PHYSICS

CURRENT ELECTRICITY

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IIT-JEE SYLLABUS

Electric current Ohm's law Series and parallel arrangements of resistances and cells Kirchhoff's laws and simple applications Heating effect of current RC circuits with d.c. source.

The present chapter is about motion of charges under effect of external field. The term Electric current or simply current is used to describe the flow of charge through some region of space. In most common situation flow of charge takes place in a conductor. It is also possible, however, for current to exist outside a conductor.

In this chapter we shall start with basic laws which is used for analysis of circuit consisting of resistors and batteries. Currents have many effect, including heating effect of current which we shall study about in this chapter. The measurement of current and voltage in a circuit is to be discussed in this chapter. We conclude the chapter by analysing circuit consisting of resisters, capacitors and batteries (RC circuit).

CURRENT ELECTRICITY

1. ELECTRIC CURRENT

Electric current across an area held perpendicular to the direction of flow of charge is defined to be the amount of charge flowing across the area per unit time. If charge ΔQ passes through the area in time interval Δt at uniform rate, the current I is defined by

$$I = \frac{\Delta Q}{\Delta t} \qquad \text{...(i)}$$

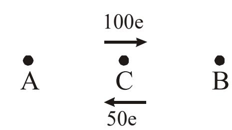
If rate of flow of charge is not steady, then instantaneous current is given by

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad \text{...(ii)}$$

The S. I. unit of current is ampere (A). Smaller currents are more conveniently expressed in milliampere ($1mA = 10^{-3}$ A) or microampere $\left(1\mu A = 10^{-6} A\right)$.

Illustration: 1

Two boys A and B are sitting at two points in a field. Both boys are sitting near assemblence of charged balls each carrying charge +3e. A throws 100 balls per second towards B while B throws 50 balls per second towards A. Find the current at the mid point of A and B.



Solution:

Let mid point be C as shown

Charge moving to the right per unit time = $100 \times 3e = 300e$

Charge moving to the left per unit time = $50 \times 3e = 150e$

Movement of charge per unit time is $300e - 150e = 150e$ towards right

$$I = 150e = 150 \times 1.6 \times 10^{-19} A = 2.4 \times 10^{-17} A.$$

Illustration: 2

Flow of charge through a surface is given as $Q = 4t^2 + 2t$ (for 0 to 10 sec.)

(a) Find the current through the surface at $t = 5$sec.

(b) Find the average current for (0 – 10 sec)

Solution:

(a) Instantaneous current

$$I = \frac{dQ}{dt} = \frac{d}{dt}(4t^2 + 2t) = 8t + 2$$

at t = 5 sec;

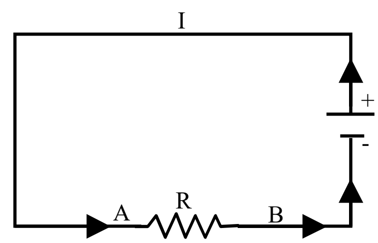
$$I = 8 \times 5 + 2 = 42 \text{ Amp}$$

(b) Average current

$$I = \frac{\Delta Q}{\Delta t} = \frac{Q}{t} = \frac{4 \times (10)^2 + 2 \times 10}{10} = \frac{420}{10} = 42 \text{ Amp.}$$

2. ELECTROMOTIVE FORCE (EMF)

To maintain a steady electric current, the conductor cannot be isolated; it must be part of a closed circuit that includes an external agency or device (figure). This device is required to transport the positive charge from B back to A, i.e., from lower to higher potential and thus maintain the potential difference between A and B. The external device will need to do work for transporting positive charge from lower to higher potential. Such a device is the source known as source of *electromotive force* abbreviated as emf. It is the analogue of the pump in the water flow.



The external source, as said above, does work on taking a positive charge from lower to the higher potential.
A natural way of characterizing the external source of energy is in terms of the work that it needs to do per unit positive charge in transporting it from lower to higher potential. This is known as electromotive force or emf of the device, denoted by ε.

$$\varepsilon = V_{open}$$

The emf of a source is thus the potential difference between its two terminals in open circuit i.e. when no external resistances are connected.

3. DRIFT VELOCITY AND CURRENT DENSITY

In any lattice of a metal, free electrons are moving randomly, colliding with the latice following a zig zag path.
However, in absence of any electric field, no of electrons crossing from left to right is equal to no. of electrons crossing from right to left. So, the net current through a cross section is zero.
When an electric field is applied, inside the conductor, path of electrons becomes directed opposite to the electric field. Due to this, the random motion of electrons get modified and there is a net transfer of electrons across a cross section resulting in a net current.
So, drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it.
If v_d = average drift speed in a metal of cross section S and n = number of free electrons per unit volume each having charge e

$$I = \frac{dq}{dt} = nev_dS$$

If J = current flowing through a unit area of cross section of the current carrying conductor.

$$J = \frac{I}{S} = nev_d$$

$$v_d = \frac{J}{ne}$$

Illustration : 3

Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2 carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/m^3, and its atomic mass is 63.5 amu

Solution:

