screw rule- If the direction of linear motion of a right handed screw gives the direction of current in a conductor, then the direction of rotation of screw, gives the magnetic field produced by that current.

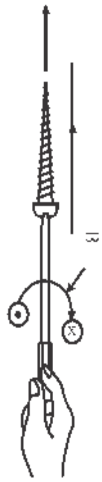


Fig 7.6 Maxwell's screw rule

7.4 Magnetic Field Due to a Long and Straight Current Carrying Conductor

7.4.1 Magnetic Field of a Straight Current Carrying Wire of Finite Length

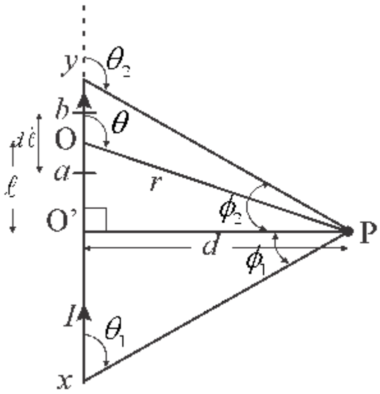


Fig. 7.7 Magnetic field due to long conductor

As per the figure 7.7, a straight, wire X Y lies in the plane of the paper. It carries current from x to y end, consider a point P, at a perpendicular distance d from it. The magnetic field due to an arbitrary length element $d\vec{\ell}$ (ab) whose mid point is O at a. Distance OP = r. From Biot-Savart's law

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\theta}{r^2} \qquad \ldots (7.11)$$

here there are three variables, l, r and θ. We can change l and r in terms of θ, by the geometry in the figure.

From $\Delta OO'P$ &

$$\frac{OO'}{O'P} = \frac{\ell}{d} = \cot\angle(POO') = \cot(180-\theta) = -\cot\theta$$

$$\ell = -d\cot\theta \qquad \ldots (7.12)$$

$$\frac{d\ell}{d\theta} = -d(-\text{cosec}^2\theta)$$

$$d\ell = d\,\text{cosec}^2\theta\, d\theta \qquad \ldots (7.13)$$

Again from $\Delta OO'P$

$$\text{cosec}(180-\theta) = \frac{OP}{OO'} = \frac{r}{d}$$

$$\text{cosec}\,\theta = \frac{r}{d}$$

$$r = d\,\text{cosec}\,\theta \qquad \ldots (7.14)$$

Substituting in equation 7.13

$$dB = \frac{\mu_0}{4\pi}\frac{I(d\,\text{cosec}^2\theta\, d\theta)\sin\theta}{(d\,\text{cosec}\,\theta)^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I(d\,\text{cosec}^2\theta)\sin\theta d\theta}{d^2\,\text{cosec}^2\theta}$$

$$dB = \frac{\mu_0 I}{4\pi d}\sin\theta\, d\theta \qquad \ldots (7.15)$$

since the angle θ, changes from θ_1 to θ_2 for conductor XY- so to obtain the magnetic field due to wire at P_1 on integrating dB between limits θ_1 to θ_2

$$B = \frac{\mu_0 I}{4\pi d}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = \frac{\mu_0 I}{4\pi d}[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$B = \frac{\mu_0 I}{4\pi d}[\cos\theta_1 - \cos\theta_2] \qquad \ldots (7.16)$$

Again from geometry of the fig 7.7; $\theta_1 = 90° - \phi_1$
$(\because \theta_1 + \phi_1 = 90°)$

$$\theta_2 = \phi_2 + 90°$$

Substituting θ_1 and θ_2 in equation 7.16.

$$B = \frac{\mu_0 I}{4\pi d}[\cos(90°-\phi_1) - \cos(90°+\phi_2)]$$

$$B = \frac{\mu_0 I}{4\pi d}[\sin\phi_1 + \sin\phi_2] \qquad \ldots (7.17)$$

Here ϕ_1 and ϕ_2 angles subtended by ends *x* and *y* at P with O^1P.

7.4.2 Magnetic Field Due to Straight Current Carrying Conductor of Infinite Length

Since the length of the conductor is infinite, so the angles $\phi_1 = \phi_2 = \pi/2$. Using equation (7.17)

We get

$$B = \frac{\mu_0 I}{4\pi d}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$$

$$= \frac{\mu_0 I}{4\pi d}(1+1) \qquad \left[\because \sin\frac{\pi}{2} = 1\right]$$

or

$$B = \frac{\mu_0 I}{2\pi d} \qquad \ldots (7.18)$$

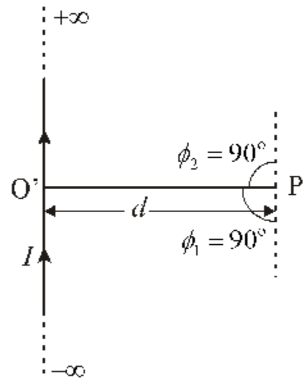


Fig 7.8 Magnetic field due to infinitely long conductor

Special Condition

(i)Magnetic field at a distance d from one end of the finite conductor.

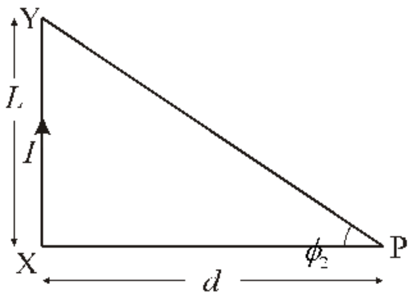


Fig 7.9 Half infinite wire

$$B = \frac{\mu_0 I}{4\pi d}(\sin O + \sin\phi_2)$$

$$B = \frac{\mu_0 I}{4\pi d}\sin\phi_2 \qquad \ldots (7.19)$$

From fig 7.9 $\sin\phi_2 = \frac{L}{YP} = \frac{L}{\sqrt{L^2+d^2}}$

From fig 7.9 $B = \frac{\mu_0 I}{4\pi d}\frac{L}{\sqrt{L^2+d^2}}$ . . . (7.20)

(ii) Magnetic field at a perpendicular distance d from one end of the infinite wire.

For this condition $\phi_2 = \frac{\pi}{2}$ and $\phi_1 = 0°$; hence from eqn. (7.17)

$$B = \frac{\mu_0 I}{4\pi d}\left[\sin 0 + \sin\frac{\pi}{2}\right]$$

$$B = \frac{\mu_0 I}{4\pi d} \quad . . . (7.21)$$

(iii) Magnetic field due to a conductor of finite length when the point P is situated at a perpnedicular distance d from its mid point - here $\phi_1 = \phi_2 = \phi$

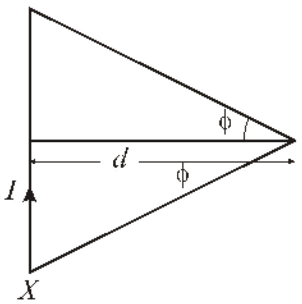


Fig 7.10 Wire of finite length

From eqn. (7.17)

$$B = \frac{\mu_0 I}{4\pi d}(\sin\phi + \sin\phi)$$

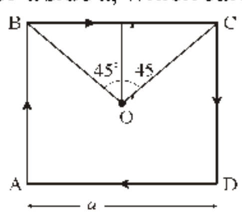
$$B = \frac{\mu_0 I}{4\pi d}(2\sin\phi)$$

$$B = \frac{\mu_0 I}{2\pi d}\sin\phi \quad . . . (7.22)$$

(iv) At the axial position of point P; $\phi = 0$ hence $B = 0$.

Example 7.1 : Find the magnetic field at the center O of square ABCD of a side a, which carries a current I A.

Solution :



The ends of each side makes an angle 45° at the center. Hence magnitude of magnetic field is same for all sides, from right hand palm rule the direction is also same, and downwards, hence $B_1 = B_2 = B_3 = B_4$ and. Total magnetic field at the center will be 4 times B_1

$$B = \frac{\mu_0 I}{4\pi d}(\sin\phi_1, + \sin\phi_2)$$

here, $B_1 = B_2 = B_3 = B_4 = \frac{\mu_0 I}{4\pi(a/2)}(\sin 45° + \sin 45°)$

$$= \frac{\mu_0 I}{2\pi a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{2\pi a}\left(\frac{2}{\sqrt{2}}\right)$$

$$|\vec{B}| = \frac{8\sqrt{2}\times 10^{-7} I}{a} Tesla$$

Example 7.2 : Find the net magnetic field at point P due to two perpendicular current carriers, in two situations given in fig (A) and fig (B).

Solution :

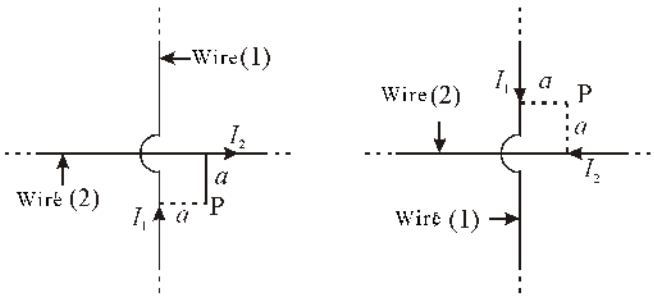


Fig. (A) Fig. (B)

In fig (A), the magnetic field due to I_1 is $B_1 = \frac{\mu_0 I_1}{2\pi a}$

The direction of B_1 is perpendicular to page and downwards. The magnetic field due to I_2 at P is $B_2 = \frac{\mu_0 I_2}{2\pi a}$; again the direction of B_2 is same as that of B_1 according to right hand rule. Hence the net field at P is $|\dot{B}| = B_1 + B_2 = \frac{\mu_0}{2\pi a}(I_1 + I_2)$ the direction is to page downwards.

Again for fig (B) the magnetic field at P, due to I_1 is $B_1 = \frac{\mu_0 I_1}{2\pi a}$ the direction of B_1 is perpendicular to page upwards.

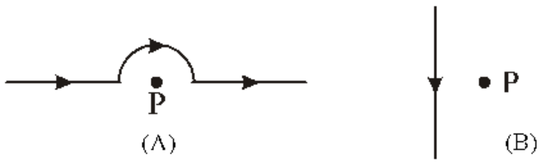
The magnetic field at P, due to I_2 is $B_2 = \frac{\mu_0 I_2}{2\pi a}$ the direction of B_2 is perpendicular to the page downwards.

hence the net magnetic field at P is

$$\vec{B} = \vec{B}_1 - \vec{B}_2$$

$$|\vec{B}| = B_1 - B_2 = \frac{\mu_0}{2\pi a}(I_1 - I_2)$$

Example 7.3 : Show the direction of magnetic fields at point P, as $\otimes$ and $\odot$.



Solution : In fig. (A) the direction of $\vec{B}$ will be downwards and given as $\otimes$. In fig (B) the direction of $\vec{B}$ at P is upwards, and given as $\odot$.

7.5 Magnetic Field Due to a Current Carrying Circular Coil

7.5.1 Magnetic Field at the Centre of Coil

To find the magnetic field at the center of a coil of radius R, having a current I, we consider the contribution of small lenght element $\delta\vec{\ell}$ at center O.

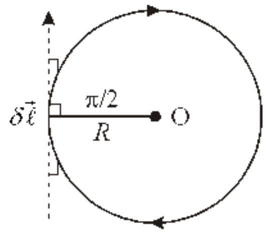


Fig 7.11 Magnetic field at center of a coil

(i) $\delta\vec{\ell}$ is at a distance R from the center.

(ii) $\delta\vec{\ell}$ is perpnedicular to R; i.e $\theta = \pi/2$.

hence from Biot and Savart's law

$$\delta B_{Centre} = \frac{\mu_0}{4\pi}\frac{I\delta\ell\sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\delta\ell}{R^2}\sin\left(\frac{\pi}{2}\right) \qquad \because r = R \text{ and } \theta = \frac{\pi}{2}$$

$$\delta B_{Centre} = \frac{\mu_0}{4\pi}\frac{I\delta\ell}{R^2} \qquad ...(7.23)$$

Since all the length elements contribute in the same direction. The net magnetic field is sum of all contributions

$$B_{Centre} = \frac{\mu_0 I}{4\pi R^2}\Sigma\delta\ell \qquad ...(7.24)$$

$$B_{Centre} = \frac{\mu_0 I}{4\pi R^2}(2\pi R)$$

$$B_{Centre} = \frac{\mu_0 I}{2R} = \frac{\mu_0}{4}\frac{(2I)}{R} \qquad ...(7.25)$$

If the coils has N turns, the magnetic field at center is

$$B_{Centre} = \frac{\mu_0 N I}{2R} \qquad ...(7.26)$$

Dependance of B on radius of the coil.

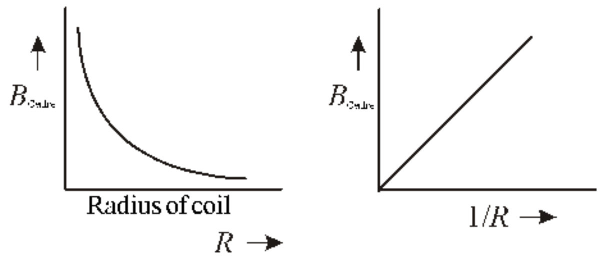


Fig 7.12 Dependance of magnetic field on radius of coil

As it is evident from equation (7.26), hence the graph between B and R is hyperbolic, and between B and 1/R it is straight line, as shown in fig 7.12.

Special : We can find B at the center of coil by another method. Let the angle subtended by $\delta\vec{\ell}$ at center be δα, then

$$\delta\alpha = \frac{arc\,\delta\ell}{radius\,R} = \frac{\delta\ell}{R} \quad \text{or} \quad \delta\ell = R\delta\alpha$$

using this relation in equation (7.24) we get

$$B_{Centre} = \frac{\mu_0 I}{4\pi R^2}\times 2\pi R = \frac{\mu_0 I}{2R}$$

since $\sum\delta\ell = R\Sigma\delta\alpha = R(2\pi)$ for whole loop.

Magnetic field at center due to one fourth of coil will be and

$$\sum\delta\ell = \Sigma R\delta\alpha = R\times\frac{1}{4}(2\pi) = R\times\frac{\pi}{2}$$

$$B = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right) \qquad ...(7.27)$$

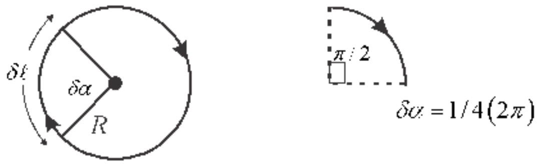


Fig 7.13 : Magnetic field due to a segment at centre

7.5.2 Magnetic Field Due to a Circular Current Carrying Coil at an Axial Point

A coil of radius R with a current I is considered in Y - Z plane with center at origin O. Consider a point P on the axis of the coil (as X-axis). Consider a small length element $LM - \delta\vec{\ell}$.

