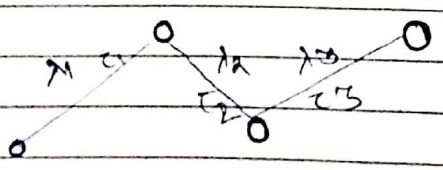
Kinetic theory of Gases

Introduction:
↳ John Dalton discovered atom in 1803
↳ Kinetic theory in 1873
↓
Atoms / molecules constantly moves (MOTION)

Given by Maxwell & Boltzmann

Assumption:
i) Intermolecular force is absent in ideal gases
* We study avg. properties of gases

Mean Free Path:
Avg. distance travelled by a molecule b/w 2 successive coll^{n}s.



$$\lambda_m = \frac{\lambda_1 + \lambda_2 + \dots + \lambda_n}{n}$$

Avg relaxation time (τ)

Avg time period b/w 2 successive coll^{n}s.

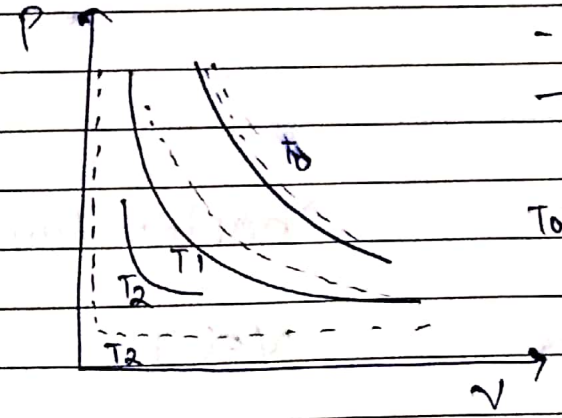
$$\tau_m = \frac{\tau_1 + \tau_2 + \cdots \tau_n}{n}$$

IDEAL GAS Laws:

I] Boyle's law: (1661)

At, constant temp of gas, [isothermal]

$$V \propto \frac{1}{P}$$



----- → experimental

——— → Law/theoretical.

$T_0 > T_1 > T_2$

High temp: (Real gas → ideal gas)

Gas law agrees experiment.

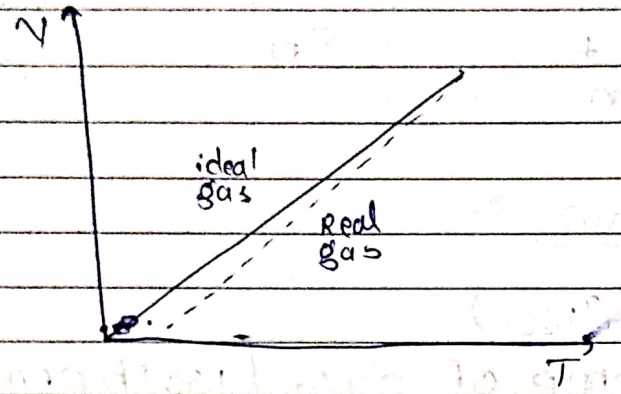
$$VP = \text{constant}$$

$$\boxed{P_1V_1 = P_2V_2} = \text{constant.}$$

II) 1783: Charles Law

At constant P

$V \propto T$



At high temp. law holds for all gases.

$\frac{V}{T} = \text{constant}$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

*

Boyle's Law	Charles' Law
$V \propto \frac{1}{P}$	$V \propto T$
T = constant	P = constant