The direction of drift velocity of conduction electrons is opposite to the direction of electric field i.e., electrons drift in the direction of increasing potential. The drift speed υ_d is given by

$$\upsilon_d = (I/neA).$$

Now, $e = 1.6 \times 10^{-19}$ C, $A = 1.0 \times 10^{-7}$ m^2, $I = 1.5$ A. The density of conduction electrons, n is equal to the number of atoms per cubic metre (assuming one conduction electron per Cu atom as is reasonable from its valence electron count of one). A cubic metre of copper has a mass of 9.0×10^3 kg. Since 6.0×10^{23} copper atoms have a mass of 63.5 g.

$$n = \frac{6.0 \times 10^{23}}{63.5} \times 9.0 \times 10^6 = 8.5 \times 10^{28}\ \text{m}^{-3}$$

which gives

$$\upsilon_d = \frac{1.5}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 1.0^{-7}} = 1.1 \times 10^{-3}\ \text{m s}^{-1}.$$

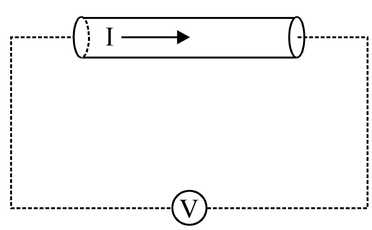
4. OHM'S LAW

In case of certain substances (such as iron, platinum etc) at constant temperature the current which flows is directly proportional to the applied voltage.

If a potential difference V causes current I in a substance then,

$$V \propto I \text{ or } V = IR$$

Where R is a constant called as resistance of the substance. This law is called ohm's law and the substances which obey it are called ohmic or linear substances. The inverse of resistance is called as conductance, represented by G. Unit of conductance is mho or siemen.



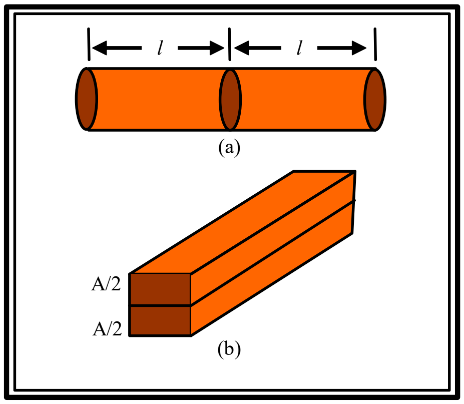
5. MORE ABOUT RESISTANCE

The resistance of a resistor (an element in a circuit with some resistance R) depends on its geometrical factors (length, cross-sectional area) and also on the nature of the substance of which the resistor is made of. It is convenient to separate out the 'size' factors from the resistance R so that we can define a quantity that is characteristic of the material and is independent of the size or shape. Consider a rectangular slab of length l and area of cross section A. For a fixed current I, if the length of the slab is doubled, the potential drop across the slab also doubles. (It is the electric field that drives the current in the conductor and potential difference is electric field times the distance). This means that resistance of the slab doubles with the doubling of its length. That is, $R \propto l$. Next, imagine the slab as being made of two parallel slabs, each of area $\frac{A}{2}$. If for a given voltage V, the current I flows across the full slab, it is clear that through each half-slab, the current flowing is $\frac{I}{2}$. Thus, the resistance of each half-slab is twice that of the full slab. That is, $R \propto \frac{l}{A}$. Combining the two dependences, we get

$$R \propto \frac{l}{A} \qquad \text{...(iv)}$$

or $$R = \frac{\rho l}{A} \qquad \text{...(v)}$$

where ρ is a constant of proportionality called *resistivity*. It depends only on the nature of the material of the resistor and its physical conditions such as temperature and pressure. The unit of resistivity is ohm m (Ω m). The inverse of ρ is called conductivity, and is denoted by σ. The unit of σ is $(\Omega m)^{-1}$ or mho m^{-1} or siemen m^{-1}.



A perfect conductor would have zero resistivity and a perfect insulator would have infinite resistivity. Though these are ideal limits, the electrical resistivity of substances has a very wide range. Metals have low resistivity of 10^{-8} Ωm to 10^{-6} Ωm, while insulators like glass or rubber have resistivity, some 10^{18} times (or even more) greater. Generally, good electrical conductors like metals are also good conductors of heat, while insulators like ceramic or plastic materials are also poor thermal conductors.

6. TEMPERATURE DEPENDENCE OF RESISTANCE

The resistance of most conductors and of all pure metals increases with temperature. But in carbon the resistance decreases with temperature. There are also some alloys where there is no change of resistance with temperature. If R_0 and R be the resistance of a conductor at 0°C and θ°C, then it is found that

$$R = R_0(1 + \alpha\theta)$$

where α is a constant called the temperature coefficient of resistance.

$$\alpha = \frac{R - R_0}{R_0.\theta}$$

and the unit of α is K^{-1} or $°C^{-1}$.

If R_1 and R_2 be the resistance of a conductor at temperatures θ_1°C and θ_2°C, then

$$R_1 = R_0(1 + \alpha\theta_1) \text{ and } R_2 = R_0(1 + \alpha\theta_2)$$

and $$\alpha = \frac{R_2 - R_1}{R_1\theta_2 - R_2\theta_1}$$

Illustration: 4

The current in a conductor is 5 A when the voltage between the ends of the conductor is 12V.

(i) What is the resistance of the conductor?

(ii) What will be the current in the same conductor if the voltage is increased to 42 V?