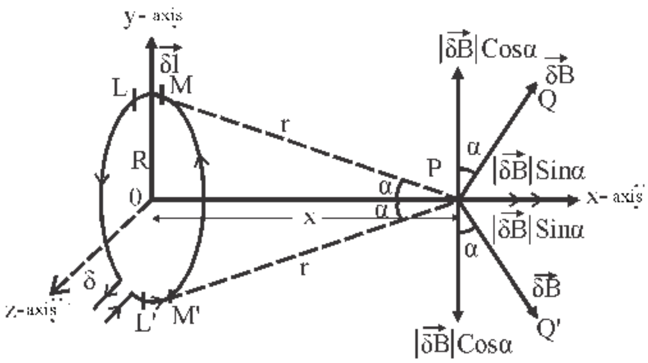


Fig 7.14 Magnetic field due to circular current at axial point

The distance of $\delta\vec{\ell}$ from P is r and angle between $\delta\vec{\ell}$ and $\vec{r}$ is 90^0 (i.e. 90^0) . From Biot and Savart's law the magnetic field at point P due to the above length element $\delta\vec{\ell}$ is

$$\delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\delta\vec{\ell}\times\hat{r}}{r^2}$$

$$|\,\delta\vec{B}\,| = \frac{\mu_0}{4\pi}\frac{I\delta\ell\sin 90^\circ}{r^2}$$

$$(\because |\,\delta\vec{\ell}\times\hat{r}\,| = \delta\ell\sin\theta \text{ and } \theta = 90^\circ)$$

$$|\,\delta\vec{B}\,| = \frac{\mu_0}{4\pi}\frac{I\,\delta\ell}{r^2} \qquad \ldots (7.28)$$

The direction of $\delta\vec{B}$ will be always perpendicular to the plane of $\vec{r}$ and $\delta\vec{\ell}$, according to right hand rule, as shown by $\vec{PQ}$. Similarly the contribution due to diametrically opposite length element L'M' = $\delta\vec{\ell}$ will be -

$$|\,\delta\vec{B}'\,| = \frac{\mu_0}{4\pi}\frac{I\,\delta\ell}{r^2}$$ and the direction will be the direction of $\vec{PQ}'$. As shown in the diagram. Now resolving, both $\delta\vec{B}\vec{PQ}$ into components, as parallel and perpendicular components, $\delta\vec{B}$ and $\delta\vec{B}'$ we see that cosine components being equal and opposite cancels each other, and the sine component, due to whole of coil contribute to the magnetic field at axial point P. Hence

$$B = \sum |\,\delta\vec{B}\,| \sin\alpha = \sum \frac{\mu_0}{4\pi}\frac{I\,\delta\ell}{r^2}\sin\alpha$$

$$= \frac{\mu_0}{4\pi} I \sum \frac{\delta\ell}{r^2}\left(\frac{R}{r}\right)$$ (From the diagram for whole coil $\sin\alpha = \frac{R}{r}$)

$$= \frac{\mu_0}{4\pi}\frac{I\,R}{r^3}\sum\delta\ell = \frac{\mu_0}{4\pi}\frac{I\,R}{r^3}(2\pi R)$$

$$(\because \sum\delta\ell = \text{ circumfrence of the coil } = 2\pi R$$

$$= \frac{\mu_0}{4\pi}\frac{I(2\pi R^2)}{r^3}$$

$$B = \frac{\mu_0}{4\pi}\frac{I(2\pi R^2)}{r^3} \qquad \ldots (7.31)$$

from pythogorous theorem

$$r^2 = R^2 + x^2$$

$$r = (R^2 + x^2)^{\frac{1}{2}}$$

$$r^3 = (R^2 + x^2)^{3/2} \qquad \ldots (7.32)$$

By putting value or r in eqn. (7.31) and eqn. (7.32)

$$B = \frac{\mu_0}{4\pi}\frac{2I\,(\pi R^2)}{(R^2 + x^2)^{3/2}} \qquad \ldots (7.33)$$

If the coil has N number of turns

$$B = \frac{\mu_0}{4\pi}\frac{2I\,(N\pi R^2)}{(R^2 + x^2)^{3/2}} \qquad \ldots (7.34)$$

or $$B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}} \qquad \ldots (7.35)$$

In vector notation

$$\vec{B}_{axis} = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}\hat{x} \qquad \ldots (7.36)$$

Due to the direction of current shown in the figure, the direction of $\vec{B}$ will be in positive x direction.

Special Conditions -

(i) Magnetic field at the center of the coil

In this case $x = 0$; hence $\vec{B}$ will be maximum

$$B_{centre} = \frac{\mu_0 N I R^2}{2(R^2+0)^{3/2}}$$

$$B_{centre} = \frac{\mu_0 N I}{2R} = B_{maximum} \qquad \ldots (7.37)$$

(It is same as obtained earlier)

(ii) If the point P is at a large distance compared to R, i.e. $x >> R$,

hence $\frac{R^2}{x^2}$ is negligible

$$B = \frac{\mu_0 N I R^2}{2(o+x^2)^{3/2}} = \frac{\mu_0 N I R^2}{2x^3} \qquad \ldots (7.38)$$

(iii) If the point P is at $x = R/2$

$$B_{x=R/2} = \frac{\mu_0 N I R^2}{2\left[R^2+\left(\frac{R}{2}\right)^2\right]^{3/2}}$$

$$= \frac{\mu_0 N I R^2}{2\left(R^2+\frac{R^2}{4}\right)^{3/2}} = \frac{\mu_0 N I R^2}{2\left(\frac{5R^2}{4}\right)^{3/2}}$$

$$B_{x=R/2} = \frac{4}{5\sqrt{5}}\frac{\mu_0 N I}{R} \qquad \ldots (7.39)$$

Comparing with magnetic field at center we get

$$B_{x=R/2} = \frac{8}{5\sqrt{5}} B_{centre} = 0.72 B_{centre} \qquad \ldots (7.40)$$

The Variation of B with Distance on Axis

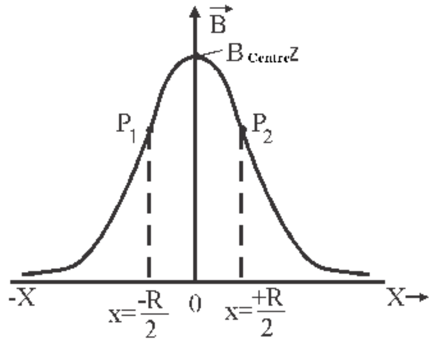


Fig 7.15 Variation B due to axial distance

The variation of B with distance is given by equation (7.36) and x is shown by figure (7.15). It is evident from the figure that B is maximum at center. B varies with distance on both sides, non-linearly and becomes zero at $x = \infty$. At a particular distance $x = \pm R/2$, we get two points on the curve P_1 and P_2, at these points $B \,\alpha\, I/R$ and linear. At these two points, the sign of the slope of the curve changes from positive to negative. Hence the points are called the points of inflection.

At these points $\frac{dB}{dx}$ = constant; and $\frac{d^2B}{dx^2} = 0$;

The distance between these points is equal to R.

7.5.3 Comparision of Small Current Loop with a Magnetic Dipole

The magnetic field due to a circular coil at an axial point is given by

$$B = \frac{\mu_0}{4\pi}\frac{2I\,(N\pi R^2)}{(R^2+x^2)^{3/2}}$$

If the loop is small $R^2 << x^2$; $\frac{R^2}{x^2}$ is negligible,

also $A = \pi R^2 =$ is area of the current loop, we get

$$B = \frac{\mu_0}{4\pi}\frac{2NIA}{x^3}$$

or $$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3} \qquad \ldots\ldots(7.41)$$

where M = NIA is the magnetic moment of current loop. This expression is exactly similar to the magnetic field produced by a small bar magnet at the distance on its axis from the center of the magnet.

Hence a small current loop is equivalent to a bar magnet (magnetic dipole).

7.5.4 Helmholtz Coils

Two identical coaxial coils held in vertical plane, such that their center are at a distance equal to the radius of the coil. These coils are called Helmholtz coils.

The plane of the coils is parallel to each other. The coils are connected in series, they produce exactly same magnetic field. Fig 7.16. The coils are used to produce uniform magnetic field, in the area between the coils.

Fig 7.16 Helmholtz coils

The Magnetic Field Between the Space of Coils

The magnetic field between the space of the coils is the vector sum of the fields produced by the two coils. The center of space is the area about the points of inflection.

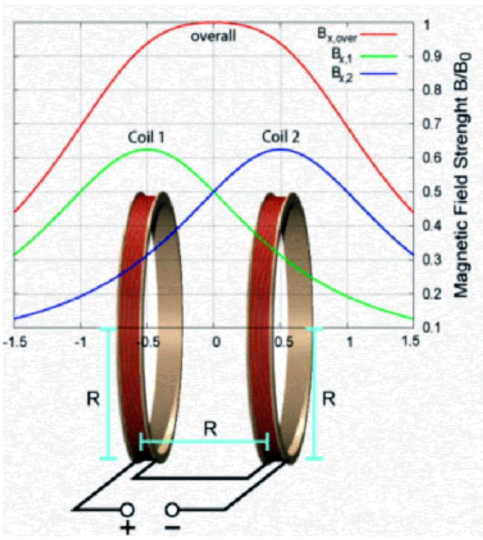


Fig 7.17 Uniform magnetic field between the coils

The magnetic field at the center of space i.e. point of inflection of both the coils, hence the magnetic field will be

$$B_1 = \frac{4\mu_0 NI}{5\sqrt{5}R} \qquad (B_x = R/2)$$

$$B_1 = B_2 = \frac{4\mu_0 NI}{5\sqrt{5}R}$$

$$B = B_1 + B_2 \; [\because \vec{B}_1 \text{ and } \vec{B}_2 \;]$$

$$= 2B_1$$

$$= 2 \times \frac{4\mu_0 NI}{5\sqrt{5}R} = 0.716\frac{\mu_0 NI}{R}$$

$$B = 1.432\frac{\mu_0 NI}{2R} \qquad \ldots (7.42)$$

$$B = 1.432 B_{Centre} \qquad \ldots (7.43)$$

Which means the uniform magnetic field in the space between the coils is 1.432 times the maximum magnetic field produced at the center of each coil.

7.5.5 The Direction of Magnetic Field Due to Straight Current

(1) The form of magnetic field due to stright current can be understood by the following experiment -

Consider a straight current carryiing wire PQ passing through a cardboard ABCD, whose plane is perpendicular to the wire. Put some iron fillings on the cardborad, and establish a current in the wire. By taping the card board you will notice the iron filling to align like circular rings around the wire, as shown in the figure.

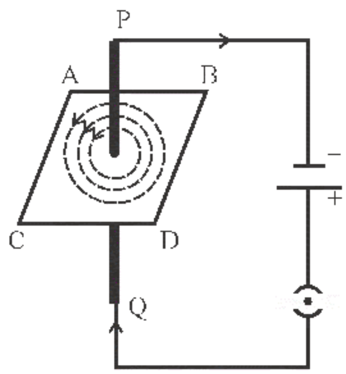


Fig 7.18 Magnetic field due to straight current

(2) Magnetic field in a current carrying coil. We look at the face of the coil. If the direction of current is N - wise (anti clock wise) then the face will behave like north pole of the magnet. And if the current in the face is clock wise i.e S-wise, the face (end) will behave like south pole.

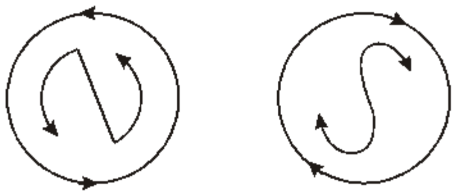


Fig 7.19 Deciding the pole of current coil

Example 7.3 : A circular coil of radius 10 cm, has 100 tightly wound turns. Find the magnetic field at the center of loop if the current in the coil is 1A.

Solution : given R = 10 cm = 0.1 m

N = 100, I = 1A,

$$B = \frac{\mu_0 N I}{2R} = \frac{4\pi \times 10^{-7} \times 100 \times 1}{2 \times 0.1}$$

$$= 2\pi \times 10^{-4} \text{ T}$$

$$= 6.28 \times 10^{-4} \text{ T}$$

Example 7.4 : A helium nucleus revolves in a circular path of radius 0.8 m, in 2 s. Find the magnetic field at the center of the circle.

Solution : The charge on *He* nucleus is q = + 2e. The magnetic field at the center of circle *r* is

$$B = \frac{\mu_0 I}{2r}$$

$$I = \frac{2e}{t} \qquad \left| \because I = \frac{q}{t} \right|$$

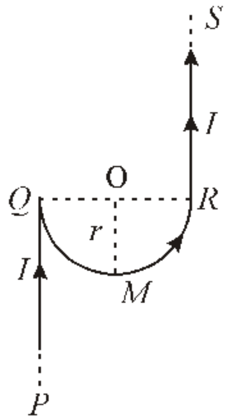
t = time taken in one revolution

$$B = \frac{\mu_0 (2e)}{2rt} = \frac{\mu_0 e}{rt}$$

$$= \frac{4\pi \times 10^{7} \times 1.6 \times 10^{19}}{0.8 \times 2}$$

$$= 12.56 \times 10^{26} \text{ T}$$

Example 7.5 : Find the magnetic field at point O in the given figure.



$$\vec{B} = \vec{B}_{PQ} + \vec{B}_{QMR} + \vec{B}_{RS}$$

Solution : Magnetic field at O is due to the current in PQ, QMR and RS. The contribution of PQ is

$$|\vec{B}_{PQ}| = |\vec{B}_{RS}| = \frac{\mu_0 I}{4\pi R} \quad \text{downwards}$$

Contribution of RS is up-wards both cancel each other, being of same magnitude.

The only contribution is due to semi circle QMR, which is

$$|\vec{B}| = |\vec{B}_{QMR}| = \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) \text{which is upwards.}$$

Example 7.6 : At what distance from the center of the coil of radius R the magnetic field will be 1/27 of the field at center.

Solution : Magnetic field at the axis of a circular current carrying coil is :

$$B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$$

Magnetic field at center

$$B_{center} = \frac{\mu_0 N I}{2R}$$

given-

$$B = \frac{1}{27} B_{center}$$

or $$\frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}} = \frac{1}{27} \frac{\mu_0 N I}{2R}$$

or $$\frac{R^2}{(R^2+x^2)^{3/2}} = \frac{1}{3^3}\frac{1}{R}$$

or $$(R^2+x^2)^{3/2} = 3^3 R^3 = (3R)^3$$

or $$(R^2+x^2)^{\frac{1}{2}} = 3R$$

or $$R^2+x^2 = 9R^2$$

or $x^2 = 8\ R^2$

or $$x = 2\sqrt{2}R$$

Example 7.7 : In Helm holtz coils, each coil has 20 turns and radius 10 cm. If the current in the coil is 0.1 A, find the magnetic field in area between the coils.

