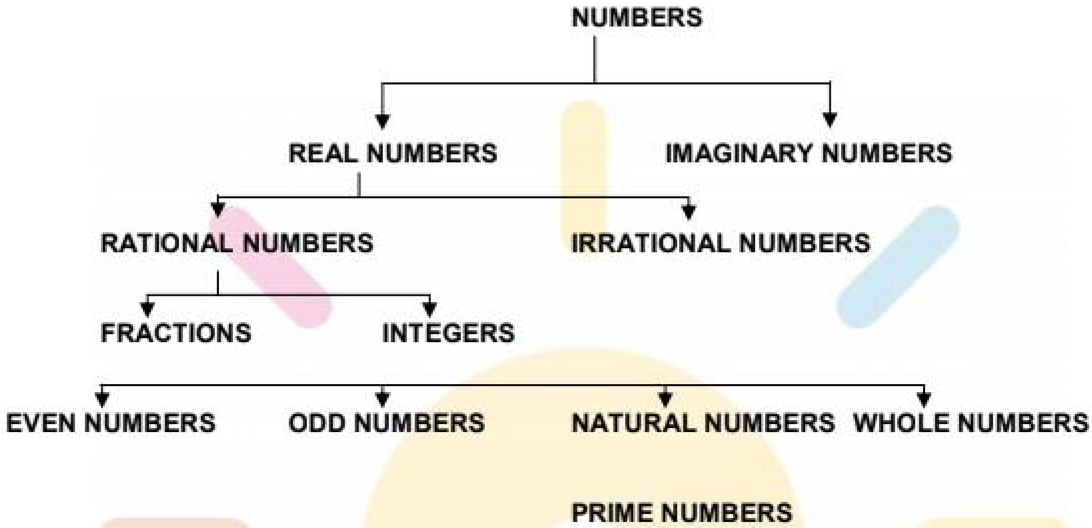
NUMBER SYSTEM

INTRODUCTION: - In our day to life we deal wit different types of numbers which can be broady classified as follows

CLASSIFICATIN OF NUMBERS



(i) **Natural numbers (N) :** Set of all non-fractional numbers from 1 to ∞ **N = {1,2,3,4..... ∞ }**

(ii) **Whole numbers(W) :** Set of all non-fractional numbers from zero to ∞ , **W = {0,1,2,3,4..... ∞ }**

(iii) **Integers (I or Z) :** Set of all non-fractional numbers from **{- ∞ ..... to ..... + ∞ }**

I or Z = { - ∞ .... -3, -2, -1, 0, +1, +2, +3.... + ∞ }

Positive integers : {0, 1, 2 ,3....}, **Negative integers :** {..... -4, -3, -2, -1}

(iv) **Prime numbers :** All natural numbers that have one & itself as their factor are prime numbers.

Ex : 2, 3, 5, 7

(v) **Composite numbers :** All natural numbers which are not prime numbers are composite numbers.

Ex : 4, 6, 8, 9

(1 is neither prime nor composite number.)

A. RATIONAL NUMBERS :-

Rational numbers : - These are real numbers which can be expressed in the form of $\frac{p}{q}$. Where p and q are integers and $q \neq 0$.

Note:

(i) whole numbers and integers are rational numbers.

(ii) Types of rational :- (a) Terminating decimal numbers and **(b)** Non-terminating repeating (recurring) decimal numbers are rational numbers.

Ex. $\frac{2}{3}, \frac{37}{15}, \frac{-17}{19}$, -3 0, 10, 4.33, 7.123123123.....

A-1. FINDING RATIONAL NUMBERS BETWEEN TWO NUMBES :-

Ex -1 Find 4 rational numbers between 4 and5.

Method (i) **a = 4, b= 5, n = 4**

$$\frac{x \times (n+1)}{n+1} = \frac{4 \times (4+1)}{4+1} = \frac{4 \times 5}{5} = \frac{20}{5}$$

$$\frac{b \times (n+1)}{n+1} = \frac{5 \times (4+1)}{4+1} = \frac{5 \times 5}{5} = \frac{25}{5}$$

$$\frac{20}{5}, \left[\frac{21}{5}, \frac{22}{5}, \frac{23}{5}, \frac{24}{5}\right], \frac{25}{5}$$

Ex-2 Find 3 rational number between $\frac{6}{5}, \frac{7}{5}$

Method (ii) $a = \frac{6}{5}, b = \frac{7}{5}, n = 3$

$$\textbf{d} = \left(\frac{n-a}{n+1}\right) = \frac{\frac{7}{5} - \frac{6}{5}}{3+1} = \left(\frac{\frac{7-6}{5}}{4}\right) = \left(\frac{1}{20}\right)$$

$$\textbf{a + d} = \frac{6}{5} + \frac{1}{20} = \frac{24+2}{20} = \frac{25}{20}$$

$$\textbf{a + 2d} = \frac{6}{5} + 2 \times \frac{1}{20} = \frac{24+1}{20} = \frac{25}{20}$$

$$\textbf{a + 3d} = \frac{6}{5} + \frac{3}{20} = \frac{24+3}{20} = \frac{27}{20}$$

$$\frac{24}{20}, \left(\frac{25}{20}, \frac{26}{20}, \frac{27}{20}\right), \frac{28}{20}$$

A-2. RATIINAL NUMBER IN DECIMAL REPRESENTATION :-

(i) Terminating decimal : In this a finits number of digit occours after decimal.

