Chapter : 8. TRIGONOMETRIC IDENTITIES

Exercise : 8A

Question: 1 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 - \cos^2\theta) \times \text{cosec}^2\theta$

= $(\sin^2\theta) \times \text{cosec}^2\theta$ ($\because \sin^2\theta + \cos^2\theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 1 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 + \cot^2\theta) \times \sin^2\theta$

= $(\text{cosec}^2\theta) \times \sin^2\theta$ ($\because 1 + \cot^2\theta = \text{cosec}^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(\sec^2\theta - 1) \times \cot^2\theta$

= $(\tan^2\theta) \times \cot^2\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

= $(\tan^2\theta) \times \cot^2\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 C

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 - \cos^2 \theta) \sec^2 \theta$

= $(\sin^2\theta) \times (1/\cos^2\theta)$ ($\because \sin^2 \theta + \cos^2 \theta = 1$

= $\tan^2 \theta$

= R.H.S.

Hence, proved.

Question: 3 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$

= $(\sin^2\theta) + (1/\sec^2 \theta)$ ($\because 1 + \tan^2 \theta = \sec^2 \theta$)

= $(\sin^2\theta) + (\cos^2\theta)$ ($\because \sin^2 \theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 3 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{1}{1 + \tan^2 \theta} + \frac{1}{1 + \cot^2 \theta}$

= $(1/\sec^2\theta) + (1/\text{cosec}^2 \theta)$ ($\because 1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \tan^2 \theta = \sec^2 \theta$)

= $(\cos^2 \theta) + (\sin^2\theta)$ ($\because \sin^2 \theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 4 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 + \cos \theta)(1 - \cos \theta)(1 + \cot^2 \theta)$

= $(1 - \cos^2 \theta) \times \text{cosec}^2 \theta$ ($\because 1 + \cot^2 \theta = \text{cosec}^2 \theta$)

= $(\sin^2 \theta) \times \text{cosec}^2 \theta$ ($\because \sin^2\theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 4 B

Prove each of the

Solution:

To prove: (cosec θ)(1 + cos θ)(cosec θ – cot θ) = 1**Proof:**Consider the left – hand side:

(cosec θ) (1 + cos θ)(cosec θ – cot θ)⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) = (cosec θ + cosec θ cos θ)(cosec θ – cot θ)since cosec θ = 1/sin θ⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ)= $\left(cosec\ \theta+\frac{\cos\theta}{\sin\theta}\right)(\text{cosec}\theta-\cot\theta)$ Also $\cot\theta=\frac{cos\theta}{sin\theta}$ So,⇒ (cosec θ) (1 + cos θ) (cosec θ – cot θ) =(cosec θ + cot θ)(cosec θ – cot θ)

Use the formula (a + b)(a - b) = a^2 - b^2

⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) =($cosec^2$ θ - cot^2 θ)

Since $cosec^2$ θ - cot^2 θ = 1⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) =1

= R.H.S.

Hence, proved.

Question: 5 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

= (– $\sin^2$ θ) × $\sin^2$ θ (∵ $\sin^2\theta + \cos^2\theta = 1$)

= – 1

= R.H.S.

Hence, proved.

Question: 5 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\tan^2\theta - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta}$

= (– $\cos^2$ θ) × $\cos^2$ θ (∵ $\sin^2\theta + \cos^2\theta = 1$)

= – 1

= R.H.S.

Hence, proved.

Question: 5 C

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\cos^2 \theta + \frac{1}{1 + \cot^2 \theta}$

= $\cos^2 \theta + \frac{1}{\text{cosec}^2 \theta}$ ($\because 1 + \cot^2 \theta = \text{cosec}^2 \theta$)

= $\cos^2 \theta + \sin^2 \theta$

= $(- \cos^2 \theta) \times \cos^2 \theta$ ($\because \sin^2\theta + \cos^2 \theta = 1$)

= – 1

= R.H.S.

Hence, proved.

Question: 6

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

= $\frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$

= $\frac{2}{\cos^2\theta}$

= $2 \sec^2 \theta$

= R.H.S.

Hence, proved.

Question: 7 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\sec \theta(1 - \sin \theta)(\sec \theta + \tan \theta)$

= $\left(\frac{1}{\cos\theta}\right) \times (1 - \sin \theta) \times \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$

= $\left(\frac{1}{\cos\theta}\right) \times (1 - \sin \theta) \times \left(\frac{1+\sin\theta}{\cos\theta}\right)$

= $\frac{1-\sin^2\theta}{\cos^2\theta}$

= $\frac{\cos^2\theta}{\cos^2\theta}$

= 1

= R.H.S.

Hence, proved.

Question: 7 B

Prove each of the

Solution:

To prove: $\sin\theta\,(1 + \tan\theta) + \cos\theta\,(1 + \cot\theta) = (\sec\theta + \operatorname{cosec}\theta)$**Proof:**Consider the left – hand side:

L.H.S. = $\sin\theta\,(1 + \tan\theta) + \cos\theta\,(1 + \cot\theta)$

$= \sin\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1 + \frac{\cos\theta}{\sin\theta}\right)$

$= \sin\theta\left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right) + (\cos\theta) \times \left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right)$

$= (\cos\theta + \sin\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= (\cos\theta + \sin\theta)\left(\frac{(\cos^2\theta + \sin^2\theta)}{\cos\theta\sin\theta}\right)$We know $\cos^2\theta + \sin^2\theta = 1$

$= \left(\frac{\cos\theta + \sin\theta}{\cos\theta\sin\theta}\right)$

$= \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)$

$= \operatorname{cosec}\theta + \sec\theta$

= R.H.S.

Hence, proved.

Question: 8 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. $= 1 + \frac{\cot^2\theta}{1 + \operatorname{cosec}\theta}$

$= 1 + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1 + \frac{1}{\sin\theta}}$

$= 1 + \frac{\cos^2\theta}{1 + \sin\theta} \times \frac{\sin\theta}{\sin^2\theta}$

$= 1 + \frac{\cos^2\theta}{(1 + \sin\theta)\sin\theta}$

$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\sin\theta + \sin^2\theta}$

$= \frac{\sin\theta + 1}{\sin\theta(1 + \sin\theta)}$

$= 1/\sin\theta$

$= \operatorname{cosec}\theta$

= R.H.S.

Hence, proved.

Question: 8 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. $= 1 + \frac{\tan^2\theta}{1+\sec\theta}$

$= 1 + \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{1}{\cos\theta}}$

$= 1 + \frac{\sin^2\theta}{1+\cos\theta} \times \frac{\cos\theta}{\cos^2\theta}$

$= 1 + \frac{\sin^2\theta}{(1+\cos\theta)\cos\theta}$

$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{\cos\theta + \cos^2\theta}$

$= \frac{\cos\theta+1}{\cos\theta(1+\cos\theta)}$

$= 1/\cos\theta$

$= \sec\theta$

= R.H.S.

Hence, proved.

Question: 9

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. $= \frac{(1+\tan^2\theta)\cot\theta}{\mathrm{cosec}^2\theta}$

$= \frac{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\times\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$

$= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} \times \sin^2\theta$

$= 1 \times \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

= R.H.S.

Hence, proved.

Question: 10

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. $= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$

$= \frac{\sin^2\theta}{1+\sin^2\theta} + \frac{\cos^2\theta}{1+\cos^2\theta}$

$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{(1+\sin^2\theta)(1+\cos^2\theta)}$

$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}$

= 1

= R.H.S.

Hence, proved.

Question: 11

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$

Adding both the fractions, we get

$= \frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{\sin\theta(1+\cos\theta)}$

As $\sin^2\theta + \cos^2\theta = 1$, we have

$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

= 2/sin θ

= 2cosec θ

= R.H.S.

Hence, proved.

Question: 12

Prove :

Solution:

Consider L.H.S. = $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{(\sin\theta-\cos\theta)\cos\theta}+\frac{\cos^2\theta}{(\cos\theta-\sin\theta)\sin\theta}$

$= \frac{1}{(\sin\theta-\cos\theta)}\left(\frac{sin^2\theta}{cos\ \theta}-\frac{\cos^2\theta}{\sin\ \theta}\right)$

$= \frac{1}{(sin\theta-cos\theta)}\left(\frac{sin^3\theta-\cos^3\theta}{cos\theta\sin\theta}\right)$

$= \frac{\sin^3\theta-\cos^3\theta}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$

$= \frac{\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$

$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\,.\cos\theta}+\frac{\sin\theta\,.\cos\theta}{\sin\theta\,.\cos\theta}$

$= \frac{1}{\sin\theta\,.\cos\theta}+1$

= sec θ cosec θ + 1

= R.H.S.

Hence, proved.

Question: 13

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. $= \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$

$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$= \frac{(\cos\theta-\sin\theta)\left(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta\right)}{\cos\theta-\sin\theta}$

$= \cos^2\theta + \sin^2\theta + \sin\theta\cos\theta$

$= 1 + \cos\theta\sin\theta$

= R.H.S.

Hence, proved.

Question: 14

Prove each of the

Solution:

$\frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta} = \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$

$= \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)}$

$= \cos\theta + \sin\theta$

= R.H.S.

Hence, proved.

Question: 15

Prove each of the

Solution:

Consider L.H.S. $= (1 + \tan^2\theta)(1 + \cot^2\theta)$

$= (\sec^2\theta)(\operatorname{cosec}^2\theta)$

$= \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta}$

$= \frac{1}{1-\sin^2\theta} \times \frac{1}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta - \sin^4\theta}$

= R.H.S.

Hence, proved.

Question: 16

Prove each of the

Solution:

Consider L.H.S. $= \frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)^2}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}\right)^2}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(\frac{1}{\sin^2\theta}\right)^2}$

$= \left(\frac{\sin\theta}{\cos\theta} \times \cos^4\theta\right) + \left(\frac{\cos\theta}{\sin\theta} \times \sin^4\theta\right)$

$= \sin\theta\,(\cos^3\theta) + \cos\theta\,(\sin^3\theta)$

$= \sin\theta\cos\theta\,(\cos^2\theta + \sin^2\theta)$

$= \sin\theta\cos\theta$

= R.H.S.

Hence, proved.

Question: 17 A

Prove each of the

Solution:

Consider L.H.S. $= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

[Using $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$]

$= (\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= [\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta) - \sin^2\theta\cos^2\theta]$

$(\because (a^2 + b^2) = (a+b)^2 - 2ab))$

$= [1 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$

$= 1 - 3\sin^2\theta\cos^2\theta$

= R.H.S.

Hence, proved.

Question: 17 B

Prove each of the

Solution:

Consider L.H.S. = $\sin^2\theta + \cos^4\theta$

$= (\sin^2\theta) + (\cos^2\theta)^2$

$= (\sin^2\theta) + (1 - \sin^2\theta)^2$

$= (\sin^2\theta) + 1 + \sin^4\theta - 2\sin^2\theta$

$= 1 - \sin^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^4\theta$

= R.H.S.

Hence, proved.

Question: 17 C

Prove each of the

Solution:

Consider L.H.S. = $\text{cosec}^4\theta - \text{cosec}^2\theta$

$= (\text{cosec}^2\theta)^2 - (\text{cosec}^2\theta)$

$= (1 + \cot^2\theta)^2 - (\text{cosec}^2\theta)$

$= 1 + \cot^4\theta + 2\cot^2\theta - (\text{cosec}^2\theta)$

$= 1 + \cot^4\theta + \cot^2\theta - (\text{cosec}^2\theta - \cot^2\theta)$

$= 1 + \cot^4\theta + \cot^2\theta - 1$

$= \cot^4\theta + \cot^2\theta$

= R.H.S.

Hence, proved.

Question: 18 A

Prove each of the

Solution:

Consider L.H.S. = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$

$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta}$

$= \cos^2\theta - \sin^2\theta$

= R.H.S.

Hence, proved.