Solution : Magnetic field in the required area is

$$B = \frac{8}{5\sqrt{5}}\frac{\mu_0 N I}{R}$$

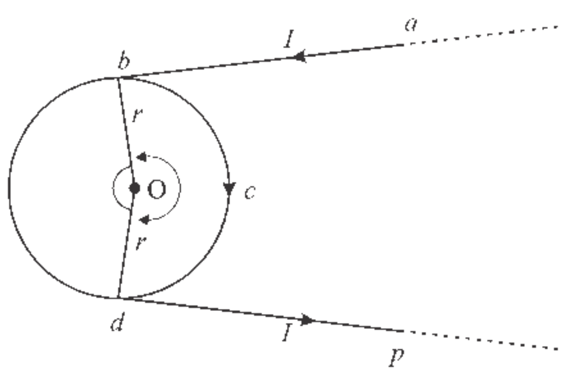
given - $N = 25$, $R = 10$ cm $= 0.1$ m, $I = 0.1$ A

$$B = \frac{8}{5\sqrt{5}} \times \frac{4\pi\times10^{-7}\times25\times0.1}{0.1} T$$

$= 2.25 \times 10^{-5}$ T

Example 7.8 : A wire of infinite length is curved as shown in the figure. If the current is I, then find the angle , for which the magnetic field at center O is zero.

Solution :



The total magnetic field at center is

$$\vec{B}_0 = \overrightarrow{B_{ab}} + \overrightarrow{B_{bcd}} + \overrightarrow{B_{dp}}$$

$$B_{ab} = \frac{\mu_0 I}{4\pi r} \text{ (upwards)}$$

$$B_{dp} = \frac{\mu_0 I}{4\pi r} \text{ (upwards)}$$

If we take + *ve* sign for an up wards magnetic field and - *ve* sign for downwards magnetic field then -

$$B_{bcd} = \frac{\mu_0 I}{4\pi r^2} \text{ x (length of arc bcd)}$$

$$B_{bcd} = \frac{\mu_0 I}{4\pi r^2}(2\pi-\theta)r \text{ (downwards)}$$

Total magnetic field at O is

$$|\vec{B}_0| = \frac{\mu_0 I}{4\pi r} - \frac{\mu_0 I}{4\pi r}(2\pi-\theta) + \frac{\mu_0 I}{4\pi r}$$

$$\frac{\mu_0 I}{4\pi r} - \frac{\mu_0 I}{4\pi r}(2\pi-\theta) + \frac{\mu_0 I}{4\pi r} = 0$$

As given in question

$$\frac{\mu_0 I}{4\pi r}[1-(2\pi-\theta)+1] = 0$$

$$2-(2\pi-\theta) = 0$$

$$\theta = 2\pi - 2 = 2(\pi-1)rad.$$

7.6 Motion of a Charge in Magnetic Field

If a charge q is moving in both electric field $\vec{E}$ and magnetic field $\vec{B}$, then the net force on the praticle is

$[\vec{F}_e = q\vec{E}\ ;\ \vec{F}_m = q(\vec{v}\times\vec{B})]$

hence $\vec{F} = \vec{F}_e + \vec{F}_m = q[\vec{E} + \vec{v}\times\vec{B}]$ . . . (7.44)

This force was given by H.A. Lorentz, and hence the name, Lorentz force.

The magnetic force on a moving charge is given by

$\vec{F}_m = q(\vec{v}\times\vec{B}) = qvB\sin\theta\,\hat{n}$ . . . (7.45)

$\left|\vec{F}_m\right| = qvB\sin\theta$ . . . (7.46)

The direction of force is given by $\hat{n}$, which is a unit vector perpendicular to the plane of v & B according to right hand rule. If the charge is negative, the force is opposite to that on + ve charge.

Special Cases :

(7.5.1) If the charge is stationary; $|\vec{v}| = 0$; $|\vec{F}| = 0$, only a moving charge experiences magnetic force. 7.5.2. If $\theta = 0°$ or $180°$ i.e. the charge is moving parallel or antiparallel to the field; $\sin\theta = 0$ $|\vec{F}| = 0$, hence the charge continue to move in staight line.

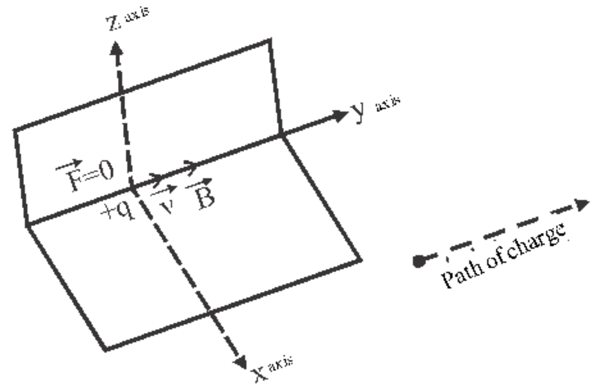


Fig 7.20 (A) Froce on charge in magnetic field
Fig 7.20 (B) Motion of charge parallel to $\vec{B}$

7.6.1 Motion of Charge in Perpendicular Magnetic Field

If $\vec{v}$ and $\vec{B}$ are mutualy perpendicular, then $\theta = 90^0$. the force on the charge will be maximum and equal to $F = qvB\sin 90°$; where is to the plane of $\vec{v}$ and $\vec{B}$

$$F = qvB = F_{max} \quad \ldots (7.47)$$

The direction of this force is shown in Z direction in fig 7.21 (A) hence the charge will have a circular motion in X-Z plame. In fig 7.21 (B). The $\vec{B}$ is perpendicularly down-wards to the page.

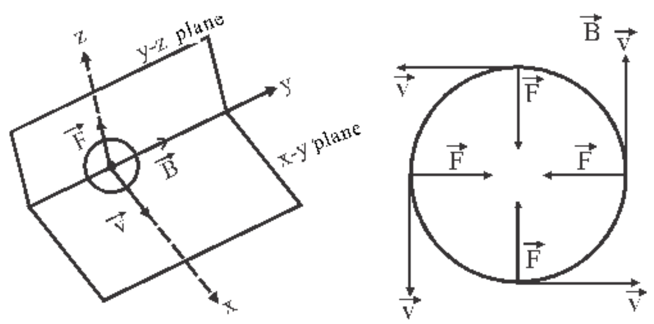


Fig 7.21 (A) Force on a charge in magnetic field
Fig 7.21 (B) Motion of charge in magnetic field

If the charged particle has a mass m, and moving on a circular path of radius r, then the magnetic force will act as centripetal force, hence

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} = \frac{p}{qB} \quad \ldots (7.48)$$

If kinetic energy of particle is E_k, than $p = \sqrt{2mE_K}$

The radius of circular path is

$$r = \sqrt{\frac{2m\,E_K}{qB}} \quad \ldots (7.49)$$

It means that radius of circular path is proportional to the linear momentum of the particle. Since r is E_k will be constant. It menas the works done by this force, on particle is zero.

The circular motion of a charged particle behave like a current loop, and produces its magnetic field which affect the existing magnetic field.

The time period T of this circular motion is

$$T = \frac{2\pi\,m}{qB} \quad \ldots (7.50)$$

and the frequency

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m} \quad \ldots (7.51)$$

The angular frequency

$$\omega = 2\pi\nu$$

$$\omega = \frac{qB}{m} \quad \ldots (7.52)$$

From equation (7.50) it is clear that time period T and frequency w or ν is independent of speed and kinetic energy E_k. It is also independent of momentum.

This important concept is used in the design of cyclotron. T depends on B and specific charge of the particle q/m. $T \propto 1/B$ and $T \propto m/q$.

7.6.2 Motion of charged particle when $0° < \theta < 90°$

If the veloity of the particle makes an angle θ with

$\vec{B}$, then velocity v has, two components, v_{11} and $v_{\perp}$. The component $v_{11} = v\cos\theta$ is in the direction of B, and $F_m = 0$. The particle will move in a straight line with constant velocity. The other component $v_{\perp} = v\sin\theta$ make the particle to have a uniform circular motion.

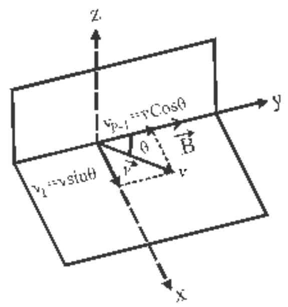


Fig 7.22 (A) Force on a charge when $0^0 < \theta < 90^0$

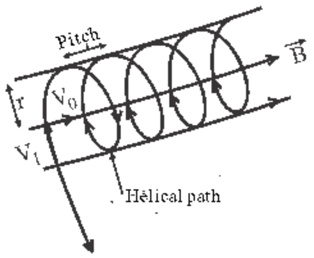


Fig 7.22 (B) Helical motion of the particle

The combination of these two motions is a helical motion, which is shown in fig 7.22(B). Radius of the helical path is

$$r = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB} \qquad \ldots (7.53)$$

and time period $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$ ... (7.54)

The linear distance, between the consecutive revolutions is called pitch, which is

$$y = v_{11}T = (v\cos\theta)\frac{2\pi m}{qB}$$

$$y = \frac{2\pi mv\cos\theta}{qB} \qquad \ldots (7.55A)$$

Or $$y = \frac{2\pi r}{\tan\theta} \qquad \ldots (7.55B)$$

In the polar region, for example in Northern Canada and Alaska, sometimes a spectacular pattern of coloured light polar aura is seen, which is called AURORA BOREALIS scientists explained it a s a phenomem due to motion of charged particles from cosmic rays in the earth's magnetic field, which is strong at poles.

Example 7.8 : An electron of energy $10eV$ is moving on a circular path in perpendicular magnetic field B = 10^{-5}T. Find the velocity of electron and radius of circular path.

Solution : $E_K = \frac{1}{2}mv^2 = 10eV$

$$= 10 \times 1.6 \times 10^{19} J$$

$$v = \sqrt{2E_k m}$$

$$v = \sqrt{2 \times 10 \times 1.6 \times 10^{19} \times 9.1 \times 10^{31}}$$

1 eV = 1.6 × 10^{19}J. m = 9.1 × 10^{31} kg

$$v = 1.88 \times 10^{-6}\ m/s$$

radius of circular path

$$r = \frac{mv}{qB} = \frac{mv}{eB} \quad (\because q = e = 1.6 \times 10^{-19} c)$$

$$= \frac{9.1 \times 10^{31} \times 1.88 \times 10^{6}}{1.6 \times 10^{-19} \times 10^{-5}}$$

r = 1.07 m

Example 7.9 : A beam of proton with velocity 4 x 10^5 m/s is moving in a uniform magnetic field 0.3 T at an angle 60^0 with B. Find (i) radius of the path and (ii) pitch.

Solution : The path is helical hence

$$v_{11} = v\cos\theta = 4 \times 10^5 \cos 60°$$

$$= 4 \times 10^5 \times \frac{1}{2}$$

$$v_{11} = 2 \times 10^5\ m/s$$

$$v_{\perp} = v\sin\theta = 4 \times 10^5 \times \frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3} \times 10^5\ m/\sec$$

$$r = \frac{mv_{\perp}}{qB}$$

$$= \frac{(1.67\times10^{-27})\times(2\times\sqrt{3}\times10^{5})}{1.6\times10^{-19}\times0.3}$$

$$r = 12\times10^{-5}\,m$$

(ii) $$y = v_{11}T = v_{11}\times\frac{2\pi m}{qB}$$

$$y = \frac{2\times10^{5}\times2\times3.14\times1.67\times10^{-27}}{1.6\times10^{-19}\times0.3}$$

The pitch $= 43.7\times10^{-3}\,m = 43.7\,mm$

Example 7.10 : An electron is moving with speed 3 x 10^7 m/s in a perpendicular uniform magneic field B = 6 x 10^{-4}T. Find (i) radius of path (ii) frequency (iii) energy in KeV. ($m_e = 9\times10^{-31}$ kg ; $e = 1.6\times10^{-19}$ C) (1eV = 1.6×10^{-19} J)

Solution : $$r = \frac{mv}{qB} = \frac{9\times10^{-31}\times3\times10^{7}}{1.6\times10^{-19}\times6\times10^{-4}}$$

$= 2.81\times10^{-1}$m

$= 28\times10^{-2}$m = 28 cm

frequency $$v = \frac{qB}{2\pi m}$$

$$= \frac{1.6\times10^{-19}\times6\times10^{-4}}{2\times3.14\times9.0\times10^{-31}}$$

$= 17\times10^{6}$Hz = 17 MHz

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}\times9\times10^{-31}\times(3\times10^{7})^2$$

$$= \frac{1}{2}\times9\times10^{-31}\times9\times10^{14}$$

$$= 40.5\times10^{-17}\,J$$

$$= \frac{40.5\times10^{-17}}{1.6\times10^{-19}}eV$$

$= 25.3\times10^{2}$eV = 2.53 KeV

7.7 Cyclotron

It is an electromagnetic device, used to accelarate, massive + *ve ly* charged particles like α – particles proton, deutron, at high velocities.

It was invented by E.O. Lawrance and M.S. Livingston, to investigate the structure of nucleus (1934).

7.7.1 Principle of Cyclotron

(i) The charged particles are compelled to move in a perpendicular magnetic field, with constant freqeucny/ time period.

(ii) The electric potential (AC potential of high frequency) provide energy twice in one cycle.

7.7.2 Construction

Two hollow, D shaped metalic containers called "Dees" are placed between poles of magnet such that B is perpendicular to "Dees". These "Dess" are placed in vaccum chamber to avoid collision of charged particles with air molecules.

An AC source of cyclotron frequency $v = \frac{qB}{2\pi m}$ is connected to "Dees".

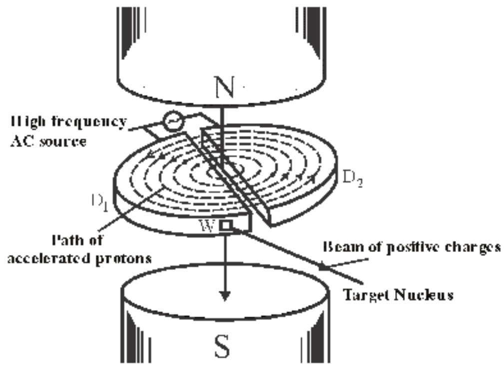


Fig 7.23 Cyclotron

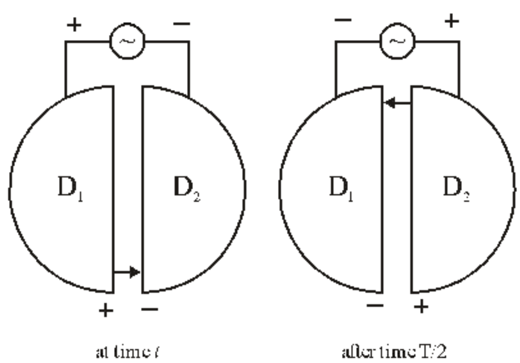


Fig 7.24 AC source at "Deed"

Working of Cyclotron-

The source of ions/particles (to be accelarated) is placed at the center of the circle made by the two "Dees". As soon as the particle is ejected from the source by its own velocty, it enters perpendicular magnetic field and starts circular motion inside Dee. After completing half circle in D_1, its enters the space between D_1 and D_2 where it is exposed to electric field of potential V. It experiences a kick and gains energy qV. The particle enters D_2 with increased velocity and moves in larger half circle in D_2. When the particle leaves D_2 and enters the space between Dees, the polarity of the applied voltage is reversed. The particle again gains energy qV and pre-enters D_1 with increased velocity. The particle gain an energy $2qV$ from electric field in one revolution.

