Straight Lines

Question 1.
In a ΔABC, if A is the point (1, 2) and equations of the median through B and C are respectively $x + y = 5$ and $x = 4$, then B is
(a) (1, 4)
(b) (7, -2)
(c) none of these
(d) (4, 1)

Answer: (b) (7, -2)
The equation of median through B is $x + y = 5$
The point B lies on it.
Let the coordinates of B are $(x_1, 5 – x_1)$
Now CF is a median through C,
So coordiantes of F i.e. mid-point of AB are
$((x_1 + 1)/2, (5 – x_1 + 2)/2)$
Now since this lies on $x = 4$
$\Rightarrow (x_1 + 1)/2 = 4$
$\Rightarrow x_1 + 1 = 8$
$\Rightarrow x_1 = 7$
Hence, the cooridnates of B are (7, -2)

Question 2.
The equation of straight line passing through the point (1, 2) and perpendicular to the line $x + y + 1 = 0$
(a) $y – x + 1 = 0$
(b) $y – x – 1 = 0$
(c) $y – x + 2 = 0$
(d) $y – x – 2 = 0$

Answer: (b) $y – x – 1 = 0$
Given straight line is: $x + y + 1 = 0$
$\Rightarrow y = -x – 1$
Slope = -1

Now, required line is perpendicular to this line.
So, slope = -1/-1 = 1
Hence, the line is
$y - 2 = 1 \times (x - 1)$
$\Rightarrow y - 2 = x - 1$
$\Rightarrow y - 2 - x + 1 = 0$
$\Rightarrow y - x - 1 = 0$

Question 3.
The points (-a, -b), (0, 0), (a, b) and (a^2, ab) are
(a) vertices of a square
(b) vertices of a parallelogram
(c) collinear
(d) vertices of a rectangle

Answer: (c) collinear
Let the four points are P(-a, -b), O(0, 0), Q(a, b) and $R(a^2, ab)$
Now,
m_1 = slope of OP = b/a
m_2 = slope of OQ = b/a
m_3 = slope of OR = b/a
Since $m_1 = m_2 = m_3$
So, the points O, P, Q, R are collinear.

Question 4.
The equation of the line through the points (1, 5) and (2, 3) is
(a) $2x - y - 7 = 0$
(b) $2x + y + 7 = 0$
(c) $2x + y - 7 = 0$
(d) $x + 2y - 7 = 0$

Answer: (c) $2x + y - 7 = 0$
Given, points are: (1, 5) and (2, 3)
Now, equation of line is
$y - y_1 = \{(y_2 - y_1)/(x_2 - x_1)\} \times (x - x_1)$
$\Rightarrow y - 5 = \{(3 - 5)/(2 - 1)\} \times (x - 1)$
$\Rightarrow y - 5 = (-2) \times (x - 1)$
$\Rightarrow y - 5 = -2x + 2$
$\Rightarrow 2x + y - 5 - 2 = 0$
$\Rightarrow 2x + y - 7 = 0$

Question 5.
The slope of a line which passes through points (3, 2) and (-1, 5) is
(a) 3/4
(b) -3/4
(c) 4/3
(d) -4/3

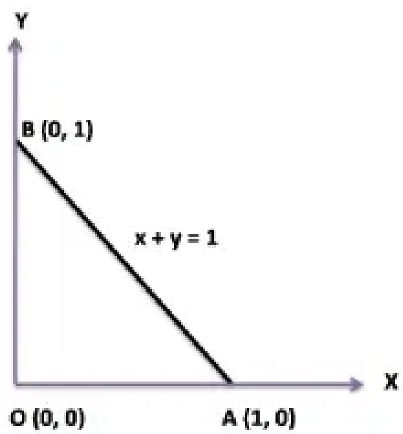
Answer: (b) -3/4
Given, points are (3, 2) and (-1, 5)
Now, slope m = (5 – 2)/(-1 – 3)
⇒ m = -3/4
So, the slope of the line is -3/4

Question 6.
The ratio of the 7th to the (n – 1)th mean between 1 and 31, when n arithmetic means are inserted between them, is 5 : 9. The value of n is
(a) 15
(b) 12
(c) 13
(d) 14

Answer: (d) 14
Let the A.P. are 1, A_1, A_2, A_3 …… A_m, 31
a = 1, a_n = 31 and n = m + 2
Now, a_n = a + (n – 1)d
⇒ 31 = 1 + (m + 2 – 1)d
⇒ 30 = (m + 1)d
⇒ d = 30/(m + 1)
Again, A_7 = a + 7d = 1 + 7[30/(m + 1)] …………….. 1
and A_{m-1} = a + (m – 1)d = 1 + (m – 1)[30/(m + 1)] ………. 2
From equation 1 and 2, we get
A_7/A_{m-1} = 5/9
⇒ 1 + 7[30/(m + 1) / 1 + (m – 1)[30/(m + 1)] = 5/9
⇒ [m + 1 + 7(30)] / [m + 1 + 30 m – 30] = 5/9
⇒ [m + 211] / [31 m – 29] = 5/9
⇒ 9[m + 211] = 5[31 m – 29]
⇒ 9 m + 1899 = 155 m – 145
⇒ 146 m = 2044
⇒ m = 2044/146