Ex. $\frac{1}{4} = 0.25, \frac{1}{2} = 0.5$

```
4) 1.0000 ( 0.25
    8
   ----
    20
    20
   ----
     0
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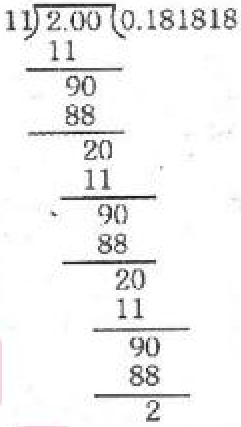
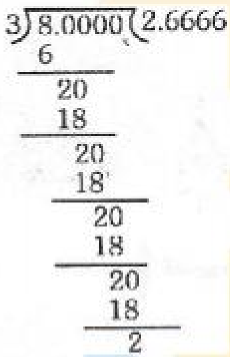
(ii) **Non terminating & Repeating (Recurring decimal) :-** The remainder never become zero.

Ex. $\frac{2}{11} = 0.181818 = \overline{0.18}$



$\therefore \frac{2}{11} = 0.181818...... = 0.\overline{18}$

Ex. $\frac{8}{3} = 2.666666 = 2.\overline{6}$

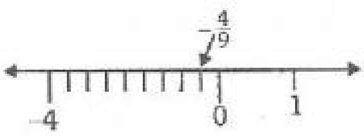


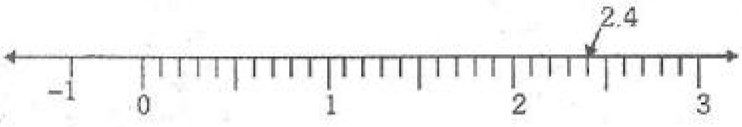
$\therefore \frac{8}{3} = 2.6666..... = 2.\overline{6}$

A-3. **REPRESENTATION OF RATIIONAL NUMBERS ON A NUMBER LINE:-**

(i) **Positive fraction :** $\frac{3}{6}$ Divide a unit into 6 equal parts.

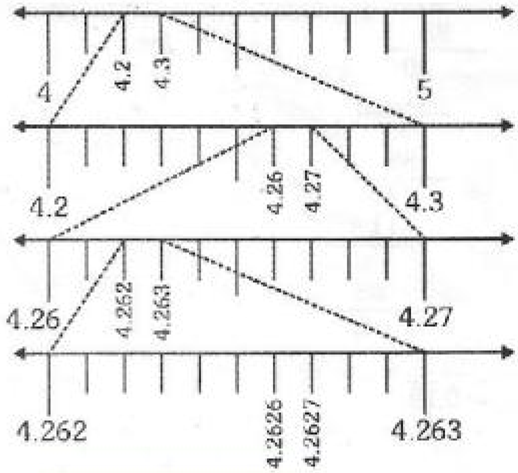
(ii) **Negative fraction:** $-\frac{4}{9}$ divide a unit into 9 equal parts



(iii) **Terminating decimals :** 2.4 

(iv) **Non terminating & repeating decimals :** Visualize $4.\overline{26}$ on the number line, up to 4 decimals places.

$4.\overline{26} = 4.262626....$



A-4. **PROPERTIES OF RATIONAL NUMBERS :-** If $\frac{a}{b}$ and $\frac{c}{d},\frac{e}{f}$ are three rational numbers then.

(i) $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$ [commutative law of additional]

(ii) $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$ [associative law of additional]

(iii) $\left(\frac{a}{b}+\frac{-a}{b}\right)=\left(\frac{-a}{b}+\frac{a}{b}\right)=0,-\frac{a}{b}$ is called the additive inverse of $\frac{a}{b}$

(iv) $\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$ [commutative law of multiplication]

(v) $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\left(\frac{c}{d}\times\frac{e}{f}\right)$ [associative law of multiplication]

(iv) $\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right)$ [distributive law]

A.5. **CONVERSION OF NON TERMINATING & REPEATING DECIMALS NUMBERS TO THE FORM P q :-**

Non-terminating repeating decimals, are basically two types.

(i) **Pure recurring decimals :-** A decimal in which all the digits after the decimal point are repeated these types of decimals are known as pure recurring decimals.

Ex. $0.\overline{6}\,.0.\overline{16}\,0.\overline{123}$ are pure recurring decimals.

(ii) **Mixed recurring decimals : -** A decimal in which at least one of the digits after the decimal point is not repeated and then some digit or digits are repeated. These type of decimals are known as mixed recurring decimals.

Ex. $2.1\bar{6}, 0.3\bar{5}, 0.7\overline{85}$ are mixed recurring decimals.

(a) In order to convert a pure recurring decimal to the form $\frac{p}{q}$, we follow the following steps :-

Step (i) Obtain the repeating decimal and put it equal to x (say).

(ii) Write the number in decimal from by removing bar from the top of repeating digit and listing repeating digits at least twice.

Ex. x = $0.\bar{8}$ Has x = 0.888

(iii) Determine the number of digits having bar on their heads.

(iv) If the repeating decimal has 1 place repetition, multiply by 10; a two place repetition. multiply by 100; a three place repetition, multiply by 1000 and so on.

(v) Subtract the number in step (ii) from the number obtained in step (iv).

(vi) Divide both sides of the equation by the coefficient of x.

(vii) Write the rational number in its simplest form.

Ex. 1 Express of the $0.\bar{3}$ in the form $\frac{p}{q}$:

Sol. Let x = $0.\bar{3}$ Then,

x 0.33333.... ...... (i)

Here we have only one repeating digit after decimal so we multiply both sides by 10.

$\Rightarrow$ 10x = 3.33333... ....(ii)

On subtracting (i) from (ii), we get

10x - x = (3.33333.....) - (0.33333....)

9x = 3

$\Rightarrow \quad x = \frac{3}{9}$

$\Rightarrow \quad 0.\bar{3} = \frac{9}{9}$ i.e., $0.33333..... = \frac{3}{9}$

Ex.2 Show that 1.272727 = $1.\overline{27}$ can be expressed in the form $\frac{p}{q}$ where p and q are integers and q $\neq$ 0.

Sol. Let x = $1.\overline{27}$ Then,

x = 1.27272727.... ..... (i)

Here we have only two repeating digit after decimal so we multiply both sides by 100.

$\Rightarrow$ 100 × = 127.272727....

On subtracting (i) from (ii), we get

99x = (127.272727...) - (1.272727....)

$\Rightarrow$ 99x = 126

$\Rightarrow \quad x = \frac{126}{99} = \frac{14}{11}$

Hence, $1.\overline{27} = \frac{14}{11}$

Ex. 3 Convert the $23.\overline{43}$ number in the form $\frac{p}{q}$.

Sol. Let x = $23.\overline{43}$ Then1

$\Rightarrow$ x = 23.434343..... ....(i)

Here we have only two repeating digit after decimal so we multiply both sides by100.

Multiplying both sides of (i) by 100, we get

100 x = 2343.4343.... ....(ii)

Subtracting (i) from (ii), we get

100x - x = (2343.4343...) - (23.4343....)

$\Rightarrow$ 99x = 2320

$\Rightarrow$ $x = \frac{2320}{99}$

Aliter method :

We have,

$23.\overline{43} = 23 + 0.\overline{43}$

$\Rightarrow$ $23.\overline{43} = 23 + \frac{43}{99}$ [Using the above rule, we have $0.\overline{43} = \frac{43}{99}$]

$\Rightarrow$ $23.\overline{43} = \frac{23 \times 99 + 43}{99}$

$= \frac{2277 + 43}{99} = \frac{2320}{99}$

Ex.4 If $\frac{1}{7} = 0.\overline{142857}$, write the decimal expression of $\frac{2}{7}.\frac{3}{7}.\frac{4}{7}$ and $\frac{5}{7}$ without actually doing the long division.

Sol. Thus, we have

$\frac{2}{7} = 2 \times \frac{1}{7} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}$

(b) In order to convert a mixed recurring decimal to the form $\frac{p}{q}$, we follow the following steps:

(i) Obtain the mixed recurring decimal and write it equal to x (say).

(ii) Determine the number of digits after the decimal point which do not have bar on them. Let there be n digits without bar just after the decimal point.

(iii) Multiply both sides of x by 10^n so that only the repeating decimal is on the right side of the decimal point.

(vi) Use the method of converting pure recurring decimal to the form $\frac{p}{q}$ and obtain the value of x.

Ex. Show that 0.2353535... = $0.2\overline{35}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol. Let x = $0.2\overline{35}$. Then.

10x = $0.2\overline{35}$

$\Rightarrow$ 10x = 2 + $0.\overline{35}$

$\Rightarrow \quad 10x = 2 + \frac{35}{99} \quad \left[\therefore 0.\overline{35} = \frac{35}{99}\right]$

$\Rightarrow \quad 10x = \frac{2 \times 99 + 35}{99}$

$\Rightarrow \quad 10x = \frac{198 + 35}{99} \Rightarrow 10x = \frac{233}{99} \Rightarrow x = \frac{233}{990}$