If the particle had N revolutions before coming out of cyclotron its energy is increased by $2NqV$, which appears as kinetic energy of the particle.

Mathematical Analysis-

Let m, q an v be the mass, charge and the velocity of the charged particle. When particle enters a perpendicular magnetic field B,

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} = \frac{p}{qB} \qquad \ldots (7.57)$$

$$(p = mv = \ )$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \quad \text{(here v is velocity)} \ \ldots (7.58)$$

$$\frac{T}{2} = \frac{\pi m}{qB} \quad \text{and} \quad v = \frac{1}{T} = \frac{qB}{2\pi m} \qquad \ldots (7.59)$$

which is called cyclotron frequency. It is independent of v and r. Note that the frequency of the applied AC voltage to "Dees" is equal to cyclotron frequency $v = \frac{1}{T} = \frac{qB}{2\pi m}$, then the cyclotron is said to be in condition of resonance. And the particle gains maximum energy from the system.

$$\omega = 2\pi v = \frac{qB}{m} \qquad \ldots (7.61)$$

Kinetic energy of the particle is $E_K = \frac{1}{2}mv^2$

$$= \frac{1}{2}m\frac{q^2B^2r^2}{m^2} \qquad \left(7.57 \ \ v = \frac{qBr}{m}\right)$$

$$E_k = \frac{1}{2}\frac{q^2B^2r^2}{m} \qquad \ldots (7.62)$$

When the particle is about to come out from cyclotron, $r - R$; The kinetic energy will be maximum

$$E_{\max} = \frac{1}{2}\frac{q^2B^2R^2}{m} \qquad \ldots (7.63)$$

If the particle had completed N resovtions before coming out, the energy obtained from electric field is

$$E = (2qV)N \qquad \ldots (7.64)$$

Since energy is changed into kinetic energy, we get

$$(2qV)N = \frac{1}{2}\frac{q^2B^2R^2}{m}$$

$$N = \frac{1}{4}\frac{qB^2R^2}{mV} \qquad \ldots (7.65)$$

7.7.4 Limitations of Cyclotron

(i) It can't accelarate light particles like electrons, because to obtain required kinetic energy, we have to provide very high velocity to electrons. At such relativistic velocity, the mass of electron no more remain constant, and the cyclotron frequency changes, which disturbs the resonance of the cyclotron. To accelarate electrons, another device called Betatron is used.

(ii) Neutral particles like neutrons can not be accelarated by cyclotron.

Uses of Cyclotron

(i) The particles accelarated by cyclotron are used to study the structure of neucleus.

(ii) The accelarated ions are impregnated by bombarding into another materials to improve quality or synthesis of new materials.

(iii) To obtain new radio active materials, which has applications in several fields like research and medical sciences.

Example 7.11 : The cyclotron frequency is 10 MHz. To accelarate protons, what will be the value of magnetic field? Radius of Dees is 60 cm. also find the maximum kinetic energy of accelate protons in MeV.

($e = 1.6 \times 10^{19}$c, $m_p = 1.57 \times 10^{27}$kg, 1 MeV = 1.6×10^{-13} sJ)

Solution : The cyclotron freuqency is $\nu = \dfrac{qB}{2\pi m}$

hence $B = \dfrac{2\pi m\nu}{q}$

$$= \frac{2 \times 3.14 \times 1.67 \times 10^{27} \times 10^{7}}{1.6 \times 10^{-19}}$$

$B = 0.66$ T

The maximum velocity of protons is

$$v = \frac{qBr}{m} = \frac{1.6 \times 10^{19} \times 0.66 \times 0.60}{1.67 \times 10^{-27}}$$

$= 3.78 \times 10^{7}$ m/s

The maximum kinetic energy

$$E_k = \frac{1}{2}mv^2 = \frac{1.67 \times 10^{27} \times (3.78 \times 10^{7})^2}{1.6 \times 10^{-13}}$$

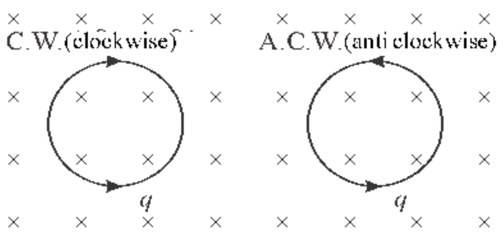
= 7 MeV

Example 7.12 : Discuss the path of a charge q (charged particle) entering a uniform magnetic field.

Solution : Case (i) If the particle enters the field, parallel or anti parallel to the field, then $F_m = o$, The path of the particle is a straight line.

Case (ii) When the particle enters the field perpendicularly $\theta = 0°$ then the force $F_m = qVB$ will be normal to V. The path will be a circle. It will move clock or anticlock wise according to direction of B.

Case (iii) When the particle enters magnetic field at an angle $\theta \neq 0°$, 180, 90. The path of the particle will be helical.



7.8 Force on Current Carrying Conductor in Magnetic Field

When a current carrying conductor is placed in a uniform magnetic field, the charge carriers, (the free electrons) moving with drift velocity v_d experience a force $\vec{F} = q(\vec{v}_d \times \vec{B})$. Hence, there will be a net force on the conductor.

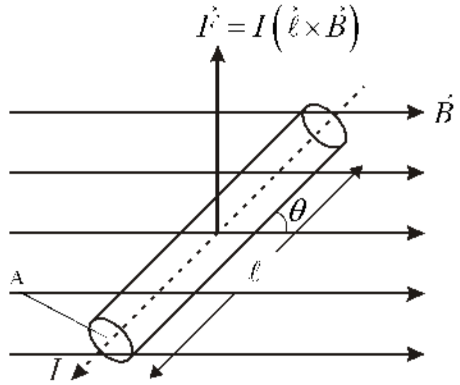


Fig 7.25 A current carrier in magnetic field

As in fig 7.25 conductor of cross section A, number of free electrons per unit volume 'n' carries a current I. Its $\vec{\ell}$ length l is placed in uniform magnetic field B at an angle θ. The total amount of charge of free dectrons will be $q - neAl$. Since the velocity of this charge is v_d , the net force on the conductor will be

$$|\vec{F}| = q\, v_d B \sin\theta$$

$$= neA\ell\, v_d\, B \sin\theta \qquad (\because q = neA\ell)$$

$$= (neA\, v_d)\ell\, B \sin\theta \qquad (\because I = neA\, v_d)$$

$$|\vec{F}| = I\ell\, B \sin\theta \qquad \ldots (7.66)$$

$$\vec{F} = I(\vec{\ell} \times \vec{B}) \qquad \ldots (7.67)$$

Here the direction of $\vec{\ell}$ is in the direction of current. Direction of the force will be perpendicular to the plane of $\vec{\ell}$ and $\vec{B}$, according to right hand rule.

7.8.1 Direction of Force on a Current Carrying Conductor in Magnetic Field

For this, two laws are in use -

7.8.1.1 Fleming's Left Hand Rule

Make thumb, index finger and middle finger of yoar left hand perpendicular to each other. If index finger indicates the direction of $\vec{B}$ (magnetic field) middle finger, the direction of I; then the thumbs will indicate the direction of force on the conductor.

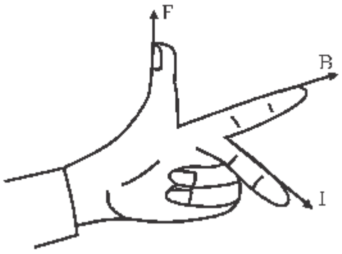


Fig 7.26 Fleming's left hand rule

7.8.1.2 Right Hand Palm Rule

If we spread our right hand in suh away that the finger are in the direction of magnetic field B, the thumb is in the direction of I, then the force on the conductor will be upward and perpendicular to the palm.

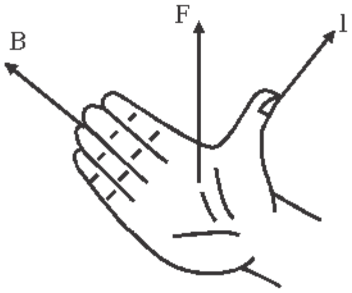


Fig 7.27 Right hand palm rule

7.9 Magnetic Force Between Two Parallel Current Carrying Conductors

Let two parallel conductors carrying currents I_1 and I_2 are in the plane of the paper at a distance d in air/vacuum. $\vec{B}_1$ and $\vec{B}_2$ are the magnetic fields produced by currents I_1 and I_2, at the location of II and I conductor. The force on the dl_1 length of the first conductor carrying currenr I in magnetic field $\vec{B}_2$ will be -

direction shown if fig (7.29) here $B_2 = \frac{\mu_0 I_2}{2\pi d}$

$$\delta \vec{F}_{21} = I_2 \left(\delta \vec{\ell}_2 \times \vec{B}_1\right)$$

$$\left|\delta \vec{F}_{21}\right| = I_2 \left|\delta \vec{\ell}_2\right| \left|\vec{B}_1\right| \sin 90^\circ$$

$$\left|\delta \vec{F}_{21}\right| = I_2 \delta \ell_2 B_1$$

$$\left|\delta \vec{F}_{21}\right| = \frac{\mu_0 I_1 I_2 \delta \ell_2}{2\pi d} \qquad \ldots (7.69)$$

similarly the force on the length dl_2 due to B_2 and I_2

$$\delta \vec{F}_{12} = I_1 \left(\delta \vec{\ell}_1 \times \vec{B}_2\right)$$

$$\left|\delta \vec{F}_{12}\right| = I_1 \left|\delta \vec{\ell}_1\right| \left|\vec{B}_2\right| \sin 90^\circ$$

$$= I_1 \delta \ell_1 B_2$$

hence $$\left|\delta \vec{F}_{12}\right| = \frac{\mu_0 I_1 I_2}{2\pi d} \delta \ell_1 \qquad \ldots (7.70)$$

(1) If the direction of current in the two conductors is same, they experience a force of attraction.

(2) If the direction of current in the two conductors is opposite to each other, they will experience a force of repulsion (Fig. 7.28).

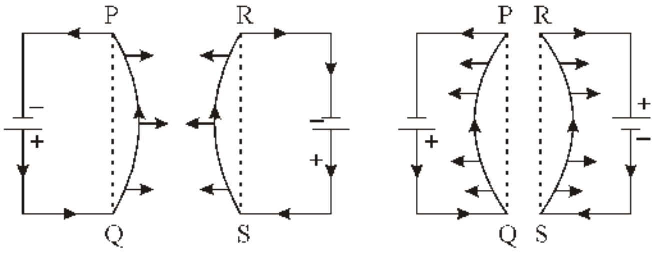


Fig 7.28 Force between two parallel currents

The direction of both forces is as per right hand palm rule, and shown in the figure 7.28. These forces are action reaction pairs, $\delta \vec{F}_{12}$ and $\delta \vec{F}_{21}$, and are opposite in direction.

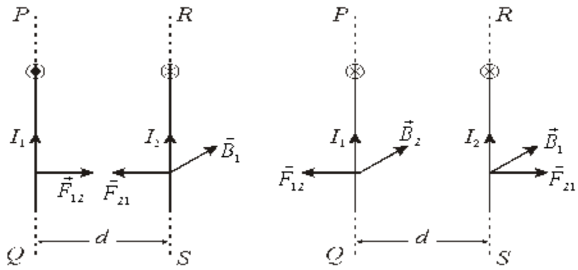


Fig 7.29 Force between two parallel currents

The force per unit length of the conductors is given

by $$\frac{\left|\delta \vec{F}_1\right|}{\left|\delta \ell_1\right|} = \frac{\left|\delta \vec{F}_2\right|}{\left|\delta \ell_2\right|} = \frac{\mu_0 I_1 I_2}{2\pi d} \text{ N/m} \qquad \ldots (7.72)$$

7.9.1 Defination of Standard Ampere in S.I. Units

From equation 7.72, the force per unit length on two parallel currents in air/vacuum, is $\frac{\delta F}{\delta \ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$ N/m

If we put the condition that $I_1 = I_2 = 1\,\text{A}$ and

$$d = 1\,\text{m} \text{ in air } \frac{\delta F}{\delta \ell} = \frac{\mu_0 \times 1 \times 1}{2\pi \times 1} = \frac{4\pi \times 10^{-7}}{2\pi}$$

$$= 2 \times 10^{-7}\ \text{N/m}$$

From the above condition we can define 1 A. 1 A is that current maintained in two parallel conductors placed at a distance of 1m in air, if it exerts a force per unit length equal to 2×10^{-7} N/m then the current in each conductor is 1 A. The latest definition of Ampere in SI units effective from 20-5-2019 can be searched at (http://physics.nist.gov).

7.10 Force and Torque on a Rectangular Current Loop in Uniform Magnetic Field

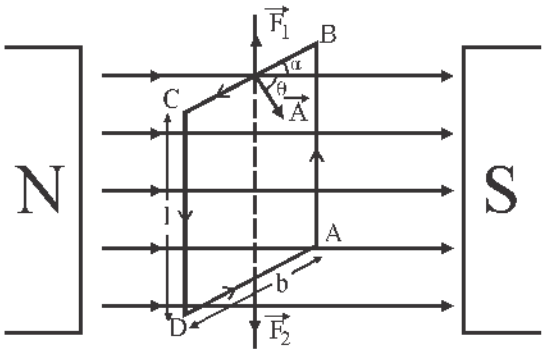


Fig 7.30 Torque on a current loop in magnetic field

Consider rectangular current loop ABCD of length l and breadth b and area A. The current in the loop is I. It is placed in a uniform magnetic field B. To find net force on the coil, we consider the force on each side of rectangle and just sum up. At any instant if the area vector $\vec{A}$ makes an angle θ with $\vec{B}$, the force on side BC is $\vec{F}_1 = I\left(\vec{b}\times\vec{B}\right)$, the direction is upward in the plane of the paper. Similarly the force $\vec{F}_2$ on DA is $\vec{F}_2 = I\left(\vec{b}\times\vec{B}\right)$, the direction is downward in the plane of the paper. They are equal, opposite and collinear hence get cancelled. The force on side CD and AB is $\left|\vec{F}_3\right| = I\ell B\sin 90^\circ = I\ell B$, again their sum is zero. But they are not colinear, hence they produce a torque on the coil. Net force on the loop is $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$

Calculation of Torque

As shown in the figure 7.31, the forces $\vec{F}_3$ and $\vec{F}_4$ are equal and opposite. They act on two different points, hence produce a torque $\vec{\tau}$.

