Oscillations

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$ (b) $a = -200x^2$

(c) $a = -10x$ (d) $a = 100x^3$

2. Which of the following examples represent periodic but not simple harmonic motion?

(a) The rotation of earth about its axis.

(b) Motion of an oscillating mercury column in a U-tube.

(c) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

(d) All of the above

3. The function $\sin\omega t - \cos\omega t$ represents

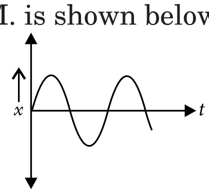
(a) a simple harmonic motion with a period $\frac{\pi}{\omega}$.

(b) a simple harmonic motion with a period $\frac{2\pi}{\omega}$.

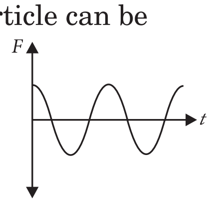
(c) a periodic, but not simple harmonic motion with a period $\frac{\pi}{\omega}$.

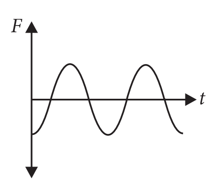
(d) a periodic, but not simple harmonic motion with a period $\frac{2\pi}{\omega}$.

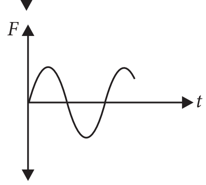
4. Displacement-time graph of a particle executing S.H.M. is shown below.

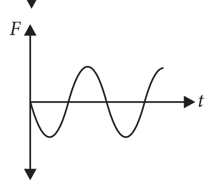


The corresponding force-time graph of the particle can be

(a) 

(b) 

(c) 

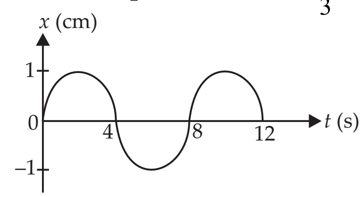
(d) 

5. The maximum speed of a particle executing S.H.M. is 10 m/s and maximum acceleration is 31.4 m/s^2. Its periodic time is

(a) 2 s (b) 4 s

(c) 6 s (d) 1 s

6. The x-t graph of a particle undergoing simple harmonic motion is shown in figure. The acceleration of the particle at $t = \frac{4}{3}$ s is



(a) $\frac{\sqrt{3}}{32}\pi^2$ cm s^{-2} (b) $\frac{-\pi^2}{32}$ cm s^{-2}

(c) $\frac{\pi^2}{32}$ cm s^{-2} (d) $\frac{-\sqrt{3}}{32}\pi^2$ cm s^{-2}

7. A particle executes SHM of type $x = A\sin\omega t$. It takes time t_1 from $x = 0$ to $x = \frac{A}{2}$ and t_2 from $x = \frac{A}{2}$ to $x = A$. The ratio $t_1 : t_2$ will be

(a) 1 : 1 (b) 1 : 2

(c) 1 : 3 (d) 2 : 1

8. A particle is executing S.H.M. according to the equation $x = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$ in SI units. The displacement and acceleration of the particle at $t = 1.5$ s is

(a) –3.0 m, 100 m s^{-2} (b) +2.54 m, 200 m s^{-2}

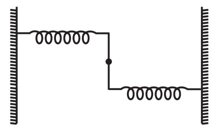
(c) –3.54 m, 140 m s^{-2} (d) +3.55 m, 120 m s^{-2}

9. A particle executes a linear S.H.M. with an amplitude of 4 cm. At the mean position the velocity of the particle is 10 cm/s. What is the displacement of the particle when its speed becomes 5 cm/s?

(a) $2(\sqrt{3})$ cm (b) $2(\sqrt{5})$ cm

(c) $\sqrt{5}$ cm (d) $\sqrt{3}$ cm

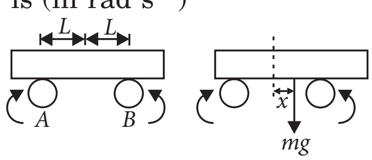
10. A uniform rod of length l and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constant k. The springs are fixed to rigid support as shown in figure and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{6M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

11. A uniform bar with mass m lies symmetrically across two rapidly rotating fixed rollers, A and B with distance L = 2.0 cm between the bar's centre of mass and each roller. The rollers , whose directions of rotation are shown in figures slip against the bar with coefficient of kinetic friction μ_k = 0.40. Suppose the bar is displaced horizontally by a distance x as shown in figure and then released. The angular frequency ω of the resulting horizontal simple harmonic motion of the bar is (in rad s^{-1})



(a) 14 (b) 15

(c) 16 (d) 17

12. A particle performs simple harmonic motion with a period of 2 seconds. The time taken by it to cover a displacement equal to half of its amplitude from the mean position is

(a) 1/2 s (b) 1/3 s

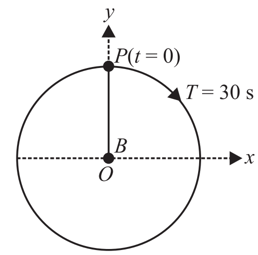
(c) 1/4 s (d) 1/6 s

13. A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 100 N m^{-1}. The block is pulled to a distance x = 10 cm from its equilibrium position at x = 0 on a frictionless surface from rest at t = 0. The kinetic energy and potential energy of the block when it is 5 cm away from the mean position is

(a) 0.375 J, 0.125 J (b) 0.125 J, 0.375 J

(c) 0.125 J, 0.125 J (d) 0.375 J, 0.375 J

14. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x-projection of the radius vector of the rotating particle P is



(a) $x(t) = A\sin\left(\frac{2\pi}{30}t\right)$

(b) $x(t) = A\cos\left(\frac{\pi}{15}t\right)$

(c) $x(t) = A\sin\left(\frac{\pi}{15}t + \frac{\pi}{2}\right)$

(d) $x(t) = A\cos\left(\frac{\pi}{15}t + \frac{\pi}{2}\right)$

15. The displacement of a particle varies with time according to the relation
$y = a \sin \omega t + b \cos \omega t$.

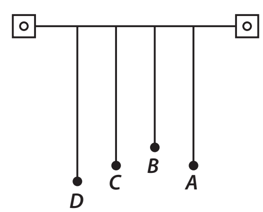
(a) The motion is oscillatory but not SHM.

(b) The motion is SHM with amplitude $a + b$.

(c) The motion is SHM with amplitude $a^2 + b^2$.

(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

16. Four pendulums A, B, C and D are suspended from the same elastic support as shown in figure. A and C are of the same length, while B is smaller than A and D is larger than A.



If A is given a transverse displacement then

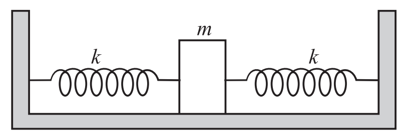
(a) D will vibrate with maximum amplitude

(b) C will vibrate with maximum amplitude

(c) B will vibrate with maximum amplitude

(d) all the four will oscillate with equal amplitude.

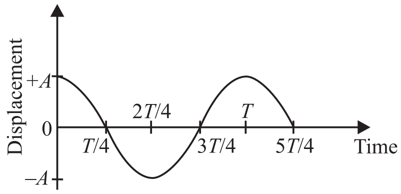
17. Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in the figure. The time period of oscillation is



(a) $2\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$

(c) $2\pi\sqrt{\frac{2m}{k}}$ (d) $\pi\sqrt{\frac{m}{2k}}$

18. The displacement-time graph for a particle executing SHM is as shown in figure.



Which of the following statements is correct?

(a) The velocity of the particle is maximum at $t = \frac{3}{4}T$.

(b) The velocity of the particle is maximum at $t = \frac{T}{2}$.

(c) The acceleration of the particle is maximum at $t = \frac{T}{4}$.

(d) The acceleration of the particle is maximum at $t = \frac{3}{4}T$.

19. Which of the following examples represent periodic motion as well as simple harmonic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its *N-S* direction and released.

(c) A hydrogen molecule rotating about its centre of mass.

(d) An arrow released from a bow.

20. A particle executing SHM has a maximum speed of 30 cm s^{-1} and a maximum acceleration of 60 cm s^{-2}. The time period of oscillation is

(a) π s (b) $\frac{\pi}{2}$ s

(c) $\frac{\pi}{4}$ s (d) 2π s

21. A particle executing SHM is described by the displacement function $x(t) = A\cos(\omega t + \phi)$, If the initial ($t$ = 0) position of the particle is 1 cm, its initial velocity is π cm s^{-1} and its angular frequency is π s^{-1}, then the amplitude of its motion is

(a) π cm (b) 2 cm

(c) $\sqrt{2}$ cm (d) 1 cm

22. When the displacement of a particle executing SHM is one-fourth of its amplitude, what fraction of the total energy is the kinetic energy?

(a) $\frac{16}{15}$ (b) $\frac{15}{16}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

23. If the frequency of human heart beat is 1.25 Hz, the number of heart beats in 1 minute is

(a) 80 (b) 65

(c) 90 (d) 75

24. A point particle oscillates along the x-axis according to the law $x = x_0\cos(\omega t - \frac{\pi}{4})$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then

(a) $A = x_0\omega^2$, $\delta = 3\pi/4$ (b) $A = x_0$, $\delta = -\pi/4$

(c) $A = x_0\omega^2$, $\delta = \pi/4$ (d) $A = x_0\omega^2$, $\delta = -\pi/4$

25. The length of a seconds pendulum on the surface of earth is 1 m. Its length on the surface of the moon is

(a) $\frac{1}{6}$ m (b) 1 m

(c) $\frac{1}{36}$ m (d) 36 m

26. A block of mass 200 g executing SHM under the influence of a spring of spring constant k = 90 N m^{-1} and a damping constant b = 40 g s^{-1}. The time elapsed for its amplitude to drop to half of its initial value is

(Given ln (1/2) = –0.693)

(a) 7 s (b) 9 s

(c) 4 s (d) 11 s

27. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then

(a) $T_1 < T_2$ (b) $T_1 > T_2$

(c) $T_1 = T_2$ (c) $T_1 = 2T_2$

28. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy ?

(a) $\frac{1}{6}$s (b) $\frac{1}{4}$s
(c) $\frac{1}{3}$s (d) $\frac{1}{12}$s

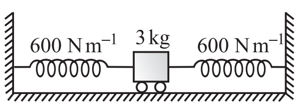
29. Consider a pair of identical pendulums, which oscillate with equal amplitude independently such that when one pendulum is at its extreme position making an angle of 2° to the right with the vertical, the other pendulum makes an angle of 1° to the left of the vertical. The phase difference between the pendulums is

(a) $\frac{\pi}{2}$ (b) $\frac{2}{3}\pi$
(c) $\frac{3}{2}\pi$ (d) π

30. A simple pendulum executing SHM with a period of 6 s between two extreme positions B and C about a point O. If the length of the arc BC is 10 cm, how long will the pendulum take the move from position C to a position D towards O exactly midway between C and O?