A.6. DIRECT METHOD :-

$$\left(\frac{p}{q}\right) \text{form} = \frac{(\text{complete number} - \text{number formed by Non-repeating digit})}{\text{No. of 9 as no. of repeating digits after that write no. of 0 as no. of non repeating digits}}$$

Ex. **(i)** $0.\overline{35} = \frac{35 - 0}{99} = \frac{35}{99}$

(ii) $0.4\overline{35} = \frac{435 - 4}{990} = \frac{431}{990}$

A-7. DETERMINING THE NATURE OF THE DECIMAL EXPANSIONS OF RATIONAL NUMBERS:-

(a) Let x be a rational number whose decimal expansion terminates. Then we can expressed x in the form $\frac{p}{q}$, where p and q are co-primes, and the prime factorization of q is of the form $2^m \times 5^n$. where m, n are non-negative integers.

(b) Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^m \times 5^n$ where m, n are non-negative integers. Then, a has a decimal expansion which terminates.

(c) Let $x = \frac{p}{q}$ be rational number, such that the prime factorisation of q is not of the form $2^m \times 5^n$. where m, n are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating.

Ex. **(i)** $\frac{189}{125} = \frac{189}{5^3} = \frac{2^3 \times 189}{2^3 \times 5^3} = \frac{8 \times 189}{(2 \times 5)^3} = \frac{1512}{10^3}$

(ii) $\frac{17}{6} = 2.83333....$

(we observe that the prime factorisation of the dominators of these rational numbers are not of the form $2^m \times 5^n$. where m,n are non-negative integers.)

(iii) $\frac{17}{8} = \frac{17}{2^3 \times 5^0}$

(So, the denominator 8 of $\frac{17}{8}$ is of the form $2^m \times 5^n$. where m,n are non-negative integers.)

Hence $\frac{17}{8}$ has terminating decimal expansion.

(iv) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

(Clearly, 455 is not the $2^m \times 5^n$. So, the decimal expansion $\frac{64}{455}$ is non-terminating repeating.)

TEST OF DIVISIVILITY : A positive integer N is divisible by

(i) 2 if and only if the last digit (nit's digit) is even.
(ii) 3 if and only if the sum of all the digits is divisibly by 3.
(iii) 4 if and only if the number formed by last two digits is divisibly by 4.
(iv) 5 if and only if the last digit is either 0 or 5.
(v) 8 if and only if the number formed by last three digits is divisibly by 8.
(vi) 9 if and only if the sum of all the digits is divisibly by 9.
(vii) 11 if and only if the difference between the sum of digits in the odd places (starting from right) and sum of the digits in the even places (starting from right) is a multiple of 11.
(viii) 25 if and only if the number formed by the last two digits is divisibly by 25.

B. IRRATIONAL NUMBER :-

Irrational numbers:- A number is called irrational number. If it can not be written in the form $\frac{p}{q}$, where & q are integers and $q \neq o$

Ex. $\sqrt{2}, \sqrt{3}, 3\sqrt{2}, 2+\sqrt{3}, \sqrt{2+\sqrt{3}}$.....etc.

(i) Non-terminating & non-repeating decimal numbers are Irrational numbers.

Ex.1 Prove that $\sqrt{2}$ is not a rational number.

Sol. Let us find the square root of 2 by long division methods as shown below.

1.414215

1	2.000000000000
+1	1
24	100
4	96
281	400
+1	281
2824	11900
+4	11296
28282	60400
+2	56564
282841	383600
+1	282841
2828423	10075900
3	8485269
28284265	159063100
+5	141421325
28284270	11641775

$\sqrt{2}$ **= 1.414215**

Clearly, the decimal representation of $\sqrt{2}$ is neither terminating nor repeating.

(a) We shall prove this by the method of contradiction If possible, let us assume that $\sqrt{2}$ is a rational number.

Then- $\sqrt{2} = \frac{a}{b}$ where a, b are integers having no common factor other then 1.

$\Rightarrow \left(\sqrt{2}\right)^2 = \left(\frac{a}{b}\right)^2$ (squaring both sides)

$2 = \frac{a^2}{b^2}$

$a^2 = 2b^2$

$\Rightarrow$ 2 divides a^2

$\Rightarrow$ 2 divides a

Therefore let a = 2c for some integer c.

$\Rightarrow$ $a^2 = 4c^2$

$\Rightarrow$ $2b^2 = 4c^2$

$\Rightarrow$ $b^2 = 2c^2$

$\Rightarrow$ 2 divides b^2

$\Rightarrow$ 2 divides b

Thus, 2 is common factor of a and b.

But, it contradicts out assumption that a and be have no common factor other than 1.

So, our assumption that $\sqrt{2}$ is a rational, is wrong.

Hence, $\sqrt{2}$ is irrational.

Ex.2 Prove that $\sqrt[3]{3}$ is irrational.

So. Let $\sqrt[3]{3}$ be rational $= \frac{p}{q}$ where p and q $\in$ z and p. q have no common factor except 1 also q > 1.

$\therefore \frac{p}{q} = \sqrt[3]{3}$

Cubing both side

$$\frac{p^3}{q^3} = 3$$

Multiply both sides by q^2

$\frac{p^3}{q} = 3q^2$ Hence L.H.S. is rational since p. q have no common factor.

$\therefore p^3$, q also have no common factor while R.H.S. is an integer.

$\therefore$ L.H.S. $\neq$ R.H.S. which contradicts our assumption that $\sqrt[3]{3}$ is Irrational.

Ex.3 Prove that 2 + $\sqrt{3}$ is irrational.

Sol. Let 2 + $\sqrt{3}$ be a rational number equal to r

$\therefore 2 + \sqrt{3} = r$

$\sqrt{3} = r - 2$

Here L.H.S. is an irrational number while R.H.S. r - 2 is rational $\therefore$ L.H.S. $\neq$ R.H.S.

Hence it contradicts our assumption that 2 + $\sqrt{3}$ is rational.

$\therefore$ 2 + $\sqrt{3}$ is irrational.

B-1 PROPERTIES FO IRRATIIONAL NUMBERS : -

(i) Negative of an irrational number is an irrational number. E.g.:- $\sqrt{3}, -\sqrt[4]{5}$ **are irrational.**

(ii) Sum and difference of a rational and an irrational number is an irrational number.

Two number's are 2 and $\sqrt{3}$

Sum = 2 + $\sqrt{3}$ is a irrational number.

Difference = 2 - $\sqrt{3}$, is an irrational number.

Also $\sqrt{3}-2$ is an irrational number.

Ex. Two number's are 4 and $\sqrt[3]{3}$

Sum = 4 + $\sqrt[3]{3}$ is an irrational number.

Difference = 4 - $\sqrt[3]{3}$ is an irrational number.

(iii) Sum and difference of two irrational number is not necessarily an irrational number.

Ex. Two irrational numbers are $\sqrt{3}.\ 2\sqrt{3}$

Sum = $\sqrt{3}+2\sqrt{3}=3\sqrt{3}.$ is an irrational.

Difference = $2\sqrt{3}-\sqrt{3}=\sqrt{3}$ is an irrational.

Ex. Two irrational number are $\sqrt{3}.\ \sqrt{5}$

Sum = $\sqrt{3}+\sqrt{5}.$ is an irrational number.

Difference = $\sqrt{5}-\sqrt{3}$ is an irrational number.

Ex. Two irrational numbers are $\sqrt{3},-\sqrt{3}$

Sum = $\sqrt{3}+\left(-\sqrt{3}\right)=0$, which is rational.

Difference $=\sqrt{3}-\left(-\sqrt{3}\right)=2\sqrt{3}.$ which is irrational

Ex. Two irrational numbers are 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$

Sum = $\left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right)=4$. a rational number.