$V \propto \frac{T}{P}$

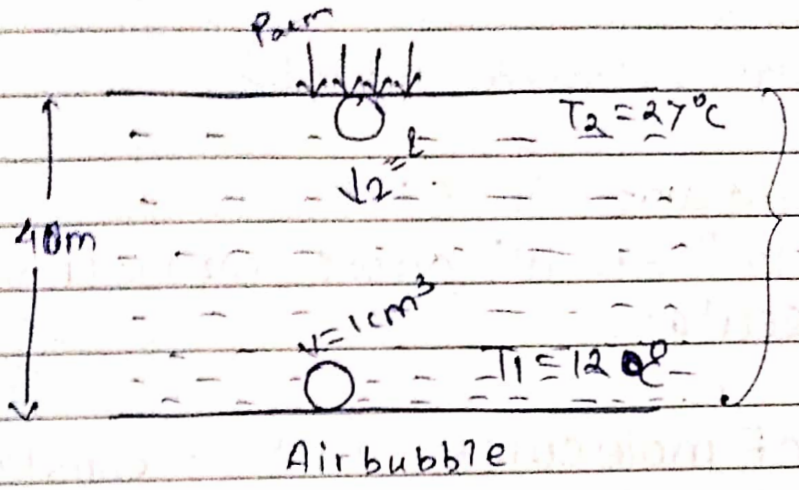
$\frac{PV}{T} = \text{constant}$

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

$P_{atm} = 1 atm$

$= 1.01 \times 10^5 \frac{N}{m^2}$

Q



$P_1V_1 = P_2V_2$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{(1.01 \times 10^5 + 10^3 \times 10 \times 40) \times 1cm^3}{285} = \frac{(1.01 \times 10^5) \times V_2}{300}$

$\frac{10^5 + 4 \times 10^5}{285} = \frac{10^5 \times V_2}{300}$

$10^5(5)$

$V_2 = \frac{300}{60}$

$V_2 = 5 cm^3$

III) Avogadro's hypothesis (1811)

At constant T & P:
Equal vol^m of all gases contains equal no. of molecules.

$V \propto$ no. of molecules (P & T → constant)
$V \propto$ no. of moles
$\boxed{V \propto n}$

Ideal Gas Equation:

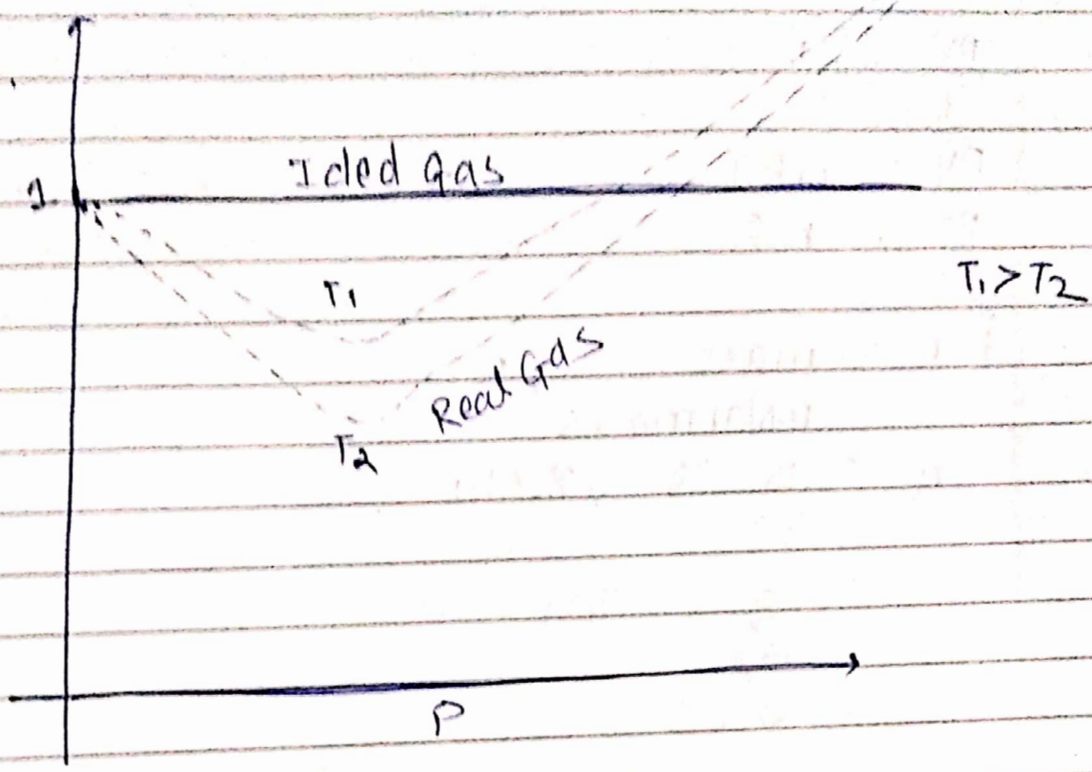
$V \propto \frac{1}{P}$ $\quad V \propto T$ $\quad V \propto n$