Question: 18 B

Prove each of the

Solution:

Consider L.H.S. = $\frac{1-\tan^2\theta}{\cot^2\theta-1}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1}$

$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta / \cos^2\theta$

$= \tan^2\theta$

= R.H.S.

Hence, proved.

Question: 19 A

Prove each of the

Solution:

Consider L.H.S. = $\frac{\tan\theta}{\sec\theta-1} + \frac{\tan\theta}{\sec\theta+1}$

$= \frac{\tan\theta(\sec\theta+1)+\tan\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$

$= \frac{\tan\theta\sec\theta+\tan\theta+\tan\theta\sec\theta-\tan\theta}{\sec^2\theta-1}$

$= \frac{2\tan\theta\sec\theta}{\tan^2\theta}$

$= \frac{2\sec\theta}{\tan\theta}$

$= [2(1/\cos\theta)]/[\sin\theta/\cos\theta]$

$= [2/\sin\theta]$

$= 2\operatorname{cosec}\theta$

= R.H.S.

Hence, proved.

Question: 19 B

Prove each of the

Solution:

Consider L.H.S. = $\frac{\cot\theta}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta}$

$= \frac{\cot^2\theta+(\operatorname{cosec}\theta+1)^2}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{\cot^2\theta+\operatorname{cosec}^2\theta+1+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{\operatorname{cosec}^2\theta+\operatorname{cosec}^2\theta+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{2\operatorname{cosec}^2\theta+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{2\operatorname{cosec}\theta(\operatorname{cosec}\theta+1)}{(\operatorname{cosec}\theta+1)}$

= 2 cosec θ/cot θ

= 2 (1/sin θ)/(cos θ/sin θ)

= 2/ cos θ

= 2 sec θ

= R.H.S.

Hence, proved.

Question: 20 A

Prove each of the

Solution:

Consider L.H.S. = $\frac{\sec\theta-1}{\sec\theta+1}$

Multiply and divide by (sec θ + 1):

$= \frac{\sec\theta-1}{\sec\theta+1} \times \frac{\sec\theta+1}{\sec\theta+1}$

$= \frac{\sec^2\theta-1}{(\sec\theta+1)^2}$

$= \frac{\tan^2\theta}{(1+\sec\theta)^2}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\left(\frac{1+\cos\theta}{\cos\theta}\right)^2}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{(1+\cos\theta)^2}{\cos^2\theta}}$

$= \frac{\sin^2\theta}{(1+\cos\theta)^2}$

= R.H.S.

Hence, proved.

Question: 20 B

Prove each of the

Solution:

Consider L.H.S. = $\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}$

Multiply and divide by (sec θ + tan θ):

$= \frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta} \times \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

$= \frac{\sec^2\theta-\tan^2\theta}{(\sec\theta+\tan\theta)^2}$

$= \frac{1}{\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)^2}$

$= \frac{1}{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{\cos^2\theta}{(1+\sin\theta)^2}$

= R.H.S.

Hence, proved.

Question: 21 A

Prove each of the

Solution:

Consider L.H.S. = $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

Multiply and divide by (1 + sin θ):

$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

= (1 + sin θ)/cos θ

= (1/cos θ) + (sin θ/cos θ)

= sec θ + tan θ

= R.H.S.

Hence, proved.

Question: 21 B

Prove each of the

Solution:

Consider L.H.S. = $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

Multiply and divide by (1 – cos θ):

$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

= (1 – cos θ)/sin θ

= (1/sin θ) – (cos θ/sin θ)

= cosec θ – cot θ

= R.H.S.

Hence, proved.

Question: 21 C

Prove each of the

Solution:

Consider L.H.S. = $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

Multiply and divide by (1 + cos θ) in first part and (1 – cos θ) in the second part:

$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

= [(1 + cos θ)/sin θ] + [(1 – cos θ)/sin θ]

= [(1/sin θ) + (cos θ/sin θ)] + [(1/sin θ) – (cos θ/sin θ)]

= [cosec θ + cot θ] + [cosec θ – cot θ]

= 2 cosec θ

= R.H.S.

Hence, proved.

Question: 22

Prove each of the

Solution:

Consider L.H.S. $= \frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta} + \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

Using identities $(a^3 + b^3) = (a + b)(a^2 + b^2 – ab)$ and $(a^3 – b^3) = (a – b)(a^2 + b^2 + ab)$

$\therefore$ L.H.S. $= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)}{(\cos\theta+\sin\theta)} + \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{(\cos\theta-\sin\theta)}$

$= (\cos^2\theta + \sin^2\theta – \cos\theta\sin\theta) + (\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)$

= (1 – cos θ sin θ) + (1 + cos θ sin θ)

= 2

= R.H.S.

Hence, proved.

Question: 23

Prove each of the

Solution:

Consider L.H.S. $= \frac{\sin\theta}{(\cot\theta+\operatorname{cosec}\theta)} - \frac{\sin\theta}{(\cot\theta+\operatorname{cosec}\theta)}$

$= \frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}\right)} - \frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)}$

$$\begin{aligned}
&= \frac{\sin^2\theta}{1+\cos\theta} - \frac{\sin^2\theta}{\cos\theta-1} \\
&= \frac{\sin^2\theta}{1+\cos\theta} + \frac{\sin^2\theta}{1-\cos\theta} \\
&= \sin^2\theta\left(\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta}\right) \\
&= \sin^2\theta\left(\frac{1-\cos\theta+1+\cos\theta}{1-\cos^2\theta}\right) \\
&= \sin^2\theta \times \frac{2}{\sin^2\theta} \quad [As, \sin^2\theta+\cos^2\theta = 1] \\
&= 2
\end{aligned}$$

= R.H.S.

Hence, proved.

Question: 24 A

Prove each of the

Solution:

Consider L.H.S. $= \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$

$= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{1 - 2\sin\theta\cos\theta + 1 + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$

$= \frac{2}{2\sin^2\theta - 1}$

= R.H.S.

Hence, proved.

Question: 24 B

Prove each of the

Solution:

Consider L.H.S. $= \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$

$= \frac{(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2}{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{1 + 2\sin\theta\cos\theta + 1 - 2\sin\theta\cos\theta}{1 - \cos^2\theta - coss^2\theta}$

$= \frac{2}{1 - 2\cos^2\theta}$

= R.H.S.

Hence, proved.

Question: 25

Prove each of the

Solution:

Consider L.H.S. $= \frac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{(1 - \sin^2\theta) + \cos\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{(\cos^2\theta) + \cos\theta}{\sin\theta(1 + \cos\theta)}$

$= \frac{\cos\theta(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)}$

= $\cos\theta/\sin\theta$

= $\cot\theta$

= R.H.S.

Hence, proved.

Question: 26 A

Prove each of the

Solution:

Consider L.H.S. = $\frac{\text{cosec}\,\theta+\cot\theta}{\text{cosec}\,\theta-\cot\theta}$

Multiply and divide by (cosec θ + cot θ):

$= \frac{\text{cosec}\,\theta+\cot\theta}{\text{cosec}\,\theta-\cot\theta} \times \frac{\text{cosec}\,\theta+\cot\theta}{\text{cosec}\,\theta+\cot\theta}$

$= \frac{(\text{cosec}\,\theta+\cot\theta)^2}{\text{cosec}^2\,\theta-\cot^2\theta}$

$= (\text{cosec}\,\theta + \cot\theta)^2$

Thus, proved.

Also, consider $(\text{cosec}\,\theta + \cot\theta)^2 = \text{cosec}^2\,\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$

$= 1 + \cot^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$ ($\because 1 + \cot^2\theta = \text{cosec}^2\,\theta$)

$= (1 + 2\cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta)$

= R.H.S.

Hence, proved.

Question: 26 B

Prove each of the

Solution:

Consider L.H.S. = $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

Multiply and divide by (sec θ + tan θ):

$= \frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} \times \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

$= \frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$

$= (\sec\theta + \tan\theta)^2$

Thus, proved.

Also, consider $(\sec\theta + \tan\theta)^2 = \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

$= (1 + 2\tan^2\theta + 2\sec\theta\tan\theta)$

= R.H.S.

Hence, proved.

Question: 27 A

Prove each of the

Solution:

Consider L.H.S. = $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

Multiply and divide by ((1 + cos θ) + sin θ):

$= \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta} \times \frac{1+\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}$

$= \frac{(1+\cos\theta+\sin\theta)^2}{(1+\cos\theta)^2 - \sin^2\theta}$

$=\frac{1+\cos^2\theta+\sin^2\theta+2\cos\theta+2\sin\theta+2\cos\theta\sin\theta}{1+\cos^2\theta+2\cos\theta-(1-\cos^2\theta)}$

$=\frac{1+1+2\cos\theta+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$

$=\frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos\theta(1+\cos\theta)}$

$=\frac{2(1+\cos\theta)(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$

$=\frac{1+\sin\theta}{\cos\theta}$

= R.H.S.

Thus, proved.

Question: 27 B

Prove each of the

Solution:

Consider L.H.S. $=\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}$

Multiply and divide by (cos θ + 1) + sin θ):

$=\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}\times\frac{\cos\theta+1+\sin\theta}{\cos\theta+1+\sin\theta}$

$=\frac{(1+\sin\theta)^2-\cos^2\theta}{(\sin\theta+\cos\theta)^2-1}$

$=\frac{1+\sin^2\theta+2\sin\theta-(1-\sin^2\theta)}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}$

$=\frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$

$=\frac{1+\sin\theta}{\cos\theta}$

= R.H.S.

Thus, proved.

Question: 28

Prove each of the

Solution:

Consider L.H.S. $=\frac{\sin\theta}{\sec\theta+\tan\theta-1}+\frac{\cos\theta}{\operatorname{cosec}\theta+\cot\theta-1}$

$=\frac{\sin\theta}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-1}+\frac{\cos\theta}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}-1}$

$=\frac{\sin\theta\cos\theta}{1+\sin\theta-\cos\theta}+\frac{\cos\theta\sin\theta}{1+\cos\theta-\sin\theta}$

$=\sin\theta\cos\theta\times\left(\frac{1}{1+\sin\theta-\cos\theta}+\frac{1}{1+\cos\theta-\sin\theta}\right)$

$=\sin\theta\cos\theta\times\left(\frac{1+\sin\theta-\cos\theta+1+\cos\theta-\sin\theta}{(1+\sin\theta-\cos\theta)(1+\cos\theta-\sin\theta)}\right)$

$=\sin\theta\cos\theta\times\frac{2}{1-(\sin\theta-\cos\theta)^2}$

$=\sin\theta\cos\theta\times\frac{2}{1-(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}$

$=\sin\theta\cos\theta\times\frac{2}{1-1+2\sin\theta\cos\theta}$

= sin θ cos θ/ sin θ cos θ

= 1

= R.H.S.

Question: 29

Prove each of the

Solution:

Consider L.H.S. = $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

= $\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

= $\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

= $\frac{2}{\sin^2\theta-\cos^2\theta}$

Thus, prove.

Also, consider $\frac{2}{\sin^2\theta-\cos^2\theta}=\frac{2}{\sin^2\theta-(1-\sin^2\theta)}$

= $\frac{2}{(2\sin^2\theta-1)}$

= R.H.S.

Hence, proved.

Question: 30

Prove each of the

Solution:

Consider L.H.S. = $\frac{\cos\theta\,\text{cosec}\,\theta-\sin\theta\sec\theta}{\cos\theta+\sin\theta}$

= $\frac{\cos\theta\left(\frac{1}{\sin\theta}\right)-\sin\theta\left(\frac{1}{\cos\theta}\right)}{\cos\theta+\sin\theta}$

= $\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta(\cos\theta+\sin\theta)}$

= $\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta\sin\theta(\cos\theta+\sin\theta)}$

= $\frac{\cos\theta-\sin\theta}{\cos\theta\sin\theta}$

= (1/sin θ) – (1/cos θ)

= cosec θ – sec θ

= R.H.S.