Two irrational number are $\sqrt{3}+3,\sqrt{3}-3$

Difference $=\sqrt{3}+3-\sqrt{3}+3=6$. a rational number.

Ex. Two irrational numbers are $\sqrt{3}-\sqrt{2}.\sqrt{3}+\sqrt{2}$

Sum = $\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$ an irrational.

(iv) Product of a rational number with an irrational number is not always irrational.

Ex. 2 is a rational number and $\sqrt{3}$ is an irrational

$2\times\sqrt{3}=2\sqrt{3}$, an irrational.

Ex. 0 a rational and $\sqrt{3}$ an irrational

$0\times\sqrt{3}=0$ a rational.

(v) Product of a non-zero rational number with an irrational number is always irrational.

Ex. $\frac{4}{3}\times\sqrt{3}=\frac{4}{3}\sqrt{3}=\frac{4}{\sqrt{3}}$ is an irrational

(vi) Product of an irrational with an irrational is not always irrational.

Ex. **(i)** $\sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3} = \sqrt{9} = 3$ a rational number.

(ii) $2\sqrt{3} \times 3\sqrt{3} = 2 \times 3\sqrt{3 \times 2} = 6\sqrt{6}$ an irrational number.

(iii) $\sqrt[3]{3} \times \sqrt[3]{3^2} = \sqrt[3]{3 \times 3^2} = \sqrt[3]{3^3} = 3$ a rational number

(iv) $\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right) = (2)^2 - \left(\sqrt{3}\right)^2$ = 4 - 3 = 1 a rational number.

(v) $\left(2 + \sqrt{3}\right)\left(2 + \sqrt{3}\right) = \left(2 + \sqrt{3}\right)^2 - (2)^2 + \left(\sqrt{3}\right)^2 + 2(2) \times \left(\sqrt{3}\right) = 4 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3}$ an irrational number.

IMPORTANT NOTE :-

(a) $\sqrt{-2} \neq -\sqrt{2}$ it is not a irrational number.

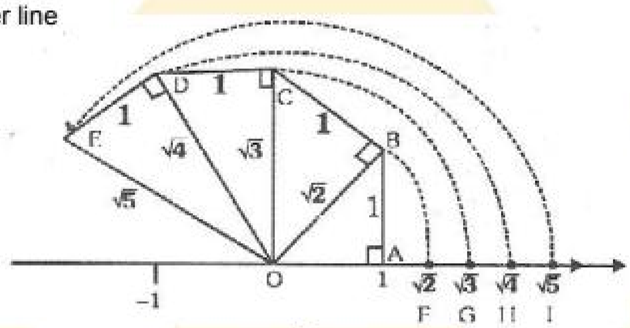
(b) $\sqrt{-2} \times \sqrt{-5} \neq \left(\sqrt{-2 \times -5} = \sqrt{10}\right)$

$\sqrt{-2}, \sqrt{-5}, \sqrt[5]{-2}$ are called imaginary numbers.

$\sqrt{-2} = i\sqrt{2}$ where i(iota) $\sqrt{-1}$

B-2. REPRESENTATION OF IRRATIONAL NUMBERS ON A NUMBER LINE :=

$\sqrt{2}, \sqrt{3}, \sqrt{5}$ on a number line



OB = $\sqrt{2}$ = OF OC = $\sqrt{3}$ = OG

OD = $\sqrt{4}$ = OH OE = $\sqrt{5}$ =OI

Ex.1 Insert a rational and an irrational number between 2 and 3.

Sol. If a and b are two positive rational numbers such that ab is not perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

Also, if a, b are rational numbers, then $\frac{a + b}{2}$ is a rational number between them.

$\therefore$ A rational number between 2 and 3 is $\frac{2 + 3}{2} = 2.5$

An irrational number between 2 and 3 is $\sqrt{ab} = \sqrt{2 \times 3} = \sqrt{6}$

Ex.2 Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Sol. We known that, if a and b are two distinct positive irrational number, than $\sqrt{ab}$ is an irrational number lying between a and b.

$\therefore$ irrational number between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2} \times \sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$

irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2} \times 6^{1/6}} = 2^{1/*4} \times 6^{1/8}$

Hence, required irrational numbers are $6^{1/4}$ and $2^{1/4} \times 6^{1/8}$

Ex.3 Identify $\sqrt{45}$ as rational number or irrational number.

Sol. We have,

$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

Since 3 is a rational number and $\sqrt{5}$ is an irrational number. Therefore, the product $3\sqrt{5} = \sqrt{45}$ is an irrational number.

C REAL NUMBER :-

Real numbers :- Rational numbers together with irrational numbers are known as real numbers.

Thus a real number is either rational or irrational but can not be simultaneously both. If a real number is not rational, it has be to be irrational and vice versa. $3, \frac{7}{5}$, -1.5, 2.3, 5.76245 etc. are some of the rational number. whereas $\sqrt{2}. \sqrt{3}. \sqrt[3]{7}. \sqrt[5]{11}. \pi$ are some examples of irrational numbers.

(i) These are the numbers which can represent actual physical quantities in a meaningful way. These can be represented on the number line. Number line is geometrically straight line with arbitrarily defined zero(origin).

C-1. GEOMETRICAL REPRESENTATION OF REAL NUMBERS :-

To represent only real numbers on numbers line we follow the following algorithm.

ALGORITHM

(i) Obtain the positive real number x (say)

(ii) Draw a line and mark a point A on it.

(iii) Mark point B on the line such that AB = X units.

(iv) From point B mark a distance of 1 unit and mark the new point as C.

(v) Find the mid-point of AC and mark the points as O.

(vi) Draw a circle with centre O and radius OC.

(vii) Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Length BD is equal to $\sqrt{x}$

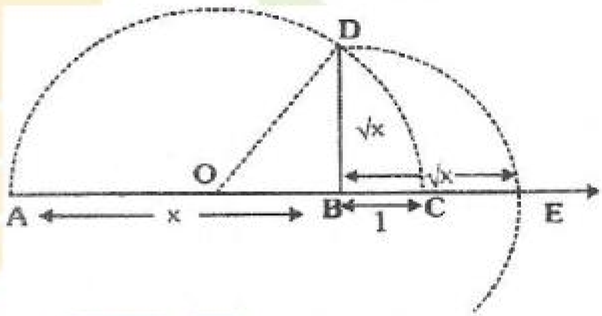
Justification : We have.

AB = x units and BC = 1 units.

$\therefore$ **AC** = (x+1) units

$\Rightarrow$ **OA** = OC = $\frac{x+1}{2}$ units.

$\Rightarrow$ **OD** = $\frac{x+1}{2}$ units.



Now, OB = AB - OA = x - $\frac{x+1}{2} = \frac{x-1}{2}$

Using Pythagoras theorem in ΔOBD, we have **[$\therefore$ OA = OC= OD]**

$OD^2 = OB^2 + BD^2$

$\Rightarrow$ $BD^2 = OD^2 - OB^2$

$\Rightarrow$ $BD^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$

$\Rightarrow$ $BD = \sqrt{\frac{(x^2+2x+1)-(x^2-2x+1)}{4}} = \sqrt{\frac{4}{x4}} = \sqrt{x}$

This shows hat $\sqrt{x}$ exists for all real numbers x > 0.

C-2. EXPONENTS OF REAL NUMBERS :-

(i) Positive exponent :-

For any real number a and a positive integer 'n' we define a^n as

a^n = a x a xa x....... xa (n times)

a^n is called the n^{th} power of a. The real number 'a' is called the base and 'n' is called the exponent of the n^{th} power of a.

Ex. $3^3 = 3 \times 3 \times 3 = 27$

For any non - zero real number 'a' we define $a^0 = 1$.

Thus, $4^0 = 1$, $6^0 = 1$, $\left(\frac{3}{2}\right)^0 = 1$ and so on.

(ii) Negative exponent :-

For any non - zero real number 'a' and a positive integer 'n' we define $a^{-n} = \frac{1}{a^n}$

Thus we have defined a^n for all integral values of n. positive. zero or negative. a^n is called that n^{th} power of a.

Ex. $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$

(iii) Rational power [Exponents] :-

For any positive real number a and a rational number $\frac{p}{q}$, where q > 0, we define $a^{p/q} = (a^p)^{1/q}$

i.e. $a^{p/q}$ is the principal q^{th} root of a^p .

C-3. RATIONAL EXPONENTS OF A REAL NUMBER :-

(i) n^{th} root of a positive real number

If a is a positive real number and 'n' is a positive integer, then the principal n^{th} root of a is the unique positive real number x such that $x^n = a$.