$\tau =$ Force x perpendicular distance between forces

$$\tau = \left(I\ell B\right)\left(b\sin\theta\right)$$

$$= I\left(\ell b\right)B\sin\theta$$

$$\tau = IAB\sin\theta \qquad \ldots(7.73)$$

If the loop has number of N turns

$$\tau = NIAB\sin\theta \qquad \ldots(7.74)$$

$$\tau = MB\sin\theta \qquad \ldots(7.75)$$

Here $\vec{M} = NI\vec{A}$ $\qquad \ldots(7.76)$

is the magnetic moment of the current loop.

In vector form $\vec{\tau} = \vec{M}\times\vec{B}$ $\qquad \ldots(7.77)$

or $\vec{\tau} = NI\vec{A}\times\vec{B}$ $\qquad \ldots(7.78)$

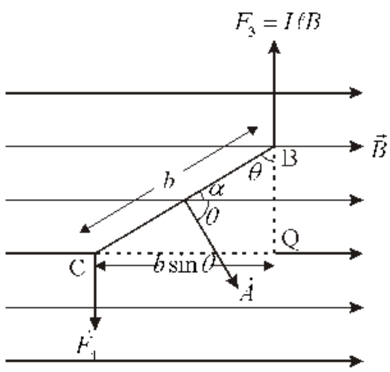


Fig 7.31 Torque on a current loop

Note - When we consider the angle between the plane of the coil and as the equation (7.75) will be

The direction of $\vec{\tau}$ is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ as per right hand screw rule.

Comparing $\vec{\tau}$ on an electric dipole in uniform electric field, we see that it is $\left(\tau = PE\sin\theta\right)$

Special Conditions

(i) When the plane of the coil is perpendicular to $\vec{B}$, i.e. $\theta = 0^\circ$, 180° and $\alpha = 90^\circ$ from eq. 7.75, the

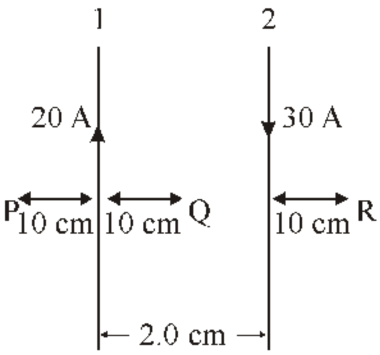
$\tau = \tau_{\min} = MB\sin\theta = 0 =$ (minimum).

(ii) When the plane of the coil is parallel to i.e $\theta = 90^\circ$; $\alpha = 0^\circ$ the torque will be maximum

$$\tau = \tau_{\max} = MB\sin 90^\circ = MB \qquad \ldots (7.79)$$

Electric motor and moving coil meters work on this principle.

Example 7.13 : Find the magnetic field $\vec{B}$ at points P, Q and R, due to two parallel currents as given in diagram.



Solution : Field $\vec{B}$ due to a straight current I

d at distance is given by $B = \dfrac{\mu_0 I}{2\pi d}$

A point P the net $B_P = B_{P_1} \sim B_{P_2}$

$$= \frac{\mu_0(20)}{2\pi(0.1)} \sim \frac{\mu_0(30)}{2\pi(0.3)}$$

$$= \frac{\mu_0}{2\pi}[200-100] = \frac{4\pi\times10^{-7}}{2\pi}\times100$$

$$= 2\times10^{-5}T$$

of B_P is given by as perpendicular to page upwards.

(ii) At point Q; net $\vec{B_Q} = \vec{B_{Q_1}} + \vec{B_{Q_2}}$ since both are in same direction.

$$B_Q = \frac{\mu_0(20)}{2\pi(0.1)} + \frac{\mu_0(30)}{2\pi(0.1)}$$

$$= \frac{\mu_0}{2\pi}[200+300]$$

$$= 2\times10^{-7}\times500$$

$$= 10\times10^{-5}T = 10^{-4}T$$

given by i.e. to page downwards.

(iii) Similarly on point R. The net $\overrightarrow{B_R} = \overrightarrow{B_{R_1}} + \overrightarrow{B_{R_2}}$

$$B_R = B_{R_1} \sim B_{R_2} = \frac{\mu_0}{2\pi}\left[\frac{I_1}{d_1} \sim \frac{I_2}{d_2}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{30}{0.1} - \frac{20}{0.3}\right]$$

$$= 2\times10^{7}\times2.33\times10^{2}$$

$$= 4.66\times10^{5}\ \mathrm{T}$$

The direction will to page upwards.

Example 7.14 : A 10m wire carries a current of 10A. It is placed at , with B. Find force per unit on the wire, if B = $5.5\times10^{-4}\,T$.

Solution : The force on whole wire is given by

$$\vec{F} = I\left(\vec{\ell}\times\vec{B}\right)$$

The for per unit length is;

$$\left|\vec{F}\right| = I\ell B\sin\theta$$

$$I = 10\,A,\; B = 5.0\times10^{-4}\,T$$

$$\ell = 10\,\text{m} \text{ तथा } \theta = 30^\circ$$

$$\left|\vec{F}\right| = 10\times10\times5\times10^{4}\times\sin30^\circ$$

$$= 10\times10\times5\times10^{-4}\times\frac{1}{2}N$$

$$= 250\times10^{4}\,N$$

$$\frac{\left|\vec{F}\right|}{\ell} = \frac{250\times10^{4}}{10} = 25\times10^{-4} = 0.025\ \text{N/m}$$

7.11 Galvanometer

In previous chapters, we have studied about the physical quantities like electric current and potential. In this section we will study about the devices that measure these quantities.

Galvanometer is a device used to detect current in a circuit or potential difference between two points. It can be converted into a voltmeter and Ammeter. It is based on the principle of torque on a current in magnetic field. They are of two types - (i) Moving coil (ii) Moving magnet type.

In this section we will study only moving coil type galvanometer, which are again of two types -

(i) Suspended coil glavanometer (ii) Pivoted coil galvanometer. Both types are based on same principle, but differ in their construction and working.

7.11.1.1 Suspended Coil Galvanometer

As shown in the diagram 7.32, a rectangular or circular coil of insulated copper wire wounded over an alluminium frame, is suspended by a phospher bronze fiber between the poles of a horse shoe magnet. The alluminium frame of the coil is free to rotate about a fixed iron core. One end of the coil is connected to terminal T_1 via phospher bronze fiber and the other to terminal T_2 via an elastic spring.

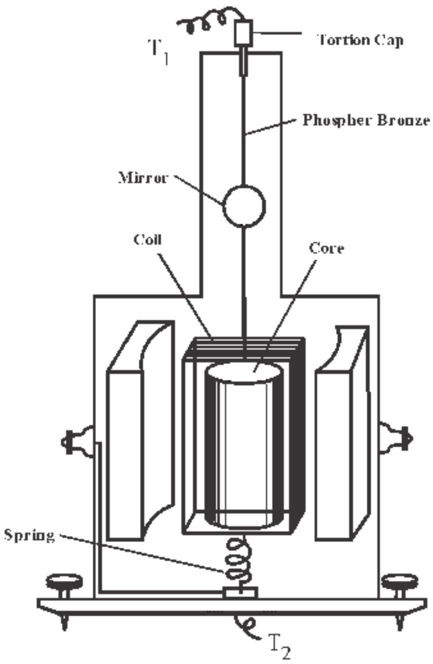


Fig 7.32 Suspended coil galvanometer

Principle - It is based on te principle of, torque on a current loop in magnetic field. This torque deflect (rotate) the coil, which is measured on a suitable scale.

A coil of N turns and area A, having current I experiences a torque $\tau_{def} = NAIB\sin\theta$ in magnetic field B. To aviod the monlinearity of, the poles of the magnet are made concave in shape, so that the field B is always radial and $\theta = 90^\circ$. So $\tau_{def} = NAIB$, which has a linear relation with I.

The role of phospher bronze fiber is to produce a counter torque (Restoring Torque) due to its tortion; so that the coil comes in equilibrium after a rotation of θ . θ is measured by lamp and scale arrangement, for that, a small mirror is attached (fixed) to phaspher bronze fiber.

If the restoring torque per degree due to phospher bronze fiber is C, then the restoring torque for deflection ϕ will be $\tau_R = C\phi$ . . . (7.80)

In equilibrium $\tau_{def} = \tau_R$

or $NAIB = C\phi$

$$I = \left(\frac{C}{NAB}\right)\phi \quad \ldots (7.81)$$

$$I = k\phi \quad \ldots (7.82)$$

here $k = \dfrac{C}{NAB}$ is a constant called reduction factor of the galvanometer. Hence $I \propto \phi$

7.11.1.2 Radial Field and Role of Iron Core

As shown in the fig 7.33, when the poles of the magnet are made concave in shape, then magnetic field is always radical and perpendicular to area vector $\vec{A}$ of the coil, the torque is maximum. $\tau_{def} = NAIB$ The soft iron core intensifies the effective magnetic field B, so that k is reduced and sensitivity of the galvanometer is increased.

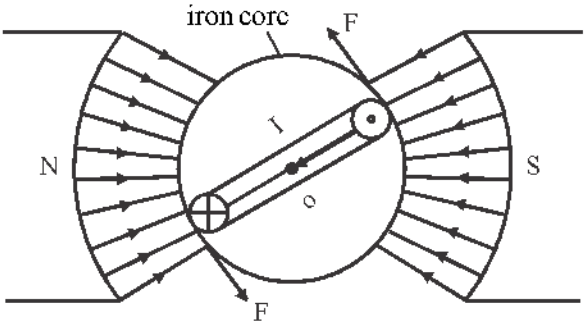


Fig 7.33 Radical magnetic field

7.11.1.3 Working

In this device the deflection is measured using lamp and scale arrangement. If the perpendicular distance of mirror from scale is D, and deflection of light spot on scale is d. $\tan(2\phi) = \frac{d}{D}$ and for small deflection (tan x ≃ x), $2\phi = \frac{d}{D}$; $\phi = \frac{d}{2D}$. (Here we have taken 2ϕ, because deflection of mirror by ϕ, deflects the reflected ray by angle 2ϕ) and $I \propto d \propto \phi$

7.11.1.4 Sensititvity of Galvanometer

If a small current causes large deflection in galvanometer it is said to be more sensitive. The sensitivity is inverse of figure of merit/reduction factor

From equ. 7.81 $I = \left(\frac{C}{NAB}\right)\phi = k\phi$

Hence, the current sensitivity

$$S_i = \frac{\phi}{I} = \frac{NAB}{C} = \frac{1}{k} \quad \ldots (7.84)$$

To increases sensitivity we can increase N, B and A to certain practical limit. C can be decreased by taking the fiber of phospher bronze as long and thin. Again to practical limit for ruggedness of galvanometer.

Voltage Sensitvity - It is defined as deflection per volt in galvanometer $V_s = \frac{\phi}{V}$

If the resistance of galvanometer coil is G, and current is I ; $V = IG$ and

$$V_s = \frac{\phi}{IG}$$

$$V_s = \frac{NAB}{CG} \quad \ldots (7.86)$$

From equation 7.84 and 7.86

$$V_s = \frac{S_i}{G} \quad \ldots (7.87)$$

7.11.2 Figure of Merit of Galvanometer

It is defined as the current required for unit deflection in galvanometer. Hence it is inverse of current sensitivity

$$X = \frac{1}{S_i} = \frac{I}{\phi}$$

$$X = \frac{C}{NAB} = k \quad \ldots (7.88)$$

7.11.3 Pivoted Coil Galvanometer

All the arrangements of coil, frame, iron core and concave shape of magnetic poles is same as that in suspended coil galvanometer, except that its coil is pivoted on two sharp point (instead of suspension) so that it is free to rotate. Coil is connected to terminals T_1 and T_2 via two hair springs, which provide restoring torque for equlibrium. A very light indicator needle of aluminimum is attached to the coil. The other end of the indicator needle gives deflection on a graduated dial. Such galvanolmeter is also called weston galvanometer. In spite of its less sensitivity compared to suspended coil galvanometer, it is most used because of convinience in use.

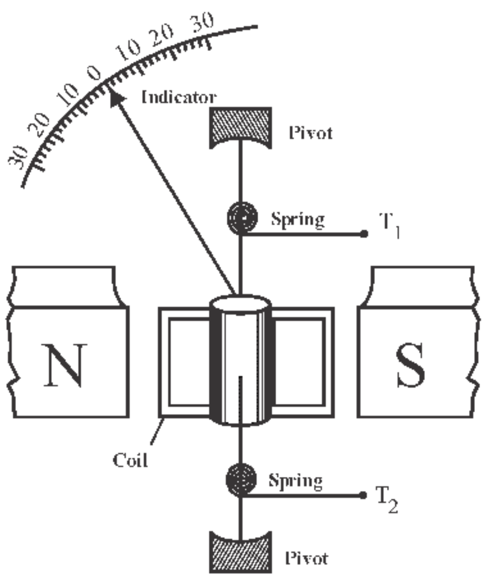


Fig 7.34 Pivoted coil galvanometer

Use of Shunt

Galvanometer gets damaged, due to excessive current. A copper wire (shunt) is connected between the terminals T_1 and T_2 which by passes the extra current and protect the galvanometer from damage. Fig 7.35 shows a shunted galvanometer.

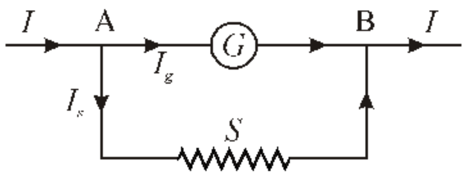


Fig 7.35 Shunted galvanometer

If G and R are resistances of galvanometer and shunt respectively and I, I_g and I_s are the main current, current in galvanometer and current in shunt. Then

$I_g G = I_s S$ (potential between A & B is same for both)

$$\frac{I_s}{I_g} = \frac{G}{S}$$

$$\frac{I_s}{I_g} + 1 = \frac{G}{S} + 1$$

$$\frac{I_g + I_s}{I_g} = \frac{G+S}{S}$$

$$\frac{I}{I_g} = \frac{G+S}{S} \qquad (\because\ I = I_g + I_S)$$

which gives $$I_g = \left(\frac{S}{G+S}\right) I \qquad \ldots (7.89)$$

Galvanometers of required range are fabricated using above relation.