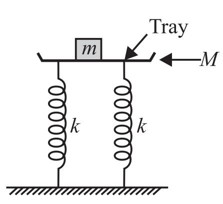
(a) 0.5 s (b) 1 s
(c) 1.5 s (d) 3 s

31. A trolley of mass 3 kg, as shown in figure, is connected to two identical springs, each of spring constant 600 N m^{-1}. If the trolley is displaced from its equilibrium position by 5 cm and released, the maximum speed of the trolley is



(a) 0.5 m s^{-1} (b) 1 m s^{-1}
(c) 2 m s^{-1} (d) 3 m s^{-1}

32. A tray of mass M = 10 kg is supported on two identical springs, each of spring constant k, as shown in figure. When the tray is depressed a little and released, it executes simple harmonic motion of period 1.5 s. When a block of mass m is placed on the tray, the period of oscillation becomes 3 s. The value of m is



(a) 10 kg (b) 20 kg
(c) 30 kg (d) 40 kg

33. The amplitude of a damped oscillator becomes $\left(\frac{1}{3}\right)^{\text{rd}}$ in 2 seconds. If its amplitude after 6 seconds is $\frac{1}{n}$ times the original amplitude, the value of n is

(a) 3^2 (b) $\sqrt[3]{3}$
(c) 2^3 (d) 3^3

34. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$. (K = 1 m s^{-2}) where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is (Take g = 10 m s^{-2})

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$
(c) 1 (d) $\frac{4}{5}$

35. An air chamber of volume V has a neck of cross-sectional area a into which a light ball of mass m just fits and can move up and down without friction. The diameter of the ball is equal to that of the neck of the chamber. The ball is pressed down a little and released. If the bulk modulus of air is B, the time period of the oscillation of the ball is

(a) $T = 2\pi\sqrt{\frac{Ba^2}{mV}}$ (b) $T = 2\pi\sqrt{\frac{BV}{ma^2}}$
(c) $T = 2\pi\sqrt{\frac{mB}{Va^2}}$ (d) $T = 2\pi\sqrt{\frac{mV}{Ba^2}}$

36. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A block of mass m is suspended from this balance, when displaced and released, it oscillates with a period 0.5 s. The value of m is (Take g = 10 m s^{-2})

(a) 8 kg (b) 12 kg
(c) 16 kg (d) 20 kg

37. A U-tube of uniform cross-section holds 1 kg of pure mercury and 0.2 kg of water in equilibrium. The diameter of cross-section is 1.2 cm. Relative density of mercury is 13.6. If the system in equilibrium is slightly disturbed, the period of oscillation of the liquid column in the tube will be (take g = 10 m s^{-2})

(a) 0.1 s (b) 1.5 s
(c) 0.50 s (d) 0.2 s

Case Based MCQs

Case I : Read the passage given below and answer the following questions from 38 to 42.

Simple Harmonic Motion

Simple harmonic motion is the simplest form of oscillation. A particular type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is termed as simple harmonic motion (S.H.M).

A body is undergoing simple harmonic motion if it has an acceleration which is directed towards a fixed point, and proportional to the displacement of the body from that point.

Acceleration $a \propto -x \Rightarrow a = -kx$ or $\frac{d^2x}{dt^2} = -kx$,
where x = displacement at any instant t.

38. Which of the following is not a characteristics of simple harmonic motion?
(a) The motion is periodic.
(b) The motion is along a straight line about the mean position.
(c) The oscillations are responsible for the energy conversion.
(d) The acceleration of the particle is directed towards the extreme position.

39. The equation of motion of a simple harmonic motion is
(a) $\frac{d^2x}{dt^2} = -\omega^2 x$ (b) $\frac{d^2x}{dt^2} = -\omega^2 t$
(c) $\frac{d^2x}{dt^2} = -\omega x$ (d) $\frac{d^2x}{dt^2} = -\omega t$

40. Which of the following expressions does not represent simple harmonic motion?
(a) $x = A\cos\omega t + B\sin\omega t$ (b) $x = A\cos(\omega t + \alpha)$
(c) $x = B\sin(\omega t + \beta)$ (d) $x = A\sin\omega t\cos^2\omega t$

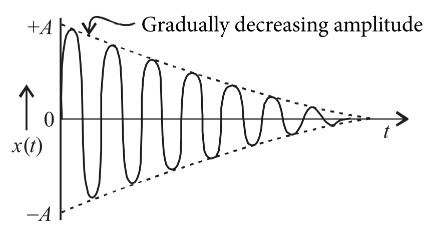
41. The time period of simple harmonic motion depends upon
(a) amplitude (b) energy
(c) phase constant (d) mass

42. Which of the following motions is not simple harmonic?
(a) Vertical oscillations of a spring
(b) Motion of a simple pendulum
(c) Motion of planet around the Sun
(d) Oscillation of liquid in a U-tube

Case II : Read the passage given below and answer the following questions from 43 to 46.

Damped Simple Harmonic Motion

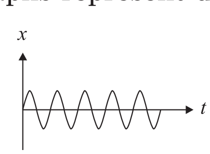
The oscillations in presence of dissipative force where the amplitude decreases gradually with the passage of time are called damped oscillations. A part of the energy of the oscillating system is lost in the form of heat, in overcoming these resistive forces, As a result, the amplitude of such oscillations decreases exponentially with time, as shown in figure. Eventually, these oscillations die out. In these oscillations, the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, etc. Due to decrease in amplitude, the energy of the oscillator also goes on decreasing exponentially.

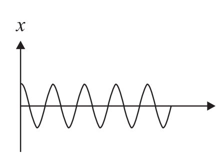


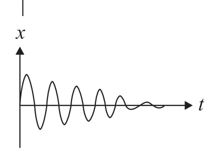
The force producing a resistance to the oscillation is called damping force.

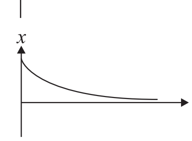
43. A particle oscillating under a force $\vec{F} = -k\vec{x} - b\vec{v}$ is a (k and b are constants)
(a) simple harmonic oscillator
(b) linear oscillator
(c) damped oscillator
(d) forced oscillator

44. Which of the following displacement-time graphs represent damped harmonic oscillation?

(a) 

(b) 

(c) 

(d) 

45. In case of a force vibration, the resonance wave becomes very sharp when the
(a) applied periodic force is small
(b) quality factor is small
(c) damping force is small
(d) restoring force is small

46. The S.I. unit of damping constant is
(a) kg s
(b) kg^2s
(c) kg m/s
(d) kg/s

Assertion & Reasoning Based MCQs

For question numbers 47-56, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false and R is also false

47. Assertion (A) : Resonance is a special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency and the amplitude of forced vibration, is maximum.

Reason (R) : The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.

48. Assertion (A) : Simple harmonic motion is not a uniformly accelerated motion.

Reason (R) : Velocity is non uniform in SHM.

49. Assertion (A) : The bob of a simple pendulum is a ball full of water. If a fine hole is made at the bottom of the ball, then the time period will no more remain constant.

Reason (R) : The time period of a simple pendulum does not depend upon mass.

50. Assertion (A) : When a simple pendulum is made to oscillate on the surface of moon, its time period increases.

Reason (R) : Moon is much smaller as compared to earth.

51. Assertion (A) : If a man with a wrist watch on his hand falls from the top of a tower, its watch gives correct time during the free fall.

Reason (R) : The working of the wrist watch depends on spring action and it has nothing to do with gravity.

52. Assertion (A) : The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%.

Reason (R) : Time period is directly proportional to length of pendulum.

53. Assertion (A) : The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is $0.612\ s^{-1}$.

Reason (R) : The frequency of a second pendulum does not depend upon acceleration due to gravity.

54. Assertion (A) : Damped oscillation indicates loss of energy.

Reason (R) : The loss in damped oscillation may be due to friction, air resistance etc.

55. Assertion (A) : The value of acceleration due to gravity is low at the mountain top than at the plane.

Reason (R) : If a pendulum clock is taken to mountain top, it will gain time.

56. Assertion (A) : In real situation the amplitude of forced oscillation can never be infinite.

Reason (R) : The energy of oscillator is continuously dissipated.

SUBJECTIVE TYPE QUESTIONS

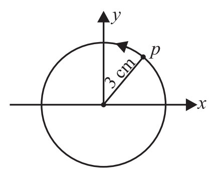
Very Short Answer Type Questions (VSA)

1. What are the basic properties required by a system to oscillate?

2. What is the : (a) distance moved
(b) displacement of a particle executing SHM in one vibration?

3. $x(t) = A\cos(\omega t + \phi)$ is the equation of simple harmonic motion. What do we call ϕ in this equation?

4. The figure shows circular motion of a reference particle to represent simple harmonic motion. What is the amplitude of simple harmonic motion?



5. A simple harmonic motion is described by $a = -16x$ where a is acceleration and x is displacement in meter. What is the time-period ?

6. Glass window panes are sometimes broken by a explosion several miles away. Explain why?

7. On what factors does the energy of a simple harmonically vibrating particle depends?

8. How is the frequency of oscillation related with the frequency of change in the of K.E and P.E of the body in S.H.M.?

9. What will be the time period of a second pendulum inside an artificial satellite?

10. Two simple pendulums of unequal length meet each other at mean position while oscillating. What is their phase difference?

Short Answer Type Questions (SA-I)

11. How is the time period of the pendulum effected when pendulum is taken to hills or in mines?

12. Two exactly identical pendulums are executing (approximiate) SHMs with amplitudes a and na respectively. Calculate the ratio of their energies of oscillation.

13. A block weighing 40 N is suspended from a spring that has a force constant of 200 Nm^{-1}. The system is undamped and is subjected to a harmonic driving force of frequency 10 Hz, resulting a forced-motion amplitude of 2 cm. What is the maximum value of driving force ?

14. A 2 kg particle undergoes SHM according to $x = 1.5 \sin\left(\frac{\pi}{4}t + \frac{\pi}{6}\right)$, when x is in metre and t in second. What is the total mechanical energy of the particle ?

15. Show that the acceleration of a particle in SHM is proportional to its displacement from the mean position.

16. Three springs of spring factor k, $2k$, k, respectively are connected in parallel to a mass m. If the mass = 0.08 kg and k = 2 N/m, then find the new time period?