The principal n^{th} root of a positive real number a is denoted by $a^{1/n}$ or $\sqrt[n]{a}$

(ii) Principal n^{th} root of a negative Real Number

If a is a negative real number and n is an odd positive integer, then the principal n^{th} root of a is defined as $-|a|^{1/n}$ i.e. the principal n^{th} root of is minus of the principal n^{th} root if | a |.

Remark : If a is negative real number and n is an even positive integer, then the principal n^{th} root if a is not defined, because an even power of a real number is always positive. Therefore $(-9)^{1/2}$ is a meaning less quantity, if we confine ourselves to the set of real number, only.

C-4. LAWS OF RATIONAL EXPONENTS :-

The following laws hold the rational exponents.

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m+n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^{-n} = \frac{1}{a^n}$

(v) $a^{m/n} = (a_m)^{1/n} = (a^{1/n})^m = a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(vi) $(ab)^m = a^m b^m$

(vii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} (b \neq 0)$

(viii) $a^{bn} = a^{b+b+b....n\text{ times}}$

(ix) $a^m \div a^m = a^0 = 1$

Where a, be are positive real numbers and m, n are rational numbers.

Ex.1 Simplify each of the following

(i) $5^2 \times 5^4$ **(ii)** $(3^2)^3$ **(iii)** $\left(\frac{3}{4}\right)^{-3}$

Sol. **(i)** $5^2 . 5^4 = 5^{2+4} = 5^6 = 15625$ $\therefore a^m \times a^n = a^{m+1}$

(ii) $(3^2)^3 = 3^{2\times3} = 3^6 - 729$ $\therefore (a^m)^n = a^{m\times n}$

(iii) $\left(\frac{3}{4}\right)^{-3} = \frac{1}{\left(\frac{3}{4}\right)^3} = \frac{1}{\frac{3^3}{4^3}} = \frac{1}{\frac{27}{64}} = \frac{64}{27}$ $\therefore a^{-n} = \frac{1}{a^n}$

Ex.2 Simplify each of the following

(i) $\left(\frac{2}{11}\right) \times \left(\frac{11}{3}\right)^2 \times \left(\frac{3}{2}\right)^3$ **(ii)** $2^{55} \times 2^{60}$ $2^{97} \times 2^{18}$ **(iii)** $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$

Sol. **(i)** $\left(\frac{2}{11}\right)^4 \times \left(\frac{11}{3}\right)^2 \times \left(\frac{3}{2}\right)^3 = \frac{2^4}{11^4} \times \frac{11^2}{3^2} \times \frac{3^3}{2^3} = \frac{2^4 \times 11^2 \times 3^3}{11^4 \times 3^2 \times 2^3} = \frac{2 \times 3}{11^2} = \frac{6}{121}$

(ii) We have $2^{55} \times 2^{60} - 2^{97} \times 2^{18} = 2^{55+60} - 2^{97+18} = 2^{115} - 2^{115} = 0$

(iii) $\frac{2^4 \times 2^{2-1} - 2^2 \times 2^n}{2^4 \times 2^{n+2} - 2 \times 2^{n+2}} = \frac{2^{n+5} - 2^{n+2}}{2^{n+6} - 2^{n-3}} = \frac{2^{n+5} - 2^{n+2}}{2.2^{n-5} - 2.2^{n+2}} = \frac{2^{n+5} - 2^{n+2}}{2(2^{n+5} - 2^{n+2})} = \frac{1}{2}$

Ex.3 Assuming that x is a positive real number and a, b, c are rational numbers, show that :

$$\left(\frac{x^a}{x^b}\right)^{1/ab}\left(\frac{x^b}{x^c}\right)^{1/bc}\left(\frac{x^c}{x^d}\right)^{1/ac} = 1$$

Sol. $= \left(x^{a-b}\right)^{1/ab}.\left(x^{b-c}\right)\left(x^{c-a}\right)^{1/ac} = x^{(a-b)/ab}.x^{(b-c)}.x^{(c-a)/ac}$

$= x^{\frac{1}{b}-\frac{1}{a}}.x^{\frac{1}{c}-\frac{1}{b}}.x^{\frac{1}{a}-\frac{1}{c}} = x^{\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}} = x^0 = 1$

Ex. 4 if $\frac{9^n \times 3^2(3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$ prove that m - n = 1.

Sol. $\Rightarrow \frac{(3^3)^n \times 3^2 \times 3^{-\frac{n}{2}-2} - (3^3)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 2^3}. = \frac{1}{27}$

$\Rightarrow \frac{3^{3n}\left(3^2 - 1\right) - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{3n}.8}{3^{3m}.8} = \frac{1}{27}$

$\Rightarrow 3^{3n-3m} = \frac{1}{3^3}$

$\Rightarrow 3^{3n-3m} = 3^{-3}$ [On equating the exponent]

$\Rightarrow$ 3n - 3m = -3 $\Rightarrow$ n - m = - 1 $\Rightarrow$ m - n = 1.

Ex.5 Assuming that x is a positive real number and a,b,c are rational numbers, show that :

$$\left(\frac{x^a}{x^b}\right)^{a+b-c}\left(\frac{x^b}{x^c}\right)^{b-c-a}\left(\frac{x^c}{x^a}\right)^{c-a-b} = 1$$

Sol. $\left(x^{a-b}\right)^{a+b-c}.\left(x^{b-c}\right)^{b+c-a}.\left(x^{c-a}\right)^{c+a-b}$

$= x^{(a-b)(a+b)-c(a-b)}.x^{(b-c)(b+c)-a(b-c)}.x^{(c-a)(c+a)-b(c-a)} =$

$= x^{a^2-b^2-ca+bc}.x^{b^2-c^2-ab+ac}x^{c^2-a^2-bc+ba}$

$= x^{a^2-b^2-ca-bc-b^2-c^2-ab+bc-a^2-b+ba}$

$= x^0 = 1$

Ex.6 If $a^x = b$. by = c and $c^2 = a$. prove that xyz = 1.

Sol. We have,

$a^{xyz} = (a^x)^{yz}$

$\Rightarrow \quad a^{xyz} = (b)^{yz}$ $[\therefore x^2 = b]$

$\Rightarrow \quad a^{xyz} = (b^y)^{yz}$

$\Rightarrow \quad a^{xyz} = c^z$ $[\therefore b^y = c]$

$\Rightarrow \quad a^{xyz} = a$ $[\therefore c^z = a]$

$\therefore \quad a^{xyz} = a^1 \Rightarrow xyz = 1$

Ex. 7 If $a^x = b^y = c^z$ and $b^2 = ac$. prove that $y = \frac{2xy}{x+z}$

Sol. Let $a^x = b^y = c^z = k$. Than. $a = k1^{/x}$ $b = k^{1/y}$ and $c = k^{1/z}$

Now $b^2 = ac$

$\Rightarrow (k^{1/y})^2 = k^{1/x} \times k^{1/z} \Rightarrow k2/y = k^{1/x+1/z}$

$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$ $\Rightarrow \frac{2}{y} = \frac{x+z}{xz}$ $\Rightarrow y = \frac{2xz}{x+z}$

Ex. 8 If $25^{x-1} = 5^{2x-1} - 100$. find the value of x.

Sol. We have.

$\Rightarrow$ 25x-1 = 52x-1 - 100

$\Rightarrow (5^2)^{x-1} = 5^{2x-1} - 100$

$\Rightarrow 5^{2x-2} = 5^{2x-1} = -100$

$\Rightarrow 5^{2x-2} - 5^{2x-2}.5^1 = -100$

$\Rightarrow 5^{2x-2}(1-5) = -100$

$\Rightarrow 5^{2x-2}(-4) = -100$

$\Rightarrow 5^{2x-2} = 25$ $\Rightarrow 5^{2x-2} = 5^2$

$\Rightarrow 2x - 2 = 2$ $\Rightarrow 2x = 4$

$\Rightarrow x = 2$

D. SURDS :

Surd : - Any irrational number of the form $\sqrt[n]{a}$ is given a special name surd.

Where **'a'** is called **radicand,** it should always be a rational number. Also the symbol $\sqrt[n]{\ }$ is called the **radical** sign and the index **n** is called **order** of the surd.

$\sqrt[n]{a}$ is read as nth root of 'a' and also be written as $a^{\frac{1}{n}}$.

D-1. LAWS OF A SURDS :-

(i) $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$

(ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [Here order should be same]

(iii) $\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$

(iv) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a} = \sqrt[m]{\sqrt[n]{a}}$

(v) $\sqrt[n]{a} = \sqrt[n.p]{a^p}$ [Important for changing order of surds]

or $\sqrt[n]{a^m} = \sqrt[n.p]{a^{m.p}}$

Ex. **(i)** $\sqrt[3]{6^2}$ make its order 6

Then $\sqrt[3]{6^2} = \sqrt[3.2]{6^{2.2}} = \sqrt[6]{6^4}$

(ii) $\sqrt[3]{6}$ make its order 15

Then $\sqrt[3]{6} = \sqrt[3.5]{6^{1.5}} = \sqrt[15]{6^5}$

