Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 21

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. A letter of English alphabets is chosen at random. The probability that the letter chosen is a vowel is **[1]**

a) $\frac{2}{26}$ b) $\frac{4}{26}$

c) $\frac{1}{26}$ d) $\frac{5}{26}$

2. If the points (6, 1), (8, 2), (9, 4) and (p, 3), taken in order are the vertices of a parallelogram, then the value of 'p' is **[1]**

a) 5 b) – 7

c) 6 d) 7

3. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is **[1]**

a) 5 cm b) $\sqrt{7}$ cm

c) 10 cm d) $2\sqrt{7}$ cm

4. In $\triangle$ABC, if $\angle A = x^o$, $\angle B = 3x^o$ and $\angle C = y^o$. If 3y - 5x = 30, then $\angle B$ = **[1]**

a) 60^o b) 45^o

c) 30^o d) 90^o

5. The ratio in which the point (1, 3) divides the line segment joining the points (-1, 7) and (4, -3) is **[1]**

a) 2 : 3
b) 7: 2
c) 3 : 2
d) 2 : 7

6. If x is a positive integer such that the distance between points P (x, 2) and Q (3, -6) is 10 units, then x = **[1]**

a) 3
b) 9
c) -9
d) -3

7. A shuttlecock used for playing badminton is the combination of **[1]**

a) a cylinder and a hemisphere
b) frustum of a cone and a hemisphere
c) a cylinder and a sphere
d) a sphere and a cone

8. Cards marked with numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. The probability that the number on the card is a prime number less than 20 is **[1]**

a) $\frac{8}{25}$
b) $\frac{4}{25}$
c) $\frac{12}{25}$
d) $\frac{2}{25}$

9. A number is selected at random from 1 to 75. The probability that it is a perfect square is **[1]**

a) $\frac{10}{75}$
b) $\frac{8}{75}$
c) $\frac{6}{75}$
d) $\frac{4}{75}$

10. If p and q are the roots of the equation $x^2 + px + q = 0$, then **[1]**

a) p = 1, q = - 2
b) p = - 2, q = 1
c) b = 0, 9 = 1
d) p = - 2, q = 0

11. If α and β are the roots of the equation $3x^2 + 8x + 2 = 0$ then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ = ? **[1]**

a) $\frac{2}{3}$
b) 4

c) -4 | d) $\frac{-3}{8}$

12. If x = a cosθ and y = b sinθ, then the value of $b^2x^2 + a^2y^2$ is **[1]**

a) a + b | b) a^2b^2

c) a - b | d) ab

13. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is **[1]**

a) 5 | b) 11

c) $7 + \sqrt{5}$ | d) 12

14. The product of a rational number and an irrational number is **[1]**

a) both rational and irrational number | b) none of these

c) an irrational number only | d) a rational number only

15. If the length of the shadow of a tower is $\sqrt{3}$ times that of its height, then the angle of elevation of the sun is **[1]**

a) 75° | b) 60°

c) 30° | d) 45°

16. The least positive integer divisible by 20 and 24 is **[1]**

a) 480 | b) 240

c) 360 | d) 120

17. For any collection of n items, $(\sum x) - \overline{x} =$ **[1]**

a) (n - 1)$\overline{x}$ | b) 0

c) n$\overline{x}$ | d) (n + 1)$\overline{x}$

18. **Assertion (A):** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162 **[1]**
Reason: If a, b are two positive integers, then H.C.F. $\times$ L.C.M. = a $\times$ b

a) Both A and R are true and R is the correct explanation of A. | b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. | d) A is false but R is true.

19. One equation of a pair of dependent linear equations is – 5x + 7y = 2, then the second equation can be **[1]**

a) – 10x + 14y + 4 = 0 | b) – 10x – 14y + 4 = 0

c) $10x - 14y + 4 = 0$ d) $10x + 14y + 4 = 0$

20. **Assertion (A):** In the △ABC, AB = 24 cm, BC = 10 cm and AC = 26 cm, then △ABC is a right-angle triangle. **[1]**
Reason (R): If in two triangles, their corresponding angles are equal, then the triangles are similar.

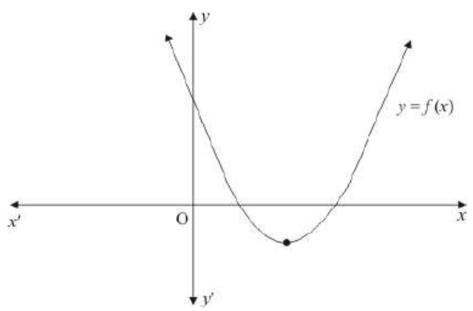
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

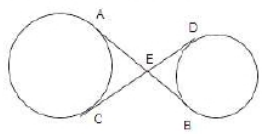
21. Find the probability that a leap year should have exactly 52 tuesday. **[2]**

22. Solve the pair of equations: $\frac{x}{3} + \frac{y}{4} = 4, \frac{5x}{6} - \frac{y}{8} = 4$ **[2]**

23. The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown below. Write the sign of c. **[2]**

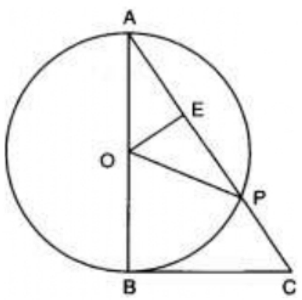


24. In the given figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD. **[2]**



OR

In figure, BC is a tangent to the circle with centre O. OE bisects AP. Prove that ΔAEO ~ ΔABC.