Solution:

(i) Given that I = 5A; V = 12 V; R = ?

$$R = \frac{V}{I} = \frac{12V}{5A} = 2.4\Omega$$

(ii) It the voltage applied becomes 42 V

$$I = \frac{V}{R} = \frac{42V}{2.4ohm} = 17.5A$$

Illustration : 5

The resistivity of a ferric-chromium-aluminium alloy is $51 \times 10^{-8}\ \Omega$-m. A sheet of the material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine resistance between (a) opposite ends and (b) opposite faces.

Solution:

(a) As seen from figure (a) in this case,

$\ell = 15\ cm = 0.15\ m$

$A = 6 \times 0.014 = 0.084\ cm^2 = 0.084 \times 10^{-4}\ m^2$

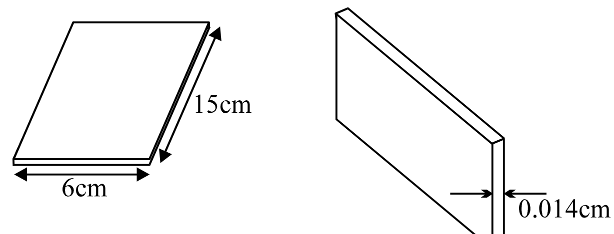
$$R = \rho\frac{\ell}{A} = \frac{51\times10^{-8}\times0.15}{0.084\times10^{-4}} = 9.1\times10^{-3}\ \Omega$$

(b) As seen from figure (b) here

$\ell = 0.014\ cm = 14 \times 10^{-5}\ m$

$A = 15 \times 6 = 90\ cm^2 = 9 \times 10^{-3}\ m^2$

$\therefore\ R = 51 \times 10^{-8} \times 14 \times 10^{-5}/9\times10^{-3} = 79.3 \times 10^{-10}\ \Omega$



(a) (b)

Illustration : 6

A potential difference of 100 V is applied to the ends of a copper wire one metre long. Calculate the average drift velocity of the electrons. Given $\sigma = 5.81\times10^{7}\ \Omega^{-1}m^{-1}$

Solution:

Since $\Delta V = 100V, l = 1m$.

$$\therefore \quad \text{electric field} = \frac{\Delta V}{l} = \frac{100}{1} = 100Vm^{-1}$$

Also, conductivity $\sigma = 5.81\times10^{7}\ \Omega^{-1}m^{-1}$

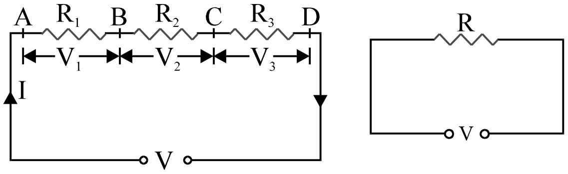
$N = 8.5 \times 10^{28}\ m^{-3}$

$$\therefore \quad \upsilon_d = \frac{\sigma}{eN}E = \frac{5.81\times10^{7}\times100}{1.6\times10^{-19}\times8.5\times10^{28}} = 0.43\ m\ s^{-1}$$

7. COMBINATION OF RESISTANCES

Resistances in Series Combination

When some conductors having resistances R_1, R_2 and R_3 etc. are joined end-on-end as Figure, they are said to be connected in series. It can be proved that the equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that (i) current is the same through all the three conductors (ii) but voltage drop across each is different due to its different resistances and is given by Ohm's Law and (iii) sum of the three voltage drops is equal to the voltage applied across the three conductors. There is a progressive fall in potential as we go from point A to D as shown in figure.



$\therefore \quad V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$ (Ohm's Law)

But $\quad V = IR$

Where R is the equivalent resistance of the series combination.

$\therefore \qquad IR = IR_1 + IR_2 + IR_3 \quad$ or $\quad R = R_1 + R_2 + R_3$

Also $\qquad \dfrac{1}{G} = \dfrac{1}{G_1} + \dfrac{1}{G_2} + \dfrac{1}{G_3}$

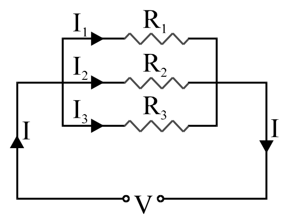
As seen from above, the main characteristics of a series circuit are :

1. Same current flows through all parts of the circuit.
2. Different resistors have their individual voltage drops.
3. Voltage drops are additive.
4. Applied voltage equals the sum of different voltage drops.
5. Resistances are additive.

Resistances in Parallel Combination

Three resistances, joined as shown in figure are said to be connected in parallel. In the case (i) p.d. across all resistances is the same (ii) current in each resistor is different and is given by Ohm's Law and (iii) the total current is the sum of the three separate currents.

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$



Now, $\qquad I = \dfrac{V}{R}$ where V is the applied voltage.

R = equivalent resistance of the parallel combination.

$\therefore \qquad \dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2} + \dfrac{V}{R_3}$ or $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

Also $\qquad G = G_1 + G_2 + G_3$

Resistances in Mixed Combination

If resistances are arranged in series-parallel mixed grouping, we apply method of successive reduction to find equivalent resistance.