7.11.4 Ammeter

An ammeter is a current measuring device, so it is always connected in series in the circuit. Resistance of an ideal ammeter should be zero, so that it does not effect the current in the circuit, (to be measured). But a practical ammeter has certain non-zero resistance (i.e. resistance of coil).

To make the effective resistance of the galvanometer as low as possible, a very low resistance is connected between the terminals T_1 and T_2 of the galvanometer it is called shunt. The value of shunt resistance is determined as per requirement of the range.

As shown in the fig 7.36, a shunt is connected between its terminals T_1 and T_2 whose value is determined by the relation $I_g G = (I - I_g) S$

$$S = \frac{I_g G}{(I - I_g)} \qquad \ldots (7.90)$$

Here I = range of a ammeter; Ig = current for full scale deflection.

$$I_g = \left(\frac{I - I_S}{G}\right) S \qquad \ldots (7.91)$$

The effective resistance of the ammeter is given by

$$R_A = \frac{GS}{G+S} \qquad \ldots (7.92) \text{ (law of parallel combination)}$$

Since $S << G$

$$R_A = \frac{GS}{G} \approx S \qquad \ldots (7.93)$$

The converted ammeter is first calibrated. It is zero is marked to extreme left on the dial.

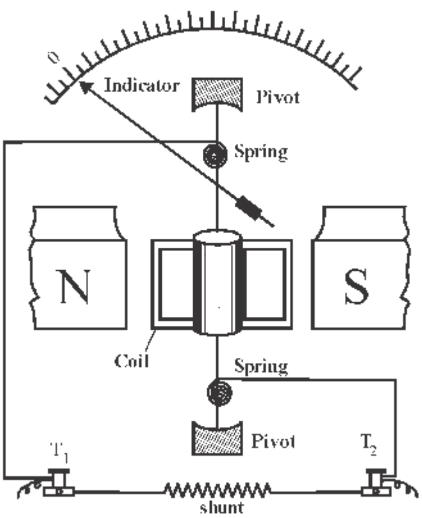


Fig 7.36 Conversion of galvanometer into ammeter

7.11.5 Voltmeter

It is a device to measure potential differnce between two points. A volt meter is always connected in parallel to the points. A voltmeter should not draw any current for it self (not to change the potential difference to be measure).

An ideal voltmeter should have infinite resistance. But an infinite resistance give I=0, and coil of voltmeter will not rotate. So for a particle moving coil galvanometer, it should as high resistance as possible (as per its range).

To convert a galvanometer to a voltmeter a very high resistance is connected in series with galvanometer.

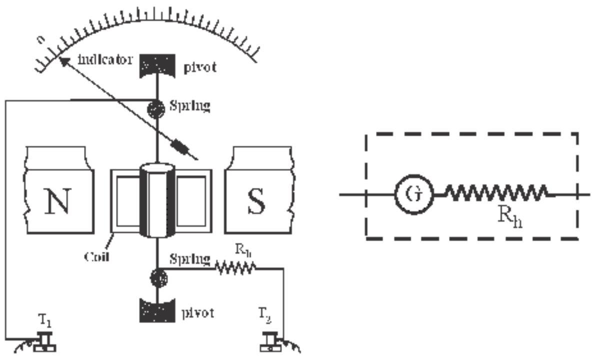


Fig 7.37 Conscrsion of a galvanometer to a voltmeter

As per diagram the potential across the series combinaion of G and R_H (applied potential) is -

$$V = I_g\left(R_H + G\right)$$

$$R_H + G = \frac{V}{I_g}$$

$$R_H = \frac{V}{I_g} - G \qquad \ldots(7.94)$$

$$R_V = G + R_H \qquad \ldots(7.95)$$

$$R_H >> G$$

Since $R_H >> G$. The effective resistance of the combination $R_V \approx R_H$.

Here V = range of voltmeter; G = resistance of galvanometer coil

and Ig = Current for full scale deflection of galvanometer

The practical voltmeter discussed above is unable to meaure potential difference very accuratly. For this we use potentiometer.

Example 7.15 : A deflection for certain current is 50. When it is short circuited by a reistance of $12\,\Omega$, the deflection reduces to 10. Find resistance of galvanometer.

Solution : For shunted galvanometer $I \propto \phi$

$$\frac{I_g}{I} = \frac{10}{50} = \frac{1}{5}$$

$$I_g = \frac{I}{5}$$

$$(I - I_g)S = I_g G$$

$$\left(I - I/5\right) \times 12 = \left(I/5\right)G$$

$$G = 4 \times 12 = 48\,\Omega$$

Example 7.16 : For a galvanometer the current for full scale deflection is 5mA, and resistance of its coil is $99\,\Omega$. Find the value of required resistance to convert it to (i) Ammeter of range 5 A. (ii) Volt meter of range 5V.

Solution : Given $I_g = 5\,mA$, $G = 99\,\Omega$, $I = 5\,A$

(i) convert to an ammeter the value of shunt

$$S = \frac{I_g G}{I - I_g} = \frac{5\times10^{-3}}{5 - 0.05}\times 99$$

$$= \frac{5\times10^{-3}}{4.95}\times 99 = 0.1\,\Omega$$

hence a resistance of $0.1\,\Omega$ is to be connected in parallel to galvanometer.

(ii) To convert to a volt meter

$$R_H = \frac{V}{I_g} - G$$

$$R_H = \frac{5}{5\times10^{-3}} - 99 = 1000 - 99$$

$$R_H = 901\,\Omega$$

be connected in series with galvanometer.

7.12 Ampere's Circuital Law

Just as Gauss's law in electrostatics help in tackling the problems regarding $\vec{E}$ due to a symetrical charge distributions, where Coulomb's law can't; there is a law called Ampere's law, which tackles problems regarding $\vec{B}$ due to symetric currents where Biot and Savarts law can't.

According to this law;

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma I \qquad \ldots(7.96)$$

It states that the line integral of magnetic field produced by electric currents in air/vacuum, over a closed path (loop) enclosing an area, is equal to the product of μ_0 and alzebric sum of the currents passing through that area. $\oint \vec{B}\cdot d\vec{\ell}$ is also called circulation of magnetic field.

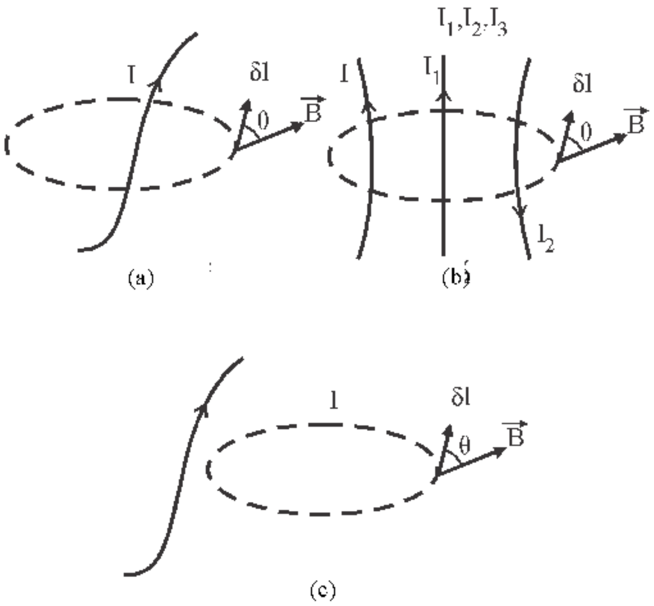


Fig 7.38 Ampere's Law

Here alzebric sum of currents $\Sigma i = I$, means I can be taken as +*ve* ore -*ve*; according to one convention. If integration is taken in anticlock-wise direction, the upward currents are taken as +*ve*; and vice-versa. For fig (a) $\Sigma i = I$; for fig (b) $\Sigma i = I_1 + I_2 + (-I_3)$; for fig (c) $\Sigma i = 0$; For $\oint \vec{B}\cdot d\vec{\ell} = 0$; it does not mean that there is no magnetic field it states that the loop choosen, contains no currents.

$B = \mu_0 H$ here H is magnetizing field or magnetic intensity.

The ampere's law in terms H is

$\oint \vec{H}\cdot d\vec{\ell} = \Sigma i$ and i.e. circulation of H is called mmf (magnetio motive force).

Ampere's law is same as Biot and Savarts law and both can be deduced from each other. While tackling problems, the Ampere's loop is selected in such a way that -

(i) B is same on each part of the loop.

(ii) $\vec{B}$ and $d\vec{\ell}$ should be parallel so that $\vec{B}\cdot d\vec{\ell} = Bd\ell$

(iii) $\vec{B}$ and $d\vec{\ell}$ should be so that $\vec{B}\cdot d\vec{\ell} = 0$

7.12.1 Magnetic Field Due to Infinity Long Current

The magnetic field produced by current I in conductor CD is in the form of concentric loops around the conductor, the conductor is the center of these loops. The loops are all along the length of the conductor.

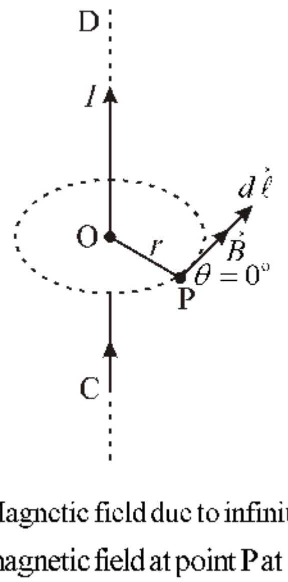


Fig 7.39 Magnetic field due to infinitly long wire

To find magnetic field at point P at a distance r from the conductor, we construct a circular Apmere loop taking wire as center. Now take a small element $d\vec{\ell}$ and find $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$ along the loop. From Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \sum i$$

$$\oint B d\ell \cos\theta = \mu_0 \sum i$$

$$\theta = 0^\circ,\ \cos\theta = 1$$

$$\sum i = I$$

$$\oint B d\ell = \mu_0 I$$

$$B\oint d\ell = \mu_0 I$$

$$\oint d\ell = \ell = 2\pi r$$

$$B(2\pi r) = \mu_0 I$$

hence $B = \dfrac{\mu_0 I}{2\pi r}$ . . . (7.98)

The result is same as that obtained earlier by using Biot and Savart's law and long mathematical process. The magnetic field in this case is $B \propto I$ and $B \propto 1/r$.

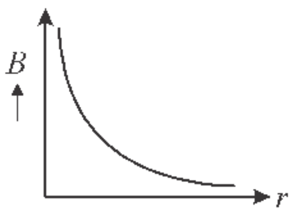


Fig 7.40 Variation of B with distancer

7.12.2 Magnetic field due to current carrying long cylendical conductor

The magnetic field due to the solid cylinder in which current I is uniformly distributed through whole cross section A, will be in the from of concentric circles, as the cylinder at the center. The loops of magnetic field lines will be there thoughout the whole length of cylinder -

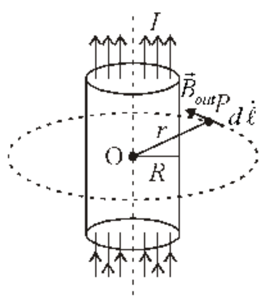


Fig 7.41 B due to cylindrical conductor

(i) To find magnetic field B at a point P, at a distance *r*, we construct a circular Ampere's loop of radius *r*, passing through point P. From Ampere's law

$$\oint \vec{B}_{out}\cdot d\vec{\ell} = \mu_0 \sum i$$

$$\oint B_{out} d\ell \cos\theta = \mu_0 \sum I$$

$$\theta = 0^\circ, \cos\theta = 1\ ;\ \sum i = I$$

$$\oint B d\ell = \mu_0 I$$

$$\oint d\ell = (2\pi r)$$

$$B_{out}\cdot 2\pi r = \mu_0 I$$

$B_{out} = \dfrac{\mu_0 I}{2\pi r}$ . . . (7.99)

Here again $B_{out} \propto \dfrac{1}{r}$ as expected.

(ii) To find $\vec{B}$ at the surface of the consuctor; $r = R$. by same consideration we obtain

$B_S = \dfrac{\mu_0 I}{2\pi R}$ . . . (7.100)

(iii) To find $\vec{B}$ at a point inside the cylinder; i.e. r < R; The same procdure is again adopted but right hand side of the Ampere's law should be taken care of. From Ampere's law

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \sum I$$

$$\oint B_{in} d\ell \cos\theta = \mu_0 \sum i$$

$$\theta = 0^\circ,\quad \cos\theta = 1$$

We consider an Ampers loop passing through that point, inside the cylinder $\sum i$ is the total currenr passing though the area of Ampers loop. From unitary method -

$$\sum i = \frac{I}{\pi R^2}\cdot \pi r^2$$

$$\sum i = \frac{I r^2}{R^2}$$

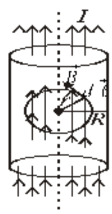


Fig. 7.42

$$\oint B_{in} d\ell = \frac{\mu_0 I r^2}{R^2}$$

$$B_{in}(2\pi r) = \frac{\mu_0 I r^2}{R^2}$$

$$B_{in} = \frac{\mu_0 I r}{2\pi R^2}$$

$$B_{in} = \left(\frac{\mu_0 I}{2\pi R}\right)\frac{r}{R}$$

hence $B_{in} = B_s \frac{r}{R}$ . . . (7.101)

$B_{in} \propto r$ and at center (undefined)

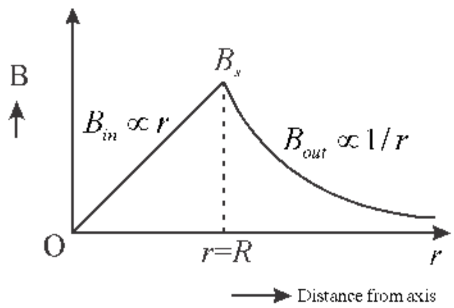


Fig 7.43 Variation of B for cylinder with r

(iv) If the conductor is a pipe (hollow cylinder) the field inside at all points is zero.

Example 7.17 : The magnetic field due to an infinitly long current carrying conductor at distance 10 cm is 10^{-5} T. Find the current in the conductor.

Solution : For infinitly long conductor

$r = 10$ cm $= 0.1$ m

$B = 10^{-5}$ T

$$B = \frac{\mu_0 I}{2\pi r}$$

$$10^{-5} = \frac{2\times10^{-7}\times I}{0.1}$$

$$I = \frac{10^{-5}\times0.1}{2\times10^{-7}} = 0.5\times10 = 5\text{ A}$$

7.13 Solenoid

A tightly wounded coil of insulated copper wire over a insulator pipe, where the turns are very close to each other is called solenoid.