(iii) $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

(iv) $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

(v) $\sqrt[3]{2} \times \sqrt[3]{6} = \sqrt[3]{2 \times 6} = \sqrt[3]{12}$

But $\sqrt[3]{3} \times \sqrt[4]{6} \neq \sqrt{3 \times 6}$ (Because order is not same)

1[st] make their order same & then you can multiple.

(vi) $\sqrt{\sqrt{\sqrt{2}}} = \sqrt[8]{2}$

D-2. IDENTITY OF A SURD :-

(i) These are not a surd,

$\sqrt[3]{8}$ because $\sqrt[3]{8} = \sqrt[3]{2^3}$ which is a rational number.

(ii) $\sqrt{7-4\sqrt{3}}$ is a surd as 7-4 $\sqrt{3}$ is a perfect square of (2 - $\sqrt{3}$)

Ex : $\sqrt{7+4\sqrt{3}}, \sqrt{9-4\sqrt{5}}, \sqrt{9+4\sqrt{5}}$......

(iii) $\sqrt{2+\sqrt{3}}$ because 2 + $\sqrt{3}$ is not a perfect square.

(iv) $\sqrt[3]{1+\sqrt{3}}$ because radicand is an irrational number.

(v) $\sqrt[3]{4}$ is a surd as radicand is a rational number.

Ex : $\sqrt[3]{5}, \sqrt[4]{12}, \sqrt[5]{7}$......

(vi) $\sqrt[3]{\sqrt{3}}$ is a surd as $\sqrt[3]{\sqrt{3}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{3}} = 3^{\frac{1}{6}} = \sqrt[3]{3}$

Ex : $\sqrt[3]{\sqrt[3]{3}}, \sqrt[4]{\sqrt[5]{6}}$,.....

(vii) 2 + $\sqrt{3}$ is a surd (as surd + rational number will give a surd)

Ex : $\sqrt{3} - \sqrt{2}, \sqrt{3} + 1, \sqrt[3]{3} + 1$,....

D-3. TYPES OF SURDS :-

(a) Simplest form of a surds :-

Ex. **(i)** $\sqrt[3]{135}$ it's simplest form is $3\sqrt[3]{5}$

(ii) $\sqrt[4]{1875}$ it's simplest form is $5\sqrt[4]{3}$

(iii) $\sqrt[4]{8} = \sqrt[6]{2^3} = \sqrt{2}$ Simplest form

(b) Quadratic surds : Surds of order 2

Ex. $\sqrt{2}.\sqrt{3}$.....

(c) Biquadratic surds : Surd of order 4

Ex : $\sqrt[4]{8}$

(d) Cubic surds : Surd of order 3

Ex : $\sqrt[3]{3}.\sqrt[3]{15}$

(e) Like surds : Two or more surds are called like if they have or can be reduced to have the same irrational or surds factor.

Ex : $\sqrt{2}\,.\,3\sqrt{2}$

(f) Unlike surds : Two or more surds are called unlike, if they are not similar, (i.e. radicand s well as index are different).

Ex : $\sqrt{5}\,.\sqrt{3}\,.\sqrt{6}$

(g) Pure surds : A Surds which has unity only as its rational factor, the other being irrational, is called pure surd.

Ex : $\sqrt{3}\,.\sqrt{15}, \sqrt[4]{1875}, \sqrt[6]{8}$

(h) Mixed surds : A surd which as a rational factor other than unity, the other factor being irrational, is called a mixed surd.

Ex : $2\sqrt{3},\ 5\sqrt[4]{3}$

(i) Simple surds : A surd consisting of a single term is called a simple surd.

Ex : $\sqrt{3}, 3\sqrt[4]{5}\,.\frac{7}{2}\sqrt[3]{6}$

(j) Compound surds : An algebraic sum of two or more surds is called as compound surd. are simple surd

Ex : $\sqrt{3} + \sqrt{5} - \sqrt[3]{4}, \sqrt{3} - \sqrt[3]{5}$ etc are compound surds.

(k) Monomial surds : Single surd is called monomial surds.

Ex : $\sqrt[3]{2}, \sqrt{2}.\frac{4}{2}\sqrt[3]{3}$.....

(1) Binomial surds : An algebraic sum of two simple surds or a rational number and a simple surds is known as a binomial surd.

Ex. : $2+\sqrt{3}, \sqrt{3}+\sqrt{2}, 2+\sqrt[3]{3}, \sqrt[3]{2}+\sqrt[3]{3}$

(m) Trinomial surds: An algebraic sum of three simple surds or the sum of a rational number and two simple surds is known as a trinomial surd.

Ex : $1+\sqrt{2}+\sqrt{3}, 2+\sqrt{3}-\sqrt{5}, \sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}$

(n) Equiradical surds : - Surds of the same order are called equiradical surds.

Ex : $\sqrt{2}, \sqrt{a}, \sqrt{5}$

(o) Non-equiradical surds : Surds of the different orders are known as non-equiradical surds.

Ex : $\sqrt{2}\ \sqrt[3]{4}, \sqrt[4]{b}$

Ex.1 Express following as a pure surds and mixed surds.

(i) $\frac{3}{4}\sqrt[3]{128}$ **(ii)** $\sqrt[5]{96}$

Sol. **(i)** $\frac{3}{4}\sqrt[3]{128} = \sqrt[3]{\frac{3^3}{4^3}} \times \sqrt[3]{128} = \sqrt[3]{\frac{27}{64} \times 128} = \sqrt[3]{54}$

(ii) $\sqrt[5]{96} = \sqrt[5]{32 \times 3} = \sqrt[5]{2^5 \times 3} = \sqrt[5]{2^5} \times \sqrt[5]{3} = 2\sqrt[5]{3}$

D-4. OPERATION OF SURDS :-

(a) Addition and subtraction of surds :- Addition & subtraction of surds are possible only when order and radicand are same i.e. only for like surds.

Ex.(i) Simplify the $15\sqrt{16} - \sqrt{216} + \sqrt{96}$

$= 15\sqrt{6} - \sqrt{6^2 \times 6} + \sqrt{16 \times 6}$

$= 15\sqrt{6} - 6\sqrt{6} + 4\sqrt{6}$

$= (15 - 6 + 4)\sqrt{6}$

$= 13\sqrt{6}$

(b) Multiplication and division of surds :

Ex. **(i)** $\sqrt[3]{4} \times \sqrt[3]{22} = \sqrt[3]{4 \times 22} = \sqrt[3]{2^3 \times 11} = 2\sqrt[3]{11}$

Ex. **(ii)** $\sqrt[3]{2}$ by $\sqrt[4]{3}$

$\sqrt[3]{2} \times \sqrt[4]{3} = \sqrt[12]{2^4} \times \sqrt[12]{3^3} = \sqrt[12]{2^4 \times 3^3} = \sqrt[12]{16 \times 27} = \sqrt[12]{432}$

(c) Comparison of surds : It is clear that if x > y > 0 and n > 1 is a positive integers then $\sqrt[n]{x} > \sqrt[n]{y}$.

$\sqrt[3]{16} > \sqrt[3]{12}, \sqrt[5]{36} > \sqrt[5]{25}$ and so on.

Ex. Which is greater in each of the following:

(i) $\sqrt[3]{6}$ and $\sqrt[5]{8}$ **(ii)** $\sqrt{\frac{1}{2}}$ and $\sqrt[3]{\frac{1}{3}}$

Sol. **(i)** $\sqrt[3]{6}$ and $\sqrt[5]{8}$

L.C.M. of 3 and 5 is 15

$\sqrt[3]{6} = \sqrt[3.5]{6^5} = \sqrt[15]{7776}$

$\sqrt[5]{8} = \sqrt[3.5]{8^3} = \sqrt[15]{512}$

$\therefore \sqrt[15]{7776} > \sqrt[15]{512}$

$\Rightarrow \quad \sqrt[3]{6} > \sqrt[5]{8}$

Sol. **(ii)** $\sqrt{\frac{1}{2}}$ and $\sqrt[3]{\frac{1}{3}}$

L.C.M. of 2 and 3 is 6

$\sqrt[6]{\left(\frac{1}{2}\right)^3}$ and $\sqrt[6]{\left(\frac{1}{3}\right)^2}$

$\sqrt[6]{\frac{1}{8}}$ and $\sqrt[6]{\frac{1}{9}}$ as 8 < 9 $\therefore \frac{1}{8} > \frac{1}{9}$

so $\sqrt[6]{\frac{1}{8}} > \sqrt[6]{\frac{1}{9}} \Rightarrow \sqrt{\frac{1}{2}} > \sqrt[3]{\frac{1}{3}}$

Ex.2 Which is greater $\sqrt{7} - \sqrt{3}$ or $\sqrt{5} - 1$?

Sol. $\sqrt{7} - \sqrt{3} = \frac{\left(\sqrt{7} - \sqrt{3}\right)\left(\sqrt{7} + \sqrt{3}\right)}{\left(\sqrt{7} + \sqrt{3}\right)} = \frac{7-3}{\sqrt{7} + \sqrt{3}} = \frac{4}{\sqrt{4} + \sqrt{3}}$

and $\sqrt{5} - 1 = \frac{\left(\sqrt{5} - 1\right)\left(\sqrt{5} + 1\right)}{\left(\sqrt{5} + 1\right)} = \frac{5-1}{\sqrt{5} + 1} = \frac{4}{\sqrt{5} + 1}$

Now, we known that $\sqrt{7} > \sqrt{5}$ and $\sqrt{3} > 1$

add $\quad \sqrt{7} + \sqrt{3} > \sqrt{5} + 1$

$$\frac{1}{\sqrt{7} + \sqrt{3}} < \frac{1}{\sqrt{5} + 1}$$

$$\frac{4}{\sqrt{7} + \sqrt{3}} < \frac{4}{\sqrt{5} + 1} \Rightarrow \sqrt{7} - \sqrt{3} < \sqrt{5} - 1$$

So $\quad \sqrt{5} - 1 > \sqrt{7} - \sqrt{3}$