To calculate the equivalent resistance between the points a and b, the network shown in figure, may be successively reduced as described below :

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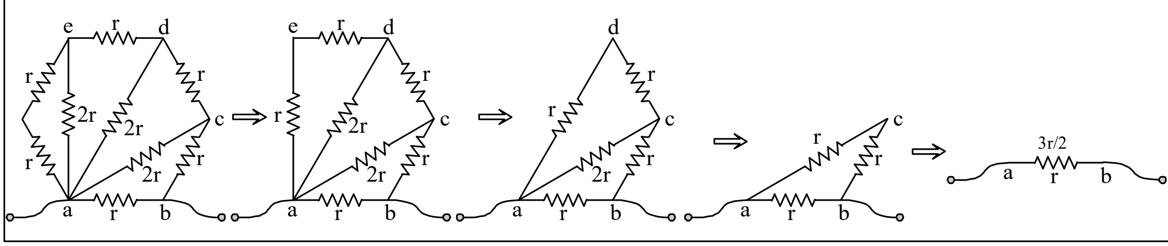
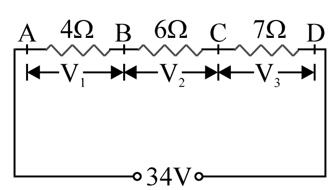


Illustration: 7

Three resistors of values 4 ohm, 6 ohm and 7 ohm are in series and a potential difference of 34 V is applied across the grouping. Find the potential drop across each resistor.



Solution:

The current through the circuit $= \dfrac{34V}{(4+6+7)ohm} = 2A$

potential difference across 4 ohm resistor = IR = 2 A × 4 ohm = 8 V potential difference across 4 ohm resistor = 2 A × 6 ohm = 12 V

potential difference across 4 ohm resistor = 2 A × 7 ohm = 14 V

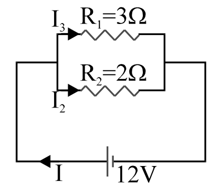
Illustration: 8

Two resistance 3 ohm and 2 ohm are in parallel connection and a potential difference of 12 V is applied across them. Find :

(a) the equivalent resistance of the parallel combination,

(b) the circuit current and

(c) the branch currents.



Solution:

(a) Two resistors R_1 and R_2 are in parallel. Their equivalent resistance R is given by

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

or $$R=\frac{R_1R_2}{R_1+R_2}=\frac{2\times3}{2+3}=\frac{6}{5}=1.2\ \Omega$$

(b) The circuit current = Circuit voltage/Circuit resistance

$$\frac{12V}{1.2\Omega}=10\,A$$

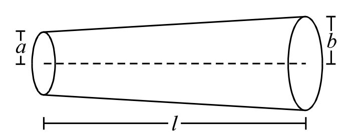
(c) The current through 2 ohm resistor

$$I_2=I\times\frac{3}{2+3}=10\times\frac{3}{5}=6A$$

The current through 3 ohm resistor

$$I_3=I\times\frac{2}{2+3}=10\times\frac{2}{5}=4A$$

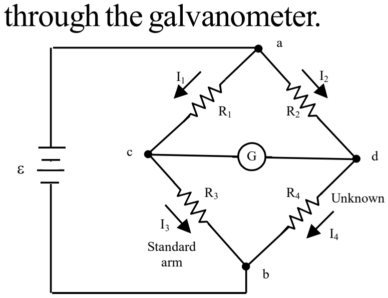
(Also $I_3 = I - I_2$ = 10 A – 6 A = 4 A)



8. WHEATSTONE BRIDGE

Figure shows the fundamental diagram of wheatstone bridge. The bridge has four resistive arms, together with a source of emf (a battery) and a galvanometer. The current through the galvanometer depends on the potential difference between the point c and d. The bridge is said to be balanced when the potential difference across the galvanometer is 0 V so that there is no current through the galvanometer.

This condition occurs when the potential difference from point c to point a, equals the potential difference from point d to point a; or by referring to the other battery terminal, when the voltage from other point c to point b equals the voltage from point d to point b. Hence, the bridge is balanced when



$$I_1R_1=I_2R_2 \qquad \ldots(i)$$

if the galvanometer current is zero, the following conditions also exist:

$$I_1=I_3=\frac{\varepsilon}{R_1+R_3} \qquad \ldots(ii)$$

and $$I_2=I_4=\frac{\varepsilon}{R_2+R_4} \qquad \ldots(iii)$$

Combining Eqs. (i), (ii) and (iii) and simplifying, we obtain

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4} \qquad \ldots(\text{iv})$$

from which we get

$$R_1 R_4 = R_2 R_3 \quad \text{or} \quad \frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad \ldots(\text{v})$$

Equation (v) is the well known expression for balance of the wheatstone bridge. If three of the resistances have known values, the fourth may be determined from Equation (v). Hence, if R_4 is the unknown resistor, its resistance can be expressed in terms of remaining resistors

$$R_4 = R_3 \frac{R_2}{R_1} \qquad \ldots(\text{vi})$$

Resistance R_3 is called the standard arm of the bridge and resistors R_2 and R_1 are called the ratio arms.

