

INTEGERS

1 CHAPTER

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➤ INTRODUCTION

Whole numbers with + or – signs are called **integers**.

Eg : -17, -5, 0, 1, 3,

Note :

- (1) Decimal numbers are not include in integers, like 0.3, $-\frac{5}{7}$, -11.97, 0.03, $\sqrt{5}$ etc.
- (2) The set of integers is denoted by I and $I = \{..... -3, -2, -1, 0, 1, 2, 3.....\}$

◆ Types of Integers

- (1) **Positive Integers** : The numbers 1, 2, 3, 4, 5, i.e., the natural numbers are called **positive integers**.
- (2) **Negative Integers** : The numbers -1, -2, -3, -4, -5, are called **negative integers**.
- (3) **Zero Integers** : The number 0 is simply an integer. It is neither positive nor negative.

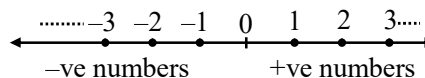
◆ EXAMPLE ◆

Ex.1 Write the predecessor and successor of the following numbers 4, -4, 6, 1, b, n^2

Predecessor	3	-5	5	0	$b-1$	n^2-1
Number	4	-4	6	1	b	n^2
Successor	5	-3	7	2	$b+1$	n^2+1

➤ INTEGERS ON NUMBER LINE

Positive numbers are always on right side of zero & negative numbers are on left side of zero.



or we can say all integers are in ascending order from left to right.

◆ EXAMPLE ◆

Ex.2 Fill the square by '<', '>' or '='

- | | | | |
|--------|-------|--------------------------|-------|
| (i) | 0 | <input type="checkbox"/> | -2 |
| (ii) | -31 | <input type="checkbox"/> | -21 |
| (iii) | -3 | <input type="checkbox"/> | 8 |
| (iv) | -7 | <input type="checkbox"/> | 7 |
| (v) | 11 | <input type="checkbox"/> | -6 |
| (vi) | 3 | <input type="checkbox"/> | 3 |
| (vii) | -1132 | <input type="checkbox"/> | -2 |
| (viii) | -1039 | <input type="checkbox"/> | -2138 |

Sol. (i) > (ii) < (iii) < (iv) < (v) >
 (vi) = (vii) < (viii) >

➤ ADDITION OF INTEGERS

In order to add two integers on a number line, we follow the following steps :

Step 1 : On the number line, mark one of the given integers.

Step 2 : Move as many units as the second number to the :

- (i) right of the first, if the second integer is positive.
- (ii) left of the first, if the second integer is negative.

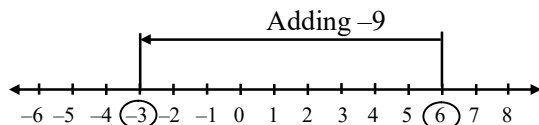
Step 3 : The point thus we reach represents the sum of two given integers.

❖ EXAMPLES ❖

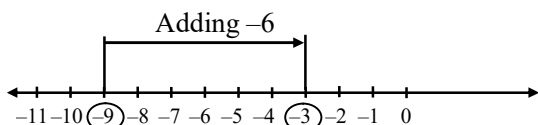
Ex.3 Add the following integers :

- (i) 6 and -9
- (ii) -3 and -4

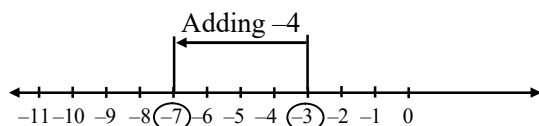
Sol.(i) First we draw a number line and mark the integer 6 on it.



To add -9 we move 9 steps to the left from 6. Thus, we reach at a point representing -3. Hence the sum of 6 and -9 is -3. That is, $6 + (-9) = -3$. Note that if we represent the number -9 on the number line then to find $6 + (-9)$ we shall move 6 units to the right of -9. Obviously, we reach at -3.



(ii) Draw a number line and mark the integer -3 on it.



To add -4 and -3 we have to move 4 steps to the left of -3. Thus, we arrive at -7. Hence, the required sum is -7. That is, $(-3) + (-4) = -7$.

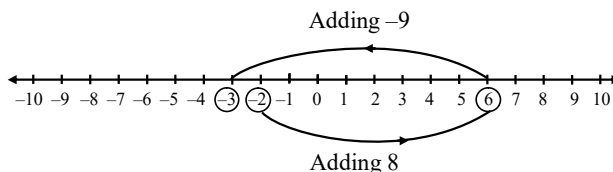
Note :

No matter which number you choose as first and the other as second number, because in both the conditions you will get the same answer.

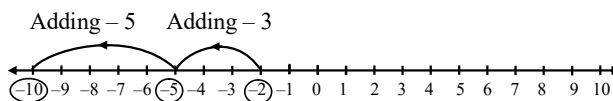
Ex.4 Draw a number line and represent each of the following on it :

- (i) $-2 + 8 + (-9)$
- (ii) $-2 + (-3) + (-5)$

Sol. (i) $-2 + 8 + (-9) = -3$



(ii) $-2 + (-3) + (-5) = -10$



➤ SUBTRACTION OF INTEGERS

We know that in the subtraction fact $7 - 2 = 5$, 7 is the **minuend**, 2 is the **subtrahend** and 5 is the **difference**.

Step 1 :

First we draw a number line and mark (label) the minuend on it.

Step 2 :

- (i) To subtract a positive integer, we move to the left from the minuend as many steps as the second integer is.
- (ii) To subtract a negative integer, we move to the right (not left) as many steps as the second integer is.