⇒ m = 14
So, the value of m is 14

Question 7.
The ortho centre of the triangle formed by lines xy = 0 and x + y = 1 is :
(a) (0, 0)
(b) none of these
(c) (1/2, 1/2)
(d) (1/3, 1/3)

Answer: (a) (0, 0)
Given lines are:
xy = 0 and x + y = 1
⇒ x = 0, y = 0 and x + y = 1


In a triangle OAB, OA and OB are the altitudes which intersect at O.
So, the required orthocentre is (0, 0)

Question 8.
Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if
(a) $a_1 /a_2 = b_1 /b_2 \neq c_1 /c_2$
(b) $a_1 /a_2 \neq b_1 /b_2 = c_1 /c_2$
(c) $a_1 /a_2 \neq b_1 /b_2 \neq c_1 /c_2$
(d) $a_1 /a_2 = b_1 /b_2 = c_1 /c_2$

Answer: (a) $a_1 /a_2 = b_1 /b_2 \neq c_1 /c_2$
Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if
$a_1 /a_2 = b_1 /b_2 \neq c_1 /c_2$

Question 9.
If the line x/a + y/b = 1 passes through the points (2, -3) and (4, -5), then (a, b) is
(a) a = 1 and b = 1
(b) a = 1 and b = −1
(c) a = −1 and b = 1
(d) a = −1 and b = -1

Answer: (d) a = −1 and b = -1
Given equation of the line is x/a + y/b = 1
⇒ bx + ay = ab
It is given that this line passes through (2, -3)
⇒ b(2) + a(-3) = ab
⇒ 2b – 3a = ab ——— (1)
It also passes through (4, -5)
⇒ 4b – 5a = ab ——— (2)
On solving equation (1) and (2), we get
a = -1 and b = -1

Question 10.
The angle between the lines x – 2y = y and y – 2x = 5 is
(a) $\tan^{-1}$ (1/4)
(b) $\tan^{-1}$ (3/5)
(c) $\tan^{-1}$ (5/4)
(d) $\tan^{-1}$ (2/3)

Answer: (c) $\tan^{-1}$ (5/4)
Given, lines are:
x – 2y = 5 ………. 1
and y – 2x = 5 ………. 2
From equation 1,
x – 5 = 2y
⇒ y = x/2 – 5/2
Here, m_1 = 1/2
From equation 2,
y = 2x + 5
Here. m_2 = 2
Now, tan θ = $|(m_1 + m_2)/\{1 + m_1 \times m_2\}|$
= |(1/2 + 2)/{1 + (1/2) × 2}|
= |(5/2)/(1 + 1)|
= |(5/2)/2|

= 5/4

⇒ $\theta = \tan^{-1}(5/4)$

Question 11.

The points on the y-axis whose distance from the line x/3 + y/4 = 1 is 4 units is

(a) (0, 32/3) and (0, 8/3)

(b) (0, -32/3) and (0, 8/3)

(c) (0, -32/3) and (0, -8/3)

(d) (0, 32/3) and (0, -8/3)

Answer: (d) (0, 32/3) and (0, -8/3)

Given equation of line is (x/3) + (y/4) = 1

⇒ 4x + 3y = 12

⇒ 4x + 3y – 12 = 0 ……………. 1

Let (0, b) is the point of the y-axis whose distance from given line is 4 unit.

When we compare equation 1 with general form of the equation Ax + By + C = 0, we get

A = 4, B = 3, C = -12

Now perpendicular distance of a line Ax + By + C = 0 from a point (x_1, y_1) is

$d = |Ax_1 + By_1 + C|/\sqrt{(A^2 + B^2)}$

So perpendicular distance of a line 4x + 3y – 12 = 0 from a point (0 ,b) is

$4 = |4\times0 + 3\times b - 12|/\sqrt{(4^2 + 3^2)}$

⇒ $4 = |3b - 12|/\sqrt{(16 + 9)}$

⇒ $4 = |3b - 12|/\sqrt{25}$

⇒ 4 = |3b – 12|/5

⇒ 4 × 5 = |3b – 12|

⇒ |3b – 12| = 20

Now

3b – 12 = 20 and 3b – 12 = -20

⇒ 3b = 20 12 and 3b = -20 + 12

⇒ 3b = 32 and 3b = -8

⇒ b = 32/3 and b = -8/3

So the points are (0, 32/3) and (0, -8/3)

Question 12.