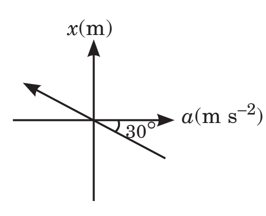
17. Three springs of constants k_1, k_2 and k_3 are connected end to end and are used to hang a mass M from the roof. Determine the elastic potential energy stored in the array.

18. The maximum velocity of a body undergoing simple harmonic motion is 0.04 m s^{-1} and its acceleration at 0.02 m from the mean position is 0.06 m s^{-2}. Determine its amplitude and time period.

19. The formula for time period T for a loaded spring, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

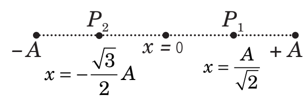
Does the time period depend on the length of the spring?

20. Figure shows the acceleration-displacement graph of a particle in SHM. Find the time period (in second).



Short Answer Type Questions (SA-II)

21. Two particles P_1 and P_2 are executing SHM along the same straight line, whose equations are given $x_1 = A\sin(\omega t + \delta_1)$ and $x_2 = A\sin(\omega t + \delta_2)$. An observer on the ground, at $t = 0$, observes particle P_1 at distance $\frac{A}{\sqrt{2}}$ moving the right from mean position O and particle P_2 at $-\frac{\sqrt{3}}{2}A$ moving to the left from mean position O, as shown in figure. Find the value of $\delta_2 - \delta_1$.



22. Define S.H.M. What are its chracteristics? At what distance from the mean position in S.H.M of amplitude r, the energy is half kinetic and half potential ?

23. A body is executing a simple harmonic motion such that its potential energy is U_1 at x and U_2 at y. When the displacement is $x + y$, calculate the potential energy.

24. Find the displacement of a simple harmonic oscillator at which its P.E. is half of the maximum energy of the oscillator.

25. A 10 kg collar is attached to a spring (spring constant 600 N/m), it slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 20 cm and released. What is the speed of the oscillation?

26. A particle is moving in a straight line with SHM of amplitude r. At a distance s from the mean position of motion, the particle receives a blow in the direction of motion which instantaneously doubles the velocity. Find the new amplitude.

27. If the ratio of the mechanical energy of a mass m whose oscillations are damped to that of its free oscillations is $e^{-3t/m}$, then determine the ratio of the amplitudes of the damped oscillations to that of free oscillations.

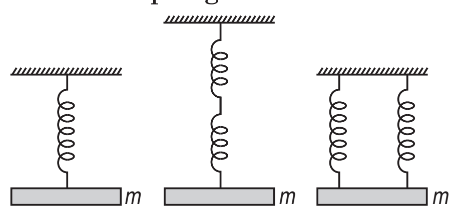
28. Draw (a) displacement time graph of a particle executing SHM with phase angle ϕ equal to zero (b) velocity-time graph and (c) acceleration-time graph of the particle.

29. Two particles are executing SHM with same amplitudes A and time period T. When one of the particles is located at extreme right, the other particle is located at mean position and moving towards left. From this instant, find the time after which both the particles will have same displacement from the equilibrium position.

30. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s)

(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right)$ (b) $x = \cos\left(\frac{\pi}{6} - t\right)$

31. Show the time period for vertical harmonic oscillations of the three system shown in figure (a), (b) and (c) in the ratio of $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$. Spring constant of each spring of k.



(a) (b) (c)

32. A particle set to be in SHM having two types of energies potential and kinetic. The potential energy is on account of displacement of particle from mean position and kinetic energy is an account of velocity of the particle. At any instant of time t, these are

$$P.E = U = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

$$K.E = K = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

and $T.E = P.E + K.E = \frac{1}{2} m\omega^2 A^2 = \text{constant}$

where, m = mass of particle.

(a) At what distance from the mean position (a fixed point) will K.E of particle would be twice of its P.E?

(b) What are the applications of this study in our day to day life ?

33. A simple pendulum is hung in a stationary lift and its periodic time is T. What will be the effect on its periodic time T if
(i) the lift goes up with uniform velocity v,
(ii) the lift goes up with uniform acceleration a, and
(iii) the lift comes down with uniform acceleration a?

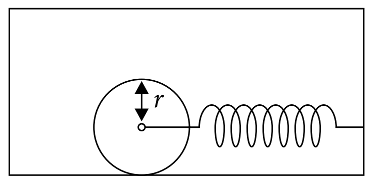
34. A horizontal spring block system of mass M executes simple harmonic motion. When the block is passing through its equilibrium position, an object of mass m is put on it and the two move together. Find the new amplitude and frequency of vibration.

Long Answer Type Questions (LA)

35. (a) Draw a graph showing the variation of kinetic energy and potential energy of a particle executing SHM with its displacement from mean position.
(b) Show that total mechanical energy of a particle executing simple harmonic motion remains conserved with time, when dissipative forces are neglected.

36. A body oscillates with SHM along the X-axis. Its displacement varies with time according to the equation : $x = (4.00 \text{ m}) \cos (\pi t + \pi/4)$. Calculate (a) displacement (b) velocity (c) acceleration at $t = 1.00$ s (d) the maximum speed and maximum acceleration and (e) phase at $t = 2.00$ s.

37. A solid cylinder of mass M is attached to a horizontal massless spring so that it can roll without slipping along a horizontal surface as shown in figure. The spring constant k is 3 N m^{-1}. If the system is released from rest at a point in which the spring is stretched by 0.25 m, find (a) the translational kinetic energy and (b) the rotational kinetic energy of the cylinder as it passes the equilibrium position. (c) Show that under these conditions, the centre of mass of cylinder executes SHM with time period, $T = 2\pi\sqrt{(3M / 2k)}$.



38. You are riding an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. (c) A particle is said to be executing simple harmonic motion, if the acceleration 'a' produced in it satisfies the following two conditions :
(i) 'a' is directly proportional to the displacement (say y) from the mean position. *i.e.* $a \propto y$.
(ii) 'a' is directed towards mean position *i.e.* acts opposite to the direction in which y increases.
i.e. Mathematically $a = -\omega^2 y$ ...(i)
where ω = angular frequency.

(a) $a = 0.7x$ does not satisfy eqn. (i), so it does not represent simple harmonic motion.
(b) $a = -200x^2$, does not satisfy equation (i) hence it does not represent simple harmonic motion.
(c) $a = -10x$, here x = displacement. It satisfies equation (i), so it represents simple harmonic motion.
(d) $a = 100x^3$ does not satisfy equation (i), hence it does not represent simple harmonic motion.

2. (a) It is periodic but not simple harmonic motion because it is not to and fro motion about a fixed point.
(b) It is simple harmonic motion.
(c) It is simple harmonic motion.

3. **(b) :** $\sin\omega t - \cos\omega t = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right)$

$$= \sqrt{2}\left(\sin\omega t\cos\frac{\pi}{4} - \cos\omega t\sin\frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

It represents S.H.M. with a period $\frac{2\pi}{\omega}$.

4. **(d) :** Displacement and force (ma) are out of phase ($\Delta\alpha = \pi$) in SHM. Therefore, the correct graph will be (d).

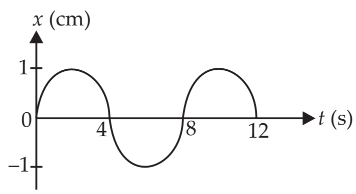
5. **(a) :** Maximum speed $v_m = \omega a$

Maximum acceleration $a_m = \omega^2 a$

$$\Rightarrow \frac{a_m}{v_m} = \omega = \frac{2\pi}{T} \Rightarrow \frac{31.4}{10} = \frac{2\times 3.14}{T}$$

$$\Rightarrow T = \frac{2\times 3.14\times 10}{31.4} \Rightarrow T = 2\text{ s}$$

6. **(d) :**



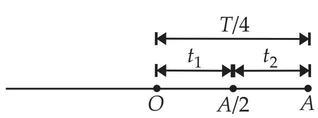
Refer figure, at time $t = 0$ s, the displacement $x = 0$. Therefore, the curve is a sinusoidal curve. It represents SHM whose amplitude $A = 1$ cm and $T = 8$ s.

As $x = A\sin\frac{2\pi}{T}t$ $\therefore$ Acceleration, $a = \frac{d^2x}{dt^2} = -\frac{4\pi^2}{T^2}A\sin\frac{2\pi}{T}t$

At $t = \frac{4}{3}$ s

$$a = -\frac{4\pi^2}{8^2}\times 1\times\sin\frac{2\pi}{8}\times\frac{4}{3} = -\frac{4\pi^2}{64}\sin\frac{\pi}{3} = -\frac{\sqrt{3}\pi^2}{32}\text{ cm s}^{-2}$$

7. **(b) :** $t_1 + t_2 = \frac{T}{4}$



or $t_2 = \frac{T}{4} - t_1$

At time, $t = t_1$, $x = \frac{A}{2}$

$$\therefore \frac{A}{2} = A\sin\omega t_1 \text{ or } \omega t_1 = \frac{\pi}{6} \text{ or } t_1 = \frac{\pi}{6\omega}$$

$$\therefore t_2 = \frac{T}{4} - \frac{\pi}{6\omega} = \frac{2\pi}{4\omega} - \frac{\pi}{6\omega} = \frac{2\pi}{6\omega} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

$\therefore t_1 : t_2 = 1 : 2$

8. **(c) :** The given equation of simple harmonic motion is

$$x(t) = 5\cos\left(2\pi t + \frac{\pi}{4}\right)$$

Compare the given equation with standard equation of SHM

$$x(t) = A\cos(\omega t + \phi)$$

we get $\omega = 2\pi\text{ s}^{-1}$

At $t = 1.5$ s

Displacement, $x(t) = 5\cos\left(2\pi\times 1.5 + \frac{\pi}{4}\right)$

$$= 5\cos\left(3\pi + \frac{\pi}{4}\right) = -5\cos\left(\frac{\pi}{4}\right)$$

$[\because \cos(3\pi + \theta) = -\cos\theta]$

$= -5 \times 0.707$ m $= -3.54$ m

Acceleration, $a = -\omega^2 \times$ displacement

$= -(2\pi\text{ s}^{-1})^2 \times (-3.54\text{ m})$

$= 140\text{ m s}^{-2}$

9. **(a) :** At the mean position, $v = A\omega$

$\therefore$ $10 = 4 \times \omega$

$\therefore$ $\omega = \frac{5}{2}$ rad/s

and $\because$ $v = \omega\sqrt{A^2 - x^2}$

$\therefore$ $v^2 = \omega^2(A^2 - x^2)$

$\therefore$ $25 = \frac{25}{4}(16 - x^2)$ $\Rightarrow$ $4 = 16 - x^2$

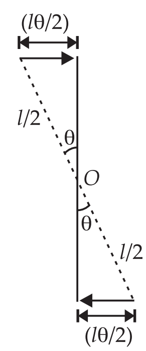
$\therefore$ $x^2 = 16 - 4 = 12$ $\Rightarrow$ $x = 2\sqrt{3}$ cm