Step 3 :

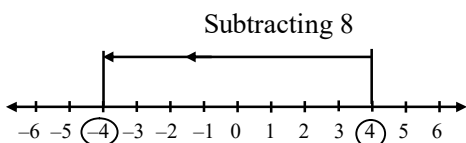
The point thus we reach represents the difference of two integers.

❖ EXAMPLE ❖

Ex.4 Subtract the following integers :

- (i) $4 - 8$
- (ii) $-5 - 4$
- (iii) $-3 - (-4)$

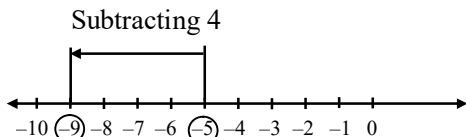
Sol. (i) First we draw a number line and mark the number 4 on it.



To subtract 8, we move 8 steps to the left of 4, thus we reach at the point representing -4 .

Hence, $4 - 8 = -4$.

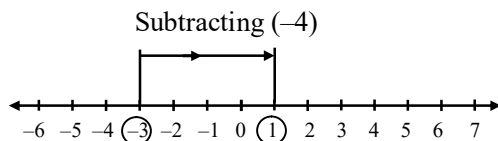
(ii) Mark the integer -5 on a number line.



To subtract 4, we move 4 steps to the left of -5 , thus we reach at the point representing -9 .

Hence, $-5 - 4 = -9$.

(iii) First we draw a number line and mark the integer -3 on it.



To subtract a negative integer -4 , we will move 4 steps to the right of -3 , thus we reach at the point representing 1.

Hence, $(-3) - (-4) = -3 + 4 = 1$

From the above example (iii)

We observe that $-3 - (-4) = 1$ which is same as $-3 + 4$.

Note : Subtracting a negative is the same as adding a positive and subtracting a positive is the same as adding a negative.

➤ LIMITATIONS OF THE NUMBER LINE

Of course, addition and subtraction of integers on a number line would not work so well if we are dealing with large numbers. Eg, $465 - 739$ or $465 + (-739)$.

➤ SUBTRACTION OF LARGER NUMBER FROM SMALLER NUMBER

We subtract smaller number from the larger number and we put a negative sign before the difference so obtained.

That is smaller natural number $-$ Larger natural number $= -$ [Larger natural number $-$ Smaller natural number]. To add two negative numbers, we add the numbers without sign and then we put the negative sign (common sign) before the sum so obtained.

❖ EXAMPLES ❖

Ex.6 Represent the following numbers as integers with appropriate signs :

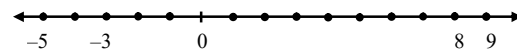
Sol.

S.No.	Statement	Signs
(i)	1500 m above sea level	+
(ii)	15°C below 0°C temperature	-
(iii)	Depth of 500 m	-
(iv)	A deposit of rupees thousand	+
(v)	Withdrawal of rupees hundred	-

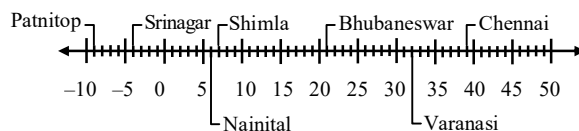
Ex.7 Represent the following numbers on a number line :

(i) $+9$ (ii) -3 (iii) $+8$ (iv) -5

Sol.



Ex.8 A number line given below shows the temperature of different cities on a particular day:



- Observe the number line and write the temperature of the cities marked on it.
- What is the difference of temperature between the hottest and the coldest places among the above ?

(iii) Can we say temperature of Bhubaneswar is more than the temperature of Nainital and Srinagar together ?

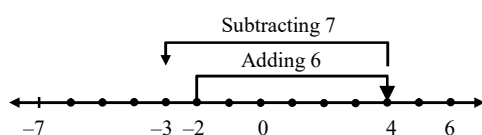
Sol. (i) Patnitop $\rightarrow -9^{\circ}\text{C}$; Srinagar $\rightarrow -4^{\circ}\text{C}$;
Nainital $\rightarrow 6^{\circ}\text{C}$; Shimla $\rightarrow 7^{\circ}\text{C}$;
Bhubaneswar $\rightarrow 21^{\circ}\text{C}$; Varanasi $\rightarrow 32^{\circ}\text{C}$;
Chennai $\rightarrow 39^{\circ}\text{C}$

(ii) 48°C (iii) Yes

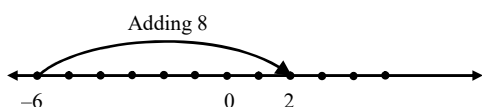
Ex.9 Draw a number line and represent each of the following :

(i) $-2 + 6 + (-7)$ (ii) $-6 + 8$

Sol. (i) $-2 + 6 + (-7) = -3$



(ii) $-6 + 8 = 2$



Ex.10 Find the difference between the following pairs of integers :

(i) -20 and -40 (ii) -19 and 30 (iii) 45 and -36

Sol. (i) $-20 - (-40) = -20 + 40 = 20$

(ii) $-19 - 30 = -49$

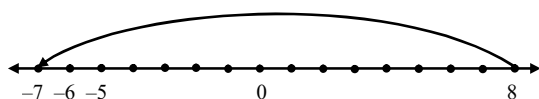
(iii) $45 - (-36) = 45 + 36 = 81$

Ex.11 Draw a number line and answer the following :

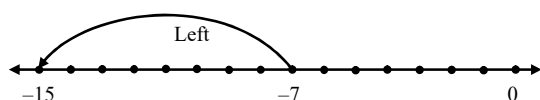
(i) Which number will we reach if we move 8 steps to the right of -15 ? Write this number with appropriate sign.