25. Determine whether the given points are vertices of a right triangle: (8, 4),(5, 7) and (–1,1) **[2]**

OR

If the points A(-1, y) and B(5, 7) lie on a circle with centre O(2, -3y), then find the values of y.

Section C

26. Given that 16 cot A = 12; find the value of $\frac{\sin A+\cos A}{\sin A-\cos A}$. **[3]**

27. 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip? **[3]**

OR

Find the LCM and HCF of 26 and 91 and verify that LCM $\times$ HCF = product of two numbers.

28. Form the pair of linear equations for the problem and find their solution by substitution method. The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball. **[3]**

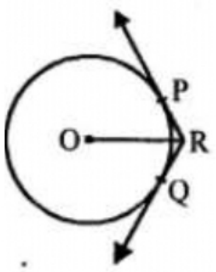
29. AD is a median of ΔABC. The bisector of $\angle$ADB and $\angle$ADC meet AB and AC in E and F respectively. Prove that $EF \| BC$. **[3]**

30. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by $\frac{\tan \alpha \tan \beta}{\tan \alpha+\tan \beta}$. **[3]**

31. In two concentric circles, a chord of length 24 cm of larger circle becomes a tangent to the smaller circle whose radius is 5 cm. Find the radius of the larger circle. **[3]**

OR

In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle$PRQ = 120°, then prove that OR = PR + RQ.



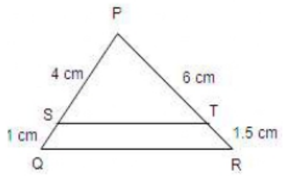
Section D

32. If x = - 2 is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal. **[5]**

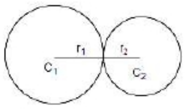
OR

The sum of squares of two consecutive multiples of 7 is 637. Find the multiples.

33. In the given figure, PS, SQ,PT and TR are 4 cm,1 cm,6 cm and 1.5 cm,respectively. Prove that ST || QR. **[5]**



34. Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers? **[5]**



OR

A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions.

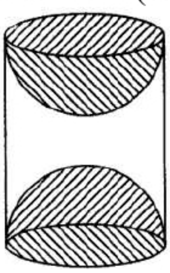
35. Find the missing frequencies in the following distribution, if the sum of the frequencies is 120 and the mean is 50. **[5]**

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	f_1	32	f_2	19

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Find the volume of the cylindrical block before the carpenter started scooping the hemisphere from it.

(ii) Find the volume of wood scooped out?

(iii) Find the total surface area of the article?

OR

Find the total surface area of cylinder before scooping out hemisphere?

37. **Read the text carefully and answer the questions:** **[4]**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.

Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 30th instalment?

(ii) If he increases the instalment by 100 rupees every month, then what amount of loan does he still have to pay after 30th instalment?

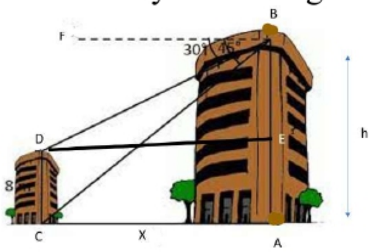
OR

If he increases the instalment by 200 rupees every month, then what amount would he pay in 40th instalment?

(iii) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 100th instalment?

38. **Read the text carefully and answer the questions:** **[4]**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^{o} and 45^{o}, respectively.



(i) Now help Vinod and Basant to find the height of the multistoried building.

(ii) Also, find the distance between two buildings.

OR

Find the distance between top of multistoried building and top of first building.

(iii) Find the distance between top of multistoried building and bottom of first building.

SOLUTION

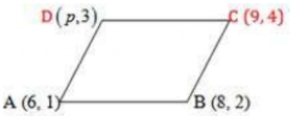
Section A

1. **(d)** $\frac{5}{26}$

   Explanation: We know that "A, E, I, O, U" are vowels
   Number of vowels = 5
   Number of possible outcomes = 5
   Number of total outcomes = 26
   $\therefore$ Required Probability = $\frac{5}{26}$

2. **(d)** 7

   Explanation: In parallelogram, AB = CD, squaring both sides

   $\Rightarrow AB^2 = CD^2$
   $\Rightarrow (8-6)^2 + (2-1)^2 = (p-9)^2 + (3-4)^2$
   $\Rightarrow 4 + 1 = p^2 + 81 - 18p + 1$
   $\Rightarrow p^2 - 18p + 77 = 0$
   $\Rightarrow (p-7)(p-11) = 0$
   $\Rightarrow p = 7$ and $p = 11$