$V \propto \frac{nT}{P}$

$V = \frac{RnT}{P}$

$\boxed{PV = nRT}$

Where,
P = Pressure
V = Vol^m
R = Gas constant : 8.314 J/mol·K.
T = Temperature (in kelvin)
n = no. of moles.

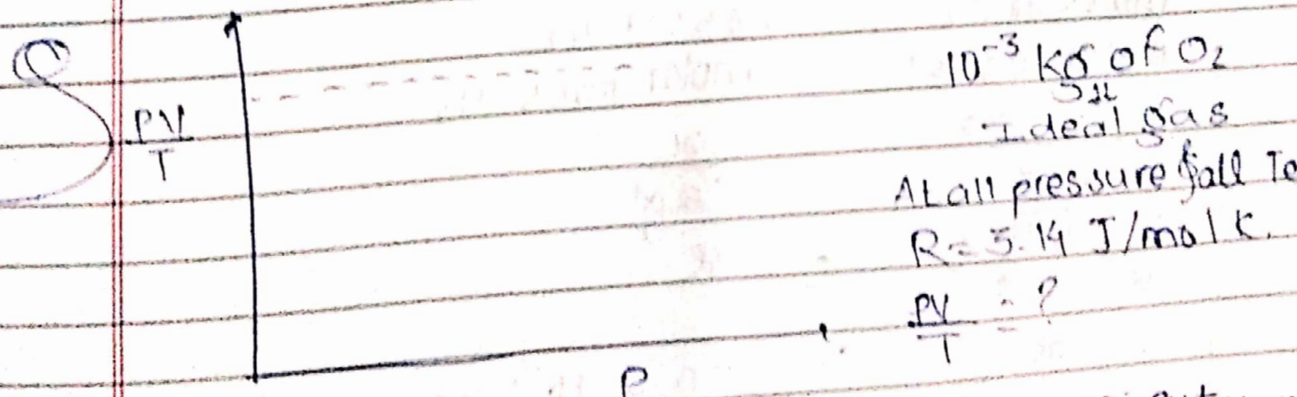


Real gas $\Rightarrow$ Ideal gas $\Rightarrow$ Low Temp Pressure
High Temp

1 mole of any Gas: $PV = RT$

ideal Gas $= \frac{PV}{RT} = 1$ (Low P & High T)

High temp ↑↑ real gas & ideal gas get closer



10^{-3} kg of O_2
Ideal gas
At all pressure fall Temp.
$R = 8.314$ J/mol K.
$\frac{PV}{T} = ?$
where ..... at y-axis.

$\frac{PV}{T} = P$

$PV = nRT$

$\frac{PV}{T} = nR$

$n = \frac{\text{mass}}{\text{molar mass}} \times R$

$n = \frac{10^{-3}\,kg}{32\,g} \times 8.314$

$= \frac{1\,g}{32\,g} \times 8.314$

$= 0.25$

For what mass of H_2 gas the value of $\frac{PV}{T}$ will be same as $\frac{PV}{T}$ for 10^{-3} kg of O_2 gas

$PV = n_{O_2} RT$

$PV = n_{H_2} RT$

$\left(\frac{PV}{T}\right)_{O_2} = \left(\frac{PV}{T}\right)_{H_2}$

$n_{O_2} R = n_{H_2} R$

$\frac{\text{mass of } O_2}{\text{molar mass } O_2} = \frac{\text{mass of } H_2}{\text{molar mass } H_2}$

$\frac{10^{-3}\,kg}{32\,g} = \frac{x}{2\,g}$

$\frac{1}{16} \times 10^{-3}\,kg = x$

$x = \frac{100}{16} \times 10^{-5}\,kg$

$= 6.25 \times 10^{-5}\,kg$

$m_{H_2} = 6.25 \times 10^{-5}\,kg$

Q Find the ratio of $\frac{\text{molecular vol}^m}{\text{Actual vol}^m \text{ occupied}}$ by O_2 molecules.
(diameter of O_2 molecule is $3A^\circ$.

