5 CHAPTER
Liquids in Rigid Motion

Q.1 A cylindrical vessel with a constant plan area of 1 m^2 is rotated about its vertical axis such that the liquid inside the vessel is about to spill. If the height of the vessel is 2 m and the height of the paraboloid is 1 m, then the volume (in m^3) of the liquid in the vessel will be
(a) 2 (b) 1.5
(c) 1.0 (d) 0.5

Q.2 An open rectangular wagon 5 m long partly filled with water is subjected to an acceleration of 1.96 m/s^2. For spilling of water, the level below the open top upto which water may be filled is
(a) 0.5 m (b) 0.6 m
(c) 0.7 m (d) 0.8 m

Q.3 An open rectangular wagon 5 m long is filled with water to a level 0.5 m below the open top. The maximum acceleration (in m/s^2) at which it can be speeded up without spilling the water is nearly
(a) 0.98 (b) 1.96
(c) 4.9 (d) 9.8

Q.4 A cylindrical vessel of radius 42.31 cm and height 1 m is open at the top. It holds water upto half its depth. Which one of the following values approximates the speed at which the cylinder is to be rotated about the vertical axis, so as to make the apex of the paraboloid just reach the centre of the bottom of the vessel?
(a) 100 rpm (b) 150 rpm
(c) 250 rpm (d) 300 rpm

Q.5 The constant angular velocity at which a liquid rotates in a cylinder about a vertical axis such that the pressure at a point on the axis is the same as at a point 2 m higher at a radius 2 m is
(a) $2g$ radians/sec
(b) g radians/sec
(c) $\sqrt{2g}$ radians/sec
(d) $\sqrt{g}$ radians/sec

Q.6 A closed tank containing water to a depth of 1.5 m falls vertically with an acceleration of 2 m/s^2. What is the pressure at the bottom of the tank?
(a) 12.5 kN/m^2 (b) 13.3 kN/m^2
(c) 11.7 kN/m^2 (d) 10.5 kN/m^2

Q.7 If a vessel containing liquid moves downward with a constant acceleration equal to 'g' then
(a) the pressure throughout the liquid mass is atmospheric
(b) there will be vacuum in the liquid
(c) the pressure in the liquid mass is greater than hydrostatic pressure
(d) None of the above

Q.8 A closed tank containing water is moving in a horizontal direction along a straight line at a constant speed. The tank also contains a steel ball and a bubble of air. If the tank is deaccelerated horizontally, then
1. the ball will move to the front
2. the bubble will move to the front
3. the ball will move to the rear
4. the bubble will move to the rear

Find out which of the above statements are correct?
(a) 1 and 2 (b) 1 and 4
(c) 2 and 3 (d) 3 and 4

Q.9 A cylindrical vessel open at the top is filled with water and rotated at a constant angular velocity about its vertical axis such that the bottom of the vessel is just exposed at the axis. The volume of water spilled as a fraction of the volume of the cylinder is

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Q.10 If a water tank, partially filled with water is being carried on a truck, moving with a constant horizontal acceleration, then the level of liquid will

(a) rise and fall alternately on the front side of the tank

(b) fall on the rear side of the tank

(c) remain the same on both sides of the tank

(d) rise on the rear side and fall on the front side of the tank

Q.11 An open rectangular box of base 2 m × 2 m contains a liquid of specific gravity 0.80 upto a height of 3.0 m. If the box is imparted a vertically upward acceleration of 4.9 m/s², what will be the pressure on the base of the tank?

(a) 11.76 kPa (b) 35.28 kPa

(c) 8.82 kPa (d) 17.64 kPa

Q.12 Read the following statements:

1. The pressure at every point of a freely falling fluid is atmospheric.
2. Crude oil is transported in small containers instead of single large container to have more stability.
3. By rotating, mass of fluids with external force about vertical axis, the pressure varies linearly with respect to radius.

The correct statements are

(a) 1 and 3 (b) 1 and 2

(c) 2 and 3 (d) 1, 2 and 3

Q.13 A closed cylinder having radius R and height H is filled with oil of density ρ. If the cylinder is rotated about its axis at the angular velocity ω, the thrust at the bottom of the cylinder is

(a) $\pi R^2(\rho g h)$

(b) $\pi R^2\left(\frac{\rho^2 w^2 R^2}{4}\right)$

(c) $\pi R^2\left(\rho w^2 R^2 + \rho g h\right)$

(d) $\pi R^2\left(\frac{\rho w^2 R^2}{4} + \rho g h\right)$

Q.14 A tank contains water upto a depth of 1.5 m. The length and width of the tank are 4 m and 2 m respectively. The tank is moving up an inclined plane with a constant acceleration of 4 m/s². The inclination of the plane with the horizontal is 30°. The angle made by the free surface of water with the horizontal is

(a) 16.35° (b) 45°

(c) 73.65° (d) 88.42°

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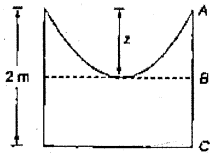
Answers Liquids in Rigid Motion

1. (b) 2. (a) 3. (b) 4. (a) 5. (d) 6. (c) 7. (a) 8. (b) 9. (c) 10. (d)
11. (b) 12. (b) 13. (d) 14. (a)

Explanations. Liquids in Rigid Motion

1. (b)



Height of paraboloid = $Z = 1\ m = \frac{r^2w^2}{2g}$

Vol. of water = $V_{AB} + V_{BC}$

V_{AB} = Volume of paraboloid

$= \frac{1}{2} \times$ Volume of circumscribing cylinder

$= \frac{1}{2} \times 1 \times 1 = 0.5\ m^3$

V_{BC} = Cross-sectional area × 1

$= 1 \times 1 = 1\ m^3$

Total volume of water = 1 + 0.5 = 1.5 m³

4. (a)

$$\frac{\omega^2 R^2}{2g} = 1$$

$$\Rightarrow \quad \omega = \frac{2\pi N}{60}$$

Solving, we get

$N = 100$ rpm

5. (d)

$$\frac{w^2 r^2}{2g} = 2$$

$$\Rightarrow \quad w = \sqrt{g}$$

6. (c)

Pressure at bottom of tank

$= \rho(g - a)h$

$= 1000 \times (9.81 - 2) \times 1.5$

$= 11.7\ kN/m^2$

11. (b)

Pressure = $\rho(g + a)h$

$= 0.8 \times 1000 \times 3 \times (9.8 + 4.9)$

$= 35.28$ kPa

13. (d)

$$\int dF = \int_0^R P(2\pi r dr)$$

$$= \int_0^R \rho g\left(H + \frac{w^2 r^2}{2g}\right) 2\pi r dr$$

$$= \frac{2\pi\rho g H R^2}{2} + \frac{2\pi\rho g w^2 R^4}{8g}$$

$$= \pi R^2\left[\rho g H + \frac{\rho w^2 R^2}{4}\right]$$

14. (a)

$\alpha = 30°$

$a = 4\ m/s^2$

$a_x = a\cos\alpha = 4\cos 30°$

$= 3.464\ m/s^2$

$a_y = a\sin\alpha = 4\sin 30° = 2\ m/s^2$

$$\therefore \quad \tan\theta = \frac{a_x}{a_y + g} = \frac{3.464}{2 + 9.81} = 0.2933$$

$$\Rightarrow \quad \theta = \tan^{-1}(0.2933) = 16.35°$$

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