10. **(c) :** If the rod is rotated through an angle θ, extension in one spring = compression in the other spring,

i.e., $x = l\theta/2$

Therefore, force acting on each of the ends of the rod,

$F = kx = k(l\theta/2)$



Restoring torque on the rod,

$\tau = -Fl = -k(l\theta/2)l = -kl^2(\theta/2)$

As $\tau = I\alpha = (Ml^2/12)\alpha$,

$-kl^2(\theta/2) = (Ml^2/12)\alpha$

or $\alpha = -\frac{6k}{M}\theta$, *i.e.*, $\alpha \propto \theta$

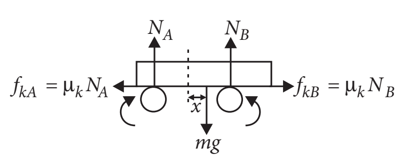
Thus, the motion of the rod is simple harmonic with

$$\omega^2 = \frac{6k}{M} \text{ or } \omega = \sqrt{\frac{6k}{M}} \text{ or } \upsilon = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

11. **(a) :** Consider horizontal forces

$\mu_k N_A - \mu_k N_B = ma$

or $a = \frac{\mu_k N_A - \mu_k N_B}{m}$ ...(i)



Taking torque about an axis perpendicular to the plane and through the contact point between bar and roller A. The bar experiences no angular acceleration about that axis.

$N_A(0) + N_B(2L) - mg(L + x) + f_{kA}(0) + f_{kB}(0) = 0$ ...(ii)

By balancing vertical forces, we have

$N_A + N_B - mg = 0$ ...(iii)

Solving eqns. (i), (ii) and (iii)

$$N_B = \frac{mg(L+X)}{2L},\ N_A = \frac{mg(L-X)}{2L},\ a = -\frac{\mu_k g}{L}X$$

Comparing it with $a = -\omega^2 X$

we get, $\omega = \sqrt{\frac{\mu_k g}{L}} = \sqrt{\frac{0.40\times 10}{2\times 10^{-2}}} \simeq 14 \text{ rad s}^{-1}$

12. (d) : The displacement in S.H.M. at any time t is given by

$$y = A\sin(\omega t)$$

Here $y = \frac{A}{2}$, Time period $T = 2$ s

$$\frac{A}{2} = A\sin\left(\frac{2\pi}{T}t\right) \text{ or } \frac{1}{2} = \sin\left(\frac{2\pi}{T}t\right)$$

$$\sin\frac{\pi}{6} = \sin\left(\frac{2\pi}{T}t\right)$$

$$\frac{2\pi}{T}t = \frac{\pi}{6}$$

$$t = \frac{T}{12} = \frac{1}{6} \text{ s} \qquad (\text{Given } T = 2 \text{ s})$$

13. (a) : Here, $m = 1$ kg, $k = 100$ N m^{-1}

$A = 10$ cm $= 0.1$ m

The block executes SHM, its angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N m}^{-1}}{1 \text{ kg}}} = 10 \text{ rad s}^{-1}$$

Velocity of the block at $x = 5$ cm (= 0.05 m) is

$$v = \omega\sqrt{A^2 - x^2} = 10\sqrt{(0.1)^2 - (0.05)^2}$$

$$= 10\sqrt{7.5\times 10^{-3}} \text{ m s}^{-1}$$

Kinetic energy of the block,

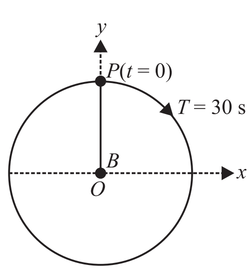
$$K = \frac{1}{2}mv^2 = \frac{1}{2}\times 1\times 0.75 = 0.375 \text{ J}$$

Potential energy of the block,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}\times 100\times (0.05)^2 = 0.125 \text{ J}$$

14. (a) : Here, $T = 30$ s

At $t = 0$, OP makes an angle of $\frac{\pi}{2}$ with the x-axis.



After a time t, it covers an angle of $\frac{2\pi}{T}t$ in the clockwise sense, and makes an angle of $\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right)$ with the x-axis.

The projection of OP on the x-axis at time t is given by

$$x(t) = A\cos\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right) = A\sin\left(\frac{2\pi}{T}t\right)$$

$$x(t) = A\sin\left(\frac{2\pi}{30}t\right) \qquad (\because T = 30 \text{ s})$$

15. (d) : Given : $x = a\sin\omega t + b\cos\omega t$ ...(i)

Let $a = A\cos\phi$ ...(ii)

and $b = A\sin\phi$ ...(iii)

Squaring and adding (ii) and (iii), we get

$a^2 + b^2 = A^2\cos^2\phi + A^2\sin^2\phi = A^2$

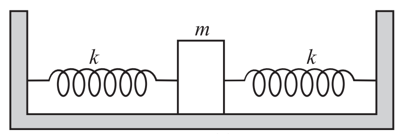
Eq. (i) can be written as

$x = A\cos\phi\sin\omega t + A\sin\phi\cos\omega t = A\sin(\omega t + \phi)$

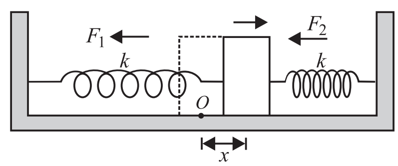
It is equation of SHM with amplitude $A = \sqrt{a^2 + b^2}$.

16. (b) : Since length of pendulums A and C is same and $T = 2\pi\sqrt{\frac{L}{g}}$, hence their time period is same and they will have same frequency of vibration. Due to it, a resonance will take place and the pendulum C will vibrate with maximum amplitude.

17. (b) :



(a)



(b)

Let the mass m be displaced by a small distance x to the right from its mean position as shown in figure (b). Due to it the spring on the left side gets stretched by a length x while that on the right side gets compressed by the same length. The forces acting on the mass are

$F_1 = -kx$ towards left hand side

$F_2 = -kx$ towards left hand side

The net force acting on the mass is

$F = F_1 + F_2 = -2kx$

Here, $F \propto x$ and –ve sign shows that force is towards the mean position, therefore the motion executed by the particle is simple harmonic.

Its acceleration is

$$a = \frac{F}{m} = -\frac{2kx}{m} \qquad \text{...(i)}$$

The standard equation of SHM is

$a = -\omega^2 x$ ...(ii)

Comparing (i) and (ii), we get

$$\omega^2 = \frac{2k}{m} \text{ or } \omega = \sqrt{\frac{2k}{m}}$$

Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

18. (a) : For the given SHM,

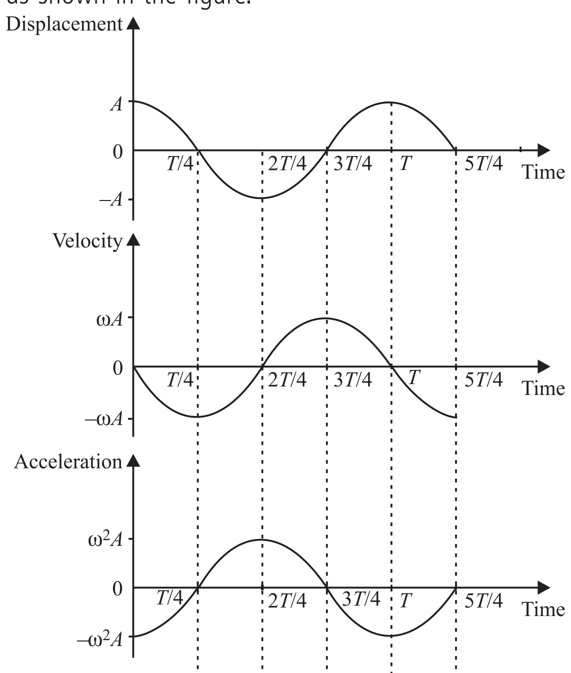
The displacement of the particle is given by

$x = A\cos\omega t$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(A\cos\omega t) = -A\omega\sin\omega t$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(-\omega A\sin\omega t) = -\omega^2 A\cos\omega t$

The corresponding velocity-time and acceleration time graph is as shown in the figure.



19. (b): (a) It is not a periodic motion as the swimmer completes only one trip but if he makes more than one trip and time for each trip is same, the motion can be categorised as periodic.

(b) It is a periodic motion because a freely suspended bar magnet if once displaced from *N-S* direction and released, it oscillates about this position. Hence it is simple harmonic motion also.

(c) It is a periodic motion only.

(d) It is not a periodic motion.

20. (a) : Maximum speed, $v_{max} = \omega A$ ...(i)

Maximum acceleration, $a_{max} = \omega^2 A$ ..(ii)

Divide (ii) by (i), we get

$$\frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \omega$$

$$\therefore \frac{a_{max}}{v_{max}} = \frac{2\pi}{T}$$

$$T = 2\pi\left(\frac{v_{max}}{a_{max}}\right)$$

Here, $v_{max} = 30$ cm s^{-1}, $a_{max} = 60$ cm s^{-2}

$$\therefore T = 2\pi\left(\frac{30 \text{ cm s}^{-1}}{60 \text{ cm s}^{-2}}\right) = \pi \text{ s}$$

21. (c) : $x = A\cos(\omega t + \phi)$ where A is amplitude.

At $t = 0$, $x = 1$ cm

$\therefore$ $1 = A\cos\phi$ ...(i)

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(A\cos(\omega t + \phi)) = -A\omega\sin(\omega t + \phi)$

At $t = 0$, $v = \pi$ cm s^{-1}

$\therefore$ $\pi = -A\omega\sin\phi$ or $\frac{\pi}{\omega} = -A\sin\phi$

$\because$ $\omega = \pi$ s^{-1}

$\therefore$ $1 = -A\sin\phi$ ...(ii)

Squaring and adding (i) and (ii), we get

$A^2\cos^2\phi + A^2\sin^2\phi = 2$

$A^2 = 2$ $(\because \sin^2\phi + \cos^2\phi = 1)$

$A = \sqrt{2}$ cm

22. (b) : In SHM,

Kinetic energy of the particle, $K = \frac{1}{2}m\omega^2(A^2 - x^2)$

where m is the mass of particle, ω is its angular frequency, A is the amplitude oscillation and x is its displacement.