3. **(d)** $2\sqrt{7}$ cm

   Explanation: Radius of the circle =6 cm
   and distance of the external point from the centre =8 cm
   Length of tangent = $\sqrt{\{(8)^2 - (6)^2\}}$
   $= \sqrt{(64-36)} = \sqrt{28}$
   $= \sqrt{(4 \times 7)} = 2\sqrt{7}cm$

4. **(d)** 90^o

   Explanation: Here, A = x, B = 3x, C = y.
   180 = 4x + y ... (i) (Sum of angles of a triangle = x + 3x + y)
   180 - 4x = y ... (ii)
   Also, 3y - 5x = 30 ... (iii)
   Substituting the value of (ii) in (iii)
   3(180 - 4x) - 5x = 30
   540 - 12x - 5x = 30
   -17x = 30 - 540
   17x = 510
   x = 30 ... (iv)
   But, angle B = 3x ... (Given)
   Therefore angle B = 3 $\times$ 30 = 90^o

5. **(a)** 2 : 3

   Explanation: Given: (x, y) = (1, 3), (x_1, y_1) = (-6, 10), (x_2, y_2) = (3, -8)
   Let $m_1 : m_2 = k : 1$

$\therefore$ x = $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

1 = $\frac{k\times4+1\times(1)}{k+1}$

k + 1 = 4k - 1

$\Rightarrow$ k = $\frac{2}{3}$

Therefore, the required ratio is 2 : 3

6. **(b)** 9

Explanation: Distance between P(x, 2) and Q(3, -6) = 10 units

$\Rightarrow \sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}=10$

$\Rightarrow \sqrt{(3-x)^2+(-6-2)^2}=10$

$\Rightarrow \sqrt{(3-x)^2+(-8)^2}=10$

$\Rightarrow \sqrt{(3-x)^2+64}=10$

Squaring both sides,

$(3 - x)^2 + 64 = 100$

$\Rightarrow 9 + x^2 - 6x + 64 - 100 = 0$

$\Rightarrow x^2 - 6x - 27 = 0$

$\Rightarrow x^2 - 9x + 3x - 27 = 0 \left\{\begin{array}{r}\because -27=-9\times3\\ -6=-9+3\end{array}\right\}$

$\Rightarrow$ x(x - 9) + 3(x - 9) = 0

(x - 9) (x + 3) = 0

Either x - 9 = 0, then x = 9 or x + 3 = 0, then x = -3

x is positive integer

Hence x = 9

7. **(b)** frustum of a cone and a hemisphere

Explanation: A shuttlecock used for playing badminton is a combination of a frustum of a cone and a hemisphere.

8. **(d)** $\frac{2}{25}$

Explanation: Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19 = 8

Number of possible outcomes = 8

Number of total outcomes = 100

$\therefore$ Required Probability = $\frac{8}{100}=\frac{2}{25}$

9. **(b)** $\frac{8}{75}$

Explanation: Number of possible outcomes = {1, 4, 9, 16, 25, 36, 49, 64} = 8

Number of Total outcomes = 75

$\therefore$ Probability (of getting a perfect square) = $\frac{8}{75}$

10. **(a)** p = 1, q = - 2

Explanation: Given sum of roots, S = p + q = – p and product pq = q

$\Rightarrow$ q(p – 1) = 0 i.e. q = 0 or p = 1

Now If q = 0 then p = 0, this implies p = q

If p = 1, then p + q = – p

q = – 2p

q = – 2(1)

q = – 2

11. **(c)** -4

Explanation: We have $\alpha + \beta = \frac{-8}{3}$ and $\alpha\beta = \frac{2}{3}$.

$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{(\alpha+\beta)}{\alpha\beta} = \frac{-8}{3} \times \frac{3}{2} = -4.$

12. **(b)** a^2b^2

Explanation: Given: x = a cosθ and y = b sinθ

$\therefore b^2x^2 + a^2y^2$

$= b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2$

$= b^2a^2\cos^2\theta + a^2b^2\sin^2\theta$

$\Rightarrow b^2x^2 + a^2y^2$

$= a^2b^2(\cos^2\theta + \sin^2\theta)$

$= a^2b^2$

$[\because \sin^2\theta + \cos^2\theta = 1]$

13. **(d)** 12

Explanation: Given: the vertices of a triangle ABC, A(0, 4), B (0, 0) and C (3, 0).

$\therefore$ Perimeter of triangle ABC = AB + BC + AC

$= \sqrt{(0-0)^2 + (0-4)^2} + \sqrt{0-3)^2 + (0-0)^2} + \sqrt{0-3)^2 + (4-0)^2}$

$= \sqrt{0+16} + \sqrt{9+0} + \sqrt{9+16}$

$= \sqrt{16} + \sqrt{9} + \sqrt{25}$

= 4 + 3 + 5 = 12 units

14. **(a)** both rational and irrational number

Explanation: The product of a rational number and an irrational number can be either a rational number or an irrational number.

e.g $\sqrt{5} \times \sqrt{2} = \sqrt{10}$ which is irrational

but $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$ which is a rational number

Thus, the product of two irrational numbers can be either rational or irrational similarly, the product of rational and irrational numbers can be either rational or irrational

$5 \times \sqrt{2} = 5\sqrt{2}$ which is irrational.

but $0 \times \sqrt{3} = 0$ which is rational.