9. KIRCHHOFF'S LAW

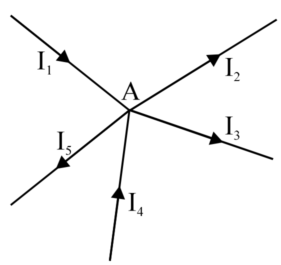
These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter. Kirchoff's laws, two in number, are particularly useful (a) in determining the equivalent resistance of a complicated network of conductors and (b) for calculating the currents flowing in the various conductors. The two laws are :

(i) Kirchhoff's Point Law or Current Law (KCL)

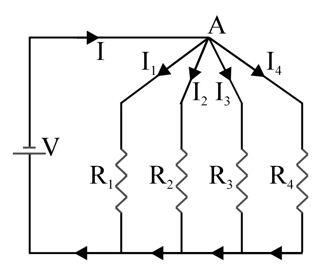
It states as follows :

In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero.

Put in another way, it simply means that the total current leaving a junction is equal to the total current entering that junction. It is obviously true because there is no accumulation of charge at the junction of the network.



(a)



(b)

Consider the case of a few conductors meeting at a point A as in figure (a). Some conductors have currents, leading towards point A, whereas some have currents leading away from point A. Assuming the incoming currents to be positive and the outgoing currents negative, we have :

$$I_1 + (-I_2) + (-I_3) + (+I_4) + (-I_5) = 0$$

or $\quad I_1 - I_2 - I_3 + I_4 - I_5 = 0$ or $I_1 + I_4 = I_2 + I_3 + I_5$

or $\quad$ incoming currents = outgoing current

Similarly, in figure(b) for node A

$$+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0 \text{ or } I = I_1 + I_2 + I_3 + I_4$$

We can express the above conclusion thus : $\sum I = 0$ ... at a junction

(ii) Kirchhoff's Mesh Law or Voltage Law (KVL)

It state as follows :

The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.fs. in that path is zero.

In other words, $\sum \text{IR} + \sum \text{e.m.f.} = 0$

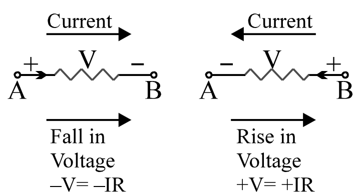
It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

Working with Kirchhoff's law

In applying Kirchhoffs laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs. otherwise results will come out to be wrong. Following sign conventions are suggested :

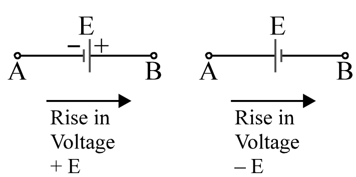
(a) Sign of Battery E.M.F.

A rise in voltage should be given a + ve sign and a fall in voltage a –ve sign. Keeping this in mind, it is clear that as we go from the –ve terminal of a battery to its +ve terminal, there is a rise in potential, hence this voltage should be given a +ve sign. If, on the other hand, we go from +ve terminal to –ve terminal, then there is a fall in potential, hence this voltage should be preceded by a –ve sign. It is important to note that the sign of the battery e.m.f. is independent of the direction of the current through that branch.



(b) Sign of IR Drop

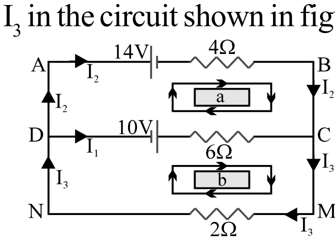
Now, take the case of a resistor as shown in figure. If we go through a resistor in the same direction as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken as –ve. However, if we go in a direction opposite to that of the current, then there is a rise in voltage. Hence, this voltage rise should be given a positive sign.



It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit under consideration.

Illustration : 9

Calculate the currents I_1, I_2 and I_3 in the circuit shown in figure.



Solution:

Junction rule at C yields

$$I_1 + I_2 - I_3 = 0 \text{ i.e., } I_1 + I_2 = I_3 \qquad ....(1)$$

while loops for meshes a and b yield respectively :

$$-14 - 4I_2 + 6I_1 - 10 = 0$$

i.e., $\quad 3I_1 - 2I_2 = 12 \qquad ....(2)$

and, $10 - 6I_1 - 2I_3 = 0$

i.e., $3I_1 + I_3 = 5$ ....(3)

Substituting I_3 from Equation (1) in (3)

$$4I_1 + I_2 = 5 \qquad ....(4)$$

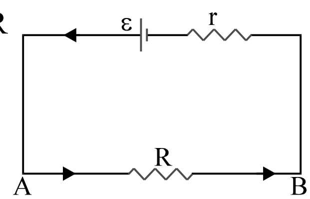
Solving equations (2) and (4) for I_1 and I_2, we find

$$I_1 = 2A \text{ and } I_2 = -3A$$

And hence equation (1) yields, $I_3 = -1A$

The fact that I_2 and I_3 are negative implies that actual direction of I_2 and I_3 are opposite to that shown in the circuit.

10. GROUPING OF CELLS



If a cell of e.m.f. ε and internal resistance r be connected with a resistance R the total resistance in the circuit is (R + r).

The current through the circuit $I = \dfrac{\varepsilon}{R + r}$

Potential difference across the ends A and B of $R = IR = \dfrac{\varepsilon R}{R + r}$

Thus, although the emf of the cell is ε, the effective potential difference it can deliver is less than ε and it is given by

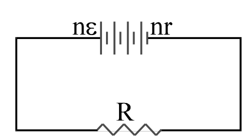
$$V_{AB} = \varepsilon - Ir$$

The quantity V_{AB} is called the terminal potential difference of the cell and this is also the potential difference across the external resistance R.