→ 1 mole of O_2 at STP
$= 22.4\,L$
$= 22.4 \times 10^{-3}\ m^3$

6.022×10^{23} molecules of O_2

$\gamma = \frac{4}{3}\pi r^3 \times 6.022 \times 10^{-23}$

$\therefore \frac{\text{Molecular vol}^m}{\text{Actual vol}^m \text{ occupied}} = \frac{4/3\,\pi r^3 \times 6.022 \times 10^{23}}{22.4 \times 10^{-3}\ m^3}$

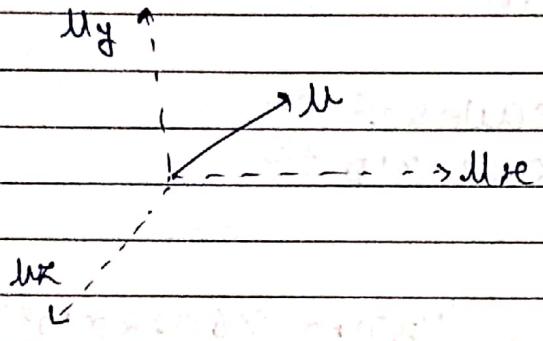
Kinetic Theory of Gases:

Postulates:

i] Gas is made up of atoms & molecules.

ii] Molecules of same gases are identical in all respects. (mass, shape, size)

iii] Molecules are constanly in random motion along straight line.



$$u = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

$$u_x = u_y = u_z$$

$$|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

$$u^2 = u_x^2 + u_y^2 + u_z^2 = 3u_x^2$$

iv] All the coll^{n}s of gas molecules among themselves & wall of container is elastic in nature

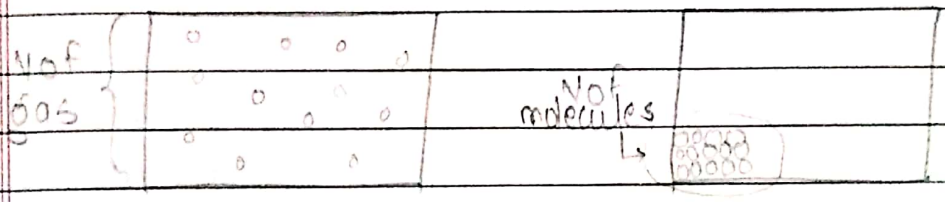
↳ Momentum
Kinetic energy
} No loss.

v] The pressure of a gas is due to coll^{n}s of molecules with wall of container.

$P \propto$ No. of coll^{n}s of molecules per unit area

vi] The kinetic energy of gas depends only upon temp (absolute) (doesn't depend on nature of gas)

vii] The vol^m occupied by gas molecules is negligible when compared to vol^m of gas.



viii] There is no inter molecular force of attraction attraction among molecules.

ix] Gravity is neglected.