Equation of the line passing through (0, 0) and slope m is

(a) y = mx + c

(b) x = my + c

(c) y = mx

(d) x = my

Answer: (c) $y = mx$
Equation of the line passing through (x_1, y_1) and slope m is
$(y - y_1) = m(x - x_1)$
Now, required line is
$(y - 0) = m(x - 0)$
$\Rightarrow y = mx$

Question 13.
The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is
(a) 3/10
(b) 2/3
(c) 3/2
(d) 7/10

Answer: (a) 3/10
Given equations are:
$3x + 4y = 9$
$\Rightarrow 3x + 4y - 9 = 0$ and
$6x + 8y = 15$
$\Rightarrow 6x + 8y - 15 = 0$
$\Rightarrow 3x + 4y - 15/2 = 0$
Now, compare these lines with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we get
$a_1 = 3, b_1 = 4, c_1 = -9$ and
$a_2 = 3, b_2 = 4, c_2 = -15/2$
Now, distance between two parallel line $= |c_1 - c_2|/\sqrt{(a_1^2 + b_1^2)}$
$= |-9 + 15/2|/\sqrt{(3^2 + 4^2)}$
$= |(-18 + 15)/2|/\sqrt{25}$
$= |(-3/2)|/5$
$= (3/2)/5$
$= 3/10$

Question 14.
What can be said regarding if a line if its slope is negative
(a) θ is an acute angle
(b) θ is an obtuse angle
(c) Either the line is x-axis or it is parallel to the x-axis.
(d) None of these

Answer: (b) θ is an obtuse angle
Let θ be the angle of inclination of the given line with the positive direction of x-axis in the

anticlockwise sense.
Then its slope is given by $m = \tan\theta$
Given, slope is positive
$\Rightarrow \tan\theta < 0$
$\Rightarrow \theta$ lies between 0 and 180 degree
$\Rightarrow \theta$ is an obtuse angle

Question 15.
Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if
(a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
(b) $a_1/a_2 \neq b_1/b_2 = c_1/c_2$
(c) $a_1/a_2 \neq b_1/b_2 \neq c_1/c_2$
(d) $a_1/a_2 = b_1/b_2 = c_1/c_2$

Answer: (a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Question 16.
The slope of a line making inclination of 30° with the positive direction of x-axis is
(a) 1/2
(b) $\sqrt{3}$
(c) $\sqrt{3}/2$
(d) $1/\sqrt{3}$

Answer: (d) $1/\sqrt{3}$
Here inclination of the line is 30°
So, slope of the line $m = \tan 30° = 1/\sqrt{3}$

Question 17.
The perpendicular distance of a line $4x + 3y + 5 = 0$ from the point (-1, 2) is
(a) 5
(b) 4
(c) 2
(d) 1

Answer: (c) 2
The perpendicular distance of a line $4x + 3y + 5 = 0$ from the point (-1, 2)
$d = |4 \times (-1) + 3 \times 3 + 5|/\sqrt{(4^2 + 3^2)}$

⇒ d = |-4 + 9 + 5|/√(16 + 9)
⇒ d = 10/√(25)
⇒ d = 10/5
⇒ d = 2

Question 18.
The inclination of the line 5x – 5y + 8 = 0 is
(a) 30°
(b) 45°
(c) 60°
(d) 90°

Answer: (b) 45°
Given line is: 5x – 5y + 8 = 0
⇒ 5y = 5x + 8
⇒ y = (5/5)x + 8/5
⇒ y = x + 8/5
Now tan θ = 1
⇒ tan θ = tan 45°
⇒ θ = 45°
So, the inclination of the line is 45°

Question 19.
The points (-a, -b), (0 , 0), (a, b) and (a^2, ab) are
(a) vertices of a square
(b) vertices of a parallelogram
(c) collinear
(d) vertices of a rectangle

Answer: (c) collinear
Let the four points are P(-a, -b), O(0, 0), Q(a, b) and R(a^2, ab)
Now,
m_1 = slope of OP = b/a
m_2 = slope of OQ = b/a
m_3 = slope of OR = b/a
Since $m_1 = m_2 = m_3$
So, the points O, P, Q, R are collinear.

Question 20.
Given the three straight lines with equations $5x + 4y = 0$, $x + 2y – 10 = 0$ and $2x + y + 5 = 0$, then these lines are
(a) none of these
(b) the sides of a right angled triangle
(c) concurrent
(d) the sides of an equilateral triangle

Answer: (c) concurrent
Since the determinant of these lines is equal to zero
i.e.

$$\begin{vmatrix} 5 & 4 & 0 \\ 1 & 2 & -10 \\ 2 & 1 & -5 \end{vmatrix} = 0$$

So, these three lines are concurrent.