(ii) If we are at -7 on a number line, in which direction should we move to reach -15 and how many steps?

Sol. (i) $8 + (-15) = -7$



(ii) $-15 - (-7) = -8$



Ex.12 Write all the integers between the given pairs in ascending and descending orders :

(i) 0 and 5 (ii) -3 and 3
(iii) -8 and -15 (iv) -40 and -32

Sol.

S.No.	Integers	Ascending Order	Descending Order
(i)	0 & 5	1, 2, 3, 4	4, 3, 2, 1
(ii)	-3 & 3	$-2, -1, 0, 1, 2$	2, 1, 0, $-1, -2$
(iii)	-8 & -15	$-14, -13, -12, -11, -10, -9$	$-9, -10, -11, -12, -13, -14$
(iv)	-40 & -32	$-39, -38, -37, -36, -35, -34, -33$	$-33, -34, -35, -36, -37, -38, -39$

Ex.13 Complete the following table :

+	3	-4	0	-12	4
-3					
-5					
4		0		-8	
7					
-19			-19		
-27					
17					
0					

Sol.

+	3	-4	0	-12	4
-3	0	-7	-3	-15	1
-5	-2	-9	-5	-17	-1
4	7	0	4	-8	8
7	10	3	7	-5	11
-19	-16	-23	-19	-31	-15
-27	-24	-31	-27	-39	-23
17	20	13	17	5	21
0	3	-4	0	-12	4

Ex.14 Write true (T) or false (F) for the following statements. Also correct those which are false :

(i) Sum of two positive integers is always positive.

- (ii) Sum of two negative integers is always positive.
- (iii) When a positive integer and a negative integer are added, the result is always a negative integer.
- (iv) The sum of an integer and its additive inverse is always zero.
- (v) When a positive integer and a negative integer are added, we take their difference and place the sign of bigger integer, ignoring the sign of both.

Sol.

- (i) T
- (ii) F (Sum of two negative integers is always negative).
- (iii) F (When a positive and a negative integers are added, the result may be a positive or a negative integer).
- (iv) T
- (v) T

Ex.15 (a) Check which of the following is a magic square. (If each row, column and diagonal have the equal sum.)

(i)

5	-1	-4
-5	-2	7
0	3	-3

(ii)

1	-10	0
-4	-3	-2
-6	4	-7

(b) Magic squares are given below, fill in the empty cells with appropriate integers:

(i)

0		4
	1	
-2		

(ii)

-5		
	-2	
	-6	1

Sol. (a) (i)

5	-1	-4
-5	-2	7
0	3	-3

\rightarrow 0 \rightarrow 0 \rightarrow 0
 \downarrow 0 \downarrow 0 \downarrow 0

\therefore No

(ii)

1	-10	0
-4	-3	-2
-6	4	-7

\rightarrow -9 \rightarrow -9 \rightarrow -9
 \downarrow -9 \downarrow -9 \downarrow -9

\therefore Yes

(b) (i)

0	-1	4
5	1	-3
-2	3	2

(ii)

-5	2	-3
0	-2	-4
-1	-6	1

Ex.16 Neena has a loan of ₹ 1200 to repay. Her brother gave ₹ 2500. Describe Neena's financial position.

Sol.

Money of loan

$$= ₹ 1200$$

Money she has from her brother = ₹ 2500

\therefore left money after paying loan

$$= ₹ 2500 - 1200$$

$$= ₹ 1300 \text{ Ans.}$$

Ex.17 Find whether the given statements are true (T) or false (F) :

- (i) The smallest integer is 0.
- (ii) The opposite of zero on a number line is zero.
- (iii) Zero is not a positive integer.
- (iv) 0 is larger than every negative integer but less than every positive integer.
- (v) A positive integer is greater than its opposite.
- (vi) Every integer is less than every natural integer.
- (vii) -1 is the greatest negative integer.
- (viii) 0 is the smallest positive integer.
- (ix) The sum of greatest negative integer and smallest positive integer is zero.
- (x) The negative of a positive integer is a negative integer.
- (xi) The negative of a negative integer is positive.
- (xii) If a and b are two integers such that $a < b$ then $(b - a)$ is always a positive integer.

Sol.

- | | | |
|---------|----------|---------|
| (i) F | (ii) T | (iii) T |
| (iv) T | (v) T | (vi) F |
| (vii) T | (viii) F | (ix) T |
| (x) T | (xi) T | (xii) T |

► PROPERTIES OF ADDITION AND SUBTRACTION

	Addition	Subtraction
(1) Closure	✓	✓
(2) Commutative	✓	×
(3) Associative	✓	×
(4) Additive Identity	✓	×
(5) Additive Inverse	✓	✓

Eg.(i) $5 + 3 = 8$ (integer), $-7 + 3 = -4$ (integer)

Eg.(ii) $3 + 7 = 10 = 7 + 3$, $4 - 5 = -1$ & $5 - 4 = 1$

Eg.(iii)
$$\left\{ \begin{array}{l} 2 + (3 + 5) = 2 + 8 = 10 \\ (2 + 3) + 5 = 5 + 5 = 10 \end{array} \right. \quad \left\{ \begin{array}{l} 1 - (7 - 9) = 1 - (-2) = 1 + 2 = 3 \\ (1 - 7) - 9 = -6 - 9 = -15 \end{array} \right.$$

❖ EXAMPLES ❖

Ex.18 Find the integer for the following integers so that sum is zero.