At $x = \frac{A}{4}$

$$K = \frac{1}{2}m\omega^2\left[A^2 - \left(\frac{A}{4}\right)^2\right] = \frac{1}{2}\left(\frac{15}{16}m\omega^2 A^2\right)$$

Energy of the particle, $E = \frac{1}{2}m\omega^2 A^2$

$$\therefore \frac{K}{E} = \frac{\frac{1}{2}\left(\frac{15}{16}m\omega^2 A^2\right)}{\frac{1}{2}m\omega^2 A^2} = \frac{15}{16}$$

23. (d) : Beat frequency of heart = 1.25 Hz

$\therefore$ Number of beats in 1 minute = 1.25 × 60 = 75

24. (a) : Given : $x = x_0\cos\left(\omega t - \frac{\pi}{4}\right)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -x_0\omega \sin\left(\omega t - \frac{\pi}{4}\right)$

Acceleration, $a = \frac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$

$$= x_0\omega^2 \cos\left[\pi + (\omega t - \frac{\pi}{4})\right]$$

or $a = x_0\omega^2 \cos\left[\omega t + \frac{3\pi}{4}\right]$ ...(i)

But $a = A\cos(\omega t + \delta)$ (given) ...(ii)

Comparing (i) and (ii), we get

$$A = x_0\omega^2, \delta = \frac{3\pi}{4}$$

25. (a) : Time period of seconds pendulum is 2 s.

$\therefore$ $2 = 2\pi\sqrt{\frac{L}{g}}$

On earth, $L = 1$ m, $g = g_e$

$\therefore$ $2 = 2\pi\sqrt{\frac{1}{g_e}}$ ...(i)

On moon, $L = L'$, $g = \frac{g_e}{6}$

$\therefore$ $2 = 2\pi\sqrt{\frac{L'}{g_e/6}} = 2\pi\sqrt{\frac{6L'}{g_e}}$ ...(ii)

From (i) and (ii), we get

$$6L' = 1 \text{ or } L' = \frac{1}{6} \text{ m}$$

26. (a) : The amplitude of the damped oscillator at any instant t is given by

$A(t) = A_0 e^{-bt/2m}$ ...(i)

where A_0 is its initial amplitude and b is the damping constant. At $t = t_{1/2}$, the amplitude drop to half of its initial value. From (i), we get

$$\frac{A_0}{2} = A_0 e^{-bt_{1/2}/2m}$$

$$\frac{1}{2} = e^{-bt_{1/2}/2m}$$

Taking natural logarithm on both sides, we get

$$\ln\left(\frac{1}{2}\right) = -\frac{bt_{1/2}}{2m}$$

$t_{1/2} = -\frac{2m\ln(1/2)}{b}$ ...(ii)

Here, $\ln\left(\frac{1}{2}\right) = -0.693$, $b = 40$ g s^{-1}, $m = 200$ g

Substituting these values in Eq. (ii), we get

$$t_{1/2} = \frac{0.693 \times 2 \times 200}{40} \text{ s} = 7 \text{ s}$$

27. (a) : Any SHM is given by the equation $x = \sin \omega t$, where x is the displacement of the body at any instant t. A is the amplitude and ω is the angular frequency.

When $x = 0$, $\omega t_1 = 0$ $\therefore$ $t_1 = 0$

When $x = A/2$, $\omega t_2 = \pi/6$, $t_2 = \pi/6\omega$

When $x = A$, $\omega t_3 = \pi/2$, $t_3 = \pi/2\omega$

Time taken from 0 to $A/2$ will be

$$t_2 - t_1 = \frac{\pi}{6\omega} = T_1$$

Time taken from $A/2$ to A will be

$$t_3 - t_2 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{2\pi}{6\omega} = \frac{\pi}{3\omega} = T_2$$

Hence, $T_2 > T_1$

28. (a) : KE of a body undergoing SHM is given by

KE = $\frac{1}{2} m\omega^2 A^2 \cos^2\omega t$ and

Total energy, $E = \frac{m\omega^2 A^2}{2}$

[symbols represent standard quantities]

From given information, $\text{KE} = E \times \frac{75}{100}$

$$\Rightarrow \frac{m\omega^2 A^2}{2}\cos^2 \omega t = \frac{m\omega^2 A^2}{2} \times \frac{3}{4} \Rightarrow \cos\omega t = \pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow \omega t = \frac{\pi}{6} \Rightarrow \frac{2\pi}{T} \times t = \frac{\pi}{6}$$

$\Rightarrow t = \frac{T}{12} = \frac{1}{6}$s [$\because$ $T = 2$ s]

29. (b) : Let θ_1 and θ_2 be the angular displacement of first and second pendulums respectively at an instant. θ_0 be the maximum angular displacement of each pendulum. Let ϕ_1 and ϕ_2 be the initial phases of the two pendulums. Then

$\theta_1 = \theta_0 \sin(\omega t + \phi_1)$ ...(i)

and $\theta_2 = \theta_0 \sin(\omega t + \phi_2)$ ...(ii)

For first pendulum, $\theta_1 = 2° = \theta_0$

From (i), we get

$2 = 2 \sin(\omega t + \phi_1)$ or $\sin(\omega t + \phi_1) = 1$

or $\omega t + \phi_1 = 90°$ ...(iii)

For second pendulum, $\theta_2 = -1°$

From (ii), we get

$-1 = 2 \sin(\omega t + \phi_2)$ or $\sin(\omega t + \phi_2) = -\frac{1}{2}$

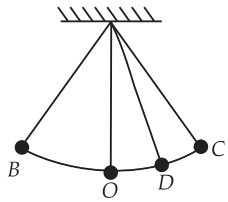
or $\sin(\omega t + \phi_2) = \sin(180° + 30°) = \sin 210°$

$\therefore$ $\omega t + \phi_2 = 210°$ ...(iv)

$\therefore$ Subtracting (iii) from (iv), we get

$$(\omega t + \phi_2) - (\omega t + \phi_1) = 210° - 90° = 120° = \frac{2\pi}{3}$$

30. (b) : O is the mean position and B and C are extreme positions.



Given : BC = 10 cm

Since B and C are the extreme positions, therefore amplitude of the SHM oscillation is

$$A = \frac{BC}{2} = 5 \text{ cm}$$

D is the midway between C and O.

$$\therefore \quad DC = \frac{5}{2} \text{ cm}$$

Since time is noted from extreme position, hence displacement x at any time t is

$x = A\cos\omega t$

For displacement CD, let t_1 be the time taken. Then

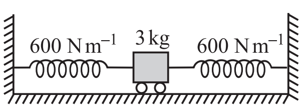
$$\frac{5}{2} = 5\cos\omega t_1$$

$$\frac{1}{2} = \cos\omega t_1 \text{ or } \cos\frac{\pi}{3} = \cos\omega t_1 \text{ or } t_1 = \frac{\pi}{3\omega}$$

$$\because \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3} \quad (\because \ T = 6 \text{ s (Given)})$$

$\therefore \quad t_1 = 1$ s

31. (b) :



In the given figure, two identical springs are connected in parallel.

$\therefore$ The effective spring constant is

$k_{eff} = 2k = 2 \times 600 \text{ N m}^{-1} = 1200 \text{ N m}^{-1}$

Here, Amplitude, A = 5 cm = 0.05 m, m = 3 kg

Angular frequency of oscillation

$$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{1200 \text{ N m}^{-1}}{3 \text{ kg}}} = 20 \text{ s}^{-1}$$

Maximum speed, $v_{max} = A\omega = (0.05 \text{ m})(20 \text{ s}^{-1}) = 1 \text{ m s}^{-1}$

$= 1 \text{ m s}^{-1}$

32. (c) : As the two identical springs are connected in parallel,

$\therefore$ The effective spring constant of the combination is

$k' = k + k = 2k$

$$\therefore \quad T = 2\pi\sqrt{\frac{M}{k'}} = 2\pi\sqrt{\frac{M}{2k}} \quad \text{...(i)}$$

When a block of mass m is placed in the tray, the period of oscillation becomes

$$T' = 2\pi\sqrt{\frac{M+m}{k'}} = 2\pi\sqrt{\frac{M+m}{2k}} \quad \text{...(ii)}$$

Divide (ii) by (i), we get

$$\frac{T'}{T} = \sqrt{\frac{M+m}{M}}$$

$$\frac{3}{1.5} = \sqrt{\frac{10+m}{10}}$$

Squaring both sides, we get

$$4 = \frac{10+m}{10} \text{ or } 40 = 10 + m \text{ or } m = 30 \text{ kg}$$

33. (d) : Amplitude of a damped oscillator at any instant t is

$A = A_0 e^{-bt/2m}$, where A_0 is the original amplitude.

When $t = 2$ s, $A = \frac{A_0}{3}$

$$\therefore \quad \frac{A_0}{3} = A_0 e^{-2b/2m} \quad \text{or} \quad \frac{1}{3} = e^{-b/m} \quad \text{... (i)}$$

When $t = 6$ s, $A = \frac{A_0}{n}$ $\therefore$ $\frac{A_0}{n} = A_0 e^{-6b/2m}$

or $\frac{1}{n} = e^{-3b/m} = (e^{-b/m})^3 = \left(\frac{1}{3}\right)^3$ $\therefore$ $n = 3^3$ (Using (i))

34. (a) : As $y = Kt^2$ $\therefore$ $v = \frac{dy}{dt} = 2Kt$

and $a = \frac{d^2y}{dt^2} = 2K = 2 \times 1 = 2 \text{ m s}^{-2}$ $(\because K = 1 \text{ m s}^{-2})$

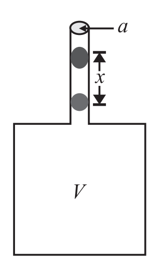
Since the point of suspension of pendulum is moving upwards with acceleration, so effective acceleration due to gravity on the pendulum is

$g' = (g + a) = 10 + 2 = 12 \text{ m s}^{-2}$

$$\therefore \quad T_1 = 2\pi\sqrt{\frac{l}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{l}{g'}}$$

Thus, $\frac{T_1^2}{T_2^2} = \frac{g'}{g} = \frac{12}{10} = \frac{6}{5}$

35. (d) : The situation is as shown in the figure. Let P be pressure of air in the chamber. When the ball is pressed down a distance x, the volume of air decreases from V to say $V - \Delta V$. Hence the pressure increases from P to $P + \Delta P$. The change in volume is



$\Delta V = ax$

The excess pressure ΔP is related to the bulk modulus B as

$$\Delta P = -B\frac{\Delta V}{V}$$

Restoring force on ball = excess pressure × cross-sectional area

or $F = -\frac{Ba}{V}\Delta V$

or $F = -\frac{Ba^2}{V}x$ $(\because \Delta V = ax)$

or $F = -kx$

where $k = \frac{Ba^2}{V}$

i.e., $F \propto -x$

Hence, the motion of the ball is simple harmonic. If m is the ball, the time period of the SHM is

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ or } T = 2\pi\sqrt{\frac{mV}{Ba^2}}$$

36. (c) : The 20 cm length of the scaler reads upto 50 kg.