Hence, proved.

Question: 31

Prove each of the

Solution:

Consider L.H.S. = (1 + tan θ + cot θ)(sin θ – cos θ)

= sin θ – cos θ + tan θ sin θ – tan θ cos θ + cot θ sin θ – cot θ cos θ

= sin θ – cos θ + tan θ sin θ – sin θ + cos θ – cot θ cos θ

= tan θ sin θ – cot θ cos θ

$$= \frac{\sin\theta}{\cos\theta} \times \sin\theta - \frac{\cos\theta}{\sin\theta} \times \cos\theta$$

$$= \frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}$$

$$\left[\text{since, } \sin\theta = \frac{1}{\text{cosec}\theta} \; and \; \cos\theta = \frac{1}{\sec\theta}\right]$$ = R.H.S.

$$= \frac{\sec\theta}{\text{cosec}^2\theta} - \frac{\text{cosec}\theta}{\sec^2\theta}$$

Hence, proved.

Question: 32

Prove each of the

Solution:

Consider L.H.S. $= \frac{\cot^2\theta(\sec\theta - 1)}{(1+\sin\theta)} + \frac{\sec^2\theta(\sin\theta - 1)}{(1+\sec\theta)}$

$$= \frac{\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\left(\frac{1}{\cos\theta} - 1\right)}{(1+\sin\theta)} + \frac{\left(\frac{1}{\cos^2\theta}\right)(\sin\theta - 1)}{\left(1+\frac{1}{\cos\theta}\right)}$$

$$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)\sin^2\theta} + \frac{(\sin\theta - 1)\cos\theta}{\cos^2\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)(1-\cos^2\theta)} + \frac{(\sin\theta - 1)\cos\theta}{(1-\sin^2\theta)(1+\cos\theta)}$$

$$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)(1-\cos\theta)(1+\cos\theta)} + \frac{(\sin\theta - 1)\cos\theta}{(1-\sin\theta)(1+\sin\theta)(1+\cos\theta)}$$

$$= \frac{\cos\theta}{(1+\sin\theta)(1+\cos\theta)} - \frac{\cos\theta}{(1+\sin\theta)(1+\cos\theta)}$$

$= 0$

= R.H.S.

Hence, proved.

Question: 33

Prove each of the

Solution:

Consider L.H.S. $= \left\{\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\text{cosec}^2\theta - \sin^2\theta}\right\}(\sin^2\theta \cos^2\theta)$

$$= \left\{\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right\}(\sin^2\theta \cos^2\theta)$$

$$= \left\{\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right\}(\sin^2\theta \cos^2\theta)$$

$$= \left\{\frac{\cos^2\theta(1-\sin^4\theta) + \sin^2\theta(1-\cos^4\theta)}{(1-\cos^4\theta)(1-\sin^4\theta)}\right\}(\sin^2\theta \cos^2\theta)$$

$$= \frac{\cos^2\theta + \sin^2\theta - \cos^2\theta\sin^4\theta - \cos^4\theta\sin^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)(1-\sin^2\theta)(1+\sin^2\theta)}(\sin^2\theta \cos^2\theta)$$

$$= \frac{1 - \cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}(\sin^2\theta \cos^2\theta)$$

$$= \frac{1 - \sin^2\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$= \frac{1-\sin^2\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$

$= \frac{1-\sin^2\theta\cos^2\theta}{1+1+\sin^2\theta\cos^2\theta}$

$= \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

= R.H.S.

Question: 34

Prove each of the

Solution:

Consider the left – hand side $= \frac{\sin A-\sin B}{\cos A+\cos B} + \frac{\cos A-\cos B}{\sin A+\sin B}$

$= \frac{(\sin A-\sin B)(\sin A+\sin B)+(\cos A-\cos B)(\cos A+\cos B)}{(\cos A+\cos B)(\sin A+\sin B)}$

$= \frac{\sin^2 A-\sin^2 B+\cos^2 A-\cos^2 B}{(\cos A+\cos B)(\sin A+\sin B)}$

$= \frac{\sin^2 A+\cos^2 A-(\sin^2 B+\cos^2 B)}{(\cos A+\cos B)(\sin A+\sin B)}$

$= \frac{1-1}{(\cos A+\cos B)(\sin A+\sin B)}$

= 0

= R.H.S.

Hence, proved.

Question: 35

Prove each of the

Solution:

Consider the L.H.S. $= \frac{\tan A+\tan B}{\cot A+\cot B}$

$= \frac{\tan A+\tan B}{\frac{1}{\tan A}+\frac{1}{\tan B}}$

$= \frac{\tan A+\tan B}{\frac{\tan A+\tan B}{\tan A\tan B}}$

$= \frac{(\tan A+\tan B)(\tan A\tan B)}{(\tan A+\tan B)}$

= tan A tan B

= R.H.S.

Hence, proved.

Question: 36 A

Show that none of

Solution:

If the given equation is an identity, then it is true for every value of θ.

So, let θ = 60°

So, for θ = 60°, consider the L.H.S. = $\cos^2 60° + \cos 60°$

$= (1/2)^2 + (1/2)$

= (1/4) + (1/2)

= 3/4 ≠ 1

Therefore, L.H.S. ≠ R.H.S.

Thus, the given equation is not an identity.

Question: 36 B

Show that none of

Solution:

If the given equation is an identity, then it is true for every value of θ.

So, let θ = 30°

So, for θ = 30°, consider the L.H.S. = $\sin^2 30° + \sin 30°$

= $(1/2)^2 + (1/2)$

= (1/4) + (1/2)

= 3/4 ≠ 2

Therefore, L.H.S. ≠ R.H.S.

Thus, the given equation is not an identity.

Question: 36 C

Show that none of

Solution:

If the given equation is an identity, then it is true for every value of θ.

So, let θ = 30°

So, for θ = 30°, consider the L.H.S. = $\tan^2 30° + \sin 30°$

= $(1/\sqrt{3})^2 + (1/2)$

= (1/3) + (1/2)

= 5/6

Consider the R.H.S. = $\cos^2 30° = (\sqrt{3}/2)^2$

= 3/4

Therefore, L.H.S. ≠ R.H.S.

Thus, the given equation is not an identity.

Question: 37

Prove that: (sin

Solution:

Consider R.H.S. = $(2\cos^3\theta - \cos\theta)\tan\theta$

= $\cos\theta(2\cos^2\theta - 1)\left(\frac{\sin\theta}{\cos\theta}\right)$

= $(2\cos^2\theta - 1)\sin\theta$

Consider L.H.S. = $(\sin\theta - 2\sin^3\theta)$

= $\sin\theta(1 - 2\sin^2\theta)$

= $\sin\theta[1 - 2(1 - \cos^2\theta)]$

= $\sin\theta[1 - 2 + 2\cos^2\theta]$

$= \sin\theta\,(2\cos^2\theta - 1)$

Therefore, L.H.S. = R.H.S.

Hence, proved.

Exercise : 8B

Question: 1

If a cos θ + b si

Solution:

Given: $a\cos\theta + b\sin\theta = m$ …….(1)

$a\sin\theta - b\cos\theta = n$ …….(2)

Square equation (1) and (2) on both sides:

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta = m^2$ …….(3)

$a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta = n^2$ ……..(4)

Add equation (3) and (4):

$[a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta] + [a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta] = m^2 + n^2$

$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta) = m^2 + n^2$

$\Rightarrow a^{2+}b^2 = m^2 + n^2$

Hence, proved.

Question: 2

If x = a sec θ +

Solution:

Given: $a\sec\theta + b\tan\theta = x$ …….(1)

$a\tan\theta + b\sec\theta = y$ …….(2)

Square equation (1) and (2) on both sides:

$a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta = x^2$ …….(3)

$a^2\tan^2\theta + b^2\sec^2\theta + 2ab\sec\theta\tan\theta = y^2$ ……..(4)

Subtract equation (4) from (3):

$[a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta] - [a^2\tan^2\theta + b^2\sec^2\theta + 2ab\sec\theta\tan\theta] = x^2 - y^2$

$\Rightarrow a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta) = x^2 - y^2$

$\Rightarrow a^2 - b^2 = x^2 - y^2$ ($\because \sec^2\theta = 1 + \tan^2\theta$)

Hence, proved.

Question: 3

If Given: $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$ …….(1)

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ …….(2)

Square equation (1) and (2) on both sides:

$\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{xy}{ab}\cos\theta\sin\theta = 1$ …….(3)

$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{xy}{ab}\cos\theta\sin\theta = 1$ ……..(4)

Add equation (3) and (4):

$\frac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 1+1$

$\Rightarrow \frac{x^2}{a^2}(1) + \frac{y^2}{b^2}(1) = 2$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Hence, proved.

Question: 4

If (sec θ + tan θ

Solution:

Given: (sec θ + tan θ) = m ....(1)

(sec θ – tan θ) = n ....(2)

Multiply equation (1) and (2):

(sec θ + tan θ) (sec θ – tan θ) = mn

$(\sec^2\theta - \tan^2\theta) = mn$

$1 = mn\ (\because 1 + \tan^2\theta = \sec^2\theta)$

Therefore, mn = 1.

Hence, proved.

Question: 5

If (cosec θ + cot

Solution:

Given: (cosec θ + cot θ) = m ....(1)

(cosec θ – cot θ) = n ....(2)

Multiply equation (1) and (2):

(cosec θ + cot θ) (cosec θ – cot θ) = mn

$(\csc^2\theta - \cot^2\theta) = mn$

$1 = mn\ (\because 1 + \cot^2\theta = \csc^2\theta)$

Therefore, mn = 1.

Hence, proved.

Question: 6

If x = a cos

Solution:

Given: $x = a\cos^3\theta$

$y = b\sin^3\theta$

Consider L.H.S. $= \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$

$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$

$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3}$

$= (\cos^2\theta + (\sin^2\theta)$

= 1 = R.H.S.

Hence, proved.

Question: 7

If (tan θ + sin θ

Solution:

Given: tan θ + sin θ = m …….(1)

tan θ – sin θ = n …….(2)

Square equation (1) and (2) on both sides:

$\tan^2\theta + \sin^2\theta + 2\sin\theta\tan\theta = m^2$ …….(3)

$\tan^2\theta + \sin^2\theta - 2\sin\theta\tan\theta = n^2$ ……..(4)

Subtract equation (4) from (3):

$[\tan^2\theta + \sin^2\theta + 2\sin\theta\tan\theta] - [\tan^2\theta + \sin^2\theta - 2\sin\theta\tan\theta] = m^2 - n^2$

$\Rightarrow 4\sin\theta\tan\theta = m^2 - n^2$

Square both sides:

$\Rightarrow 16\sin^2\theta\tan^2\theta = (m^2 - n^2)^2$

Therefore, $(m^2 - n^2)^2 = 16\sin^2\theta\tan^2\theta$

Also, 16mn = 16 × (tan θ + sin θ) × (tan θ – sin θ)

$= 16(\tan^2\theta - \sin^2\theta)$

$= 16[(\sin^2\theta/\cos^2\theta) - \sin^2\theta]$

$= 16[\sin^2\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)]$

$= 16\sin^2\theta(\sin^2\theta/\cos^2\theta)$

$= 16\sin^2\theta\tan^2\theta$

Therefore, $(m^2 - n^2)^2 = 16mn$

Hence, proved.