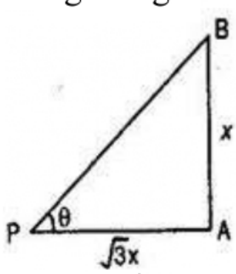
15. **(c)** 30°

Explanation: Let AB be the tree and AP be the shadow.

Let AB = x meters. Then AP = $x\sqrt{3}$ meters

Also $\angle$APB = θ

In right angled triangle ABP



$\tan\theta = \frac{AB}{AP}$

$\Rightarrow \tan\theta = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^\circ \Rightarrow \theta = 30^\circ$ Therefore, the angle of elevation of the Sun is 30°.

16. **(d)** 120

    Explanation: Least positive integer divisible by 20 and 24 is
    LCM of (20, 24).

    $20 = 2^2 \times 5$

    $24 = 2^3 \times 3$

    $\therefore$ LCM (20, 24) = $2^3 \times 3 \times 5$ = 120

    Thus 120 is divisible by 20 and 24.

17. **(a)** (n - 1)$\overline{x}$

    Explanation: For any collection of 'n' items, $(\sum x) - \overline{x}$ = n $(\overline{x})$ - $\overline{x}$ = $\overline{x}$ (n - 1)

18. **(d)** A is false but R is true.

    Explanation: $\frac{3072}{16}$ = $192 \neq 162$

19. **(c)** 10x – 14y +4 = 0

    Explanation: If the equation of a pair of dependent linear equations, then

    $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

    Given: a_1 = -5, b_1 = -5and c_1 = 2.

    For satisfying the condition of dependent linear equations, the values of a_2, b_2 and c_2 should be the multiples of the values of a_1, b_1 and c_1.

    $\therefore$ The values would be $a_2 = -5 \times (-2) = 10, b_2 = 7 \times (-2) = -14$ and

    $c_1 = 2 \times (-2) = -4$

    $\therefore$ The second equation can be 10x - 14y = - 4

20. **(b)** Both A and R are true but R is not the correct explanation of A.

    Explanation: We have,

    $AB^2 + BC^2 = (24)^2 + (10)^2$

    $= 576 + 100 = 676 = AC^2$

    $AB^2 + BC^2 = AC^2$

    ABC is a right-angled triangle.

    Also, two triangles are similar if their corresponding angles are equal. So, both A and R are true but R is not the correct explanation of A.

Section B

21. Total number of days in a leap year $= 366$,

    Total number of weeks $= 52$

    Number of tuesdays in 52 weeks = 52

    Number of days left after 52 weeks$= 366 - 52 \times 7$

    Exactly 52 tuesdays means that there should not be a tuesday in the remaining 2 days

    Possible outcome of remaining two days

    (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday) or (Sunday, Monday)

    Total possible outcome$= 7$

    Probability of not getting a Tuesday $=\frac{5}{7}$

    $\therefore$ Required probability = $\frac{5}{7}$

22. Given, pair of linear equations is $\frac{x}{3}+\frac{y}{4}=4$

$\Rightarrow 4x+3y=48 \ldots$(i)

and $\frac{5x}{6}-\frac{y}{8}=4$

$\Rightarrow 20x-3y=96 \ldots$(ii)

Now, adding Eqs. (i) and (ii), we get

24x = 144

x = 6

Now, put the value of x in Eq. (i), we get

4 × 6 + 3y = 48

3y = 24

y = 8

Hence, the required values of x and y are 6 and 8, respectively.

23. The parabola $y=ax^2+bx+c$ cuts y-axis.

Putting x = 0 in $y=ax^2+bx+c \Rightarrow y=c$

Clearly, P lies on Oy. Therefore c >0

Hence, the sign of c is positive.

24. Common tangents AB and CD to two circles intersect at E.

As we know that , the tangents drawn from an external point to a circle are equal in length.

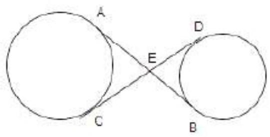
∴ EA = EC....(i)

and EB = ED...(ii)

On adding Eqs (i) and (ii), we get

EA + EB = EC + ED

⇒ AB = CD



OR

According to question we are given that BC is tangent with centre O and OE bisects AP

In ΔAOP,

OA = OP (radii) ΔAOP is an isosceles triangle. OE is a median.

Since a perpendicular to a circle from a center to a chord bisects it.

$\therefore \angle OEA=90^{\circ}$

In ΔAOE and ΔABC,

$\angle ABC=\angle OEA=90^{\circ}$

$\angle A$ is common.