If $R \to \infty$, $V_{AB} \to \varepsilon$, the emf of the cell.

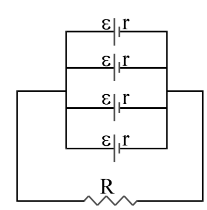
Cells in Series

Let there be n cells each of emf ε, arranged in series. Let r be the internal resistance of each cell. The total emf is nε and the total internal resistance is nr. If R be the external load, the current I through the circuit $I = \dfrac{n\varepsilon}{R + nr}$.



Cells in Parallel

In m cells each of emf ε and internal resistance r be connected in parallel and if this combination be connected to an external resistance R, then the emf of the circuit = ε.



The internal resistance of the circuit = the resistance due to m resistances each of r in parallel $= \dfrac{r}{m}$.

Now the current through the external resistor $R = \dfrac{\varepsilon}{R + \dfrac{r}{m}} = \dfrac{m\varepsilon}{mR + r}$

Mixed Grouping of Cells

Let n identical cells be arranged in series and let m such rows be connected in parallel. Obviously the total number of cells is nm. The emf of the system = nε

The internal resistance of the system = nr/m

The current through the external resistance R

$$I = \frac{n\varepsilon}{R + \dfrac{nr}{m}} = \frac{mn\varepsilon}{mR + nr}$$

Illustration : 10

Six cells are connected (a) in series, (b) in parallel and (c) in 2 rows each containing 3 cells. The emf of each cell is 1.08 V and its internal resistance is 1 ohm. Calculate the currents that would flow through an external resistance of 5 ohm in the three cases.

Solution:

(a) The cells in series.

Given that $\varepsilon = 1.08V$, $n = 6$, $r = 1$ ohm, $R = 5$ ohm

The total emf $= n\varepsilon = 6 \times 1.08$ V

The total internal resistance $nr = 6 \times 1 = 6$ ohm

The current in the circuit $I_s = \dfrac{n\varepsilon}{R + nr} = \dfrac{6 \times 1.08}{5 + 6} = 0.589 \text{ A}$

(b) The cells in parallel,

Here $\varepsilon = 1.08$ v, $m = 6$, $r = 1$ ohm, $R = 5$ ohm

$$I_p = \frac{m\varepsilon}{mR + r} = \frac{6 \times 1.08}{6 \times 5 + 1} = \frac{6.48}{31} = 0.209 \text{ A}$$

(c) The cells in multiple are with $n = 3$, $m = 2$

$$I = \frac{mn\varepsilon}{mR + nr} = \frac{6 \times 1.08}{(2 \times 5) + (3 \times 1)} = \frac{6.48}{13} = 0.498 \text{ A}.$$

Arrangement of Cells for Maximum Current

Considering the case where total number of cell (mn) is given and it is required to find the condition for maximum current.

In this case the product mn, ε, r and R are constants and m and n alone can be varied to get I maximum.

For I_{max}, denominator $(mR + nr)$ should be minimum in equation $I = \dfrac{mn\varepsilon}{mR + nr}$. This happens when $mR = nr$ or $R = nr/m$.

Hence the current through the external resistance R is a maximum when it is equal to internal resistance of the battery i.e. nr/m.

11. HEATING EFFECT OF CURRENT

When a current I flows for time t from a source of emf ε, then the amount of charge that flows in time t is $Q = I\,t$.

Electrical energy delivered $W = Q.\ V = V\,I\,t$

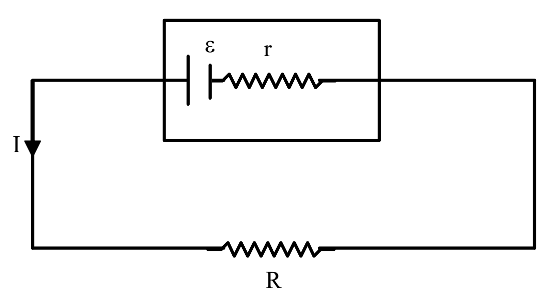
Thus, Power given to the circuit, $= W/t = VI$ or V^2/R or I^2R

In the circuit

$$\varepsilon.\ I = I^2R + I^2r, \text{ where}$$

ε I is the rate at which chemical energy is converted to electrical energy, I^2R is power supplied to the external resistance R and I^2r is the power dissipated in the internal resistance of the battery.

An electrical current flowing through conductor produces heat in it. This is known as Joule's effect. The heat developed in Joule is given by $H = I^2.R.t$



Maximum Power Theorem

Consider the arrangement in which a resistance R is connected to a battery of emf ε and internal resistance r. Power P developed in resistance is given by

$$P = \frac{\varepsilon^2 R}{(R+r)^2} \quad (\because I = \frac{\varepsilon}{R+r} \text{ and } P = I^2 R)$$

Now, for P to be maximum $\frac{dP}{dR} = 0$

$\Rightarrow \quad E^2 . \frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4} = 0$

$\Rightarrow \quad (R+r) = 2R$

or $\quad R = r$

$\Rightarrow$ The power output is maximum, when the external resistance equals the internal resistance i.e., R = r. This is called as maximum power theorem.