23, -3, 0, -1, 7, 10, 253, -497

Sol. $23 + (-23) = 0$; $-1 + (1) = 0$; $253 + (-253) = 0$
 $-3 + (3) = 0$; $7 + (-7) = 0$; $-497 + (497) = 0$
 $0 + 0 = 0$; $10 + (-10) = 0$;

Note :

- Sum of the given two integers in each of the given pairs is zero i.e. the additive identity for integers.
- To find the additive inverse, we change the + sign into - sign (except in case of 0) of the given integer and vice-versa.
- Each of the integer in such a pair is called the additive inverse of the other e.g. -8 is the additive inverse of 8.

Ex.19 Write a pair of integers whose (i) sum is -7 and (ii) difference is -9.

Sol. (i) $-9 + 2 = -4 + (-3) = -7$
(ii) $1 - 10 = 2 - 11 = -9$

Ex.20 Write a pair of integers whose difference is :

- a negative number
- an integer greater than only one of the integers.

Sol. (i) $-14 - (-5) = -9$ (Negative integer)

(ii) $(-11) - (-3) = -8$ (It is greater than - 11 and less than -3)

Ex.21 Verify : $[-a - (-b)] - c \neq -a - [-b - (c)]$:

if $a = 3$, $b = 7$, $c = -9$

Sol. LHS = $[-a - (-b)] - c$
 $= [-3 - (-7)] - (-9)$
 $= [-3 + 7] + 9$
 $= 4 + 9 = 13$

RHS = $-a - [-b - (c)]$
 $= -3 - [-7 - (-9)]$
 $= -3 - [-7 + 9]$
 $= -3 - [2]$
 $= -5$

\therefore LHS \neq RHS

Ex.22 Verify $a - (-b) = a + b$ for the following :

$a = 117$, $b = -112$

Sol. LHS = $a - (-b)$
 $= 117 - [- (-112)]$
 $= 117 - (112)$
 $= 5$

RHS = $a + b$
 $= 117 + (-112)$
 $= 117 - 112 = 5$

\therefore LHS = RHS

► MULTIPLICATION OF INTEGERS

- Two positive numbers.
- One positive and one negative number or negative to positive number.
- Two negative numbers.

Eg. (i) $5 \times 6 = 30$ (ii) $7 \times 9 = 63$
(iii) $9 \times 10 = 90$ (iv) $-3 \times 1 = -3$
(v) $-7 \times 9 = -63$ (vi) $-11 \times 11 = -121$
(vii) $13 \times -5 = -65$ (viii) $10 \times -10 = -100$
(ix) $-40 \times -20 = 800$ (x) $-5 \times -1 = 5$

◆ Sign system for multiplication

$(+) \times (+) = +$	Positive \times Positive = Positive
$(-) \times (+) = -$	Negative \times Positive = Negative
$(+) \times (-) = -$	Positive \times Negative = Negative
$(-) \times (-) = +$	Negative \times Negative = Positive

Note: (i) If negative integers are multiplied even times, product is always a positive integer.

(ii) If negative integers are multiplied odd times, product is always a negative integer.

➤ PROPERTIES OF MULTIPLICATION

- (i) Closure \checkmark
- (ii) Commutative \checkmark
- (iii) Associative identity \checkmark
- (iv) Multiplicative identity 1
- (v) Multiplicative inverse reciprocal of given number

Eg. (i) $16 \times 12 = 192$ (integer)

(ii) $17 \times 10 = 170 = 10 \times 17$ (commutative)

(iii) $2 \times (3 \times 20) = 2 \times 60 = 120$
 $(2 \times 3) \times 20 = 6 \times 20 = 120$ } (Associative)

➤ DISTRIBUTIVE PROPERTY

For any three integers a, b, c ; $a \times (b + c) = a \times b + a \times c$

Let us observe the following products :

$$(i) \quad 7 \times (2 + 5) \quad \text{and} \quad 7 \times 2 + 7 \times 5$$

$$= 49 \quad \quad \quad = 14 + 35 = 49$$

$$\text{Thus, } 7 \times (2 + 5) = 7 \times 2 + 7 \times 5$$

$$(ii) \quad -2(-3 + 1) \quad \text{and} \quad -2 \times -3 + (-2) \times (1)$$

$$= -2(-2) \quad \quad \quad = (-2) \times (-3) + (-2) \times 1$$

$$= -2 \times -2 \quad \quad \quad = 6 - 2$$

$$= 4 \quad \quad \quad = 4$$

$$\text{Thus } -2 \times (-3 + 1) = -2 \times (-3) + (-2) \times (1)$$

This property of integers is known as the distributive property of multiplication over addition.

$$(iii) \quad 7 \times (5 - 7) \quad \text{and} \quad 7 \times 5 - 7 \times 7$$

$$= 7 \times (-2) \quad \quad \quad = 35 - 49$$

$$= -14$$

$$= -14$$

$$\text{Thus, } 7 \times (5 - 7) = 7 \times 5 - 7 \times 7$$

This property of integers is known as the distributive property of multiplication over subtraction.

Note : Any number 'a' when multiply by 1 and 0, gives itself and 0 respectively.

$$\text{Eg : } 7 \times 1 = 7, \quad -3 \times 1 = -3, \quad 9 \times 0 = 0$$

◆ EXAMPLES ◆

Ex.23 In a class test containing 20 questions, 3 marks are given for every correct answer and -1 mark is given for every incorrect answer.

- (i) Ritu attempt all questions but only 11 of her answers are correct. What is her total score ?
- (ii) One of her friends attempt 8 questions but only one answer is incorrect. What is her friend's total score ?