ΔAEO ~ ΔABC .….(AA test)

25. Let the given points (8, 4), (5, 7) and (–1, 1) be denoted by A, B and C respectively.

$$AB=\sqrt{(5-8)^2+(7-4)^2}=\sqrt{9+9}=\sqrt{18}$$

$$BC=\sqrt{(-1-5)^2+(1-7)^2}=\sqrt{36+36}=\sqrt{72}$$

$$AC=\sqrt{(-1-8)^2+(1-4)^2}=\sqrt{81+9}=\sqrt{90}$$

Since $AB^2+BC^2=18+72=90=AC^2$

∴ ΔABC is right angled at B.
Hence the given points are the vertices of a right triangle.

OR

Given that points A(-1, y) and B(5, 7) lie on a circle with centre O(2, -3y).
Distance between two points $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
Now,
OA = OB (radii of a circle)
$\Rightarrow OA^2 = OB^2$
$\Rightarrow (-1 - 2)^2 + [y - (-3y)]^2 = (5 - 2)^2 + [7 - (-3y)]^2$ [using distance formula]
$\Rightarrow (-3)^2 + (y+3y)^2 = (3)^2 + (7+3y)^2$
$\Rightarrow 9 + (4y)^2 = 9 + (49 + 9y^2 + 42y)$
$\Rightarrow 16y^2 = 49 + 9y^2 + 42y$
$\Rightarrow 7y^2 - 42y - 49 = 0$
$\Rightarrow 7(y^2 - 6y - 7) = 0$
$\Rightarrow y^2 - 6y - 7 = 0$
$\Rightarrow y^2 - 7y + y - 7 = 0$
$\Rightarrow y(y-7) + 1(y-7) = 0$
$\Rightarrow (y-7)(y+1)=0$
$\Rightarrow y - 7 = 0$ or $y + 1 = 0$
$\Rightarrow y = 7$ or $y = -1$
When y = 7, Coordinate of A(-1,7) and O(2,-21).
When y = -1, Coordinate of A(-1,-1) and O(2,3).

Section C

26. We have, $16\cot A=12 \Rightarrow \cot A=\frac{12}{16} \Rightarrow \cot A=\frac{3}{4}$

Now, $\frac{\sin A+\cos A}{\sin A-\cos A}=\frac{\frac{\sin A+\cos A}{\sin A}}{\frac{\sin A-\cos A}{\sin A}}$ [Dividing Numerator Denominator by $\sin A$]

$=\frac{\frac{\sin A}{\sin A}+\frac{\cos A}{\sin A}}{\frac{\sin A}{\sin A}-\frac{\cos A}{\sin A}}$ $[\because \frac{cosA}{sinA}=cotA]$

$=\frac{1+\cot A}{1-\cot A}$

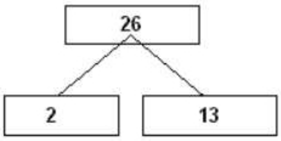
$=\frac{1+\frac{3}{4}}{1-\frac{3}{4}}=\frac{\frac{7}{4}}{\frac{1}{4}}=7\ \left[\because \cot A=\frac{3}{4}\right]$

therefore, $\frac{\sin A+\cos A}{\sin A-\cos A}=7$

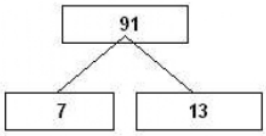
27. **Given:** Number of goats for trip = 105
Number of donkey for trip = 140
Number of cows for trip = 175
Therefore, The largest number of animals in one trip = HCF of 105. 140 and 175.
First consider 105 and 140
By applying Euclid's division lemma, we get
$140 = 105 \times 1 + 35$
$105 = 35 \times 3 + 0$
Therefore, HCF of 105 and 140 = 35
Now consider 35 and 175

Again applying Euclid's division lemma, we get
175 = 35 × 5 +0
HCF of 105, 140 and 175 is 35.
So 35 animals of same kind can go for trip in a single trip and number of trip is
105/35 +140/35 + 175/35 = 12

OR



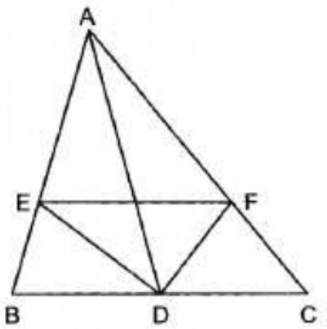
So, 26 = 2 × 13



So, 91 = 7 × 13
Therefore, LCM(26, 91) = 2 × 7 × 13 = 182
HCF(26, 91) = 13
Verification:
LCM × HCF = 182 × 13 = 2366
and 26 × 91 = 2366
i.e., LCM × HCF = product of two numbers.