Question: 8

If (cot θ + tan θ

Solution:

Given: (cot θ + tan θ) = m

(sec θ – cos θ) = n

Since, m = cot θ + tan θ

= (1/tan θ) + tan θ

$= \frac{1+\tan^2\theta}{\tan\theta}$

$= \sec^2\theta/\tan\theta$

= 1/(sin θ cos θ)

Also, n = sec θ – cos θ

= (1/cos θ) – cos θ

$= (1 - \cos^2\theta)/\cos\theta$

$= \sin^2 \theta/\cos \theta$

Now, consider the left – hand side:

$(m^2n)^{2/3} - (mn^2)^{2/3} = \left[\left(\frac{1}{\sin\theta\cos\theta}\right)^2 \times \frac{\sin^2\theta}{\cos\theta}\right]^{2/3} - \left[\left(\frac{1}{\sin\theta\cos\theta}\right) \times \left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right]^{2/3}$

$= \left[\frac{\sin^2\theta}{\sin^2\theta\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^4\theta}{\sin\theta\cos^3\theta}\right]^{2/3}$

$= \left[\frac{1}{\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^3\theta}{\cos^3\theta}\right]^{2/3}$

$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= (1 - \sin^2 \theta)\cos^2 \theta$

$= \cos^2 \theta/ \cos^2 \theta$

$= 1$

Question: 9

If (cosec θ – sin

Solution:

Given: $(\text{cosec } \theta - \sin \theta) = a^3$

$(\sec \theta - \cos \theta) = b^3$

Since, $a^3 = (\text{cosec } \theta - \sin \theta)$

$= (1/\sin \theta) - \sin \theta$

$= \frac{1-\sin^2\theta}{\sin\theta}$

$= \cos^2 \theta/\sin \theta$

Therefore, $a^2 = (a^3)^{2/3} = (\cos^2 \theta/\sin \theta)^{2/3}$

Also, $b^3 = \sec \theta - \cos \theta$

$= (1/\cos \theta) - \cos \theta$

$= (1 - \cos^2 \theta)/\cos \theta$

$= \sin^2 \theta/\cos \theta$

Therefore, $b^2 = (b^3)^{2/3} = (\sin^2 \theta/\cos \theta)^{2/3}$

Now, consider the left – hand side:

$a^2 b^2 (a^2 + b^2) = \left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{2/3} \times \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$

$= \left(\frac{\cos^2\theta\sin^2\theta}{\cos\theta\sin\theta}\right)^{2/3} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$

$= [\cos^3\theta]^{2/3} + [\sin^3\theta]^{2/3}$

$= \sin^2 \theta + \cos^2 \theta$

$= 1 =$ R.H.S.

Hence, proved.

Question: 10

If (2 sin θ

Solution:

Given: $2 \sin \theta + 3 \cos \theta = 2$

Consider $(2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 4\sin^2 \theta + 9 \cos^2 \theta + 12\sin \theta \cos \theta + 9 \sin^2 \theta + 4 \cos^2 \theta - 12 \sin \theta \cos \theta$

$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13\sin^2 \theta + 13 \cos^2 \theta$

$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13(\sin^2 \theta + \cos^2 \theta)$

$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13$

$\Rightarrow (2)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13$

$\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 13 - 4$

$\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 9$

$\Rightarrow (3 \sin \theta - 2 \cos \theta) = \pm 3$

Hence, proved.

Question: 11

If (sin θ +

Solution:

Given: $(\sin \theta + \cos \theta) = \sqrt{2} \cos \theta$ **To show:** $\cot \theta = (\sqrt{2} + 1)$ **Solution:** $(\sin \theta + \cos \theta) = \sqrt{2} \cos \theta$

Divide both sides by sin θ, $\Rightarrow \frac{sin\theta + cos\theta}{sin\theta} = \frac{\sqrt{2}\,cos\theta}{sin\theta}$

$\Rightarrow \frac{sin\theta}{sin\theta} + \frac{\cos\theta}{sin\theta} = \frac{\sqrt{2}\,cos\theta}{sin\theta}$ Since $\cot\theta = \frac{cos\theta}{sin\theta}$ $\Rightarrow 1 + \cot \theta = \sqrt{2} \cot \theta$

$\Rightarrow 1 = \sqrt{2} \cot \theta - \cot \theta$

$\Rightarrow (\sqrt{2} - 1)\cot \theta = 1$

$\Rightarrow \cot \theta = \frac{1}{\sqrt{2}-1}$

$\Rightarrow \cot \theta = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\Rightarrow \cot \theta = \frac{\sqrt{2}+1}{2-1}$

$\Rightarrow \cot \theta = \sqrt{2} + 1$

Question: 12

If (cos θ + sin θ

Solution:

Given: $\cos \theta + \sin \theta = \sqrt{2}\sin \theta$

Consider $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta -$

$2 \sin \theta \cos \theta$

$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2\sin^2 \theta + 2 \cos^2 \theta$

$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2(\sin^2 \theta + \cos^2 \theta)$

$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

$\Rightarrow (\sqrt{2}\sin \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

$\Rightarrow (\sin \theta - \cos \theta)^2 = 2 - 2\sin^2 \theta$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 2(1 - \sin^2\theta)$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 2(\cos^2\theta)$

$\Rightarrow (\sin\theta - \cos\theta) = \pm\sqrt{2}\cos\theta$

Hence, proved.

Question: 13

If $\sec\theta + \tan\theta$

Solution:

(i) Given: $\sec\theta + \tan\theta = p$ ……(1)

Then, $(\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = p$

$\Rightarrow \sec\theta - \tan\theta = (1/p)$ ……(2)

Adding equation (1) and (2), we get:

$2\sec\theta = p + (1/p)$

$\Rightarrow \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

Therefore, $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

(ii) Given: $\sec\theta + \tan\theta = p$ ……(1)

Then, $(\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = p$

$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = p$

$\Rightarrow \sec\theta - \tan\theta = (1/p)$ ……(2)

Subtracting equation (2) from (1), we get:

$2\tan\theta = p - (1/p)$

$\Rightarrow \tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

(iii) Since $\sin\theta = \tan\theta/\sec\theta$

$= \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)}$

$= \frac{\left(p - \frac{1}{p}\right)}{\left(p + \frac{1}{p}\right)}$

$= \frac{p^2 - 1}{p^2 + 1}$

Question: 14

If tan A = n tan

Solution:

Given: tan A = n tan B

Therefore, $\tan B = \dfrac{\tan A}{n}$ -

Thus, $\cot B = \dfrac{n}{\tan A}$ Squaring both sides, we get,

$\Rightarrow \cot^2 B = n^2/\tan^2 A$ ……(1)

Also, sin A = m sin B

Therefore, sin B = sin A/m

Thus, cosec B = m/sin A

$\Rightarrow \operatorname{cosec}^2 B = m^2/\sin^2 A$ ……(2)

Now, subtract equation (2) from (1):

$\operatorname{cosec}^2 B - \cot^2 B = \dfrac{m^2}{\sin^2 A} - \dfrac{n^2}{\tan^2 A}$

$\Rightarrow 1 = \dfrac{m^2}{\sin^2 A} - \dfrac{n^2 \cos^2 A}{\sin^2 A}$

$\Rightarrow 1 = \dfrac{m^2 - n^2 \cos^2 A}{\sin^2 A}$

$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$

$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$

$\Rightarrow m^2 - 1 = n^2 \cos^2 A - \cos^2 A$

$\Rightarrow (n^2 - 1)\cos^2 A = m^2 - 1$

$\Rightarrow \cos^2 A = (m^2 - 1)/(n^2 - 1)$

Hence, proved.

Question: 15

If m = (cos θ – s

Solution:

Given: m = (cos θ – sin θ)

n = (cos θ + sin θ)

Now, $\dfrac{m}{n} = \dfrac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)}$

Multiply numerator and denominator by cos θ – sin θ :

Therefore, $\dfrac{m}{n} = \dfrac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)} \times \dfrac{\cos\theta - \sin\theta}{\cos\theta - \sin\theta}$

$= \dfrac{(\cos\theta - \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}$

Now, $\dfrac{n}{m} = \dfrac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$

Multiply numerator and denominator by cos θ + sin θ :

Therefore, $\dfrac{n}{m} = \dfrac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)} \times \dfrac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}$

$= \dfrac{(\cos\theta + \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}$

Now, consider $\sqrt{\dfrac{m}{n}} + \sqrt{\dfrac{n}{m}} = \sqrt{\dfrac{(\cos\theta - \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}} + \sqrt{\dfrac{(\cos\theta + \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}}$

$= \dfrac{\cos\theta - \sin\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}} + \dfrac{\cos\theta + \sin\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}}$

$= \frac{1}{\sqrt{(\cos^2\theta - \sin^2\theta)}}(\cos\theta - \sin\theta + \cos\theta + \sin\theta)$

$= \frac{2\cos\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}}$

Divide numerator and denominator by cos θ:

$= \frac{2}{\sqrt{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}}$

$= \frac{2}{\sqrt{(1 - \tan^2\theta)}}$

Therefore, $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{2}{\sqrt{(1 - \tan^2\theta)}}$

Hence, proved.

Exercise : 8C

Question: 1

Write the value o

Solution:

Consider $(1 - \sin^2\theta) \sec^2\theta = (\cos^2\theta) \times \sec^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 2

Write the value o

Solution:

Consider $(1 - \cos^2\theta) \operatorname{cosec}^2\theta = (\sin^2\theta) \times \operatorname{cosec}^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 3

Write the value o

Solution:

Consider $(1 + \tan^2\theta) \cos^2\theta = (\sec^2\theta) \times \cos^2\theta$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

$= 1$

Question: 4

Write the value o

Solution:

Consider $(1 + \cot^2\theta) \times \sin^2\theta = (\operatorname{cosec}^2\theta) \times \sin^2\theta$

$(\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta)$

$= 1$

Question: 5

Write the value o

Solution:

Consider $\sin^2\theta + \frac{1}{1+\tan^2\theta}$

$= (\sin^2\theta) + (1/\sec^2\theta)$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

$= (\sin^2\theta) + (\cos^2\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 6

Write the value o

Solution:

Consider $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= (\cot^2\theta) - (\mathrm{cosec}^2\theta)$

$= -(\mathrm{cosec}^2\theta - \cot^2\theta)$

$(\because 1 + \cot^2\theta = \mathrm{cosec}^2\theta)$

$= -1$

Question: 7

Write the value o

Solution:

Consider $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta) = \sin\theta\sin\theta + \cos\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 8

Write the value o

Solution:

Consider $\mathrm{cosec}^2(90° - \theta) - \tan^2\theta = \sec^2\theta - \tan^2\theta$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

$= 1$

Question: 9

Write the value o

Solution:

Consider $\sec^2\theta(1 + \sin\theta)(1 - \sin\theta) = \sec^2\theta(1 - \sin^2\theta)$

$= \sec^2\theta\cos^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 10

Write the value o

Solution:

Consider $\operatorname{cosec}^2\theta\,(1+\cos\theta)(1-\cos\theta) = \operatorname{cosec}^2\theta\,(1-\cos^2\theta)$

$= \operatorname{cosec}^2\theta \sin^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Question: 11

Write the value o

Solution:

Consider $\sin^2\theta\cos^2\theta\,(1+\tan^2\theta)(1+\cot^2\theta)$

$= \sin^2\theta\cos^2\theta\,(\sec^2\theta)(\operatorname{cosec}^2\theta)$ $(\because 1+\cot^2\theta = \operatorname{cosec}^2\theta$ and $1+\tan^2\theta = \sec^2\theta)$

$= \sin^2\theta\,(\operatorname{cosec}^2\theta)\cos^2\theta\,(\sec^2\theta)$

$= 1 \times 1$

$= 1$

Question: 12

Write the value o

Solution:

Consider $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)$

$= (1+\tan^2\theta)(1-\sin^2\theta)$ $(\because \sin^2\theta+\cos^2\theta = 1$ and $1+\tan^2\theta = \sec^2\theta)$

$= (\sec^2\theta)(\cos^2\theta)$

$= 1$

Question: 13

Write the value o

Solution:

Consider $3\cot^2\theta - 3\operatorname{cosec}^2\theta = -3(\operatorname{cosec}^2\theta - \cot^2\theta)$