Sol. (i) Marks given for one correct answer = 3

$$\text{So, Marks given for 11 correct answer} = 3 \times 11 = 33$$

$$\text{Marks given for one incorrect answer} = -1$$

$$\text{So, Marks given for 9 incorrect answers}$$

$$= -1 \times 9 = -9$$

$$\text{Therefore, Ritu's total score} = 33 - 9 = 24$$

(ii) Marks given for one correct answer = 3

$$\text{So, Marks given for 7 correct answer}$$

$$= 3 \times 7 = 21$$

$$\text{Marks given for one incorrect answer}$$

$$= 1 \times -1 = -1$$

$$\text{Therefore, her friend's total score}$$

$$= 21 - 1 = 20$$

Ex.24 Complete the following multiplication :

\times	-7	-6	5	4
-7				
-6				
-5				
-4				
0				

Sol.

\times	-7	-6	5	4
-7	49	42	-35	-28
-6	42	36	-30	-24
-5	35	30	-25	-20
-4	28	24	-20	-16
0	0	0	0	0

Ex.25 Compare :

(i) $(7 + 9) \times 10$ and $7 + 9 \times 10$

(ii) $[(-4 - 6)] \times (-2)$ and $(-4) - 6 \times -7$

Sol.(i) $(7 + 9) \times 10$ and $7 + 9 \times 10$
 $= 16 \times 10 = 160$ $= 7 + 90 = 97$

(By BODMAS Rule)

$\therefore (7 + 9) \times 10 > (7 + 9 \times 10)$

(ii) $[(-4 - 6)] \times (-2) = -10 \times -2 = 20$
and $(-4) - 6 \times -7 = -4 - 6 \times -7 = -4 + 42 = 38$
 $\therefore [(-4 - 6)] \times -2 < (-4) - 6 \times -7$

Ex.26 If $a \times (-1) = -25$, is the integer a positive or negative ?

Sol. $-a = -25 \Rightarrow a = 25$ \therefore a is positive

Ex.27 Match the following :

- | | |
|--|--|
| (i) $(-7) + 9 = 9 + (-7)$ | (a) property of multiplicative identity |
| (ii) $6 + [3 + (-2)] = [(6 + 3)] + (-2)$ | (b) Commutative property of addition |
| (iii) $(-8)(-5) = (-5)(-8)$ | (c) Multiplicative property of zero |
| (iv) $4[5 \times (-5)] = (4 \times 5)(-5)$ | (d) Associative property of multiplication |
| (v) $7 \times 0 = 0$ | (e) Associative property of addition |
| (vi) $13 \times 1 = 13$ | (f) Commutative property of multiplication |

Sol.(i) b (ii) e (iii) f (iv) d (v) c (vi) a

DIVISION OF INTEGERS

Division is the reverse process of multiplication.

For example, to divide 32 by -4 means to find a number by which -4 should be multiplied such that it gives the product 32. The answer is -8 .

Eg : Observe the pattern and fill up the boxes.

Ans

(i) $6 \times 4 = 24$ $\therefore 24 \div 4 = 6$

(ii) $8 \times -5 = -40$ $\therefore -40 \div -5 = 8$

(iii) $-8 \times 3 = -24$ $\therefore \square \div 3 = -8$ -24

(iv) $7 \times 5 = 35$ $\therefore 35 \div \square = 7$ 5

(v) $-6 \times 4 = -24$ $\therefore -24 \div \square = 4$ -6

(vi) $-8 \times \square = -48$ $\therefore 48 \div \square = -8$ -6

SIGN SYSTEM FOR DIVISION

(i) The quotient of two integers involving two like signs is positive

or $(+) \div (+) = +$ and $(-) \div (-) = +$.

(ii) The quotient of two integers having opposite signs is negative

or $(+) \div (-) = -$ and $(-) \div (+) = -$.

Properties of division

(1) Closure No (divisor should be non zero)

(2) Commutative No

(3) Associative No

Eg :(i) $25 \div 5 = 5$ (integer)

(ii) $20 \div 10 = 2$ (integer)

(iii) $30 \div 7 \neq$ integer

(iv) $20 \div 5 = 4 \neq 5 \div 20$

(v) $(36 \div 9) \div 2 = 4 \div 2 = 2$

$$36 \div (9 \div 2) = 36 \div \frac{9}{2}$$

$$= 36 \times \frac{2}{9} = 4 \times 2 = 8$$

Note : Thus, division of any non-zero integer by zero is an undefined operation.

❖ EXAMPLES ❖

Ex.28 The product of two integers is -120 . If one number is -30 , what is the other.

Sol. Let the other number be 'a'

Then according to questions (a) $(-30) = -120$

$$a = -120 \div -30 = 40 \quad \text{Ans.}$$

Ex.29 In a test $+4$ marks are given for every correct answer and -2 marks are given for every incorrect answer.

(i) Neeta answered all the questions and scored 40 marks though she got 15 correct answers.

(ii) Radhey also answered all the questions and scored -16 marks though he got 5 correct answers.

How many incorrect answers had they attempted?

Sol. (i) Marks given for one correct answer = 4

So, Marks given for 15 correct answers

$$= 4 \times 15 = 60$$

Neeta's score = 40

Marks obtained for incorrect answers

$$= 40 - 60 = -20$$

Marks given for one incorrect answer = -2

Therefore, number of incorrect answers

$$= -20 \div -2 = 10$$

(ii) So, Marks given for 5 correct answers

$$= 5 \times 4 = 20$$

Radhey's score = -16

Marks obtained for incorrect answers

$$= -16 - 20 = -36$$

Marks given for one incorrect answers

$$= -2$$

Therefore, number of incorrect answers

$$= -36 \div -2 = 18$$

Ex.30 A shopkeeper earns a profit of $\text{₹ } 2$ by selling one pen and incurs a loss of 50 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of $\text{₹ } 10$. In this period, she sold 45 pens. How many pencils did she sell in this period ?