28. Let the cost of each bat and each ball be Rs.x and Rs. y respectively. Then, according to the equation, The pair of linear equations formed is
7x + 6y = 3800 .... (1)
3x + 5y = 1750 ...... (2)
From equation (2), 5y = 1750 - 3x
$y = \frac{1750-3x}{5}$ .... (3)
Substitute this value of y in equation (1), we get
$7x + 6\left(\frac{1750-3x}{5}\right) = 3800$
$\Rightarrow \quad 35x + 10500 - 18x = 19000$
$\Rightarrow \quad 17x + 10500 = 19000$
$\Rightarrow \quad 17x = 19000 - 10500$
$\Rightarrow \quad 17x = 8500$
$\Rightarrow \quad x = \frac{8500}{17} = 500$
Substituting this value of x in equation (3), we get
$y = \frac{1750-3(500)}{5} = \frac{1750-1500}{5} = \frac{250}{5} = 50$
Hence, the cost of each bat and each ball is Rs.500 and Rs.50 respectively.
Verification,
Substituting x = 500 and y = 50, we find that both the equations (1) and (2) are satisfied as shown below:
$7x + 6y = 7(500) + 6(50)$
$= 3500 + 300 = 3800$
$3x + 5y = 3(500) + 5(50)$
$= 1500 + 250 = 1750$. This verifies the solution.

29. In ΔABC, AD is the median and DE and DF are the bisectors of ∠ADB and ∠ADC respectively, meeting AB and AC in E and F respectively.



TO PROVE $EF \| BC$

PROOF:

In ΔADB, DE is the bisector of $\angle$ADB.

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EB}$...(1)

In ΔADC, DF is the bisector of $\angle$ADC.

$\therefore \quad \frac{AD}{DC} = \frac{AF}{FC}$

$\Rightarrow \quad \frac{AD}{DB} = \frac{AF}{FC}$ ...(2) $\left[\begin{array}{c} \because AD \text{ is the median} \\ \therefore BD = DC \end{array}\right]$

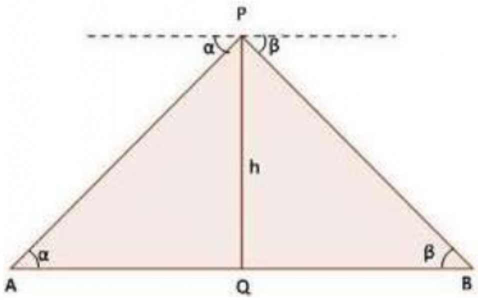
From (i) and (ii), we get

$\frac{AE}{EB} = \frac{AF}{FC}$

Thus, in ΔABC, line segment EF divides the sides AB and AC in the same ratio.

Hence, EF is parallel to BC.

30.



Let h be the height of aeroplane above the road and A and B be two consecutive milestone.

In $\Delta\ AQP\ and\ \triangle BQP$,

$\tan\alpha = \frac{h}{AQ}$ and $\tan\beta = \frac{h}{BQ}$

$\Rightarrow AQ = h\ \cot\alpha$ and $BQ = h\ \cot\beta$

$\Rightarrow AQ + BQ = h\,(\cot\alpha + \cot\beta)$

$AB = h\left(\frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}\right)$ OBJ OBJ OBJ

As, given that AB = 1 mile

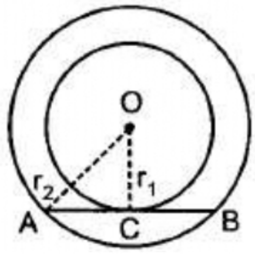
$\Rightarrow h = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$

Hence proved.

31. Given,

$r_1 = 5cm,$

$AB = 24cm$



$\because$ AB is tangent to circle $C(0,r_1)$ at C

$\therefore$ OC $\perp$ AB

In circle $C(0,r_2)$, AB is a chord and

OC $\perp$ AB

$\therefore$ AC = BC

In right $\triangle$OCA,

$OC^2 + AC^2 = AO^2$

$\Rightarrow\ 5^2 + (12)^2 = (r_2)^2$

$\Rightarrow\ 25 + 144 = (r_2)^2$

$\Rightarrow (r_2)^2 = 169$

r_2 = 13 cm

OR

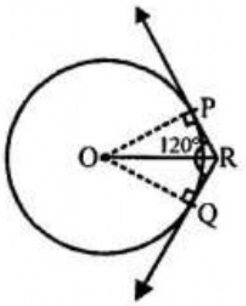
In the given figure, two tangents RQ and RP are drawn from the external point R to the circle with centre O.

$\angle$PRQ = 120°

To prove: OR = PR + RQ

Construction: Join OP and OQ.

Also join OR.



Proof: OR bisects the $\angle$PRQ

$\therefore \angle PRO = \angle QRO = \frac{120^\circ}{2} = 60^\circ$

$\because$ OP and OQ are radii and RP and RQ are tangents.

$\therefore$ OP $\perp$ PR and OQ $\perp$ QR

In right $\triangle$ OPR

$\angle POR = 180^\circ - (90^\circ + 60^\circ)$

$= 180^\circ - 150^\circ = 30^\circ$

Similarly,

$\angle QOR = 30^\circ$

and $\cos\theta = \frac{PR}{OR}$

$\Rightarrow \cos 60^\circ = \frac{PR}{OR} \Rightarrow \frac{1}{2} = \frac{PR}{OR}$

$\Rightarrow$ 2PR=OR ......(i)

Similarly, in right $\triangle$QOR

⇒ 2QR=OR ....(ii)
Adding (i) and (ii)
⇒ 2PR + 2QR = 2OR
⇒ OR = PR + RQ
Hence Proved.