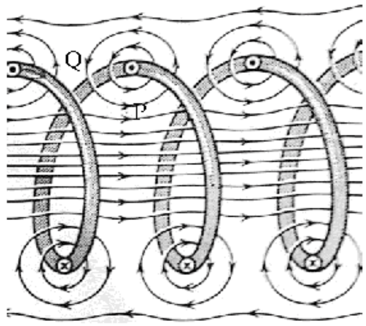


Fig 7.44 Magnetic field due to two loops

The plane of each circular loop of wire may be considered perpendicular to the axis of solenoid. To know about the magnetic field produced by solenoid we consider two current loops (1) and (2) and two points P and Q inside and outside solenoid. Fig 7.44. The constribution of both current loops at point P (inside) is in same direction, where as it is in opposite direction at Q (point outside). Super position of magnetic field due to all the loop gives B = 0 outside the solenoid; and B_{in} as strong and uniform, inside the solenoid for an ideal solenoid.

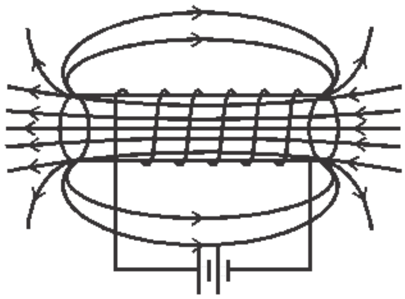


Fig 7.45 B due to loosely wounded solenoid

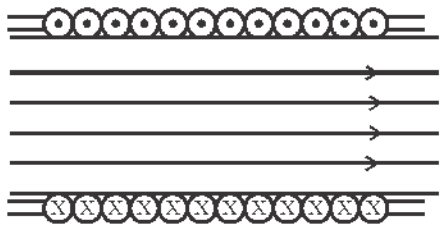


Fig 7.46 Field due to an ideal solenoid

7.13.1 Magnetic Field Inside an Infinitly Long Solenoid

To determine the field inside a soelnoid the field, we take a longitudial section (LS) of the solenoid, and show the direction of current in the loops as (x) (.) and the direction of magnetic field produced, as in fig 7.47. The current in the solenoid is I, and same in all loops. Asuming the solenoid as ideal one, we mark (show) field inside as B, nd out side as B = 0.

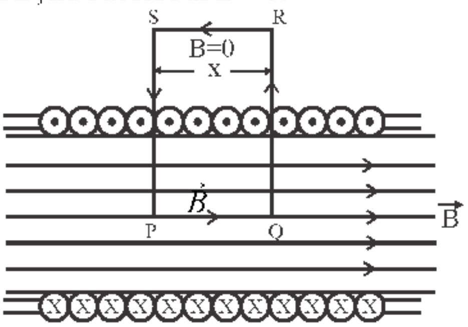


Fig 7.47 Magnetic field inside solenoid

To find B inside we construct an Amperian loop PQRS which enclose length x of th solenoid.

From Ampere's law $\oint_{PQRS} \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma i$

(= all currents enclosed by loop PQRS)

The defined quanity for solenoid is n, the turn density. (Not the total number of turns) hence the number of turns enclosed by Ampere loop PQRS is nx, hence

Again we take

$$\oint_{PQRS} \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma i$$

$$\int_P^Q \vec{B} \cdot d\vec{\ell} + \int_Q^R \vec{B} \cdot d\vec{\ell} + \int_R^S \vec{B} \cdot d\vec{\ell} + \int_S^P \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma i$$

$\int_P^Q \vec{B} \cdot d\vec{\ell} = \int_P^Q B d\ell$; (the contribution of remaining integrals it zero, due to either B=0 or $\vec{B} \cdot d\vec{\ell} = 0$)

we get, $B\int_P^Q d\ell + 0 + 0 + 0 = \mu_0 \Sigma i$ . . . (7.102)

since $\int_P^Q d\ell = x$

and $\Sigma i = nxI$

we get $Bx = \mu_0 nxI$

or $B = \mu_0 nI$ . . . (7.103)

which is uniform throught the space inside solenoid. It the solenoid is finite but its radius R << L, the result is valid at a point inside and far away from ends.

$$n = \frac{N}{L}$$

Also $B = \mu_0 \frac{N}{L} I = \mu_0 nI$ . . . (7.104)

$$H = \frac{B}{\mu_0}$$

$H = \frac{N}{L} I = nI$ . . . (7.105)

If a medium is placed inside the solenoid.

$$B_m = \mu \frac{N}{L} I$$

$B_m = \mu_r \mu_0 \frac{N}{L} I$ . . . (7.106)

where μ_r is relative magnetic permeability of the medium. For a short solenoid B at a point P is

$$B = \frac{1}{2} \mu_0 nI \left(\cos\phi_1 - \cos\phi_2\right)$$

where ϕ_1 and ϕ_2 are the angle between the axis and the line joining the point P to outer rims. Magnetic field at one end of the solenoid -

for this and

$$(\phi_1 = 90^\circ, \phi_2 = 180^\circ)$$

hence $B_{end} = \frac{1}{2} \mu_0 nI \left[\cos 90^\circ - \cos 180^\circ\right]$

$B_{end} = \frac{\mu_0 nI}{2}\left[0 - (-1)\right]$ hence $B_{end} = \frac{1}{2} \mu_0 nI$

which is half the magnetic field inside the solenoid.

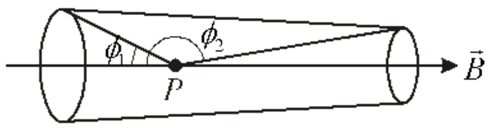


Fig 7.48 Magnetic field at the end of a solenoid

7.14 Behavioral Comparison of Bar Magnet and Current Solenoid

1. A freely suspended current carrying solenoid always stay in North-South direction. Just like a bar magnet.
2. If the ends of two current carrying solenoids are brought close to each other, they will attract or repel each other, just like magnets do.
3. Each end of solenoid behave like a North or South pole, depending upon the direction of current.
4. The ends of solenoid attracts feromagnetic materials, just like magnets do.
5. Fig 7.50 shows the relation of polarity with direction of current.

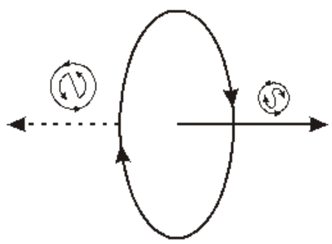


Fig 7.50 Magnetic behaviour of a current loop

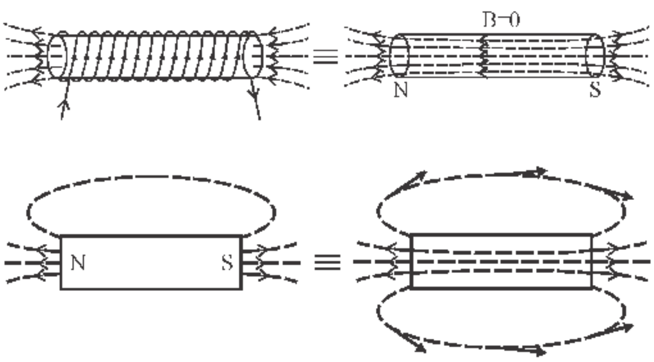


Fig 7.51 Comparison of solenoid with bar magnet

We conclude that a current carrying solenoid behave like a bar magnet. But there are some dissimilarties too -

(a) The field lines in solenoid are straight, but in a magnet some what curved.

(b) The magnetic field outside a solenoid is approximalty zero, but in case of a magnet we get magnetic field, but different at different points.

7.15 Magnetic Field on the Axis of a Toroid

If a solenoid is circularly curved and its both ends are joined it becomes Totoid. It behave like infinitly long solenoid.

It can also be fabricated by wounding insulated copper wire over a ring.

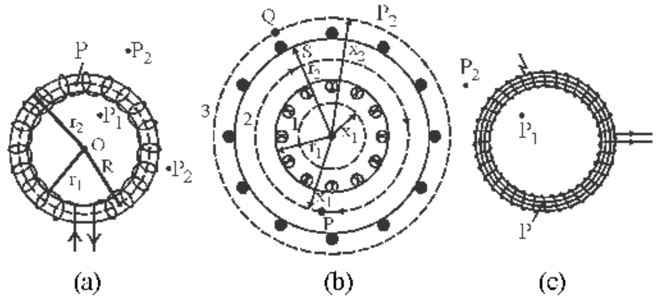


Fig 7.52 Toroid

N turns are uniformly wounded. Due to current I in each loop, they produce equal magnetic field at their centers; the contribution of all loops is in the form of concentric circles; whose center is the center of toroid. Hence the magnetic field inside (i.e. on axis) the solenoid is parallel to its axis. Tangent at a point on field line, gives the direction of magnetic field at that point.

To find magnetic field at the point on axis, we take the cross section of Toroid along/parallel to its plane. Show the direction of current as (x) and ( ) and also direction of $\vec{B}$ produced accordingly.

Now construct a circular Ampere's loop passing through the point of interest. We see that this Ampere's loop encloses all the turns and

From Ampere's law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \sum i$

$$\oint B \cdot d\ell \cos\theta = \mu_0 \sum i$$

$$\theta = 0^\circ, \quad \cos\theta = 1$$

$$\sum i = NI$$

$$\oint B d\ell = \mu_0 NI$$

$$B(2\pi r) = \mu_0 NI$$

$$B = \mu_0 \frac{N}{2\pi r} I$$

$$B = \mu_0 nI \qquad \ldots (7.107)$$

$$n = \frac{N}{2\pi r}$$, is the turn density

$$H = \frac{B}{\mu_0} \Rightarrow H = nI = \frac{N}{2\pi r} I \qquad \ldots (7.108)$$

If a emdium is present

$$B_m = \mu \frac{N}{2\pi r} I$$

or $$B_m = \mu_0 \mu_r \frac{N}{2\pi r} I \qquad \ldots (7.109)$$

Magnetic field out side Toroid is zero. Magnetic field depends on radius, current and medium inside the toroid.

Example 7.18 : A solenoid of length 0.5 m and radius 1 cm has 500 turns. It carries a current of 5A. Find magnetic field inside the toroid.

Soltuion : Given $L = 0.5m$, $r = 1cm = 0.01m$,

$N = 500$; $I = 5\,A$

$$A = \pi r^2 = 3.14 \times \left(10^{-2}\right)^2 = 3.14 \times 10^{-4} m^2$$

$L = 0.5m$. Clearly $A << L$

Where, A=Area of cross section

$$B = \mu_0 \frac{N}{L} I$$

Clearly A<<L, hence the coil can be asumed to be very long (infinitly long)

$$B = 4\pi \times 10^{\ 7} \times \frac{500}{0.5} \times 5$$

$$B = 6.28 \times 10^{-3}\, T$$

Example 7.19 : Mean radius of a toroid is 10 cm and number of turns is 500. Find the magnetic field if the current in it is 0.1 A. $\left(\mu_0 = 4\pi \times 10^{-4} Wb/Am\right)$

Solution : Magnetic field inside toroid is

$$r = 10cm = 0.1m$$

$$N = 500 \qquad B = \mu_0 \frac{N}{2\pi r} I$$

$$B = 4\pi \times 10^{\ 7} \times \frac{500 \times 0.1}{2\pi \times 0.1}$$

$$B = 10^{-4}\, Wb/m^2$$

Important Points

1. Orested found in his experiment that a current in a conductor produces magnetic field around it. This phenomenon is called magnetic effect of electric current. If a moving charge experiences a force (neglecting electric and gravitational field), there exists a magnetic at that point.
2. SI unit of magnetic field is T or Wb / m^2 or N / A.m. The force on a moving charge in magnetic field is

   $$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

   $$\left|\vec{F}\right| = qvB\sin\theta$$

   whose direction is perpendicular to the plane of $\vec{v}$ and $\vec{B}$.
3. Stationary charge produces electric field only whereas a moving charge produces, both electric and magnetic field.
4. From Biot and Savarts law, the magnetic field due a current elements Idl -

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\delta\ell\sin\theta}{r^2}\ ,\ \delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(d\vec{\ell}\times\hat{r}\right)}{r^3}$$

5. The magnetic field due a conductor of finite length $B = \frac{\mu_0 I}{4\pi d}\left[\sin\phi_1 + \sin\phi_2\right]$

magnetic field due to straight infinite current

$$B = \frac{\mu_0 I}{2\pi d}$$

6. Magnetic field due to circular current loop at

(i) Its center is $B_C = \frac{\mu_0 NI}{2R}$

(ii) On a point at distance x from its center -

$$B_{axial} = \frac{\mu_0 NIR^2}{2\left(R^2 + x^2\right)^{3/2}}$$

(iii) If R << x; $B = \frac{\mu_0 NIR^2}{2x^3} = \frac{\mu_0}{4\pi}\left(\frac{2\pi NIR^2}{x^3}\right)$

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$$

Here $M = NI\left(\pi R^2\right)$ is the magnetic moment of the coil, it shows that coil behaves like a bae magnet.

7. Helmholtz coils, has two idendical coaxial coils whose centers are at a distance equal to their radius. Magnetic field in the region, between the coils is

$B_{(\text{middle region})} = 1.432\ B_{(\text{center})}$

8. Force on a charge moving perpendicaulr to magnetic field is $F = qvB$, and path of the particle will be circular and radius of path is $\left(r = \frac{mv}{qB}\right)$, period of revolution $T = \frac{2\pi m}{qB}$ and freqeuncy $v = \frac{qB}{2\pi m}$

This freqeuncy is called cyclotron frequncy. The frequecny does not depends on velocity and radius of path. This concpet is used in cyclotron.

9. Cyclotron is a device, used to accelarate heavy positive particles. The frequency of applied AC is equal to cyclotron frequency.

10. Force on a current carrying conductor in magnetic field $\vec{F} = I\left(\vec{\ell}\times\vec{B}\right)$

$$\left|\vec{F}\right| = I\ell B\sin\theta$$

11. The force between two parallel currents is attractive if the current in them is parallel, and repulsive if the current is antiparallel. The force per unit length on such conductors is

$$\frac{F}{\delta \ell} = \frac{\mu_0 I_1 I_2}{2\pi d} \text{ N/m}$$

definition of 1 A is SI; 1 A is that current maintained in two parallel wires placed 1 meter apart in air/vacuum if a force of 2×10^{-7} N/m force act per unit length on each other, then current in both conductor is 1 Ampere.

12. A current loop in uniform magnetic field experiences no net force, but the torque on the loop is

$$\vec{\tau} = NI\vec{A} \times \vec{B}$$

$$|\vec{\tau}| = NIAB \sin\theta$$

Here N = number of turns, I = Current; A = Area, of current loop B = Magnetic field angle between $\vec{A}$ and $\vec{B}$ is θ

13. Galvanometer detects electric current. In moving coil galvanometer

(i) $I = k\phi = \frac{C}{NAB}\phi$

Here I = current, N = number of turns, A = area of coil, B = magnetic field, C = restoring torque per unit deflection, K = reduction factor of galvanometer.