$\therefore$ $F = mg = (50 \text{ kg})(10 \text{ m s}^{-2}) = 500 \text{ N}$

and $x = 20 \text{ cm} = 0.2 \text{ m}$

$\therefore$ Spring constant, $k = \frac{F}{x} = \frac{500 \text{ N}}{0.2 \text{ m}} = 2500 \text{ N m}^{-1}$

As $T = 2\pi\sqrt{\frac{m}{k}}$

Squaring both sides, we get

$$T^2 = \frac{4\pi^2 m}{k}$$

$$m = \frac{T^2 k}{4\pi^2} = \frac{(0.5 \text{ s})^2 \times (2500 \text{ N m}^{-1})}{4\times(3.14)^2} = 16 \text{ kg}$$

37. (d) : The total mass of the vibrating system

m = mass of mercury + mass of water

= 1 kg + 0.2 kg = 1.2 kg

If the system is disturbed by a small displacement x,

Restoring force = $-m\frac{d^2x}{dt^2}$

Also, the difference of levels in the two arms will be equivalent to the pressure difference of ($2x$) height of mercury (*i.e.*,) the restoring force F will be

$F = (2x)\rho Ag$ where ρ (= 13.6) is the relative density of mercury and A is area of cross-section of the tube.

Hence, $-m\frac{d^2x}{dt^2} = F = 2x\rho Ag \Rightarrow \frac{-md^2x}{dt^2} = (2\rho Ag)x$

Hence the period of the S.H.M will be

$$T = \frac{2\pi}{\omega} = 2\pi\left[\sqrt{\frac{m}{2\rho Ag}}\right]$$

Putting values, $m = 1.2$ kg, $\rho = 13.6 \times 10^3$ kg m^{-3}, $A = \pi(0.6 \times 10^{-2})^2$ m^2, $g = 10$ m s^{-2}

$$T = \sqrt{\frac{(1.2)}{2(13.6\times10^3)(3.14)(0.6\times10^{-2})^2(10)}} \simeq 0.2 \text{ s}$$

38. (d) : In SHM, the acceleration of the particle is directed towards the mean position.

39. (a) : In SHM, acceleration, $a = \frac{d^2x}{dt^2} = -\omega^2 x$

40. (d) : Simple harmonic motion is represented by a sine function or a cosine function or a linear combination of both. Hence, options (a), (b) and (c) represent simple harmonic motion while option (d) is a product of the two functions (sine and cosine) and does not represent a simple harmonic motion.

41. (d) : The time period of simple harmonic motion does not depend on amplitude, energy or the phase constant.

42. (c) : The motion of a planet around the Sun is a periodic motion but not a simple harmonic motion. All other given motions are the examples of simple harmonic motion.

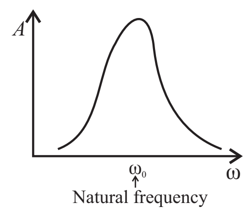
43. (c) : A particle oscillating under a force $\vec{F} = -k\vec{x} - b\vec{v}$ is damped oscillator. The first term $-k\vec{x}$ represents the restoring force and second term $-b\vec{v}$ represents the damping force.

44. (c)

45. (c) : The sharp and flat resonance will depend on damping present in the body executing resonant vibrations. Less the damping, greater will be sharpness.

46. (d)

47. (c) : Resonance occurs when the frequency of the applied force becomes nearly equal to the natural frequency of vibration of the body. During resonance the amplitude of the forced vibration reaches its maximum value.



So, if we increase the frequency of the externally impressed periodic force, the amplitude of the forced vibration does not increase but it decreases. So the given reason is false.

48. (b) : In SHM, $a = -\omega^2 y$. With change in position, acceleration (a) also changes. Thus acceleration is not uniform. Also velocity depends on displacement with change in time ($v = \omega y$).

49. (b) : As water flow out of the ball, the time period first increases and then decreases. In the beginning, when the ball is completely filled with water, the centre of gravity of the pendulum is at the centre of the ball. As water flows out, the centre of gravity of the pendulum begins to shift below the centre of the ball, thus increasing the effective length of the pendulum. Hence the time period increases. When ball is more than half empty then centre of gravity of the pendulum again rises up so that length of the pendulum decreases and time period also decreases.

50. (b) : The time period of simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. On moon, g is much smaller compared to g on earth. Therefore, T increases.

51. (a) : Wrist watch works on spring action, which is independent of gravity effect.

52. (a) : Time period of simple pendulum of length l is,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{... } (i)$$

when length of simple pendulum increases by 3% then

$$l_1 = l + \left(\frac{3}{100}\right)l = \left(\frac{103}{100}\right)l, \quad \frac{\Delta l}{l} = \frac{l_1 - l}{l} = \frac{3}{100}$$

Taking log of eqn. (i) and differentiating it, we get

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100 = 1.5\%$$

Thus increase in time period is 1.5%.

53. (c) : Second's pendulum is a simple pendulum whose time period of vibration is two seconds. Thus frequency $\upsilon = (1/2)\ s^{-1}$. When elevator is moving upwards with acceleration $g/2$, the effective acceleration due to gravity is

$$g' = g + a = g + g/2 = 3g/2$$

As $\upsilon = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ so $\upsilon^2 \propto g$

$$\therefore \quad \frac{\upsilon'^2}{\upsilon^2} = \frac{g'}{g} = \frac{3g/2}{g} = \frac{3}{2} \text{ or } \frac{\upsilon'}{\upsilon} = \sqrt{\frac{3}{2}} = 1.225$$

or, $\upsilon' = 1.225\ \upsilon = 1.225 \times (1/2) = 0.612\ s^{-1}$.

54. (a) : When a simple harmonic system oscillates with a decreasing amplitude with time, its oscillations are called damped oscillations. Energy of damped oscillator at an any instant t is given by

$E = E_0 e^{-bt/m}$ [where $E_0 = \frac{1}{2}kx_0^2$ = maximum energy]

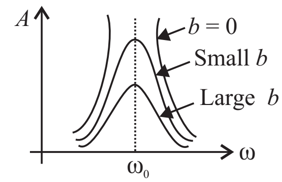
Due to damping forces, the amplitude of oscillator will go on decreasing with time whose energy is expressed by above equation.

55. (c) : Time period of a simple pendulum is inversely proportional to the square root of g, *i.e.* $T \propto 1/\sqrt{g}$. At mountain top, the value of g decreases $\left[g' = g\left(1 - \frac{2h}{R}\right)\right]$.

Hence the value of T increases, *i.e.*, the pendulum will take longer time to complete one vibration. This shows that the pendulum clock will become slow. Hence the pendulum clock will lose time on the mountain top.

56. (b) : From equation, amplitude of oscillation

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}.$$



In the absence of damping force ($b = 0$), the steady state amplitude approaches infinity as $\omega \to \omega_0$. That is, if there is no resistive force in the system and then it is possible to drive an oscillator with sinusoidal force at the resonance frequency, the amplitude of motion will build up without limit. This does not occur in practice because some damping is always present in real oscillation. Due to presence of various dissipative force in the system, the amplitude of oscillation can grow to a large value only but can never be infinite.

The graph showing amplitude as function of frequency for the forced oscillator with varying resistive force.

SUBJECTIVE TYPE QUESTIONS

1. Inertia and elasticity are the properties which are required by a system to oscillate.

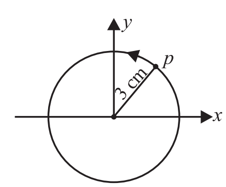
2. (a) 4 times the amplitude (b) zero.

3. The equation of simple harmonic motion is

$x(t) = A\cos(\omega t + \phi)$

Here, ϕ = Phase constant and $\omega t + \phi$ = Phase

4. The amplitude in simple harmonic motion is equal to the radius of the circle of reference. Hence, the amplitude of simple harmonic motion is 3 cm.



5. Given equation for S.H.M., $a = -16x$

$\because \quad \omega^2 = 16 \Rightarrow \omega = \pm 4$ rad s^{-1}

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = 1.57$ s

6. The disturbance set by the explosion reaches the glass panes and sets them into vibration. Sometimes, the frequency of vibration of these panes becomes equal to their natural frequency and resonance occurs. At resonance, the amplitude of vibration becomes large to such an extent that the glass panes break.

7. The energy (E) of a simple harmonically vibrating particle depends upon its : (i) mass, m (ii) frequency, υ and (iii) amplitude A. In fact, (i) $E \propto m$ (ii) $E \propto \upsilon^2$ and (iii) $E \propto A^2$.

8. P.E. or K.E. completes two vibrations in a time during which S.H.M completes one vibration or the frequency of PE. or K.E. is double than that of S.H.M

9. Time period of a second pendulum inside an artificial satellite will be infinity due to the absence of gravitation force.

10. If both pendulums are moving in the same direction, then $\phi = 0°$ and if they are moving in opposite directions, then $\phi = 180°$ or π radian.

11. On the top of a mountain or below the earth, the values of g is less than that on the surface of Earth. With a decrease in the value of g, time period of the simple pendulum increases and accordingly the pendulum loses time.