$= -3(1)$ $(\because 1+\cot^2\theta = \operatorname{cosec}^2\theta)$

$= -3$

Question: 14

Write the value o

Solution:

Consider $4\tan^2\theta - \frac{4}{\cos^2\theta} = 4\tan^2\theta - 4\sec^2\theta$

$= 4(\tan^2\theta - \sec^2\theta)$

$= 4(-1)$ $(\because 1+\tan^2\theta = \sec^2\theta)$

$= -4$

Question: 15

Write the value o

Solution:

Consider $\frac{\tan^2\theta-\sec^2\theta}{\cot^2\theta-\operatorname{cosec}^2\theta} = \frac{-1}{-1}$ $(\because 1+\tan^2\theta = \sec^2\theta$ and $1+\cot^2\theta = \operatorname{cosec}^2\theta)$

$= 1$

Question: 16

If sin θ = 1/2, w

Solution:

Give: sin θ = 1/2

Therefore cosec θ = 1/sin θ

= 2

Consider 3 $\cot^2$ θ + 3 = 3 ($\cot^2$ θ + 1)

= 3 cosec^2 θ (∵ 1 + $\cot^2$ θ = cosec^2 θ)

= 3 $(2)^2$

= 3 × 4

= 12

Question: 17

If cos θ = 2/3, w

Solution:

Give: cos θ = 2/3

Therefore sec θ = 1/cos θ

= 3/2

Consider 4 $\tan^2$ θ + 4 = 4 ($\tan^2$ θ + 1)

= 4 $\sec^2$ θ (∵ 1 + $\tan^2$ θ = $\sec^2$ θ)

= 4 $(3/2)^2$

= 4 × (9/4)

= 9

Question: 18

If cos θ = 7/25,

Solution:

Given: cos θ = 7/25

Therefore sin θ = √(1 – $\cos^2$ θ)

= √(1 –(49/625))

= √[(625 – 49)/625]

= √(576/625)

= 24/25

Thus, tan θ = sin θ/cos θ = (24/25)/(7/25)

= 24/7

Also, cot θ = 1/tan θ = 7/24

Therefore, tan θ + cot /θ = (24/7) + (7/24)

= (576 + 49)/(24 × 7)

= 625/168

Question: 19

If cos θ = 2/3, w

Solution:

Given: cos θ = 2/3

Thus, sec θ = 1/cos θ

= 3/2

Now, consider $\frac{\sec\theta-1}{\sec\theta+1} = \frac{\frac{3}{2}-1}{\frac{3}{2}+1}$

= [(1/2)/(5/2)]

= 1/5

Question: 20

If 5 tan θ = 4, w

Solution:

Given: 5 tan θ = 4

Therefore, tan θ = 4/5

Now, consider $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$ and divide numerator and denominator by cos θ:

$= \frac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$

$= \frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{1-\frac{4}{5}}{1+\frac{4}{5}}$

= (1/5)/(9/5)

= 1/9

Question: 21

If 3 cot θ = 4, w

Solution:

Given: 3 cot θ = 4

Therefore, cot θ = 4/3

Therefore, tan θ = 3/4

Now, consider $\frac{2\cos\theta+\sin\theta}{4\cos\theta-\sin\theta}$ and divide numerator and denominator by cos θ:

$= \frac{2\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{4\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$

$= \frac{2+\tan\theta}{4-\tan\theta}$

$= \frac{2+\frac{3}{4}}{4-\frac{3}{4}}$

= (11/4)/(13/4)

= 11/13

Question: 22

If cot θ = 1/√3 w

Solution:

Given: $\cot\theta = 1/\sqrt{3}$

Thus, $\tan\theta = 1/\cot\theta = \sqrt{3}$

Therefore $\sec\theta = \sqrt{(1 + \tan^2\theta)}$

$= \sqrt{(1 + 3)}$

$= \sqrt{(4)}$

$= 2$

Therefore $\sec^2\theta = 4$

Now, $\cos^2\theta = 1/\sec^2\theta = 1/4$

So, consider $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{1-\cos^2\theta}{1+1-\sin^2\theta}$

$= \frac{1-\cos^2\theta}{1+\cos^2\theta}$

$= \frac{1-\frac{1}{4}}{1+\frac{1}{4}}$

$= (3/4)/(5/4)$

$= 3/5$

Question: 23

If $\tan\theta = 1/\sqrt{5}$wr

Solution:

Given: $\tan\theta = 1/\sqrt{5}$

$\therefore \tan^2\theta = 1/5$

Consider $\frac{(\text{cosec}^2\theta-\sec^2\theta)}{(\text{cosec}^2\theta+\sec^2\theta)} = \frac{\left(\frac{1}{\sin^2\theta}-\frac{1}{\cos^2\theta}\right)}{\left(\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}\right)}$

Multiply numerator and denominator by $\sin\theta$:

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{1}{5}}{1+\frac{1}{5}}$

$= 4/6$

$= 2/3$

Question: 24

If $\cot A = 4/3$ an

Solution:

We are given that: $\cot A = 4/3$

$\Rightarrow \tan(90° - A) = 4/3$

Since $A + B = 90°$, therefore $B = 90° - A$

Therefore, $\tan(90° - A) = \tan B = 4/3$

Question: 25

If $\cos B = 3/5$ an

Solution:

We are given that: cos B = 3/5

⇒ sin (90° – B) = 3/5

Since A + B = 90°, therefore A = 90° – B

Therefore, sin (90° – B) = sin A = 3/5

Question: 26

If √3sin θ = cos

Solution:

We are given that: √3sin θ = cos θ

∴sin θ /cos θ = 1/√3

⇒ tan θ = 1/√3

⇒ tan θ = tan 30°

On comparing both sides, we get,

θ = 30°

Question: 27

Write the value o

Solution:

Consider tan 10° tan 20° tan 70° tan 80°

= tan 10° tan 20° tan (90° – 20°) tan (90° – 10°)

= tan 10° tan 20° cot 0° cot 10°

= tan 10° cot 10° tan 20° cot 20°

= 1 × 1

= 1

Question: 28

Write the value o

Solution:

Consider tan 1° tan 2°... tan 88° tan 89°

= tan 1° tan 2°... tan 44° tan 45° tan 46° ...tan 88° tan 89°

= tan 1° tan 2°...tan 44° tan 45° tan (90° – 44°) ...tan (90° – 2°) tan (90° – 1°)

= tan 1° tan 2° ... tan 44 ° tan 45° cot 44° ...cot 2° cot 1°

= tan 1° cot 1° tan 2° cot 2°... tan 44 ° cot 44 ° tan 45°

= 1 × 1 × ...× 1

= 1

Question: 29

Write the value o

Solution:

Consider cos 1° cos 2° cos 3° ... cos 180°

= cos 1° cos 2° cos 3° ...× cos 90° × ... × cos 180°

= cos 1° cos 2° cos 3° ... × 0 × ... × cos 180°

= 0 (∵ cos 90° = 0)

Question: 30

If tan A = 5/12,

Solution:

Given: tan A = 5/12

Consider (sin A + cos A) sec A = (sin A + cos A)(1/cos A)

= (sin A/cos A) + (cos A/cos A)

= tan A + 1

= (5/12) + 1

= 17/12

Question: 31

If sin θ = cos (θ

Solution:

We are given that: sin θ = cos (θ – 45°)

∴We can rewrite it as: cos (90° – θ) = cos (θ – 45°)

On comparing both sides, we get,

90° – θ = θ – 45°

⇒ θ + θ = 90° + 45°

⇒ 2θ = 135°

⇒ θ = 65.5°

Question: 32

Find the value of

Solution:

Consider $\frac{\sin 50°}{\cos 40°} + \frac{\operatorname{cosec} 40°}{\sec 50°}$ – 4 cos 50° cosec 40°

= $\frac{\sin 50°}{\cos(90°-50°)} + \frac{\operatorname{cosec} 40°}{\sec(90°-40°)}$ – 4 cos 50° cosec (90° – 40°)

= $\frac{\sin 50°}{\sin 50°} + \frac{\operatorname{cosec} 40°}{\operatorname{cosec} 40°}$ – 4 cos 50° sec 50°

= 1 + 1 – 4

= – 2

Question: 33

Find the value of

Solution:

Consider sin 48° sec 42° + cos 48° cosec 42°

= sin 48° sec (90° – 48°) + cos 48° cosec (90° – 48°)

= sin 48° cosec 48° + cos 48° sec 48°

= 1 + 1

= 2

Question: 34

If x = a sin θ an

Solution:

Given: x = a sin θ

y = b cos θ

Then $b^2x^2 + a^2y^2 = b^2(a \sin\theta)^2 + a^2(b\cos\theta)^2$

$= a^2b^2 \sin^2\theta + a^2 b^2 \cos^2\theta$

$= a^2b^2 (\sin^2\theta + \cos^2\theta)$

$= (a^2b^2) \times 1$

$= a^2b^2$

Question: 35

If 5x = sec θ and

Solution:

Given: 5x = sec θ, and 5/x = tan θ

Consider $5(x^2 - (1/x^2)) = \frac{5}{5}\left(5x^2 - \frac{5}{x^2}\right)$

$= \frac{1}{5}\left(25x^2 - \frac{25}{x^2}\right)$

$= \frac{1}{5}\left((5x)^2 - \left(\frac{5}{x}\right)^2\right)$

$= (1/5)\,[\sec^2\theta - \tan^2\theta]$

$= (1/5)[1]$

$= 1/5$ ($\because \sec^2 x - \tan^2 x = 1$)

Question: 36

If cosec θ = 2x a

Solution:

Given: 2x = cosec θ , and 2/x = cot θ

Consider $2(x^2 - (1/x^2)) = \frac{2}{2}\left(2x^2 - \frac{2}{x^2}\right)$

$= \frac{1}{2}\left(4x^2 - \frac{4}{x^2}\right)$

$= \frac{1}{2}\left((2x)^2 - \left(\frac{2}{x}\right)^2\right)$

$= (1/2)(\mathrm{cosec}^2\theta - \cot^2\theta)$

$= 1/2$ ($\because \mathrm{cosec}^2 x - \cot^2 x = 1$)

Question: 37

If sec θ + tan θ

Solution:

Given: sec θ + tan θ = x ......(1)

Then, $(\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = x$

$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = x$

$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = x$

⇒ sec θ – tan θ = (1/x) ……(2)

Adding equation (1) and (2), we get:

2 sec θ = x + (1/x)

$= (x^2 + 1)/x$

$\Rightarrow \sec\theta = (x^2 + 1)/2x$

Therefore, $\sec\theta = (x^2 + 1)/2x$

Question: 38

Find the value of

Solution:

Consider $\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$

$= \frac{\cos 38^\circ \operatorname{cosec}(90^\circ - 38^\circ)}{\tan 18^\circ \tan(90^\circ - 55^\circ) \tan 60^\circ \tan(90^\circ - 18^\circ) \tan 55^\circ}$

$= \frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \cot 55^\circ \tan 60^\circ \cot 18^\circ \tan 55^\circ}$

$= \frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \cot 18^\circ \cot 55^\circ \tan 55^\circ \tan 60^\circ}$

$= \frac{1}{1 \times 1 \times \tan 60^\circ}$

= cot 60°

= 1/√3

Question: 39

If sin θ = x, wri

Solution:

Given: sin θ = x

Therefore, cosec θ = 1/x

Using the identity $1 + \cot^2\theta = \operatorname{cosec}^2\theta$, we get

$\cot\theta = \sqrt{(\operatorname{cosec}^2\theta - 1)}$

$= \sqrt{((1/x)^2 - 1)}$

$= \sqrt{\frac{x^2 - 1}{x^2}}$

$= \frac{\sqrt{x^2 - 1}}{x}$

Question: 40

If sec θ = x, wri

Solution:

Given: sec θ = x

Using the identity $1 + \tan^2\theta = \sec^2\theta$, we get

$\tan\theta = \sqrt{(\sec^2\theta - 1)}$

$= \sqrt{(x^2 - 1)}$

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

Choos

Solution:

sec 30° = 1/cos 30°

= 1/(√3/2)

= 2/√3

cosec 60° = 1/sin 60°

= 1/(√3/2)

= 2/√3

Therefore, sec 30° /cosec 30° = (2/√3)/(2/√3)

= 1

Question: 2

Choos

Solution:

$$\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} = \frac{\tan 35^\circ}{\cot(90-35)^\circ} + \frac{\cot(90-12)^\circ}{\tan 12^\circ}$$

$$= \frac{\tan 35^\circ}{\tan 35^\circ} + \frac{\tan 12^\circ}{\tan 12^\circ}$$

= 1 + 1

= 2

Question: 3

Choose the correc

Solution:

Consider tan 10° tan 15° tan 75° tan 80°

= tan 10° tan 80° tan 15° tan 75°

= tan 10° tan (90 – 10)° tan 15° tan (90 – 15)°

= (tan 10° cot 10°) (tan 15° cot 15°)

= (1) × (1) = 1

Question: 4

Choose the correc

Solution:

Consider tan 5° tan 25° tan 30° tan 65° tan 85°

= tan 5° tan 85° tan 25° tan 65° tan 30°

= tan 5° tan (90 – 5)° tan 25° tan (90 – 25)° tan 30°

= (tan 5° cot 5°) (tan 25° cot 25°) tan 30°

= (1) × (1) × (1/√3)

= 1/√3

Question: 5

Choose the correc

Solution:

Consider cos 1° cos 2° cos 3° ... cos 180°

= cos 1° cos 2° cos 3° ...× cos 90° × ... cos 180°

= cos 1° cos 2° cos 3° ... × 0 × cos 180°

= 0 (∵ cos 90° = 0)

Question: 6

Choos

Solution:

Consider $\frac{2\sin^2 63°+1+2\sin^2 27°}{3\cos^2 17°-2+3\cos^2 73°} = \frac{2\sin^2 63°+1+2\sin^2(90°-63°)}{3\cos^2 17°-2+3\cos^2(90°-17°)}$

$= \frac{2\sin^2 63°+1+2\cos^2 63°}{3\cos^2 17°-2+3\sin^2 17°}$

$= \frac{2(\cos^2 63°+\sin^2 63°)+1}{3(\cos^2 17°+\sin^2 17°)-2}$

= (2 + 1)/(3 – 1)

= 3

Question: 7

Choose the correc

Solution:

Consider (sin 47° cos 43°) + (cos 47° sin 43°)

= (sin 47° cos (90 – 47)°) + (cos 47° sin (90 – 47)°)

= $\sin^2$ 47° + $\cos^2$ 47°

= 1

Question: 8

Choose the correc

Solution:

Consider (sec 70° sin 20°) + (cos 20° cosec 70°)

= (sec (90 – 20)° sin 20°) + (cos 20° cosec (90 – 20)°)

= (cosec 70° sin70°) + (cos 20° sec 20°)

= 1 + 1 (∵ cosec θ = 1/sin θ and sec θ = 1/cos θ)

= 2

Question: 9

Choose the correc

Solution:

We are given that: sin 3A = cos (A – 10°)

∴ We can rewrite it as: cos (90° – 3A) = cos (A – 10°)

On comparing both sides, we get,

90° – 3A = A – 10°

⇒ A + 3A = 90° + 10°

⇒ 4A = 100°

⇒ A = 25°

Question: 10

Choose the correc

Solution:

We are given that: sec 4A = cosec (A – 10°)

∴We can rewrite it as: cosec (90° – 4A) = cosec (A – 10°)

On comparing both sides, we get,

90° – 4A = A – 10 °

⇒ A + 4A = 90° + 10°

⇒ 5A = 100°

⇒ A = 20°

Question: 11

Choose the correc

Solution:

We are given that: sin A = cos B

∴We can rewrite it as: sin A = sin(90° – B)

On comparing both sides, we get,

90° – B = A

⇒ A + B = 90°

Question: 12

Choose the correc

Solution:

We are given that: cos (α + β) = 0

∴We can rewrite it as: cos (α + β) = cos (90° – 0°)

On comparing both sides, we get,

(α + β) = 0 = 90°

⇒ α = 90° – β

Therefore, sin (α – β) = sin (90° – β – β)

= sin (90° – 2β)

= cos 2β

Question: 13

Choose the correc

Solution:

Consider sin (45° + θ) – cos (45° – θ) = sin (45° + θ) – sin (90° – (45° – θ))

= sin (45° + θ) – sin (45° + θ)

= 0

Question: 14

Choose the correc

Solution:

$\sec^2 10° - \cot^2 80° = \sec^2 10° - \tan^2 (90° - 80°)$

$= \sec^2 10° - \tan^2 10°$

$= 1$

Question: 15

Choose the correc

Solution:

$\text{cosec}^2 57° - \tan^2 33° = \text{cosec}^2 57° - \cot^2 (90° - 33°)$

$= \text{cosec}^2 57° - \cot^2 57°$

$= 1$

Question: 16

Choos

Solution:

Consider $\frac{2\tan^2 30° \sec^2 52° \sin^2 38°}{\text{cosec}^2 70° - \tan^2 20°} = \frac{2\tan^2 30° \sec^2 52° \cos^2(90° - 38°)}{\text{cosec}^2 70° - \cot^2(90° - 20°)}$

$= \frac{2\tan^2 30° \sec^2 52° \cos^2 52°}{\text{cosec}^2 70° - \cot^2 70°}$

$= (2\tan^2 30° \times 1)/1$

$= 2\tan^2 30°$

$= 2(1/\sqrt{3})^2$

$= 2/3$

Question: 17

Choose the

Solution:

Consider $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2 22° + \cos^2(90° - 22°)} + \sin^2 63° + \cos 63° \sin (90° - 63°)$

$= \frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos 63° \cos 63°$

$= (1/1) + (\sin^2 63° + \cos^2 63°)$

$= 1 + 1$

$= 2$

Question: 18

Choos

Solution:

Consider $\frac{\cot(90° - \theta) \sin(90° - \theta)}{\sin \theta} + \frac{\cot 40°}{\tan 50°} - (\cos^2 20° + \cos^2 70°)$

$= \frac{\tan\theta \cos\theta}{\sin\theta} + \frac{\cot(90° - 50°)}{\tan 50°} - (\cos^2 20° + \cos^2 (90° - 70°))$

$= (\tan \theta \times \cot \theta) + (\tan 50°/\tan 50°) - (\cos^2 20° + \sin^2 20°)$

$= 1 + 1 - 1$

$= 1$

Question: 19

Choos

Solution:

Consider $\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$

$= \frac{\cos 38^\circ \operatorname{cosec}(90^\circ - 38^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan(90-18)^\circ \tan(90^\circ - 35^\circ)}$

$= \frac{\cos 38^\circ \sec 38^\circ}{= \tan 18^\circ \tan 35^\circ \tan 60^\circ \cot 18^\circ \cot 35^\circ}$

$= \frac{1}{\tan 60^\circ}$

= 1/√3

Question: 20

Choose the correc

Solution:

Given: 2 sin 2θ = √3

Therefore, sin 2θ = √3/2

⇒ sin 2θ = sin 60°

On comparing both sides, we get:

2θ = 60°

⇒ θ = 60°/2

⇒ θ = 30°

Question: 21

Choose the correc

Solution:

Given: 2 cos 3θ = 1

Therefore, cos 3θ = 1/2

⇒ cos 3θ = cos 60°

On comparing both sides, we get:

3θ = 60°

⇒ θ = 60°/3

⇒ θ = 20°

Question: 22

Choose the correc

Solution:

Given√3tan 2θ – 3 = 0

Therefore, √3tan 2θ = 3

⇒ tan 2θ = 3/√3

⇒ tan 2θ = √3

⇒ tan 2θ = tan 60°

On comparing both sides, we get:

2θ = 60°

⇒ θ = 60°/2

⇒ θ = 30°

Question: 23

Choose the correc

Solution:

Given: tan x = 3 cot x

⇒ tan x/cot x = 3

Since, cot x = 1/tan x

Therefore, tan x/cot x = 3 ⇒ $\tan^2 x = 3$

Taking square root on both sides:

⇒ tan x = √3

⇒ tan x = tan 60°

Comparing both sides:

⇒ x = 60°

Question: 24

Choose the correc

Solution:

Given: x tan 45° cos 60° = sin 60° cot 60°

⇒ x × 1 × (1/2) = (√3/2) × (1/√3)

⇒ x/2 = 1/2

⇒ x = 1

Question: 25

Choose the correc

Solution:

Given: $\tan^2 45° - \cos^2 30° = x \sin 45° \cos 45°$

⇒ $(1)^2 - (\sqrt{3}/2)^2 = x \times (1/\sqrt{2}) \times (1/\sqrt{2})$

⇒ 1 – (3/4) = x × (1/2)

⇒ x/2 = 1 – (3/4)

⇒ x/2 = 1/4

⇒ x = 2/4

⇒ x = 1/2

Question: 26

Choose the correc

Solution:

$\sec^2 60° - 1 = (1/\cos^2 60°) - 1$

= $[1/(1/2)^2] - 1$

= [1/(1/4)] – 1

= 4 – 1

= 3

Question: 27

Choose the correc

Solution:

Consider (cos 0° + sin 30° + sin 45°) (sin 90° + cos 60° – cos 45°)

= [1 + (1/2) + (1/√2)] × [1 + (1/2) – (1/√2)]

= [(3/2) + (1/√2)] × [(3/2) – (1/√2)]

= $(3/2)^2 – (1/√2)^2$

= (9/4) – (1/2)

= (9 – 2)/4

= 7/4

Question: 28

Choose the correc

Solution:

Consider $\sin^2 30° + 4 \cot^2 45° – \sec^2 60° = (1/2)^2 + 4(1)^2 – (2)^2$

= (1/4) + 4 – 4

= 1/4

Question: 29

Choose the correc

Solution:

Consider $3 \cos^2 60° + 2 \cot^2 30° – 5 \sin^2 45° = 3 (1/2)^2 + 2(√3)^2 – 5(1/√2)^2$

= 3(1/4) + 2(3) – 5(1/2)

= (3/4) + 6 – (5/2)

= (3 + 24 – 10)/4

= 17/4

Question: 30

Choose the correc

Solution:

Consider $\cos^2 30° \cos^2 45° + 4 \sec^2 60° + 2\cos^2 90° – 2 \tan^2 60°$

$$= 3\left(\frac{1}{2}\right)^2 + 2(\sqrt{3})^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3\left(\frac{1}{4}\right) + 2(3) - 5\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + 6 - \frac{5}{2}$$

$$= \frac{17}{4}$$

Question: 31

Choose the correc

Solution:

Given: cosec θ = √10

~~Therefore, sin θ = 1/√10~~

~~Since, $\sin^2 \theta + \cos^2 \theta = 1$~~

~~Therefore, $\cos \theta = \sqrt{(1 - \sin^2 \theta)}$~~

~~$= \sqrt{\left(1 - \frac{1}{10}\right)}$~~

~~$= \sqrt{\frac{(10-1)}{10}}$~~

~~$= \sqrt{\frac{9}{10}}$~~

~~$= \frac{3}{\sqrt{10}}$~~

~~Therefore, $\sec \theta = 1/\cos \theta = \frac{\sqrt{10}}{3}$~~

~~Question: 32~~

~~Choose the correc~~

~~Solution:~~

~~Given: tan θ = 8/15 = Perpendicular/Base~~

~~On comparing, we get:~~

~~Perpendicular = 8~~

~~Base = 15~~

~~Therefore, $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$~~

~~= 64 + 225~~

~~= 289~~

~~Therefore, hypotenuse = √289~~

~~= 17~~

~~Therefore cosec θ = Hypotenuse/Perpendicular~~

~~= 17/8~~

~~Question: 33~~

~~Choose the correc~~

~~Solution:~~

~~Given: sin θ = a/b~~

~~Since, $\sin^2 \theta + \cos^2 \theta = 1$~~

~~Therefore, $\cos \theta = \sqrt{(1 - \sin^2 \theta)}$~~

~~$= \sqrt{\left(1 - \frac{a^2}{b^2}\right)}$~~

~~$= \frac{\sqrt{(b^2 - a^2)}}{b}$~~

~~Question: 34~~

~~Choose the correc~~

~~Solution:~~

~~Given: tan θ = √3 = Perpendicular/Base~~

On comparing, we get:

Perpendicular = √3

Base = 1

Therefore, $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

= 3 + 1

= 4

Therefore, hypotenuse = √4

= 2

Therefore sec θ = Hypotenuse/Base

= 2/1 = 2

Question: 35

Choose the correc

Solution:

Given: sec θ = 25/7

Therefore, cos θ = 1/sec θ = 7/25 = Base/Hypotenuse

Therefore, on comparing, Base = 7 and Hypotenuse = 25

In a right – angled triangle, $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$625 = (\text{Perpendicular})^2 + 49$

Therefore, Perpendicular = √(625 – 49) = √(576)

= 24

Therefore sin θ = Perpendicular/Hypotenuse

= 24/25

Question: 36

Choose the correc

Solution:

Given: sin θ = 1/2 = Perpendicular/Hypotenuse

Therefore, on comparing, Perpendicular = 1 and Hypotenuse = 2

In a right – angled triangle, $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$4 = 1 + (\text{Base})^2$

Therefore, Base = √(4 – 1)

= √(3)

Therefore cot θ = Base/ Perpendicular

= √3/1

= √3

Question: 37

Choose the correc

Solution:

Given: cos θ = 4/5 = Base/Hypotenuse

Therefore, on comparing, Base = 4 and Hypotenuse = 5

In a right – angled triangle, $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$25 = (\text{Perpendicular})^2 + 16$

Therefore, Perpendicular = $\sqrt{(25 - 16)} = \sqrt{(9)}$

= 3

Therefore tan θ = Perpendicular/Base

= 3/4

Question: 38

Choose the correc

Solution:

Given: 3x = cosec θ , and 3/x = cot θ

Consider $3(x^2 - (1/x^2)) = \frac{3}{3}\left(3x^2 - \frac{3}{x^2}\right)$

$= \frac{1}{3}\left(9x^2 - \frac{9}{x^2}\right)$

$= \frac{1}{3}\left((3x)^2 - \left(\frac{3}{x}\right)^2\right)$

$= (1/3)(\text{cosec}^2\,\theta - \cot^2\theta)$

$= 1/3$ $(\because \text{cosec}^2 x - \cot^2 x = 1)$

Question: 39

Choose the correc

Solution:

Given: 2x = sec A, and 2/x = tan A

Consider $2(x^2 - (1/x^2)) = \frac{2}{2}\left(2x^2 - \frac{2}{x^2}\right)$

$= \frac{1}{2}\left(4x^2 - \frac{4}{x^2}\right)$

$= \frac{1}{2}\left((2x)^2 - \left(\frac{2}{x}\right)^2\right)$

$= (1/2)\,[\sec^2 A - \tan^2 A]$

= (1/2)[1]

$= 1/2$ $(\because \sec^2 x - \tan^2 x = 1)$

Question: 40

Choose the correc

Solution:

Given: tan θ = 4/3 = Perpendicular/Base

Therefore, $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

= 16 + 9

= 25

Therefore, hypotenuse = $\sqrt{25}$

= 5

Therefore sin θ = Perpendicular/Hypotenuse

= 4/5

Also, cos θ = Base/Hypotenuse

= 3/5

Thus, sin θ + cos θ = (4/5) + (3/5)

= 7/5

Question: 41

Choose the correc

Solution:

Given: tan θ + cot θ = 5

Squaring both sides, we get:

$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$

$\tan^2\theta + \cot^2\theta = 25 - 2\tan\theta\cot\theta$

= 25 – 2

= 23

Question: 42

Choose the correc

Solution:

Given: cos θ + sec θ = 5/2

Squaring both sides, we get:

$\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta = 25/4$

$\cos^2\theta + \sec^2\theta = (25/4) - 2\cos\theta\sec\theta$

= 25/4 – 2

= (25 – 8)/4

= 17/4

Question: 43

Choose the correc

Solution:

Given: tan θ = 1/√7

$\therefore \tan^2\theta = 1/7$

Consider $\frac{(\mathrm{cosec}^2\theta - \sec^2\theta)}{(\mathrm{cosec}^2\theta + \sec^2\theta)} = \frac{\left(\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}\right)}{\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)}$

Multiply numerator and denominator by sin θ:

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{1}{7}}{1+\frac{1}{7}}$

= 6/8

= 3/4

Question: 44

Choose the correc

Solution:

Given: 7tan θ = 4

Therefore tan θ = 4/7

Consider $\frac{7\sin\theta - 3\cos\theta}{7\sin\theta + 3\cos\theta}$ and divide numerator and denominator by cos θ:

$$\frac{\frac{(7\sin\theta - 3\cos\theta)}{\cos\theta}}{\frac{7\sin\theta + 3\cos\theta}{\cos\theta}} = \frac{7\tan\theta - 3}{7\tan\theta + 3}$$

$$= \frac{7\left(\frac{4}{7}\right) - 3}{7\left(\frac{4}{7}\right) + 3}$$

$$= \frac{4-3}{4+3}$$

= 1/7

Question: 45

Choose the correc

Solution:

Given: 3cot θ = 4

Therefore cot θ = 4/3

Consider $\frac{5\sin\theta + 3\cos\theta}{5\sin\theta - 3\cos\theta}$ and divide numerator and denominator by sin θ:

$$\frac{\frac{(5\sin\theta + 3\cos\theta)}{\sin\theta}}{\frac{5\sin\theta - 3\cos\theta}{\sin\theta}} = \frac{5 + 3\cot\theta}{5 - 3\cot\theta}$$

$$= \frac{5 + 3\left(\frac{4}{3}\right)}{5 - 3\left(\frac{4}{3}\right)}$$

$$= \frac{5+4}{5-4}$$

= 9

Question: 46

Choose the correc

Solution:

Given: tan θ = a/b

Consider $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ and divide numerator and denominator by cos θ:

$$\frac{\frac{(a\sin\theta - b\cos\theta)}{\cos\theta}}{\frac{a\sin\theta + b\cos\theta}{\cos\theta}} = \frac{a\tan\theta - b}{a\tan\theta + b}$$

$$= \frac{a\left(\frac{a}{b}\right) - b}{a\left(\frac{a}{b}\right) + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Question: 47

Choose the correc

Solution:

Given: $\sin A + \sin^2 A = 1$

Therefore $\sin A = 1 - \sin^2 A = \cos^2 A$ ……(1)

Now, consider $\cos^2 A + \cos^4 A = \cos^2 A(1 + \cos^2 A)$

Put the value of $\cos^2 A$ in the above equation:

Therefore, $\cos^2 A + \cos^4 A = \cos^2 A(1 + \cos^2 A)$

$= (1 - \sin^2 A)(1+1 - \sin^2 A)$

Again, from equation (1), we have $1 - \sin^2 A = \sin A$. So, put the value of sin A in the above equation:

Therefore, $\cos^2 A + \cos^4 A = (\sin A)(1+ \sin A)$

$= \sin A + \sin^2 A$

$= 1$ (given)

Therefore, $\cos^2 A + s\cos^4 A = 1$

Question: 48

Choose the correc

Solution:

Given: $\cos A + \cos^2 A = 1$

Therefore $\cos A = 1 - \cos^2 A = \sin^2 A$ ……(1)

Now, consider $\sin^2 A + \sin^4 A = \sin^2 A(1 + \sin^2 A)$

Put the value of $\sin^2 A$ in the above equation:

Therefore, $\sin^2 A + \sin^4 A = \sin^2 A(1 + \sin^2 A)$

$= (1 - \cos^2 A)(1+1 - \cos^2 A)$

Again, from equation (1), we have $1 - \cos^2 A = \cos A$. So put the value of cos A in the above equation:

Therefore, $\sin^2 A + \sin^4 A = (\cos A)(1+ \cos A)$

$= \cos A + \cos^2 A$

$= 1$ (given)

Therefore, $\sin^2 A + \sin^4 A = 1$

Question: 49

Choos

Solution:

Consider $\sqrt{\frac{1-\sin A}{1+\sin A}}$ and rationalize:

$$\sqrt{\frac{1-\sin A}{1+\sin A}} = \sqrt{\frac{1-\sin A}{1+\sin A} \times \frac{1-\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$$

$$= \frac{1-\sin A}{\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A}$$

$= \sec A - \tan A$

Question: 50

Choos

Solution:

Consider $\sqrt{\frac{1+\cos A}{1-\cos A}}$ and rationalize:

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$$

$$= \frac{1+\cos A}{\sin A}$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$$

$= \operatorname{cosec} A + \cot A$

Question: 51

Choose the correc

Solution:

Given: $\tan \theta = a/b$

Consider $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$ and divide numerator and denominator by $\cos \theta$:

$$\frac{\frac{(\cos\theta+\sin\theta)}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}} = \frac{1+\tan\theta}{1-\tan\theta}$$

$$= \frac{1+\frac{a}{b}}{1-\frac{a}{b}}$$

$$= \frac{b+a}{b-a}$$

Question: 52

Choose the correc

Solution:

Consider $(\operatorname{cosec} \theta - \cot \theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)\,(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

= R.H.S.

Hence, proved.

Question: 53

Choose the correc

Solution:

Consider (sec A + tan A)(1 – sin A) = $\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

= $\left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$

= $\frac{1 - \sin^2 A}{\cos A}$

= $\frac{\cos^2 A}{\cos A}$

= cos A

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

Consider $\frac{\cos^2 56° + \cos^2 34°}{\sin^2 56° + \sin^2 34°}$ + 3 $\tan^2$ 56° $\tan^2$ 34°

= $\frac{\cos^2 56° + \cos^2(90° - 56°)}{\sin^2 56° + \sin^2(90° - 56°)}$ + 3 $\tan^2$ 56° $\tan^2$ (90° – 56°)

= $\frac{\cos^2 56° + \sin^2 56°}{\sin^2 56° + \cos^2 56°}$ + 3 $\tan^2$ 56° $\cot^2$ 56°

= (1/1) + 3(1)

= 1 + 3

= 4

Question: 2

The value of (sin

Solution:

Consider ($\sin^2$ 30° $\cos^2$ 45° + 4 $\tan^2$ 30°+ (1/2) $\sin^2$ 90° + (1/8) $\cot^2$ 60°)

= [$(1/2)^2$ × $(1/\sqrt{2})^2$ + 4 $(1/\sqrt{3})^2$ + (1/2) × $(1)^2$ + (1/8)$(1/\sqrt{3})^2$]

= [(1/4) × (1/2)] + [(4/3)] + (1/2) + (1/24)

= (1/8) + (4/3) + (1/2) + (1/24)

= (3 + 32 + 12 + 1)/24

= 48/24

= 2

Question: 3

If cos A + cos

Solution:

Given: cos A + $\cos^2$ A = 1

Therefore cos A = 1 – $\cos^2$ A = $\sin^2$ A ......(1)

Now, consider $\sin^2$A + $\sin^4$A = $\sin^2$ A(1 + $\sin^2$A)

Put the value of $\sin^2$A in the above equation:

Therefore, $\sin^2 A + \sin^4 A = \sin^2 A(1 + \sin^2 A)$

$= (1 - \cos^2 A)(1+1 - \cos^2 A)$

Again, from equation (1), we have $1 - \cos^2 A = \cos A$. So put the value of cos A in the above equation:

Therefore, $\sin^2 A + \sin^4 A = (\cos A)(1+ \cos A)$

$= \cos A + \cos^2 A$

= 1 (given)

Therefore, $\sin^2 A + \sin^4 A = 1$

Question: 4

If $\sin \theta = \sqrt{3}/2$,

Solution:

Given: $\sin \theta = \sqrt{3}/2$

Therefore, $\operatorname{cosec} \theta = 1/\sin \theta = 2/\sqrt{3}$

$\cos \theta = \sqrt{(1 - \sin^2 \theta)}$

$= \sqrt{(1 - (3/4))}$

$= \sqrt{(1/4)}$

$= 1/2$

$\cot \theta = \cos \theta/\sin \theta = (1/2)/\sqrt{3}/2$

$= 1/\sqrt{3}$

Therefore, $(\operatorname{cosec} \theta + \cot \theta) = (2/\sqrt{3}) + (1/\sqrt{3})$

$= 3/\sqrt{3}$

$= \sqrt{3}$

Question: 5

If cot A = 4/5, p

Solution:

Given: cot A = 4/5

Consider $\frac{\sin A+\cos A}{\sin A-\cos A}$ and divide numerator and denominator by sin A:

$$\frac{\frac{(\sin A+\cos A)}{\sin A}}{\frac{\sin A-\cos A}{\sin A}} = \frac{1+\cot A}{1-\cot A}$$

$$= \frac{1+\left(\frac{4}{5}\right)}{1-\left(\frac{4}{5}\right)}$$

$$= \frac{5+4}{5-4}$$

= 9

Question: 6

If 2x = sec A and

Solution:

Given: 2x = sec A, and 2/x = tan A

Therefore, $(2x)^2 = \sec^2 A$

$$\left(\frac{2}{x}\right)^2 = \tan^2 A$$

⇒ $4x^2 = \sec^2 A$ — [1]and $\Rightarrow \frac{4}{x^2} = \tan^2 A$ [2]On subtracting [2] from [1], we get

$$4x^2 - \frac{4}{x^2} = \sec^2 A - \tan^2 A$$

$$\Rightarrow 4\left(x^2 - \frac{1}{x^2}\right) = 1 \qquad (\because \sec^2 x - \tan^2 x = 1)$$

$$\Rightarrow x^2 - \frac{1}{x^2} = \frac{1}{4}$$

Question: 7

If √3 tan θ = 3 s

Solution:

Given: √3 tanθ = 3 sinθ

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{3\sin\theta}{\sqrt{3}}$

$\Rightarrow \frac{1}{\cos\theta} = \sqrt{3}$

$\Rightarrow \frac{1}{\cos\theta} = \sqrt{3}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

Also, we know, $\sin\theta = \sqrt{1-\cos^2\theta}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\left(\frac{1}{3}\right)}$

$\Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$

Now, we need to prove that:

$\sin^2\theta - \cos^2\theta = 1/3$

$\Rightarrow$ L.H.S $= \left(\sqrt{\frac{2}{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$\Rightarrow$ L.H.S $= \frac{2}{3} - \frac{1}{3}$

⇒ L.H.S = 1/3

Question: 8

Prove that

Solution:

Consider L.H.S. $= \frac{\sin^2 73° + \sin^2 17°}{\cos^2 28° + \cos^2 62°}$

$= \frac{\sin^2 73° + \sin^2(90° - 73°)}{\cos^2 28° + \cos^2(90° - 28°)}$

$= \frac{\sin^2 73° + \cos^2 73°}{\cos^2 28° + \sin^2 28°}$

= 1/1

= 1

= R.H.S.

Hence, proved.

Question: 9

If 2 sin 2θ = √3,

Solution:

Given: 2 sin 2θ = √3

Therefore, sin 2θ = √3/2

⇒ sin 2θ = sin 60°

⇒ 2θ = 60°/2 = 30°

Therefore, θ = 30°

Question: 10

Prove that

Solution:

Consider $\sqrt{\frac{1+\cos A}{1-\cos A}}$ and rationalize:

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$$

$$= \frac{1+\cos A}{\sin A}$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$$

= cosec A + cot A

Question: 11

If cosec θ + cot

Solution:

Given: cosec θ + cot θ = p

$p^2 - 1 = (\text{cosec } \theta + \cot \theta)^2 - 1$

$= \text{cosec}^2 \theta + \cot^2 \theta + 2 \text{ cosec } \theta \cot \theta - 1$

$= \text{cosec}^2 \theta - 1 + \cot^2 \theta + 2 \text{ cosec } \theta \cot \theta$

$= \cot^2 \theta + \cot^2 \theta + 2 \text{ cosec } \theta \cot \theta$

= 2 cot θ (cot θ + cosec θ)

Also, $p^2 + 1 = (\text{cosec } \theta + \cot \theta)^2 + 1$

$= \text{cosec}^2 \theta + \cot^2 \theta + 2 \text{ cosec } \theta \cot \theta + 1$

$= \text{cosec}^2 \theta + 1 + \cot^2 \theta + 2 \text{ cosec } \theta \cot \theta$

$= \text{cosec}^2 \theta + \text{cosec}^2 \theta + 2 \text{ cosec } \theta \cot \theta$

= 2 cosec θ (cosec θ + cot θ)

Now, consider L.H.S. = $\frac{(p^2-1)}{(p^2+1)}$

$= \frac{2\cot\theta(\cot\theta + \text{cosec}\,\theta)}{2\,\text{cosec}\,\theta(\text{cosec}\,\theta + \cot\theta)}$

= cot θ/cosec θ

= cos θ

= R.H.S.

Hence, proved.

Question: 12

Prove that

Solution:

Consider R.H.S. = $\frac{1-\cos A}{1+\cos A}$ and rationalize:

$\frac{1-\cos A}{1+\cos A} = \frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$

$= \frac{(1-\cos A)^2}{1-\cos^2 A}$

$= \frac{(1-\cos A)^2}{\sin^2 A}$

$= \left(\frac{1-\cos A}{\sin A}\right)^2$

$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$

= (cosec A – cot A)^2

= L.H.S.

Hence, proved.

Question: 13

If 5 cot θ = 3, s

Solution:

Given: 5cot θ = 3

Therefore cot θ = 3/5

Consider $\frac{(5\sin\theta - 3\cos\theta)}{(4\sin\theta + 3\cos\theta)}$ and divide numerator and denominator by sin θ:

$\frac{\frac{(5\sin\theta - 3\cos\theta)}{\sin\theta}}{\frac{4\sin\theta + 3\cos\theta}{\sin\theta}} = \frac{5-3\cot\theta}{4+3\cot\theta}$

$= \frac{5-3\left(\frac{3}{5}\right)}{4+3\left(\frac{3}{5}\right)}$

$= \frac{25-9}{20+9}$

= 16/29

Hence, showed.

Question: 14

Prove that (sin 3

Solution:

Consider L.H.S. = (sin 32° cos 58° + cos 32° sin 58°)

= sin 32° cos (90° – 32°) + cos 32° sin (90° – 32°)

= $\sin^2 32° + \cos^2 32°$

= 1

= R.H.S.

Hence, proved.

Question: 15

If x = a sin θ +

Solution:

Given: a sin θ + b cos θ = x …….(1)

a cos θ – b sin θ = y …….(2)

Square equation (1) and (2) on both sides:

$a^2 \sin^2 θ + b^2 \cos^2 θ + 2ab \cos θ \sin θ = x^2$ …….(3)

$a^2 \cos^2 θ + b^2 \sin^2 θ – 2ab \cos θ \sin θ = y^2$ ……..(4)

Add equation (3) and (4):

$[a^2 \sin^2 θ + b^2 \cos^2 θ + 2ab \cos θ \sin θ] + [a^2 \cos^2 θ + b^2 \sin^2 θ – 2ab \cos θ \sin θ] = x^2 + y^2$

$⇒ a^2 (\sin^2 θ + \cos^2 θ) + b^2 (\cos^2 θ + \sin^2 θ) = x^2 + y^2$

$⇒ a^2 + b^2 = x^2 + y^2$

Hence, proved.

Question: 16

Prove that

Solution:

Consider L.H.S. = $\frac{(1+\sin θ)}{(1-\sin θ)}$

Multiply numerator and denominator by (1 + sin θ):

= $\frac{(1+\sin θ)}{(1-\sin θ)} \times \frac{(1+\sin θ)}{(1+\sin θ)}$

= $\frac{(1+\sin θ)^2}{(1-\sin^2 θ)}$

= $\frac{(1+\sin θ)^2}{\cos^2 θ}$

= $\left(\frac{1+\sin θ}{\cos θ}\right)^2$

= $[(1/\cos θ) + (\sin θ/\cos θ)]^2$

= $(\sec θ + \tan θ)^2$

= R.H.S.

Hence, proved.

Question: 17

Prove that

Solution:

Consider L.H.S. = $\frac{1}{\sec\theta-\tan\theta}-\frac{1}{\cos\theta}$

Multiply and divide the first term by (sec θ + tan θ):

$=\left(\frac{1}{\sec\theta-\tan\theta}\times\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\right)-\frac{1}{\cos\theta}$

$=\frac{\sec\theta+\tan\theta}{\sec^2\theta-\tan^2\theta}-\sec\theta$

$= \sec\theta + \tan\theta - \sec\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

$= \tan\theta$

Consider R.H.S. = $\frac{1}{\cos\theta}-\frac{1}{\sec\theta+\tan\theta}$

Multiply and divide the second term by (sec θ – tan θ):

$=\frac{1}{\cos\theta}-\left(\frac{1}{\sec\theta+\tan\theta}\times\frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}\right)$

$=\sec\theta-\frac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta}$

$= \sec\theta - \sec\theta + \tan\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

$= \tan\theta$

Therefore, L.H.S. = R.H.S.

Hence, proved.

Question: 18

Prove that

Solution:

Consider L.H.S. = $\frac{\sin A\left(1-2\sin^2 A\right)}{\cos A(2\cos^2 A-1)}$

$=\frac{\sin A\left(\sin^2 A+\cos^2 A-2\sin^2 A\right)}{\cos A\left(2\cos^2 A-\sin^2 A-\cos^2 A\right)}$

$=\frac{\sin A\left(\cos^2 A-\sin^2 A\right)}{\cos A(\cos^2 A-\sin^2 A)}$

= sin A/cos A

= tan A

= R.H.S.

Hence, proved.

Question: 19

Prove that

Solution:

Consider L.H.S. = $\frac{\tan A}{(1-\cot A)}+\frac{\cot A}{(1-\tan A)}$

$=\frac{\tan A}{\left(1-\frac{1}{\tan A}\right)}+\frac{\cot A}{(1-\tan A)}$

$=\frac{\tan^2 A}{\tan A-1}+\frac{\cot A}{(1-\tan A)}$

$=\frac{\tan^2 A}{\tan A-1}-\frac{\cot A}{(\tan A-1)}$

$=\frac{\tan^2 A-\cot A}{\tan A-1}$

$= \frac{\tan^2 A - \frac{1}{\tan A}}{\tan A - 1}$

$= \frac{\tan^3 A - 1}{\tan A\ (\tan A - 1)}$

$= \frac{(\tan A - 1)(\tan^2 A + 1 + \tan A)}{\tan A(\tan A - 1)}$

$= \frac{(\tan^2 A + 1 + \tan A)}{\tan A}$

= tan A + (1/tan A) + 1

= 1 + tan A + cot A

= R.H.S.

Hence, proved.

Question: 20

If sec 5A = cosec

Solution:

We are given that: sec 5A = cosec (A – 36°)

∴We can rewrite it as: cosec (90° – 5A) = cosec (A – 36°)

On comparing both sides, we get,

90° – 5A = A – 36°

⇒ A + 5A = 90° + 36°

⇒ 6A = 126°

⇒ A = 21°

Hence, proved.