(ii) Current sensitivity $S_i = \frac{\phi}{I} = \frac{NAB}{C} = \frac{1}{k}$

(iii) Figure of merit $X = \frac{1}{S_i} = \frac{I}{\phi} = \frac{C}{NAB} = k$

(iv) Voltage sensitivity $S_v = \frac{\phi}{V} = \frac{NAB}{CR} = \frac{S_i}{R}$

14. The value of shunt required to convert voltmeter into ammeter is $S = \left(\frac{I_g G}{I - I_g}\right)$

15. The value of reistance required for conversion of galvanometer to voltmeter is

$R_H = \frac{V}{I_g} - G$; Resistance of ideal voltmeter is infinite.

16. Ampere's circular law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i$

17. The magnetic field due to a cylendrical conductor -

(i) Outside cylender $B = \frac{\mu_0 I}{2\pi r}$; r = distance from axis.

(ii) On the surface $B = \frac{\mu_0 I}{2\pi R}$ R = radius of conductor

(iii) Inside the conductor $B_{in} = \frac{\mu_0 I}{2\pi R}\left(\frac{r}{R}\right)$; $B_{in} \propto r$

18. Infinitly long solenoid and Toroid are used to produce uniform magnetic field.

19. The magnetic field inside an ideal solenoid is $B = \mu_0 nI$ for a finite solenoid at one end $B = \frac{\mu_0 nI}{2}$.

20. Magnetic field due to current in a toroid.

(i) $B = \mu_0 nI = \frac{\mu_0 NI}{2\pi R}$; R = Radius; N = Number of turns

(ii) B = 0 magnetic field outside is zero.

Questions for Practice

Multiple Choice Questions -

1. A charge in uniform motion, produces -

(a) Only electric field

(b) Only magetic field

(c) Both electric and magnetic field

(d) EM waves along with electric & magnetic field

2. The magentic field due to a straight current at a distance r is B. If distance becomes $r/2$ keeping I constant, then magnetic field will be -

(a) 2 B (b) B / 2

(c) B (d) B / 4

3. The magnetic field at the center of a circular current carrying coil is B_O. The magnetic on the axis at a distance equal to radius of the same coil is B. The ratio B/B_0 will be -

(a) $1 : \sqrt{2}$ (b) $1 : 2\sqrt{2}$

(c) $2\sqrt{2} : 1$ (d) $\sqrt{2} : 1$

4. Helmholtz coils are used -

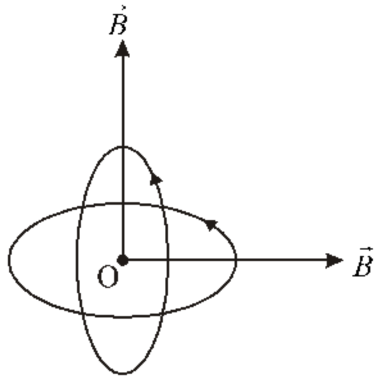
(a) To produce uniform magnetic field

(b) To measure electric current

(c) To measure magnetic field

(d) To find the direction of electric current

5. Two circular current coils are concentric and their planes are mutualy perpendicular and the magnetic field due to each coil is B, as hown in the figure: the net magnetic field at their common center will be -



(a) Zero (b) 2 B\

(c) $B/\sqrt{2}$ (d) $\sqrt{2}\,B$

6. Projected with same velocity in uniform and perpendicaulr magnetic field, which particle will experience maximum force?

(a) $_{-1}e^0$ (b) $_1H^1$

(c) $_2He^4$ (d) $_3Li^7$

7. Two wires of mains supply are at a them distance 12 cm. If the force per unit length between is equal to 4 mg weigth, the current in both the wires would be-

(a) Zero (b) 4.85 A

(c) 4.85 mA (d) 4.85×10^{-4} A

8. A proton of energy 100 eV is moving in circular path in perpendicular magnetic field of 10^{-4} T. The cyclotron frequency of the proton in Radian/s will be -

(a) 2.80×10^6 (b) 9.6×10^3

(c) 5.6×10^6 (d) 1.76×10^6

9. A galvanometer of resistance G, requires 2% of the main current as current for full scale deflection. The value of shunt will be -

(a) $\frac{G}{50}$ (b) $\frac{G}{49}$

(c) 49 G (d) 50 G

10. Magnetic field in a solenoid due to current I, is B. If the length and number of turns are doubled. To get the same magnetic field the current will be -

(a) 2 I (b) I

(c) $I/2$ (d) $I/4$

11. A toroid has a turn density n, the current is I. If the magnetic field inside (at axis) is B, the magnetic field out side will be -

(a) B (b) B / 2

(c) Zero (d) 2 B

12. A galvanometer is converted to voltmeter by connecting -

(a) High resistance in series

(b) Low resistance in series

(c) High resistance in parallel

(d) Low resistance in parallel

13. An ideal voltmeter and ideal ammeter should have -

(a) Zero and infinite resistance

(b) Infinite and zero resistance respectively

(c) Both should have zero resistance

(d) Both should have infinite resistance

Very Short Answer Questions

1. Write the name of sources, used to produce magnetic field.

2. Write the dimensions and unit of magnetic field.

3. Which field is produced by a moving charge.

4. A charge q enters a perpendicular to magnetic field B at velocity $\vec{v}$. What will be the force on the charge and path of the charge?

5. Define 1 Ampere in S.I. unit.

(Search latest definition at https://physics.nist.gov)

6. A proton is moving up word in vertical plane, it experiences a horizontal force in North direction. What will be the direction of magnetic field?

7. A charged particle is moving parallel to uniform magnetic field. What will be the path of particle?

8. A battery is connected to diametrically opposite points of circular coil. What will be the magnetic field at the center?

9. A coil of turns N and radius R is opened as a straight wire, how many times will be the magnetic field at a distance R, compared to the magnetic field at the center of the coil?

10. What will be the distance between, two points of inflations of a Helmholtz coil?

11. Write down the mathematical form of Ampere's circutal law.

12. Write down the value of magnetic field inside a copper pipe of radius R and current I.

13. Why the magnetic poles of moving coil galvanometer, made concave in shape?

14. How the current sensitivity of a galvanometer is increased?

15. At equilibrium what will be the position of coil and magnetic field in a galvanometer?

16. Why cyclotron is not used to accelarate light charged particles?

17. Which device you will use to produce uniform magnetic field?

18. How does the period of revolution of a charged particle depends on speed and radius of circular

path inside "Dee" of cyclotron.

19. Write the expression for the high resistance used to convert a galvanometer in to voltmeter.

Short Answer type question-

1. Write down the conclusions from Orested's experiment.

2. Write down, Biot and Savart's law in vector form.

3. Explain the two laws to find the direction of magnetic field produced by electric current.

4. A charge enters a uniform magnetic field at an angle ($0 < \theta < 90°$). What will be the path of the charge? Also find its pitch.

5. Find the relation between the magnetic field at R/2 on axis, and magnetic field at the center of coil. Here R is radius of the coil.

6. Show how a small current loop, behave like a bar magnet?

7. What is the circulation of magnetic field? Please explain.

8. What is the difference between a current carrying solenoid and a bar magnet?

9. Find the force per unit length on two parallel current carrying conductors.

10. Using Ampere's circutal law, find magnetic field inside a current carrying cylinder.

11. Show the period of half revolution of a positive charge in "Dee" of a cyclotron does not depend in the radius of circular path.

12. Explain the principle of cyclotron.

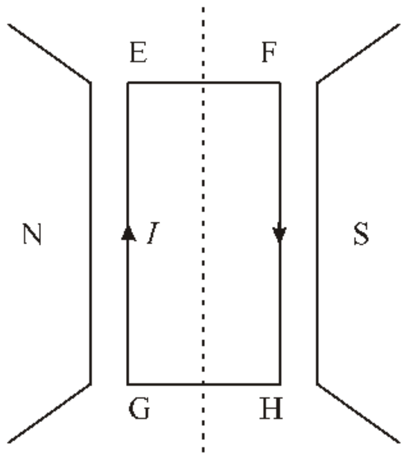
13. What is sensitivity and figure of merit of a galvanometer? How they are related to each other?

14. Find the expression for the resistance connected in parallel to convert a galvanometer to an ammeter.

15. A rectangular current loop EFGH is placed in uniform magnetic field, as shown in the figure.

(a) What will be the direction of torque on loop?

(b) When torque will be (i) maximum (ii) zero



Essay type Questions -

1. State Biot and Savant's law. Using this law find the expression for the magnetic field due to a finite straight current carrying conductor. Show that for infinitely long conductor, the field at a perpendicular distance d, is $B = \frac{\mu_0 I}{2\pi d}$

2. Using Biot and Savart's law, find the expression for magnetic field at an axial point of a circular current loop in vector form. Draw required diagram.

3. Describe the working of cyclotron. Draw the diagram showing path of particle in both "Dees". Derive expression for (i). Frequency of cyclotron (ii). Kinetic energy of ions in cyclotron.

4. Derive expression for force and torque on a current loop in uniform magnetic field. Draw required diagram. When the torque will be (i) maximum (ii) zero

5. Find the expression for force on a current carrying conductor in uniform magnetic field. Explain the right hand palm rule to explain the direction of force.

6. Write Ampere's circulate law. Find the expression for magnetic field in long current carrying solenoid.

Draw required diagram.

7. Describe construction of a toroid. Find expression for magnetic field at the axis of toroid of mean radius r number of turns N and current I. Show that the magnetic field out side, and in open area enclosed by toroid is zero.

8. What is a galvanometer? Describe the construction and working of a galvanometer using labbled diagram.

9. Describing the principle of a galvanometer, find the expression for its sensitivity and figure of merit. On what factors these depend.

Answer (Multiple Choice Questions)

(1) (C) 2 (A) 3 (B) 4 (A) 5 (D) 6 (D) 7 (B) 8 (B) 9 (B) 10 (B) 11 (C) 12 (A) 13 (B)

Very Short Answer Type -

1. Permanent magnet, current carrying conductor, moving charge, change in electric field.
2. $M^1L^0T^{-2}A^{-1}$ and Tesla.
3. Both, electric and magnetic field.
4. Force $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$; $\left|\vec{F}\right| = qvB\sin 90^\circ = qvB$ and the path will be circular.
5. If the force per unit length in air/vacuum, on two equal current carrying conductors palced at a distance 1m, is 2×10^{-7} N/m, then the current in conductor is 1 ampere.

   Search latest definition at (https//physics.nist.gov)
6. In horizontal plane towards East.
7. Rectilinear
8. Zero
9. A the center of coil $B_{centre} = \frac{\mu_0 NI}{2R}$ and due to straight wire, $B = \frac{\mu_0 I}{2\pi R}$ as required $\frac{B_{centre}}{B} = N\pi$
10. Equal to radius of coil R.
11. $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$
12. Zero
13. So that the field is radial and deflection on scale is linear.
14. By increasing number of turns, area of coil and taking soft iron core.
15. When the plane of the coil is perpendicular to B.
16. For a required energy more velocity, due to relativistic effect mass of the particle and resonance frequency changes.
17. Long solenoid.
18. Does not depend. It remains constant

    $$\left(\because T = \frac{2\pi m}{qB}\right)$$
19. $R = \frac{V}{I_g} - G$; here I_g = current for full scale deflection.

    G = Resistance of galvanometer; V = Range of voltmeter.

Numerical Problems

1. Find the magnetic field at the center of a circular coil of radius 8.0 cm and 100 turns, having a current 0.40 A.

   ($3.1\times10^{-4}\,T$)
2. A circular coil made from 6.28cm length of wire has a radius of 0.10cm and current 1.0 A. Find magent field at its center.

   ($6.28\times10^{-5}\,T$)
3. A long straight wire has a current 35 A. Find magnetic field at a distance of 10cm.

   ($3.5\times10^{-5}\,T$)
4. A wire having current of 10 A is on the plane of table. Another wire in which current is 6A is just over the wire AB, at a distance of 2 mm. find the mass per unit length of CD, such that it held there by magnetic force. What will be the direction of

current in CD, with respect to the current in AB?

(m/l $= 6 \times 10^{4}$ kg / m; opposite to AB)

5. A wire on horizontal plane has a current of 50A in south to North direction. Find the magnitude and direction of magnetic field at point 2.5m towards east.

($4 \times 10^{-6}\,T$ downwards)

6. Two long parallel wires having current I and 3I in same direction are 4m a part. Find the point where magnetic due to both is Zero.

(1 cm from I, between them)

7. A proton enters a perpendicular magnetic field of 0.2T with velocity 6.0×10^{5} m/sec. Find acceleration of proton and radius of its path.

(1.15×10^{13} m/s^2 and 0.031 m)

8. A wire with current 8 A is placed in a magnetic field of 0.15T, at an angle 30^0 form B. Find the force per unit length, and direction of force.

(0.6 N/m)

9. Two identical coils of radius 8cm and number of turns 100, are fixed coaxially at distance 12cm. If the current in them is 1 A in the same direction, find magnetic field at mid point on the axis.

($8.04 \times 10^{-4}\,T$)

10. Two wire of length 2m each are parallel and are at a distance 0.2m from each other; if the current in both is 0.2A in the same direction. Find force per unit length on them.

(2×10^{7} N/m)

11. A Square coil of side 10 cm and 20turms have a current 12 A is suspended vertically. If its area vector makes an angle 30^0 with uniform magnetic field of 0.08T. Find the torque on the coil.

(0.96 N $\times$ m)

12. Find the ratio of the radii of circular paths traced by an α particle and beam of proton, entering with equal velocity v in perpendicular magnetic field.

$$\left(\frac{r_\alpha}{r_p} = \frac{2}{1}\right)$$

13. Radius of "Dee" of cyclotron is 0.5m. A magnetic field of 1.7 T is perpendicular to it. Find the maximum energy gained by proton.

($5.53 \times 10^{12}\,J$)

14. Resistance of a galvanometer coil is 12 Ω, the current required for full scale deflection is 2mA. How you will convert it to a voltmeter of range 0-18 volt.

(By connecting 5988 resistor in series)

15. A galvanometer of resistance 99 Ω, requires 4mA current for its full scale deffection. what you will do to convert it into an ammeter of range 0-6 A?

(By connecting $6.6 \times 10^{2}\,\Omega$ resister in parallel)

16. A 1.0m long solenoid has 100 turns. Its radius is 1cm. Find magnetic filed at its axis, if current in it is 5A. Find the force on electron moving with velocity 10^4 m/s along the axis.

($B = 6.28 \times 10^{-3}\,T$ cy F = 0 N)

17. A solenoid has length 0.5m. Its winding is in double layer, each layer having 500 turns. Its radius is 1.4 cm. Find magnetic field at its center when current in it is 5A.

($12.56 \times 10^{3}\,T$)