(ii) In the next month, she earns neither profit nor loss. If she sold 80 pens, how many pencils did she sell ?

Sol. (i) Profit earned by selling one pen = $\text{₹ } 2$

Profit earned by selling 45 pens

$$= 2 \times 45 = \text{₹ } 90$$

Total loss given = 10, which we denote by $\text{₹ } 10$

Profit earned + Loss incurred = Total loss

Therefore,

Loss incurred = Total loss – Profit earned

$$= \text{₹ } (-10 - 90) = \text{₹ } -100$$

$$= -10000 \text{ paise}$$

So, Number of pencils sold = $-10000 \div -50$

$$= 200 \text{ pencils}$$

(ii) In the next month, there is neither profit nor loss.

So, Profit + Loss incurred = 0

It means profit earned = – Loss incurred

Now, profit earned by selling 80 pens

$$= 2 \times 80$$

$$= \text{₹ } 160$$

Hence, loss incurred by selling pencils = $\text{₹ } 160$

Which we indicate by $-\text{₹ } 160$ or -16000 paise

Total number of pencils sold = $(-16000) \div 50$

$$= 320 \text{ pencils}$$

➤ RULE OF BODMAS

B stands for **brackets**, O for the operation '**Of**' D for **division**, M for **multiplication**, A for **addition** and S for **subtraction**.

◆ Types of bracket

Round brackets or parenthesis ()

Curly brackets or braces { }

Square brackets []

bar or vinculum —

Vinculum or bar is used as the innermost brackets and then (), then { }, and finally [].

Eg. (i) $(8 \div \overline{2+2})$ means $8 \div 4$

$$\begin{aligned} \text{(ii)} \quad & 10 + [5 \times \{48 \div (2 \times 4)\}] \\ &= 10 + [5 \times \{48 \div 8\}] \\ &= 10 + [5 \times 6] \\ &= 10 + 30 \\ &= 40 \quad \text{Ans.} \end{aligned}$$

► THE OPERATION 'OF'

Eg. (i) 9 of half of 20 means 9 of $\frac{20}{2} = 9 \times 10 = 90$

$$\text{(ii)} \quad \text{One third of 213 means } \frac{1}{3} \times 213 = 71$$

❖ EXAMPLES ❖

Ex.31 Simplify : $57 - [28 - \{16 + (5 - \overline{3-1})\}]$.

$$\begin{aligned} \text{Sol.} \quad & 57 - [28 - \{16 + (5 - \overline{3-1})\}] \\ &= 57 - [28 - \{16 + (5 - 2)\}] \quad [\text{Removal of bar}] \\ &= 57 - [28 - \{16 + 3\}] \\ &\quad \quad \quad [\text{Innermost brackets removed}] \\ &= 57 - [28 - 19] \\ &\quad \quad \quad [\text{Next Innermost brackets removed}] \\ &= 57 - 9 = 48 \end{aligned}$$

Ex.32 Simplify : (i) $7 - \{13 - 2(4 \text{ of } -4)\}$

$$\text{(ii)} \quad 81 \text{ of } [59 - \{7 \times 8 + (13 - 2 \text{ of } 5)\}]$$

$$\begin{aligned} \text{Sol. (i)} \quad & 7 - \{13 - 2(4 \text{ of } -4)\} \\ &= 7 - \{13 - 2(4 \times -4)\} \\ &= 7 - \{13 - 2(-16)\} = 7 - \{13 - (-32)\} \\ &= 7 - \{13 + 32\} = 7 - 45 = -38 \\ \text{(ii)} \quad & 81 \text{ of } [59 - \{7 \times 8 + (13 - 2 \text{ of } 5)\}] \\ &= 81 \times [59 - \{7 \times 8 + (13 - 2 \times 5)\}] \\ &= 81 \times [59 - \{7 \times 8 + (13 - 10)\}] \\ &= 81 \times [59 - \{56 + 3\}] \\ &= 81 \times [59 - 59] \\ &= 81 \times 0 = 0 \end{aligned}$$

Ex.33 Simplify :

$$63 - (-3) \{-2 - \overline{8-3}\} \div 3\{5 + (-2)(-1)\}$$

$$\begin{aligned} \text{Sol.} \quad & 63 - (-3) \{-2 - \overline{8-3}\} \div 3\{5 + (-2)(-1)\} \\ &= 63 - (-3) \{-2 - 5\} \div 3\{5 + (-2)(-1)\} \\ &\quad \quad \quad [\text{Removal of bar}] \\ &= 63 - (-3) \{-2 - 5\} \div 3\{5 + 2\} \\ &\quad \quad \quad [(-2)(-1) = 2] \\ &= 63 - (-3) \{-7\} \div 3 \times 7 \\ &\quad \quad \quad [\text{Removal of curly brackets}] \\ &= 63 + 3 \times \left(\frac{-7}{3}\right) \times 7 = 63 - 49 = 14 \end{aligned}$$

► ABSOLUTE VALUE OF AN INTEGER

The value of an integer is numerical value with no regard to its sign and it is always positive.