D-5. RATIONALIZATION OF SURDS :-

(a) Rationalizing Factor : - If the product of two surds is a rational number then each of them is called the rationalizing factor (R.F.) of the other.

(b) Rationalization : - The process of converting a surds to a rational number by using an appropriate multiplier is known as rationalization.

(i) Rationalizing factor of $\sqrt{a}$ is $\sqrt{a}$ $\left(\because \sqrt{a} \times \sqrt{a} = a\right)$

(ii) Rationalizing factor of $\sqrt[3]{a}$ is $\sqrt[3]{a^2}$ $\left(\because \sqrt[3]{a} \times \sqrt[3]{a^2} = \sqrt[3]{a^3} = a\right)$

(iii) Rationalizing factor of $\sqrt{a} + \sqrt{b}$ is $\sqrt{a} - \sqrt{b}$ & vice versa $\left[\because \left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b\right]$

(iv) Rationalizing factor of a + $\sqrt{b}$ is $a - \sqrt{b}$ & vice versa $\left[\because \left(a + \sqrt{b}\right)\left(a - \sqrt{b}\right) = a^2 - b\right]$

(v) Rationalizing factor of $\sqrt[3]{a} + \sqrt[3]{b}$ is $\left(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}\right)\left[\because \left(\sqrt[3]{a} + \sqrt[3]{b}\right)\left(\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}\right)\right]$

$\left[\because \left(\sqrt[3]{a}\right)^3 + \left(\sqrt[3]{b}\right)^3\right]$ = a + b which is rational.

(vi) Rationalizing factor of $\sqrt{a} + \sqrt{b} + \sqrt{c}$ is $\left(\sqrt{a} + \sqrt{b} - \sqrt{c}\right)$ and $(a + b - c - 2\sqrt{ab})$.

Ex. 1 Find the following rationalizing factors :

(i) $\sqrt{10}$ **(ii)** $\sqrt{162}$

(iii) $\sqrt[3]{4}$ **(iv)** $\sqrt[3]{16}$

Sol. **(i)** $\sqrt{10}$

$[\because \sqrt{10} \times \sqrt{10} = \sqrt{10 \times 10} = 10]$ as 10 is rational number

(ii) $\sqrt{162}$

Simplest form $9\sqrt{2}$

Rationalizing factor of $\sqrt{2}$ is $\sqrt{2}$

Rationalizing factor of $\sqrt{162}$ is $\sqrt{2}$

(iii) $\sqrt[3]{4}$

$\Rightarrow \sqrt[3]{4} \times \sqrt[3]{4^2} = \sqrt[3]{4^3} = 4$

Rationalizing factor of $\sqrt[3]{4}$ is $\sqrt[3]{4^2}$

(iv) $\sqrt[3]{16}$

Simplest from of $\sqrt[3]{16}$ is $2\sqrt[3]{2}$

Now rationalizing factor of $\sqrt[3]{2}$ is $\sqrt[3]{2^2}$

$\therefore$ Rationalizing factor of $\sqrt[3]{16}$ is $\sqrt[3]{2^2}$

Ex.2 Find rationalizing factor of $\sqrt[4]{162}$

Sol. Simplest form of $\sqrt[4]{162}$ is $3\sqrt[4]{2}$

Now rationalizing factor of $\sqrt[4]{2}$ is $\sqrt[4]{2^3}$

$\therefore$ Rationalizing factor of $\sqrt[4]{162}$ is $\sqrt[4]{2^3}$

D-6. WHEN CONJUGATE SURDS AND RECIPROCALS ARE SAME :-

(a) 2 + $\sqrt{3}$ it's conjugate is 2 - $\sqrt{3}$, its reciprocal is 2+ $\sqrt{3}$ & vice versa.

(b) 5 - 2$\sqrt{6}$, it's conjugate is 5 + 2$\sqrt{6}$, its reciprocal is, 5 - 2$\sqrt{6}$ & vice versa.

Ex.1 Express the following surd with a rational denominator.

$$\frac{8}{\sqrt{15} + 1 - \sqrt{5} - \sqrt{3}}$$

$$= \frac{8}{(\sqrt{15}+1)-(\sqrt{5}+\sqrt{3})}$$

$$= \left[= \frac{8}{(\sqrt{15}+1)-(\sqrt{5}+\sqrt{3})} \times \frac{\left|\sqrt{15}+1+\sqrt{5}+\sqrt{3}\right|}{(\sqrt{15}+1)+(\sqrt{5}+\sqrt{3})}\right]$$

$$= \frac{8(\sqrt{15}+1+\sqrt{5}+\sqrt{3})}{(\sqrt{15}+1)^2-(\sqrt{5}+\sqrt{3})^2}$$

$$= \frac{8(\sqrt{15}+1+\sqrt{5}+\sqrt{3})}{15+1+2\sqrt{15}-(5+3+2\sqrt{15})}$$

$$= \frac{8(\sqrt{15}+1+\sqrt{5}+\sqrt{3})}{8}$$

$$= (\sqrt{15}+1+\sqrt{5}+\sqrt{3})$$

Ex.2 If $\frac{3+2\sqrt{5}}{3-\sqrt{2}} = a + b\sqrt{2}$. where a and b are rational.

Find the values of a and b

L.H.S. = $= \frac{3+2\sqrt{2}}{3-\sqrt{2}} = \frac{(3+2\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{9+3\sqrt{2}+6\sqrt{2}+4}{9-2} = \frac{13}{7}+\frac{9}{7}\sqrt{2}$

$\therefore \frac{13}{7}+\frac{9}{7}\sqrt{2} = a + b\sqrt{2}$

equating the rational and irrational parts

We get a $= \frac{13}{7}.b = \frac{9}{7}$

Ex.3 If x = $\frac{1}{2+\sqrt{3}}$. find the value of $x^3 - x^2 - 11x + 3$

as x = $\frac{1}{2+\sqrt{3}}$ = 2 - $\sqrt{3}$

x - 2 = - $\sqrt{3}$

squaring both sides, we get

$(x - 2)^2 = (-\sqrt{3})^2 \Rightarrow x^2 + 4 - 4x = 3 \Rightarrow x^2 - 4x + 1 = 0$

Now $x^3 - x^2 - 11x + 3 = x^3 - 4x^2 + x + 3x^2 - 12x + 3$

$x(x^2 - 4x + 1) + 3 (x^2 - 4x + 1)$

x x0 + 3 (0)

0 + 0 =0

Ex.4 If x = 1 + $\sqrt{2}+\sqrt{3}$, prove that $x^4 - 4x^3 - 4x^2 + 16 - 8 = 0$

Sol. x = 1 + $\sqrt{2}+\sqrt{3}$

Squaring both sides

$\Rightarrow (x - 1)^2 - (\sqrt{2}+\sqrt{3})^2$

$\Rightarrow x^2 + 1 - 2x = 2 + 3 + 2\sqrt{6}$

$\Rightarrow x^2 - 2x - 4 = 2\sqrt{6}$

squaring both sides

$\Rightarrow (x^2 - 2x - 4)^2 = \left(2\sqrt{6}\right)^2$

$\Rightarrow x^4 + 4x^2 + 16 - 4x^3 + 16x - 8x^2 = 24$

$\Rightarrow x^4 - 4x^3 - 4x^2 + 16x + 16 - 24 = 0$

$\Rightarrow x^4 - 4x^3 - 4x^2 + 16x - 8 = 0$

D-7. SQUARE ROOTS OF BINOMIAL QUADRATIC SURDS:-

(a) Since $\left(\sqrt{x}\sqrt{y}\right)^2 = (x + y) + 2\sqrt{xy}$ & $\left(\sqrt{x} - \sqrt{y}\right)^2 = x + y - 2\sqrt{xy}$

(b) $\therefore$ square root of $x + y + 2\sqrt{xy} = \pm\left(\sqrt{x} + \sqrt{y}\right)$

(c) Square root of $(x + y) - 2\sqrt{xy} = \pm\left(\sqrt{x} - \sqrt{y}\right)$

(d) Square root of $a^2 + b + 2a\sqrt{b} = \pm\left(a + \sqrt{b}\right)$

(e) and square root of $a^2 + b - 2a\sqrt{b} = \pm\left(a - \sqrt{b}\right)$

Ex.1 $\sqrt{7 + 4\sqrt{3}} = \sqrt{(2)^2 + \left(\sqrt{3}\right)^2 + 2 \times 2 \times \sqrt{3}}$

$\sqrt{\left(2 + \sqrt{3}\right)^2} = 2 + \sqrt{3}$

Ex.2 Simplify $\dfrac{\sqrt{5 + 2\sqrt{6}} - \sqrt{5 - 2\sqrt{6}}}{\sqrt{5 + 2\sqrt{6}} + \sqrt{5 - 2\sqrt{6}}}$

Since $\sqrt{5 + 2\sqrt{6}} = \sqrt{3} + \sqrt{2}$ & $\sqrt{5 - 2\sqrt{6}} = \sqrt{3} - \sqrt{2}$

$$\frac{\sqrt{5 + 2\sqrt{6}} - \sqrt{5 - 2\sqrt{6}}}{\sqrt{5 + 2\sqrt{6}} + \sqrt{5 - 2\sqrt{6}}} = \frac{\left(\sqrt{3} + \sqrt{2}\right) - \left(\sqrt{3} - \sqrt{2}\right)}{\left(\sqrt{3} + \sqrt{2}\right) + \left(\sqrt{3} - \sqrt{2}\right)} = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{6}$$

Ex.3 If $x = \dfrac{\sqrt{a + 2b} + \sqrt{a - 2b}}{\sqrt{a + 2b} - \sqrt{a - 2b}}$ prove that $b^2x^2 - abx + b^2 = 0$

as $x = \dfrac{\sqrt{a + 2b} + \sqrt{a - 2b}}{\sqrt{a + 2b} - \sqrt{a - 2b}} \times \dfrac{\left(\sqrt{a + 2b} + \sqrt{a - 2b}\right)}{\left(\sqrt{a + 2b} - \sqrt{a - 2b}\right)}$

$x = \dfrac{\left(\sqrt{a + 2b} + \sqrt{a - 2b}\right)^2}{(a + 2b) - (a - 2b)} = \dfrac{a + 2b + a - 2b + 2\sqrt{(a + 2b)(a - 2b)}}{4b}$

$x = \dfrac{2\left[a + \sqrt{a^2 - 4b^2}\right]}{4b} = \dfrac{a + \sqrt{a^2 - 4b^2}}{2b}$

$2bx = a + \sqrt{a^2 - 4b^2}$

$2bx - a = \sqrt{a^2 - 4b^2}$

squaring both sides $(2bx - a)^2 = \left(\sqrt{a^2 - 4b^2}\right)$