Illustration : 11

A fuse made of lead wire has an area of cross-section 0.2 mm². On short circuiting, the current in the fuse wire reaches 30 amp. How long after the short circuiting will the fuse begin to melt?

Specific heat capacity of lead = 134.3 J/kg-K.

Melting point of lead = 327°C

Density of lead = 11340 kg/m³

Resistivity of lead = 22 × 10⁻⁸ ohm-h

Initial temperature of the wire = 20°C

Neglect heat loss.

Solution:

If L be the length of the wire, its resistance

$$R = \frac{\rho L}{A} = \frac{(22\times10^{-8})L}{(0.2\times10^{-6})m^2}$$

Heat produced in the wire in one second = $I^2R = (30)^2 RJ$

Heat required to raise the temperature of the wire to 327°C

$Q = ms\Delta T$

$= (LAd)(134.4)(307)J$

The heat required to melt the wire

$$= \frac{Q}{I^2R} = \frac{LAd\times134.4\times307}{I\times\rho L}\times A \qquad = \frac{A^2 d}{I^2\rho}\times134.4\times307 = 0.0945s$$

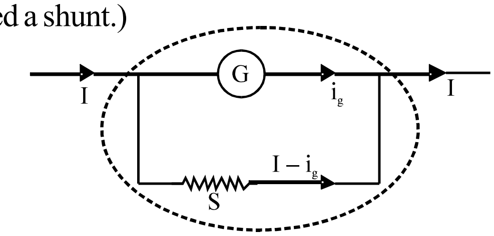
12. ELECTRICAL INSTRUMENTS

(i) Ammeter

It is an instrument used to measure currents. It is put in series with the branch in which current is to be measured. An ideal Ammeter has zero resistance. A galvanometer with resistance G and current rating i_g can be converted into an ammeter of rating I by connecting a suitable resistance S in parallel to it. (The resistance connected in parallel to the ammeter is called a shunt.)

Thus $S(I - i_g) = i_g G$

$\Rightarrow \quad S = \frac{i_g G}{I - i_g}$



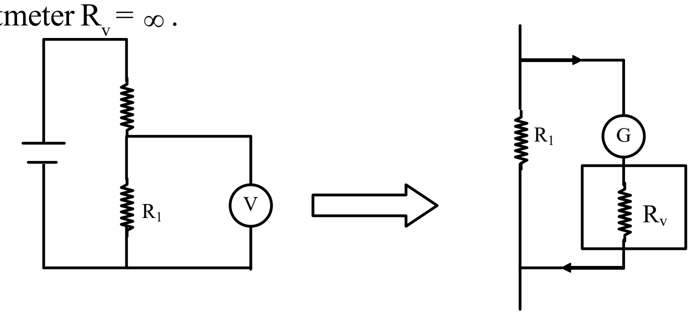
(ii) ***Voltmeter***

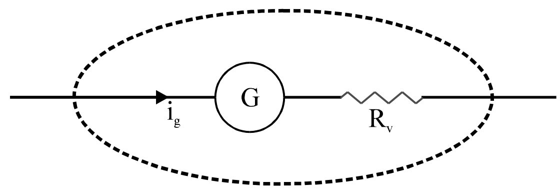
It is an instrument to find the potential difference across two points in a circuit.

It is essential that the resistance R_v of a voltmeter be very large compared to the resistance of any circuit element with which the voltmeter is connected. Otherwise, the voltmeter itself becomes an important circuit element and alters the potential difference that is measured.

$$R_v >> R$$

For an ideal voltmeter $R_v = \infty$.

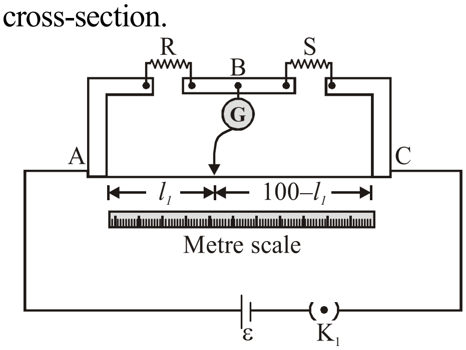




$$i_g(G+R_v) = V \quad \Rightarrow \quad R_v = \frac{V}{i_g} - G$$

(iii) ***Metre Bridge***

This is the simplest form of wheatstone bridge and is specially useful for comparing resistances more accurately. The construction of the metre bridge is shown in the Figure. It consists of one metre resistance wire clamped between two metallic strips bent at right angles and it has two points for connection. There are two gaps; in one of them a known resistance and in second an unknown resistance whose value is to be determined is connected. The galvanometer is connected with the help of jockey across BD and the cell is connected across AC. After making connections, the jockey is moved along the wire and the null point is found in accordance with the two resistances of the wheatstone bridge, wire used is of uniform material and cross-section.