$$\text{Eg : } |-27| = 27, |-31| = 31, |29| = 29, |3| = 3.$$

EXERCISE # 1

A. Multiple Choice Type Questions

- Q.1** The additive identity of integers is
(a) -1 (B) 1
(C) 0 (D) none of these
- Q.2** The smallest negative integer is
(A) 0 (B) -100
(C) -1 (D) none of these
- Q.3** The greatest positive integer is
(A) 0 (B) 100
(C) 999 (D) none of these
- Q.4** The integer which is its own additive inverse is
(A) 0 (B) -1
(C) $+1$ (D) none of these
- Q.5** The value of $5(10 - 9)$ is
(A) 5 (B) $5 \times 10 - 5 \times 9$
(C) (A) and (B) (D) none of these
- Q.6** The absolute value of -6 is
(A) 6 (B) -6
(C) 0 (D) none of these
- Q.7** If $4 + (-7) = (-7) + 4$, this property of integers is called
(A) Closure (B) Commutative
(C) Associative (D) None of these
- Q.8** If $[-7 + (-8)] + 9 = -7 + [(-8) + 9]$, this property is called
(A) Closure (B) Commutative
(C) Associative (D) None of these
- Q.9** The sum of two integers is also an integer, this property of integers is called
(A) Closure (B) Commutative
(C) Associative (D) None of these
- Q.10** If a and b are integers, then $a \div b$
(A) may or may not be an integer
(B) always an integer
(C) never be an integer
(D) none of these

B. Short Answer Type Questions

- Q.11** Evaluate the following :
(i) $(-40) \div 10$
(ii) $60 \div (-6)$
(iii) $(-49) \div (-7)$
(iv) $(-79) \div 79$
(v) $13 \div [(-4) + 3]$
(vi) $0 \div (-14)$
(vii) $(-41) \div [(-40) + (-1)]$
(viii) $[(-48) \div 12] \div 4$
(ix) Is $[(-7) + (6)] = [(-3) + 2]$?
- Q.12** Write down a pair of integers whose :
(i) sum is -7 (ii) difference is -10
(iii) sum is 0
- Q.13** Write the value of
(i) $|+22|$
(ii) $|-8|$
(iii) $|18 - 8|$
(iv) $|-5 - 4|$
(v) $-|3 - 2|$
- Q.14** Arrange the following integers in ascending order
(i) $-20, 13, 4, 0, -5, +5$
(ii) $+30, -2, 0, -6, -20, 8$
- Q.15** Which temperature is higher ?
(i) 40°C or -40°C (ii) -18°C or 12°C
(iii) -2°C or -4°C (iv) 17°C or 27°C
- Q.16** A water tank has steps inside it. A monkey is sitting on the toppest step (i.e., the first step). The water level is at the ninth step.
(i) He jumps 3 steps down and then jumps 2 steps up. If he continues in this way, in how many jumps will he reach the water level ?
(ii) After drinking water, he wants to go back, for this he jumps 4 steps up and then jumps 2 steps down in every move. In how many jumps will he reach back the top step ?
- Q.17** Match column A with column B

	Column A	Column B
(i)	$ -7 $	(a) Distributive property
(ii)	$x \times 1 = 1 \times x = x$	(b) 7
(iii)	$x \times (y + z) = x \times y + x \times z$	(c) Multiplicative identity
(iv)	$x \div 0$	(d) 20
(v)	$-2 \times (-7 - 3)$	(e) Not defined
(vi)	$-31 \times 0 \times (-1) \times 151$	(f) Commutative Property
(vii)	$a \times b = b \times a$	(g) Zero

Q.18 Match column A with column B

	Column A	Column B
(i)	$-a \times b = b \times (-a)$	(a) 1
(ii)	$[6 + 4] + (-3) = 6 + [4 + (-3)]$	(b) 0 is additive identity of integers
(iii)	$-9 - (-19)$	(c) Commutative property
(iv)	$0 + 5 = 5 + 0 = 5$	(d) Associative property
(v)	$(-709) \div (+709)$	(e) 10
(vi)	$(-1)^6$	(f) -1

Q.19 Verify and name the property used :

- (i) $-110 \times (-237) = (-237) \times (-110)$
(ii) $(-35 \times 4) \times (-152) = -35 \times [4 \times (-152)]$

Q.20 Verify and name the property used :

- (i) $-117 \times 251 + (-117) \times 249$
(ii) $156 \times 273 - 156 \times (-73)$

C. Long Answer Type Questions

Q.21 Verify that :

$a \div (b + c) \neq (a \div b) + (a \div c)$, if $a = 12$, $b = -4$, $c = 2$.

Q.22 The temperature at 12 noon was 10°C above zero. At what time the temperature will be 8°C below 0°C , if it decreases at the rate of 2°C ? If it decreases at the rate of 2°C per hour till

midnight, what would be the temperature at mid-night ?

Q.23 In a class test (+3) marks are given for every correct answer and (-2) marks are given for every incorrect answer and no marks for not attempting any questions.

- (i) Ram scored 20 marks. If he has got 12 correct answers, how many questions has he attempted incorrectly ?
(ii) Mohan scores -5 marks in the test though he has written 7 correct answers. How many questions has he attempted incorrectly ?

Q.24 A boy has ₹350 in his bank account. He deposits ₹40 everyday for 10 days. What will be the amount in his account at the end of 10 days ?

Q.25 In a class test containing 10 questions, 5 marks are answered for every correct answer and (-2) marks are awarded for every incorrect answer and 0 for questions not attempted .

- (i) Mohan gets four correct and six incorrect answers. What is his score ?
(ii) Reshma gets five correct answer and five incorrect answers. What is her score ?
(iii) Heena gets two correct and five incorrect answers out of seven questions she attempts. What is her score ?

Q.26 A cement company earns a profit of ₹11 per bag of white cement sold out a loss of ₹6 per bag of grey cement sold.