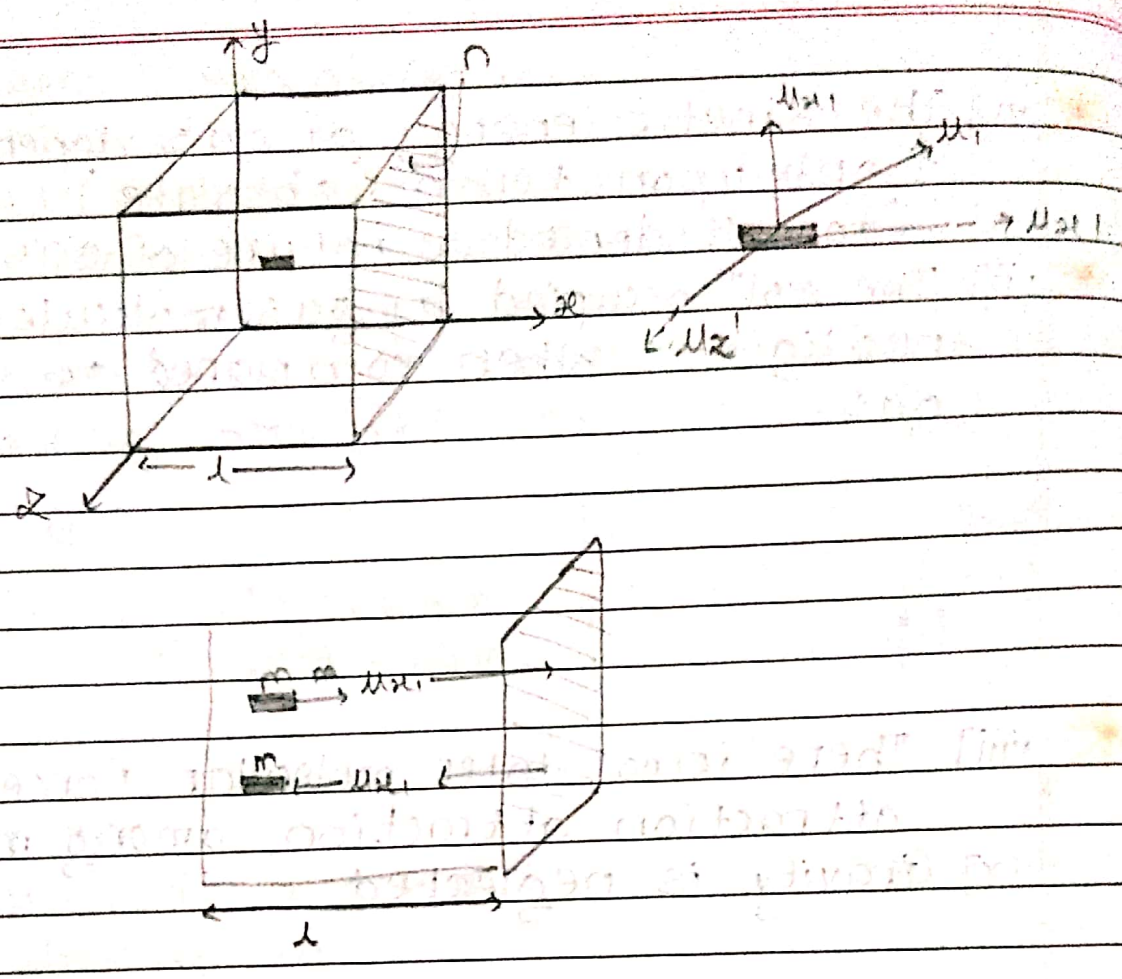
* Kinetic gas eq^n:

$$PV = \frac{1}{3} mn u_{rms}^2$$

m → mass of one molecule of gas
n → no. of molecules
u_{rms} → Root mean square velocity of molecules

$$u_{rms} = \sqrt{\frac{u_1^2 + u_2^2 + u_3^2 + \ldots u_n^2}{n}}$$

K.E of n moles of gas $= n \times \frac{3}{2} RT$



$p_i = m u_{x1}$

$p_f = -m u_{x1}$

$\Delta p = 2 m u_{x1}$

time taken for each collⁿ $= \frac{2l}{u_{x1}}$

Force exerted on wall $= \frac{\Delta p}{\Delta t}$

$\vec{F} = \frac{-2 m u_{x1}}{\frac{2l}{u_{x1}}}$

$\vec{F} = \frac{m u_{x1}^2}{l}$

$$* \; u = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$$
$$u^2 = 3u_x^2$$

$$\vec{F} = \frac{m}{l} \frac{1}{3} u_1^2$$

$$\vec{F} \text{ due to one molecule} = \frac{m}{l} u_{x_1}^2 = \frac{m}{l} \frac{1}{3} u_1^2$$