The resistance can be found with the help of the following relation :

$$\frac{R}{S} = \frac{\sigma\ell_1}{\sigma(100-\ell_1)}$$

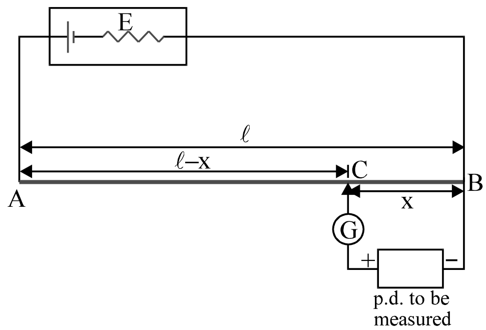
$$\frac{R}{S} = \frac{\ell_1}{100-\ell_1}$$

$$R = S\frac{l_1}{100-l_1} \qquad \ldots \qquad (1)$$

where σ is the resistance per unit length of the wire and l_1 is the length of the wire from one end where null point is obtained. The bridge is most sensitive when null point is somewhere near the middle point of the wire. This is due to end resistances.

(iv) Potentiometer

We already know that when a voltmeter is used to measure potential difference, its finite resistance causes it to draw a current from the circuit. Hence the p.d. which was to be measured is changed due to the presence of the instrument. Potentiometer is an instrument which allows the measurement of p.d. without drawing current from the circuit. Hence it acts as an infinite-resistance voltmeter.



The resistance between A and B is a uniform wire of length ℓ, with a sliding contact C at a distance x from B. The sliding contact is adjusted until the galvanometer G reads zero. The no deflection condition of galvanometer ensures that there is no current through the branch containing G and the p.d. to be measured. The length x for no deflection is called as the balancing length.

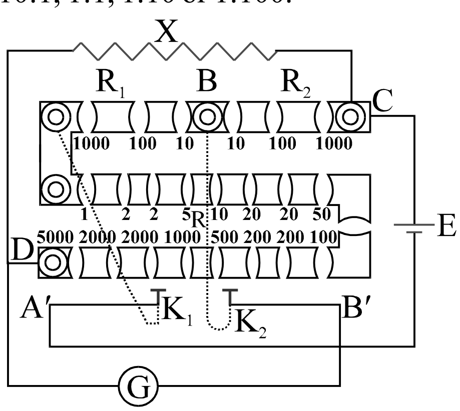
$V_{CB} = V$ p.d. to be measured.

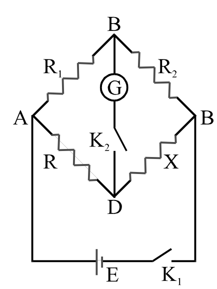
If λ is the resistance per unit length of AB.

$$V = V_{CB} = \frac{x}{\ell} V_{AB} = \left(\frac{V_{AB}}{\ell}\right) x$$

(v) The Post Office Box

It is a compact form of the Wheatstone bridge. It consist of compact resistance so arranged that different desired values of resistance may be selected in the three arms of Wheatstone bridge, as shown in figure. Each of the arm AB and BC contains three resistances of $10, 10^2$, and $10^3\,\Omega$, respectively. These are called the ratio arms. Using these resistances the ratio R_2/R_1 can be made to have any of the following values : 100:1, 10:1, 1:1, 1:10 or 1:100.





The arm AD is a complete resistance box containing resistances from 1 to 5000Ω. The tap keys K_1 and K_2 are also provided in the post office box. The key K_1 is internally connected to the point A and the key K_2 to the point B (as shown by dotted line in the figure). The unknown resistance X is connected between C and D, the battery between C and the key K_1 and the galvanometer between D and the key K_2. The circuit shown in first figure is exactly the same as that of the Wheatstone bridge shown in second figure.

Hence, the value of the unknown resistance is given by

$$X = R\left(\frac{R_2}{R_1}\right)$$

Illustration : 12

A battery of emf 1.4 V and internal resistance 2 ohm is connected to a resistor of 100 ohm resistance through an ammeter. The resistance of the ammeter is 4/3 ohm. A voltmeter has also been connected to find the potential difference across the resistor.

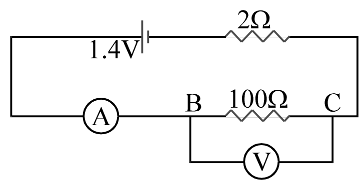
(a) Draw the circuit diagram.

(b) The ammeter reads 0.02 A. What is the resistance of the voltmeter?

(c) The voltmeter reads 1.1 V. What is the error in the reading?

Solution:

(i) The circuit diagram is shown.



(ii) Let the resistance of the voltmeter be R ohm. The rquivalent resistance of voltmeter (R ohm) and 100 ohm in parallel is $\frac{100\times R}{100+R}=\frac{100R}{100+R}$

The resistance of the ammeter $=\frac{4}{3}\Omega$

The total resistance of the circuit $=\frac{100R}{100+R}+\frac{4}{3}+2\Omega$

The current in the circuit as read by the ammeter = 0.02 A

$$\text{Now, } 0.02=\frac{1.4}{\frac{100R}{100+R}+\frac{4}{3}+2}$$

or $\quad R = 200\ \Omega$

Resistance of the voltmeter = 200 Ω

(iii) The effective resistance between B and C $=\frac{100\times 200}{100+200}=\frac{200}{3}\Omega$

The potential drop across this resistance = circuit current $\times\frac{200}{3}=0.02\times\frac{200}{3}=\frac{4}{3}V=1.33V$

The reading of the voltmeter = 1.1 V

The error in the reading of the voltmeter = 1.1 – 1.33 = – 0.23 V