Section D

32. Here x = - 2 is the root of the equation $3x^2+7x+p=0$
then, $3(-2)^2+7(-2)+p=0$
or, p=2
Roots of the equation $x^2+4kx+k^2-k+2=0$ are equal,then,
$16k^2-4\left(k^2-k+2\right)=0$
or, $16k^2-4k^2+4k-8=0$
or, $12k^2+4k-8=0$
or, $3k^2+k-2=0$
or, (3k-2)(k + 1) = 0
or, $k=\frac{2}{3},-1$
Hence, roots = $\frac{2}{3},-1$

OR

According to the question, let the consecutive multiples of 7 be 7x and 7x +7
$(7x)^2+(7x+7)^2=637$
or, $49x^2+49x^2+49+98x=637$
or, $98x^2+98x-588=0$
or, $x^2+x-6=0$
or, (x + 3)(x - 2) = 0
or, x = -3 ,2
Rejecting the value, x=2
Thus,the required multiples are, 14 and 21.

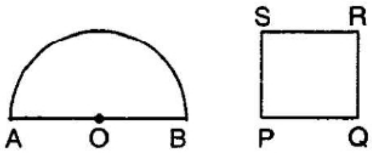
33. In $\triangle PQR$, $\frac{PS}{SQ}=\frac{4}{1}=4$
and $\frac{PT}{TR}=\frac{6}{1.5}=4$
Thus, $\frac{PS}{SQ}=\frac{PT}{TR}$
Hence, ST ∥ QR [by converse of basic proportionality theorem]

34. Let the radii of the two circular plots be r_1 and r_2, respectively.
Then, $r_1 + r_2$ = 14 [∵ Distance between the centres of two circular plots = 14 cm,given]....(i)
Also, Sum of Areas of the plots = 130π
$\therefore \pi r_1^2+\pi r_2^2=130\pi \Rightarrow r_1^2+r_2^2=130$...(ii)
Now, from equation (i) and equation (ii),
$\Rightarrow (14-r_2)^2+r_2^2=130$
$\Rightarrow 196-2r_2+2r_2^2=130$
$\Rightarrow 66-2r_2+2r_2^2=0$
Solving the quadratic equation we get,
$r_2 = 3$ or $r_2 = 11$,
but from figure it is clear that, $r_1 > r_2$

$\therefore r_1 = 11$ cm and $r_2 = 3$ cm

The value depicted by the farmers are of cooperative nature and mutual understanding.

OR



Let radius of semicircular region be r units.

Perimeter = $2r + \pi r$

Let side of square be x units

Perimeter = 4x units.

A.T.Q, $4x = 2r + \pi r \Rightarrow x = \frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$

$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$

$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$

$\Rightarrow r^2(\pi - 2)^2 = 64$

$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$

$\Rightarrow r = \frac{8}{\pi-2} = \frac{8}{\frac{22}{7}-2}$ = 7 cm

Perimeter of semicircle = $2 \times 7 + \frac{22}{7} \times 7$ = 36 cm

Perimeter of square = 36 cm

Side of square = $\frac{36}{4}$ = 9 cm

Area of square = $9 \times 9 = 81$ cm^2

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22}{2\times 7} \times 7 \times 7$ = 77 cm^2

35.

Class Interval	**Frequency f_i**	**Mid-value x_i**	**$f_i x_i$**
0-20	17	10	170
20-40	f_1	30	$30f_1$
40-60	32	50	1600
60-80	f_2	70	$70f_2$
80-100	19	90	1710
	$\sum f_i = 68 + f_1 + f_2 = 120$		$\sum f_i x_i = 3480 + 30f_1 + 70f_2$

given mean = 50

Mean= $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 50 = \frac{3480+30f_1+70f_2}{120}$

$\Rightarrow 6000 = 3480 + 30f_1 + 70f_2$

$\Rightarrow 30f_1 + 70f_2 = 252$ ...(i)

Also,$68 + f_1 + f_2 = 120$

$\Rightarrow f_1 = 52 - f_2$

Substituing in (i), we have

$3(52 - f_2) + 7f_2 = 252$

$\Rightarrow$ 4f_2 =96

$\Rightarrow$ f_2=-24

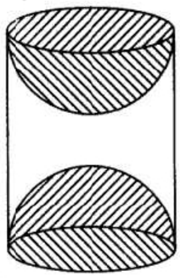
$\Rightarrow$ f_1 =52 - 24 =28

Hence, f_1=28 and f_2=24

Section E

36. **Read the text carefully and answer the questions:**

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Given:

Length of rectangle = 12 cm

Width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

Height of cylinder = 10 cm

Diameter of base = 7 cm

$\Rightarrow$ Radius of base = 3.5 cm

Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7} \times 3.5^2 \times 10$ = 385 cm^3