F due to 'n' molecules:

$$= \frac{1}{3} \frac{m}{l} u_1^2 + \frac{1}{3} \frac{m}{l} u_2^2 \ldots + \frac{1}{3} \frac{m}{l} u_n^2$$

$$F_{net} = \frac{1}{3} \frac{m}{l} \left[\frac{u_1^2 + u_2^2 + \ldots u_n^2}{n} \right] \times n$$

$$F_{net} = \frac{1}{3} \frac{mn}{l} u_{rms}^2$$

$$\text{Pressure} = \frac{F}{A} = \frac{1}{3} \frac{mn u_{rms}^2}{l \times l^2} = \frac{1}{3} \frac{mn u_{rms}^2}{l^3}$$

$$P = \frac{1}{3} \frac{mn u_{rms}^2}{V}$$ Kinetic Gas eqn

$$PV = \frac{1}{3} mn u_{rms}^2$$

$PV = nRT$

$R = 0.0821$ atm l/mol k

$K.E = \frac{3}{2} RT$

8.314 J/mol k

Kinetic energy:

$K.E = \frac{1}{2} m u_1^2$ (of 1 molecule)

K.E of n molecules $= \frac{1}{2} m u_1^2 + \frac{1}{2} m u_2^2 + \cdots + \frac{1}{2} m u_n^2$

$= \frac{1}{2} m (u_1^2 + u_2^2 + u_3^2 + \cdots + u_n^2)$

n

$$\boxed{\text{K.E of n molecules} = \frac{1}{2} m u^2_{rms} \times n}$$

$$\frac{K.E}{PV} = \frac{\frac{1}{2} n m u^2_{rms}}{\frac{1}{3} n m u^2_{rms}}$$

$$\boxed{PV = \frac{3}{2} K.E}$$

$$\boxed{K.E = \frac{3}{2} PV}$$ for 1 mole of gas (ideal) [$PV = RT$]

$$\boxed{K.E = \frac{3}{2} RT}$$

$$\boxed{PV = \frac{1}{3} n m N u^2_{rms}}$$ rms $\boxed{\text{K.E of 1 mole of ideal gas} = \frac{3}{2} RT}$

m = mass of 1 molecule; N = no. of molecules; R = Gas constant 8.314 J/mol k

$\frac{R}{N_A}$ = Boltzmann constant

K.E of n moles of ideal gas	$= n \times \frac{3}{2} RT$

K.E of 1 molecule	$= \frac{3}{2} \frac{RT}{N_A} = \frac{3}{2} KT$

K = Boltzman constant

$K = 1.38 \times 10^{-23} J/K$

Q.1 Find out K.E of 8g of CH_4 at 27°C.

→ $K.E = \frac{n \times 3}{2} RT$

$= \frac{0.5 \times 3}{2} \times 8.314 \times 300 J$

no. of moles = mass / molar mass

$= \frac{8}{16} = 0.5$

Q.2 Find out K.E of 1 molecule of oxygen (O_2) gas at 127°C.

→ $K.E = \frac{3}{2} KT$

$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 400$

$= 600 \times 1.38 \times 10^{-23} J$

Q.3 Find the change in K.E of 1 mole of ideal gas when temp changes by 50°C

→ $\Delta K.E = KE_f - K.E_i$

$= \frac{3}{2}RT_2 - \frac{3}{2}RT_1$

$= \frac{3R}{2}\Big[$

Sir

$KE_i = \frac{3}{2}RT$

$K.E_f = \frac{3}{2}R(T+50)$

$\Delta KE = KE_f - K.E_i$

$= \frac{3}{2}R(T+50-T)$

$= \frac{3}{2}R \times 50$

$= \frac{3}{2} \times 8.314 \times 50$ J

Q4 At what temp, the KE will be half of its value at ~~120~~ 127°C.