$4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$

$4b^2x^2 - 4abc + 4b^2 = 0$

$b^2x^2 - abx + b^2 = 0$

EXERCISE - 01

OBJECTIVE TYPE QUESTIONS

1. How many prime number are there between 0 and 30:-

(A) 9 (B) 10 (C) 8 (D) 11

2. Two irrational numbers between 2 and 2.5 are :-

(A) $\sqrt{5}$ and $\sqrt{2\times\sqrt{5}}$ (B) $\sqrt{5}$ and $\sqrt{2\times5}$ (C) $\sqrt{5}$ and $\sqrt{2\times\sqrt{7}}$ (D) None of these

3. The exponential form of $\sqrt{\sqrt{2}\sqrt{3}}$ is :-

(A) $6^{1/2}$ (B) $6^{1/3}$ (C) $6^{1/4}$ (D) 6

4. The rational form of -25.6875 is :-

(A) $-\frac{411}{16}$ (B) $-\frac{421}{16}$ (C) $-\frac{431}{16}$ (D) $-\frac{441}{16}$

5. The rational form of $2.74\overline{35}$ is :-

(A) $\frac{27161}{999}$ (B) $\frac{27}{99}$ (C) $\frac{27161}{9900}$ (D) $\frac{27191}{9000}$

6. The value of $0.4\overline{23}$ is :-

(A) $\frac{423}{1000}$ (B) $\frac{479}{1000}$ (C) $\frac{423}{990}$ (D) $\frac{419}{990}$

7. Which of the following is not a rational number :-

(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{9}$ (D) $\sqrt{16}$

8. $1+\cfrac{1}{1+\cfrac{1}{1+1/3}}$ is equal to :-

(A) 1/3 (B) 11/7 (C) 3 (D) $1\frac{1}{3}$

9. The number $\frac{3-\sqrt{3}}{3+\sqrt{3}}$ is :-

(A) Rational (B) Irrational (C) Both (D) Can't say

10. If $x-\frac{1}{x}=\sqrt{3}$ then $x^3-\frac{1}{x^3}$ equal :-

(A) $6\sqrt{3}$ (B) $3\sqrt{3}$ (C) 3 (D) $\sqrt{3}$

11. The value of $5.\overline{2}$:-

(A) $\frac{45}{9}$ (B) $\frac{46}{9}$ (C) $\frac{47}{9}$ (D) None

12. $\dfrac{(x^{a+b})(x^{b+c})^2(x^{c+a})^2}{(x^a.b^b.x^c)^4}$

(A) -1 (B) 0 (C) 1 (D) None

13. The value of $\dfrac{(0.6)^0-(0.1)^{-1}}{(3/2)^{-1}(3/2)^3+\left(-\frac{1}{3}\right)^{-1}}$ is :-

(A) 3/2 (B) -3/2 (C) 2/3 (D) -1/2

14. If $2^x = 4^y = 8^z$ and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{4z}=4$, then the value of x is :-

(A) $\frac{7}{16}$ (B) $\frac{7}{32}$ (C) $\frac{7}{48}$ (D) None of these

15. If $9^{x-1} = 3^{2x-1} - 486$ then the value of x is :-

(A) 3.5 (B) 2.5 (C) 1.5 (D) 0

16. If $a = \frac{1}{3-2\sqrt{2}}, b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^2 + b^2$ is :-

(A) 34 (B) 35 (C) 36 (D) 37

17. $\dfrac{2^{n+4}-2(2^n)}{2(2^{n+3})}+2^{-3}$ is equal to :-

(A) 2^{n+1} (B) $-2^{n+1}+\frac{1}{8}$ (C) $\frac{9}{8}-2^n$ (D) 1

18. If $2^{2x-y} = 32$ and 2^{x+y} 16 then $a^2 + y^2$:-

(A) 9 (B) 10 (C) 11 (D) 13

19. The value of $\dfrac{(25)^{5/2}\times(243)^{2/5}}{(16)^{3/4}\times(8)^{5/3}}$ is :-

(A) $\frac{5625}{128}$ (B) $\frac{5615}{256}$ (C) $\frac{5625}{256}$ (D) None

20. The value of $\left[(x^{a-a^{-1}})^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}}$ =

(A) x (B) 1/x (C) x^a (D) $1/x^a$

21. $\sqrt[4]{\sqrt[3]{x^2}}$ =

(A) x (B) $x^{1/2}$ (C) $x^{1/3}$ (D) $x^{1/6}$

22. The value of $5\sqrt{3}-3\sqrt{12}+2\sqrt{75}$ on simplifying is :-

(A) $5\sqrt{3}$ (B) $6\sqrt{3}$ (C) $\sqrt{3}$ (D) $9\sqrt{3}$

23. If $\sqrt{3}$ = 1.732, $\sqrt{5}$ = 2.236, then the value of $\frac{6}{\sqrt{3}-\sqrt{3}}$ is :-

(A) 10.905 (B) 11.904 (C) 11.905 (D) None

24. The product of $4\sqrt{6}$ and $3\sqrt{24}$ is :-

(A) 124 (B) 134 (C) 144 (D) 154

25. If a = $\frac{2+\sqrt{3}}{2-\sqrt{3}}, b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$ then the value of a + b is :-

(A) 14 (B) -14 (C) $8\sqrt{3}$ (D) $-\sqrt{3}$

26. If x = $\frac{1}{2-\sqrt{3}}$ find the value of $a^3 - 2x^2 - 7x + 5$ is :-

(A) 2 (B) 1 (C) 0 (D) 3

27. The surd $3\sqrt[4]{\sqrt[3]{5}} - \sqrt[3]{\sqrt[4]{5}}$ in its simplest form is equal to :-

(A) $2\sqrt[12]{5}$ (B) $\sqrt[12]{5}$ (C) $\sqrt[2]{5}$ (D) None of these

28. Simplify $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+2}$:-

(A) 1 (B) 0 (C) 10 (D) 100

29. If $\frac{5\sqrt{3}}{7-4\sqrt{3}}$ $4a + \sqrt{3}b$ the value of a and b is :-

(A) a = 47, b = 27 (B) a = 27, b = 47 (C) a = 15, b = 35 (D) a = 35, b = 25

30. The value of $\sqrt[3]{24} + \sqrt[3]{81} - \sqrt[3]{192}$ is :-

(A) $\sqrt[3]{3}$ (B) $\sqrt{3}$ (C) 3 (D) None of these

ANSWERS KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	C	A	C	D	A	B	B	A	C	C	B	A	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	D	B	D	A	D	D	B	C	A	D	A	B	C	A

EXERCISE - 02

SUBJECTIVE TYPE QUESTIONS

1. Find 5 rational number between $\frac{3}{5}$ and $\frac{4}{5}$.

2. Represent 3.765 on the number line.

3. Find the decimal representation of 22/7.

4. Express the following decimal in the form $\frac{p}{q}$.

(a) $0.3\bar{2}$ (b) $0.12\bar{3}$

5. Prove that $\sqrt{3}$ is irrational.

6. Find two rational and two irrational numbers between 0.15 and 0.16.

7. Virsualize $4.\overline{26}$ on the number line, upto 4 decimal places.

8. Simplify $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$.

9. Prove that $\frac{a^{-1}}{a^{-1}+b^{-1}} + \frac{a^{-1}}{a^{-1}-b^{-1}} = \frac{2b^2}{b^2-a^2}$

10. If $\frac{3+2\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$, where a and b are rationals. Find the values of a and b.

11. If $a = x^{q+r} y^p$, $b = x^{r+p} y^q$, $c = x^{p+q} y^r$. Prove that $a^{q-r} \times b^{r-p} \times c^{p-q} = 1$.

12. If x is a positive real number and the exponents are rational numbers, show that :

(i) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$

(ii) $\left(\frac{x^b}{x^c}\right)^a \left(\frac{x^c}{x^a}\right)^b \left(\frac{x^a}{x^b}\right)^c = 1$

13. Simplify $\frac{6}{2\sqrt{3}-\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$.

14. If $x = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $y = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, find the value of $3x^2 + 4xy - 3y^3$.

15. $x = 3 + 2\sqrt{2}$, find the value of $x^4 + \frac{1}{x^4}$.

16. If x = $\sqrt{3+2\sqrt{2}}$ find the values of :-

(i) $x+\frac{1}{x}$ (ii) $x^2+\frac{1}{x^2}$

17. In the following determine rational numbers a and b.

$$\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}=a+7\sqrt{5}b.$$

18. If x = $2^{\frac{1}{3}}+2^{\frac{2}{3}}$ show that $x^3 - 6x = 6$.

NUMBER SYSTEM	ANSWER KEY	EXERCISE – 2 (IX) - CBSE

- **Subjective type answers**

1. $\frac{19}{30},\frac{20}{30},\frac{21}{30},\frac{22}{30}$ and $\frac{23}{30}$ **3.** 3.14285714 **4. (a)** $\frac{29}{90}$, **(b)** $\frac{1221}{9900}$

6. 0.155 and 0.1525 irrational number 0.1549 and d0.1524 **8.** 1

10. $a=\frac{13}{7}$ and $b=\frac{9}{7}$ **13.** 0 **14.** $4+\frac{56}{3}\sqrt{10}$

15. 1154 **16. (i)** $2\sqrt{2}$ **(ii)** 6 **17.** $a=0, b=\frac{1}{11}$