(ii) Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

$\Rightarrow$ radius of base = 3.5 cm

Volume of wood scooped out = 2 $\times$ volume of hemisphere

$\Rightarrow$ Volume of wood scooped-out = $2 \times \frac{2}{3} \times \pi \times r^3$

$\Rightarrow$ Volume of wood scooped out = $\frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.66 \text{ cm}^3$

(iii)Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

$\Rightarrow$ radius of base = 3.5 cm

Total surface area of the article

$= 2\pi(3.5)(10) + 2\left[2\pi(3.5)^2\right]$

$= 70\pi + 49\pi = 119\pi$

$= 119 \times \frac{22}{7} = 17 \times 22$

= 374 cm^2

OR

Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

$\Rightarrow$ radius of base = 3.5 cm

T.S.A of cylinder = $2\pi r(r + h)$

$\Rightarrow$ T.S.A of cylinder = $2 \times \frac{22}{7} \times 3.5(3.5 + 10) = 99 \text{ cm}^2$

37. **Read the text carefully and answer the questions:**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.

Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month

therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200,

fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
Now, amount paid in the 30th installment,
a_{30} = 1000 + (30 - 1)100 = 3900 {a_n = a + (n - 1)d}

(ii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month=1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
Amount paid in 30 instalments,
$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100] = 73500$
Hence, remaining amount of loan that he has to pay = 118000 - 73500 = 44500 Rupees

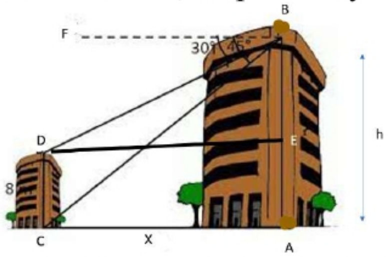
OR

Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
If he increases the instalment by 200 rupees every month, amount would he pay in 40th instalment
Then a = 1000, d = 200 and n = 40
a_{40} = a + (n - 1)d
⇒ a_{40} = 1000 + (40 - 1)200
⇒ a_{40} = 880

(iii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
Amount paid in 100 instalments
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_n = \frac{100}{2}[2 \times 1000 + (100 - 1)100]$
⇒ S_n = 100000 + 9900
⇒ 109900

38. **Read the text carefully and answer the questions:**
Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are

30^{o} and 45^{o}, respectively.



(i) Let h is height of big building, here as per the diagram.

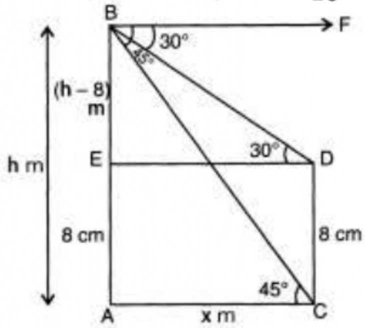
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^{\circ}$

$\angle FBC = \angle BCA = 45^{\circ}$



In $\triangle$ACB, $\angle A = 90^{\circ}$

$\tan 45^{\circ} = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^{\circ}$

$\tan 30^{\circ} = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h($\sqrt{3}$ - 1) = 8$\sqrt{3}$

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m

(ii) Let h is height of big building, here as per the diagram.

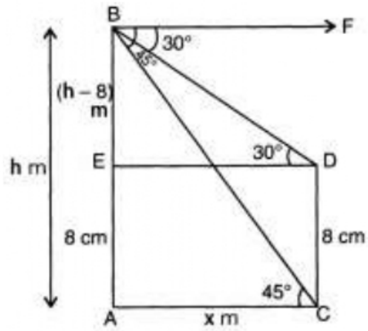
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^{\circ}$

$\angle FBC = \angle BCA = 45^{\circ}$



In △ACB, $\angle A = 90^\circ$
$\tan 45^\circ = \frac{AB}{AC}$
⇒ x = h, ...(i)
In △BDE, $\angle E = 90^\circ$
$\tan 30^\circ = \frac{BE}{ED}$
$\Rightarrow x = \sqrt{3}(h-8)$.(ii)
From (i) and (ii), we get
$h = \sqrt{3}h - 8\sqrt{3}$
h($\sqrt{3}$ - 1) = 8$\sqrt{3}$
h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84)$ = 18.92 m
Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.

OR

In △BDE
cos 30° = $\frac{ED}{BD}$
$\Rightarrow BD = \frac{ED}{\cos 30^0}$
$\Rightarrow BD = \frac{\frac{8\sqrt{3}}{\sqrt{3}-1}}{\frac{\sqrt{3}}{2}} = \frac{16}{\sqrt{3}-1}$
$\Rightarrow BD = 8\left(\sqrt{3}+1\right)$ = 21.86 m
Hence, the distance between top of multistoried building and top of first building is 21.86 m.

(iii) In △ABC
$\sin 45^\circ = \frac{AB}{BC}$
$\Rightarrow BC = \frac{AB}{\sin 45^0}$
$\Rightarrow BC = \frac{18.92}{\frac{1}{\sqrt{2}}}$
⇒ BC = 26.76 m
Hence the distance between top of multistoried building and bottom of first building is 26.76 m.