12. According to question, for two identical pendulums

$E_1 = \frac{1}{2} k a^2$; $E_2 = \frac{1}{2} k (na)^2$

$$\therefore \quad \frac{E_1}{E_2} = \frac{a^2}{n^2a^2} = \frac{1}{n^2}$$

13. Mass of block is $m = \frac{40}{9.8} = 4.082\,\text{kg}$

Angular frequency of driving force is $\omega_d = 2\pi \times 10$
$= 62.8$ rad s^{-1}

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{4.082}} \simeq 7\ \text{rad s}^{-1}$$

$b = 0$ and $A = 0.020$ m (given)

Now, $$A = \frac{F_0/m}{\sqrt{(\omega_d^2 - \omega_0^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

$$\Rightarrow \quad 0.02 = \frac{F_0}{4.082\sqrt{[(62.8)^2 - 7^2]^2 + 0}} \Rightarrow F_0 = 318\ \text{N}$$

14. Comparing the given equation with $x = A\sin(\omega t + \delta)$

$A = 1.5$ m, $\omega = \frac{\pi}{4}$ rad s^{-1}, $m = 2\,\text{kg}$ (given)

$\therefore$ Total energy is

$$E_r = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}(2)\left(\frac{\pi}{4}\right)^2 (1.5)^2 = 1.36\ \text{J}$$

15. A particular type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is termed as simple harmonic motion (SHM).

A body is undergoing simple harmonic motion if it has an acceleration which is

(i) directed towards a fixed point, and

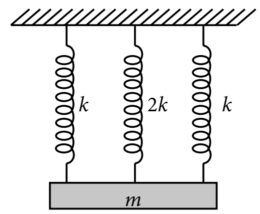
(ii) proportional to the displacement of the body from that point.

acceleration $a \propto -x$ or, $a = -kx$ (where k is any constant)

or $\frac{d^2x}{dt^2} = -kx$, where x = displacement at any instant t

Simple harmonic motion can also be represented as the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

16. Given that, mass $m = 0.08$ kg. Spring factor, $k = 2$ N/m. Effective spring constant for the given parallel combination of springs can be calculated as



$K = k + 2k + k = 4k = 8$ N/m

Hence, new time period is

$$T = 2\pi\sqrt{\frac{m}{K}} = 2 \times 3.14\sqrt{\frac{0.08}{8}} = 0.628\ \text{s}$$

17. $y = \frac{Mg}{k}, U = \frac{1}{2}ky^2 = \frac{1}{2}k\frac{M^2g^2}{k^2} = \frac{1}{2}\cdot\frac{M^2g^2}{k}$

$\therefore$ Total potential energy is $U = U_1 + U_2 + U_3$

or $$U = \frac{1}{2}M^2g^2\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)$$

18. $V_{max} = A\omega = 0.04$ m s^{-1} ...(i)

$x = A\sin\omega t \Rightarrow a = -A\omega^2 \sin\omega t$

$\therefore$ $a = -\omega^2 x$ ...(ii)

At $x = 0.02$ m, $|a| = 0.06$ m s^{-2} (given)

$\therefore$ $\omega^2(0.02) = 0.06$

$\Rightarrow$ $\omega = \sqrt{3}$

$\therefore$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}} = 3.63\ \text{s}$ ...(iii)

From (i) and (ii)

$$A = \frac{0.04}{\omega} = \frac{0.04}{\sqrt{3}} = 2.31 \times 10^{-2}\text{m}$$

19. Although length of the spring does not appear in the expression for the time period, yet the time period depends on the length of the spring. It is because, force constant of the spring depends on the length of the spring.

20. The slope of given graph is

$$-\tan 30° = -1/\sqrt{3}$$

$\therefore$ $\frac{x}{a} = -\frac{1}{\sqrt{3}}$

We know in SHM, $a = -\omega^2 x \Rightarrow \frac{x}{a} = -\frac{1}{\omega^2}$

$\therefore$ $\frac{1}{\omega^2} = \frac{1}{\sqrt{3}} \Rightarrow \omega = 3^{1/4}$ $\quad\therefore$ $T = \frac{2\pi}{3^{1/4}}$

21. $x_1 = A\sin(\omega t + \delta_1)$

At $t = 0$, $x = \frac{A}{\sqrt{2}}$

$\Rightarrow \quad \frac{A}{\sqrt{2}} = A\sin[\omega(0) + \delta_1]$

$\sin\delta_1 = \frac{1}{\sqrt{2}} \Rightarrow \delta_1 = \frac{\pi}{4}$

$x_2 = A\sin(\omega t + \delta_2)$

At $t = 0$,

$\Rightarrow \quad -\frac{\sqrt{3}}{2}A = A\sin[\omega(0) + \delta_2]$; $\sin\delta_2 = -\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \delta_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$$\delta_2 - \delta_1 = \frac{4\pi}{3} - \frac{\pi}{4} = \left(\frac{16-3}{12}\right)\pi = \frac{13}{12}\pi$$

22. A particular type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is termed as simple harmonic motion (SHM).

A body is undergoing simple harmonic motion if it has an acceleration which is

– directed towards a fixed point, and

– proportional to the displacement of the body from that point.

acceleration $a \propto -x \Rightarrow a = -kx$

or $\frac{d^2x}{dt^2} = -kx$, where x = displacement at any instant t.

Energy, $E_k = \frac{1}{2}m\omega^2(a^2 - x^2)$,

(Here a is amplitude of motion)

$E_p = \frac{1}{2}m\,\omega^2 x^2$

As, $E_k = E_p$

$\therefore \quad \frac{1}{2}m\,\omega^2(a^2 - x^2) = \frac{1}{2}m\,\omega^2 x^2$

or $2x^2 = a^2$ or $x = \pm\frac{a}{\sqrt{2}}$

23. At displacement x, $U_1 = \frac{1}{2}m\omega^2 x^2$

$\Rightarrow \quad x = \sqrt{\frac{2U_1}{m\omega^2}}$

Similarly, $y = \sqrt{\frac{2U_2}{m\omega^2}}$

When displacement is $(x + y)$, potential energy is

$$U = \frac{1}{2}m\omega^2(x+y)^2 = \frac{1}{2}m\omega^2(x^2 + y^2 + 2xy)$$

$$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2y^2 + m\omega^2xy$$

$$= U_1 + U_2 + 2\sqrt{U_1U_2}$$

24. PE of the oscillator at displacement y,

$U = \frac{1}{2}m\omega^2y^2$

Maximum energy of the oscillator, $E = \frac{1}{2}m\omega^2A^2$

According to question, $U = \frac{1}{2}E$

$\frac{1}{2}m\omega^2y^2 = \frac{1}{4}m\omega^2A^2 \Rightarrow y^2 = \frac{A^2}{2}$

$\Rightarrow y = \pm\frac{A}{\sqrt{2}}$

25. Angular frequency of spring – block system is given by

$\omega = \sqrt{k/m}$

Maximum speed in oscillation,

$v_{max} = A\omega = A\sqrt{k/m}$

Here, A = 20 cm = 0.2 m,

k = 600 N m^{-1}, m = 10 kg, v_{max} = ?

$v_{max} = 0.2\sqrt{\frac{600}{10}} = 0.2\sqrt{60}$ m s^{-1}

26. Velocity, $v = \omega\sqrt{r^2 - y^2}$

At, $y = s$, let $v = v_0$, then $v^2_0 = \omega^2(r^2 - s^2)$ ...(i)

Due to blow, the new velocity at $y = s$ is $v = 2v_0$

Let $r = r'$, therefore, from (i)

$(2v_0)^2 = \omega^2(r'^2 - s^2)$ ...(ii)

Dividing (ii) by (i), we have, $\frac{r'^2 - s^2}{r^2 - s^2} = 4$

On solving, $r' = \sqrt{4r^2 - 3s^2}$

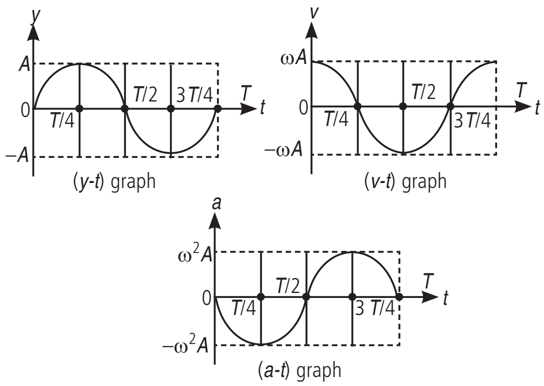
27. If E and E_0 represent the mechanical energy of the damped and free oscillations respectively, then as per question $\frac{E}{E_0} = e^{-3t/m}$

As mechanical energy $\propto$ (amplitude)^2

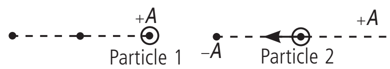
$\therefore$ The ratio of the amplitudes of the damped oscillations to that of free oscillations is

$$\frac{A}{A_0} = \left(\frac{E}{E_0}\right)^{1/2} = (e^{-3t/m})^{1/2} = e^{-3t/2m}$$

28.



29. If this instant is taken as $t = 0$. Then, initial phase for particle (1) is $\delta_1 = \dfrac{\pi}{2}$



and initial phase for particle (2) is $\delta_2 = \pi$.

$\therefore \quad x_1 = A\sin\left(\omega t + \dfrac{\pi}{2}\right)$

and $x_2 = A\sin(\omega t + \pi)$

When both of them have same displacement,

$$\therefore \quad A\sin\left(\omega t + \frac{\pi}{2}\right) = A\sin(\omega t + \pi)$$

or $\quad \cos \omega t = -\sin\omega t \quad$ or $\tan\omega t = -1$

or $\quad \dfrac{3\pi}{4} = \omega t \quad$ or $\dfrac{2\pi}{T}t = \dfrac{3\pi}{4} \Rightarrow t = \dfrac{3T}{8}$

30. (a) Here, $x = -2\sin\left(3t + \dfrac{\pi}{3}\right)$

$$= 2\cos\left[\frac{\pi}{2} + \left(3t + \frac{\pi}{3}\right)\right] = 2\cos\left(3t + \frac{5\pi}{6}\right)$$

$\therefore \quad A = 2$ cm, $\phi = 5\pi/6$ and $\omega = 3$ rad s^{-1}

Therefore, at $t = 0$, the particle is at the point P, such that, $\phi = \angle POX = \dfrac{5\pi}{6}$ as shown in figure (i).

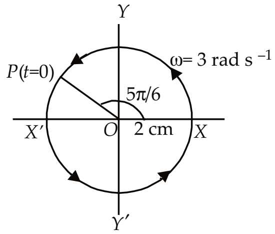


Figure (i)

(b) Here, $x = \cos(\pi/6 - t) = \cos(t - \pi/6)$

$[\because \cos(-\theta) = \cos\theta]$

$\therefore \quad A = 1$ cm, $\phi = -\pi/6$, $\omega = 1$ rad s^{-1}

Therefore, at $t = 0$, the particle is at the point P, such that $\phi = \angle POX = -\dfrac{\pi}{6}$ as shown in figure (ii).

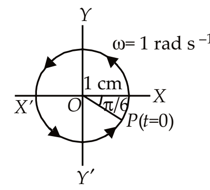


Figure (ii)

31. In the given figures,

For (a) $\quad T_a = 2\pi\sqrt{\dfrac{m}{k}}$

For (b) $\quad T_b = 2\pi\sqrt{\dfrac{m}{k_{eq}}} \qquad \left\{\text{Here } k_{eq} = \dfrac{k^2}{2k}\right\}$

$$T_b = 2\pi\sqrt{\frac{m}{k}2} = \sqrt{2} \times 2\pi\sqrt{\frac{m}{k}}$$

For (c) $\quad T_c = 2\pi\sqrt{\dfrac{m}{k_{eq}}} \qquad$ {Here $k_{eq} = 2k$}

$$T_c = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}} \times 2\pi\sqrt{\frac{m}{k}}$$

$\therefore \quad$ Ratio $T_a : T_b : T_c$ is $1 : \sqrt{2} : 1/\sqrt{2}$

32. (a) As conditions is $KE = 2PE$

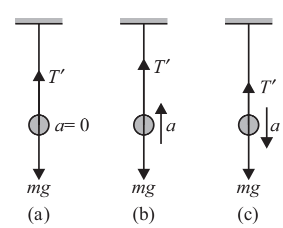
$$\frac{1}{2}m\omega^2(A^2 - x^2) = 2 \times \frac{1}{2}m\omega^2 x^2$$

$$A^2 - x^2 = 2x^2 \Rightarrow x = \pm\frac{A}{\sqrt{3}}$$

The ± signs represent the distance of particle on either side.

