Maxwell's Equations

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Introduction

- Stationary charges → Electrostatic field
- Steady currents → Magnetostatic field
- Time-varying currents → Electromagnetic field (or waves)

Faraday's Law

- The induced emf (V_{emf}), in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt}$$

where, λ = flux linkage
N = number of turns in the circuit
ψ = flux through each turn

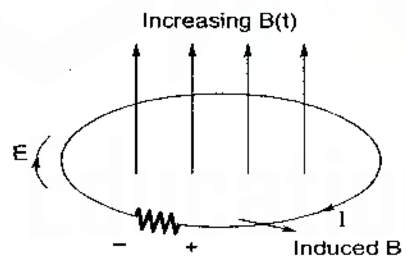
- The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it.

Transformer and motional Electromotive Forces

Stationary loop in time-varying magnetic field

(i) Transformer emf

$$V_{emf} = \oint_L \vec{E}\, d\vec{l} = -\frac{d}{dt}\int_S \vec{B}.d\vec{S}$$



(ii) Maxwell's equation for time-varying field

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Note: Time varying electric field E is not conservative ($\nabla \times E \neq 0$).

Moving loop in static magnetic field

(i) Motional electric field

$$\vec{E}_m = \frac{\vec{F}_m}{Q} = \vec{u} \times \vec{B}$$

where, F_m = Force on charge moving with uniform Velocity u, in a magnetic field

(ii) Motional emf or flux-cutting emf

$$V_{emf} = \oint \vec{E}_m \cdot d\vec{l} = \oint_L \left(\vec{u} \times \vec{B}\right) \cdot dl$$

Moving loop in time-varying field

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint_L \left(\vec{u} \times \vec{B}\right) \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{u} \times \vec{B}\right)$$

Displacement Current

Displacement current density

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

Maxwell's Equations in Final Forms

Maxwell's equation for static electric and magnetic fields

Differential (or Point form)	Integral Form	Remarks
$\vec{\nabla} \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{S} = \int_v \rho_v dv$	Gauss's law
$\vec{\nabla} \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{S} = 0$	Nonexistence of magnetic monopole
$\vec{\nabla} \times \vec{E} = 0$	$\oint_L \vec{E} \cdot d\vec{l} = 0$	Conservative of electrostatic field
$\vec{\nabla} \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S}$	Ampere's Law

Maxwell's equation for time-varying field

Differential Form	Integral Form	Remarks
$\vec{\nabla}\cdot\vec{D} = \rho_v$	$\oint_S \vec{D}\cdot d\vec{S} = \int_v \rho_v dv$	Gauss's law
$\vec{\nabla}\cdot\vec{B} = 0$	$\oint_S \vec{B}\cdot d\vec{S} = 0$	Non-existence of isolated magnetic charge
$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{dt}$	$\oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{S}$	Faraday's law
$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$	$\oint_L \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) d\vec{S}$	Ampere's circuit law

Time-Varying Potentials

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

Lorentz condition for potentials

$$\nabla \cdot A = -\mu \in \frac{\partial V}{\partial t}$$

Time Harmonic Field

- A time harmonic field is one that varies periodically or sinusoidally with time.
- Phasor form of vector A is A_s (x, y, z)

$$A = R_e\{A_s e^{j\omega t}\}$$

Maxwell's equation for time harmonic field

Point Form	Integral Form
$\vec{\nabla}\cdot\vec{D}_s = \rho_{vs}$	$\oint \vec{D}_s\cdot d\vec{S} = \int \rho_{vs} dv$
$\vec{\nabla}\cdot\vec{B}_s = 0$	$\oint \vec{B}_s\cdot d\vec{S} = 0$
$\vec{\nabla}\times\vec{E}_s = -j\omega\vec{B}_s$	$\oint \vec{E}_s\cdot d\vec{l} = -j\omega\int \vec{B}_s\cdot d\vec{S}$
$\vec{\nabla}\times\vec{H}_s = \vec{J}_s + j\omega\vec{D}_s$	$\oint \vec{H}_s\cdot d\vec{l} = \int(\vec{J}_s + \omega\vec{D}_s)\cdot d\vec{S}$

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