- (i) The company sells 4,000 bags of white cement and 8,000 bags of grey cement in a month. What is its profit or loss ?
(ii) What is the number of white cement bags it must sell to have neither profit nor loss, if the number of grey bags sold is 1,100 bags.

Q.27 Find

- (i) a pair of negative integers whose difference gives 6.
- (ii) a pair of negative integers whose difference gives -9 .
- (iii) a negative integer and a positive integer whose difference is -15 .
- (iv) a negative integer and a positive integer whose difference is 16.

Q.28 An elevator descends into a mineshaft at the rate of 7m/min. If the descent starts from 15 m above the ground level, how long will it take to reach 475 m ?

Q.29 Find the value of :

$$72 - [3 + \{18 - \overline{19 - 2}\}] \div \{1 + 5 \text{ of } 7 - (3 - 1)\}$$

Q.30 Find the value of :

$$5 \div [5 + \{5 - (5 + \overline{5 - 5})\}]$$

ANSWER KEY

1. C 2. D 3. D 4. A 5. C 6. A 7. B 8. C 9. A 10. A
11. (i) -4 (ii) -10 (iii) 7 (iv) -1 (v) -13 (vi) 0 (vii) 1 (viii) -1 (ix) yes
12. (i) -1, -6 or -9, 2 (ii) -11, -1 or 35, -45 (iii) -1, 1 or 20, -20
13. (i) 22 (ii) 8 (iii) 10 (iv) 9 (v) -1
14. (i) -20, -5, 0, 4, 5, 13 (ii) -20, -6, -2, 0, 8, 30
15. (i) 40°C (ii) 12°C (iii) -2°C (iv) 27°C
16. (i) 11 times (ii) 5 times
17. (i) b (ii) c (iii) a (iv) e (v) d (vi) g (vii) f
18. (i) c (ii) d (iii) e (iv) b (v) f (vi) a
19. (i) Commutative property of multiplication. (ii) Associative property of multiplication.
20. (i) Distributive property of multiplication over addition
(ii) Distributive property of multiplication over subtraction
22. 9 pm, -14°C 23. (i) 8 (ii) 13
24. ₹ 750 25. (i) 8 marks (ii) 15 marks (iii) 0 mark
26. (i) Loss of ₹ 4,000 (ii) 600 bags
27. (i) -3, -9 (ii) -10, -1 (iii) -8, 7 (iv) 1, -15
28. 1 hr 10 min.
29. 2
30. 1

EXERCISE # 2

A. Multiple Choice Type Questions

- Q.1** Every integer is also a
(A) natural number (B) whole number
(C) (A) and (B) both (D) none of these
- Q.2** When 0 is multiplied by any negative integer, their product will be
(A) a positive integer
(B) a negative integer
(C) zero
(D) none of these
- Q.3** The sum of two integers is always
(A) a natural number (B) a whole number
(C) an integer (D) none of these
- Q.4** The multiplicative identity of integers is
(A) 0 (B) +1
(C) -1 (D) none of these
- Q.5** Every positive integer is greater than
(A) zero
(B) every negative integer
(C) both (A) and (B)
(D) none of these

B. True/False Type Questions

- Q.6** The greatest positive integer is 100.
- Q.7** The smallest negative integer is not defined.
- Q.8** Zero is positive integer.
- Q.9** Every negative integer is greater than every positive integer.
- Q.10** Zero is less than every positive integer.
- Q.11** Division in integers is closed.
- Q.12** On dividing any integer by zero, the result is always zero.
- Q.13** $18 - |-12|$ is equal to 6.
- Q.14** $|-50|$ is equal to -50.
- Q.15** Additive inverse of zero is 1.

C. Fill in the Blanks Type Questions

- Q.16** The product of 200 negative integers is integer.
- Q.17** The product of 999 negative and 999 positive integers is
- Q.18** The absolute value of $-35 + 25 =$
- Q.19** Which temperature is greater -110°C or 110°C ?
- Q.20** At noon the temperature was 30°C . One hour later it was 6°C lower. The temperature after an hour was
- Q.21** The additive inverse of 0 is
- Q.22** The product of 58 negative integers is a integer.
- Q.23** By which integer $-(700)$ must be divided to get the quotient 700 ?
- Q.24** When we divide a negative integer by a positive integer, the result so obtained is
- Q.25** A negative integer is always than its additive inverse.
- Q.26** The value of $|28 - 18|$ is
- Q.27** The smallest negative integer is
- Q.28** $(-111) \div \dots\dots\dots = 1$
- Q.29** $8 + (9 + 7) = (8 + 9) + \dots\dots\dots$
- Q.30** Additive inverse of negative integer is... always.
- Q.31** Julius Caesar was born in 100 B.C. and was 66 years old when he died. In which year did he die?
- Q.32** A certain freezing process requires that room temperature be lowered from 43°C at the rate of 8°C every hour. What will be the room temperature 6 hours after the process begins ?
- Q.33** The temperature at 12 noon was 15°C above zero. If it decreases at the rate of 3°C per hour until midnight, at what time would the temperature be 9°C below zero ?

ANSWER KEY

1. D	2. C	3. C	4. B	5. C
6. F	7. T	8. F	9. F	10. T
11. F	12. F	13. T	14. F	15. F
16. positive	17. negative	18. 10	19. 110°C	20. 24°C
21. 0	22. positive	23. -1	24. negative integer	25. less
26. 10	27. does not exist	28. -111	29. 7	
30. positive integer	31. 34 B.C.	32. -5°C	33. 8 pm	