(b) This study shows that sum total of $P.E$ and $K.E$ of a particle in SHM stays constant at all positions and at all times. However, $P.E$ and $K.E$ both keep on changing with position or time. The same is true in day to day life. We can acquire one form of energy by spending some other form of energy and vice-versa.

33. (i) When the lift goes up figure (a) with uniform velocity v, tension in the string, $T' = mg$. The value of g remains unaffected.



The period T remains same as that in stationary lift, *i.e.*,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(ii) When the lift goes up with acceleration a as shown in figure (b), the net upward force on the bob is $T' - mg = ma \Rightarrow T' = m(g + a)$

The effective value of g is $(g + a)$ and the time period is

$$T_1 = 2\pi\sqrt{\frac{l}{g+a}}$$

Clearly, $T_1 < T$, *i.e.*, time period decreases.

(iii) When lift comes down with acceleration a figure (c), the net downward force on the bob is

$$mg - T' = ma \Rightarrow T' = m(g - a)$$

The effective value of g becomes $(g - a)$ and the time period is

$$T_2 = 2\pi\sqrt{\frac{l}{g-a}}$$

Clearly, $T_2 > T$, *i.e.*, time period increases.

34. Original frequency,

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

Let A = Initial amplitude of oscillation

v = Velocity of mass M when passing through mean position.

Maximum kinetic energy = Total energy

or $\frac{1}{2}Mv^2 = \frac{1}{2}kA^2 \quad \therefore \quad v = \sqrt{\frac{k}{M}}\,A$

When mass m is put on the system,

total mass = $(M + m)$. If v' is the velocity of the combination in equilibrium position, then by the conservation of linear momentum,

$$Mv = (M + m)v' \quad \text{or} \quad v' = \frac{Mv}{M+m}$$

If A' is the new amplitude, then

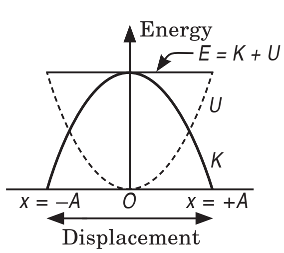
$$\frac{1}{2}(M+m)v'^2 = \frac{1}{2}kA'^2$$

or $$A' = \sqrt{\frac{M+m}{k}} \times v' = \sqrt{\frac{M+m}{k}} \times \frac{Mv}{M+m}$$

$$= \sqrt{\frac{M+m}{k}} \times \frac{M}{M+m} \times \sqrt{\frac{k}{M}}A = \sqrt{\frac{M}{M+m}}\,A$$

New frequency, $\upsilon' = \frac{1}{2\pi}\sqrt{\frac{k}{M+m}}$

35. (a) The variations of kinetic energy K, potential energy U and total energy E with displacement x. The graphs for K and U are parabolic while that for E is a straight line parallel to the displacement axis. At $x = 0$, the energy is all kinetic and for $x = \pm A$, the energy is all potential.



(b) The energy of a harmonic oscillator is partly kinetic and partly potential. When a body is displaced from its equilibrium position by doing work upon it, it acquires potential energy. When the body is released, it begins to move back to equilibrium position, thus acquires kinetic energy.

At any instant, the displacement of a particle executing SHM is given by

$$x = A\cos(\omega t + \phi_0)$$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi_0)$

Hence, kinetic energy of the particle at any time t is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi_0)$$

But $A^2\sin^2(\omega t + \phi_0) = A^2[1 - \cos^2(\omega t + \phi_0)]$

$= A^2 - A^2\cos^2(\omega t + \phi_0) = A^2 - x^2$

or $K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$

When the displacement of a particle from its equilibrium position is x, the restoring force acting on it is

$$F = -kx$$

If we displace the particle further through a small distance dx, then work done against the restoring force is given by

$$dW = -Fdx = +\,kxdx$$

The total work done in moving the particle from mean position ($x = 0$) to displacement x is given by

$$W = \int dW = \int_0^x kxdx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

The work done against the restoring force is stored as the potential energy of the particle. Hence potential energy of a particle at displacement x is given by

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi_0)$$

At any displacement x, the total energy of a harmonic oscillator is given by

$$E = K + U = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}kx^2$$

or $E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2 = 2\pi^2m\upsilon^2A^2 \quad (\because \omega = 2\pi\upsilon)$

Thus the total mechanical energy of a harmonic oscillator is independent of time or displacement.

Hence in the absence of any frictional force, the total energy of a harmonic oscillator is conserved.

36. By comparing the given equation with the general equation for SHM along X-axis, *i.e.*,

$x = A\cos(\omega t + \phi_0)$, we get

$A = 4.00\ m$, $\omega = \pi$ rad/s, $\phi_0 = \pi/4$

(a) Displacement at t = 1.00 s, *i.e.*,
$x = (4.00\ m) \cos(\pi \times 1 + \pi/4)$
$= (4.00)(-\cos \pi/4) = (4.00)(-0.707) = -2.8$ m

(b) Velocity at t = 1.00 s, *i.e.*, $v = -\omega A \sin(\omega t + \phi_0)$

or $v = -(\pi)(4.00) \sin[\pi \times 1 + \pi/4]$m/s

$= -(\pi)(4.00) \sin\left(\frac{5\pi}{4}\right)$m/s

$= (4\pi) \times 1/\sqrt{2} = 8.87$ m/s

(c) $a = -\omega^2 A \cos(\omega t + \phi_0)$
$= -\pi^2 \times 4.00 \cos(\pi \times 1 + \pi/4)$
$= -(4.00\ \pi^2)(-\cos \pi/4)$ m/s^2
$= 4.00 \times (3.14)^2 \times 0.707$ m/s^2 = 27.9 m/s^2

(d) Maximum velocity,
$v_{max} = \omega A = \pi \times 4.00 = 12.56$ m/s
Maximum acceleration, $a_{max} = \omega^2 A$
$= \pi^2 \times 4.00 = 39.4$ m/s^2

(e) Phase, $(\omega t + \phi_0) = (\pi/s) \times 2s + \frac{\pi}{4}$

$$= 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$$

37. Let at any position, x be the extension in the spring and v be the velocity of centre of mass of the cylinder. Then,

P.E. of spring, $U = \frac{1}{2}kx^2$

K.E. of translation, $K_T = \frac{1}{2}Mv^2$

K.E. of rotation, $K_R = \frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$ and $v = \omega r$

$$\left[\because I = \frac{1}{2}Mr^2\right]$$

Total mechanical energy of the system is
$E = U + K_T + K_R =$

$$\frac{1}{2}kx^2 + \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{1}{2}kx^2 + \frac{3}{4}Mv^2 \quad \text{...(i)}$$

As per question, $v = 0$ if $x = 0.25$ m, then

$$E = \frac{1}{2} \times 3 \times (0.25)^2 + 0 = \frac{3}{32}\text{ J}$$

At equilibrium position, $U = 0$ [as $x = 0$],
so from (i)

$$\frac{3}{32} = 0 + \frac{3}{4}Mv^2 \quad \text{or} \quad Mv^2 = \frac{1}{8}\text{ J}$$

At equilibrium position

(a) Translational K.E. = $\frac{1}{2}Mv^2 = \frac{1}{2}\left(\frac{1}{8}\right) = \frac{1}{16}$ J

(b) Rotational K.E. = $\frac{1}{4}Mv^2 = \frac{1}{4}\left(\frac{1}{8}\right) = \frac{1}{32}$ J

(c) In SHM, the total energy is conserved, so $\frac{dE}{dt} = 0$

From (i), we have

$$0 = \frac{1}{2}k.2x\frac{dx}{dt} + \frac{3}{4}M2v\frac{dv}{dt}$$

or $\frac{3}{4}M\frac{d^2x}{dt^2} = -\frac{1}{2}kx$ $\left[\because \frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = \frac{d^2x}{dt^2}\right]$

or $\frac{d^2x}{dt^2} = \frac{-2k}{3M}x$ ...(ii)

From (ii), we note that acceleration $(d^2x/dt^2) \propto x$ and is directed towards equilibrium position. Hence, the cylinder will execute SHM. Comparing (ii) with the equation $\frac{d^2x}{dt^2} = -\omega^2 x$, we have

$$\omega^2 = \frac{2k}{3M} \quad \text{or} \quad \omega = \sqrt{\frac{2k}{3M}}$$

Time period of SHM, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3M}{2k}}$

38. (a) Here mass supported by each wheel, M = 750 kg and x =15 cm = 0.15 m

If k is spring constant of the spring, then restoring force developed on being compressed through a distance x

$F = -kx$

If M is mass supported by each wheel, then

$\therefore$ $kx = Mg$

or $k = \frac{Mg}{x} = \frac{750 \times 9.8}{0.15} = 4.9 \times 10^4$ N m^{-1}

(b) If b is damping constant for the spring and shock absorber system, then damped amplitude of oscillation is given by

$A = A_0 e^{-bT/2M}$

where T is period of oscillation of the spring, A_0 the initial amplitude of the oscillations and M, the mass supported by it. From the above relation we have

$A = A_0 e^{-bt/2M}$

As the amplitude of oscillation decreases by 50% during one complete oscillation,

$$\frac{1}{2}A_0 = A_0 e^{-bt/2M}$$

or $2 = e^{bt/2M}$

$$\log_e 2 = \frac{bt}{2M}\log_e e = \frac{bt}{2M}$$

Now,

$$t = 2\pi\sqrt{\frac{M}{4k}} = 2 \times \frac{22}{7} \times \sqrt{\frac{3000}{4 \times 4.9 \times 10^4}} = \frac{44}{70}\sqrt{\frac{15}{9.8}}$$

$$\therefore b = \frac{2M\log_e 2}{t} = \frac{2 \times 750 \times 0.6931}{\frac{44}{70}\sqrt{\frac{15}{9.8}}} = 1336.99 \text{ kg s}^{-1}$$