→ $K.E_{127} = n \times \frac{3}{2} \times R \times 400$

$K.E_T = \frac{K.E_{127°C}}{2}$

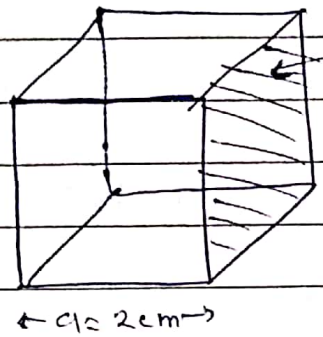
$n \times \frac{3}{2} \times R \times T = \frac{n \times 3/2 \times R \times 400}{2}$

$\boxed{T = 200K}$

1amu = 1.67 × 10^{-27} kg

Short cut
KE ∝ T

Q5



They make 500 coll^{n}s with the wall in each sec.

Find the temp.

→ Time for 1 coll^n = $\frac{1}{500}$ s ...(1)

time taken for 1 coll^n = $\frac{2a}{u_{rms}}$

$\frac{1}{500} = \frac{2a}{u_{rms}}$

$\frac{1}{500} = \frac{2 \times 2 \times 10^{-2}}{u_{rms}}$

$u_{rms} = 20$ m/s

K.E of 1 molecule = $\frac{1}{2} m u_{rms}^2$

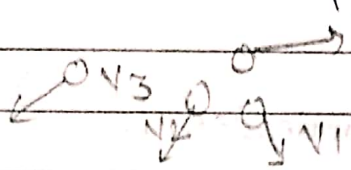
$= \frac{3}{2} kT$

$\frac{1}{2} m u_{rms}^2 = \frac{3}{2} kT$

1 molecule of He = 4amu
$= 4 \times 1.67 \times 10^{-27}$ kg

$M = m N_A$
(molar mass) (1 molecule mass)

RMS Velocity
[Root Mean Square]



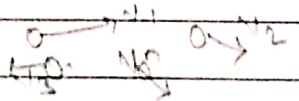
$$v_{RMS} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$$

$$v_{RMS\ (m/s)} = \sqrt{\frac{3RT}{M}}$$

$R = 8.314\ J/mol\ K$

T = in Kelvin

M = Mass i.e molar mass of gas (in kg)



$$K.E_{Total} = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + \dots + \frac{1}{2} m v_n^2$$

$$= \frac{1}{2} m \left(\frac{v_1^2 + v_2^2 + \dots v_n^2}{n} \right) \times n$$

$$= \frac{1}{2} m v_{rms}^2 \times n$$

K.E of 1 mole of gas $= \frac{1}{2} m v_{rms}^2 \times N_A$

$$\frac{3}{2} RT = \frac{1}{2} M v_{rms}^2$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$ (in kg)

Q. V_{RMS} for H_2 molecules at 27°C.

→ $V_{RMS} = \sqrt{\frac{3RT}{M}}$

$= \sqrt{\frac{3 \times 8.314 \times 300}{2 \times 10^{-3}}}$

$= \sqrt{3 \times 4 \times 300 \times 10^3}$

$= \sqrt{36 \times 10^5}$

$= \sqrt{3.6 \times 10^6}$

$= 1.3 \times 10^3$

Q if V_{RMS} for H_2 is x at a given Temp. Find V_{RMS} for O_2 at given Temp.

→

$$\frac{V_{O_2}}{V_{H_2}} = \frac{\sqrt{\frac{3RT}{M_{O_2}}}}{\sqrt{\frac{3RT}{M_{H_2}}}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$\frac{V_{O_2}}{x} = \sqrt{\frac{2 \times 10^{-3}}{32 \times 10^{-3}}} = \sqrt{\frac{1}{16}}$$

$$V_{O_2} = \frac{x}{4}$$

$V_{RMS} \propto \sqrt{T}$

Q At 300°K, V_{rms} of H_2 is same as V_{rms} of O_2 at T (in kelvin). Find T.

→

$$V_{rms\,H_2} = V_{rms\,O_2}$$

$$\sqrt{\frac{3RT}{M_{H_2}}} = \sqrt{\frac{3RT}{M_{O_2}}}$$

$$\sqrt{\frac{300}{2\times10^{-3}}} = \sqrt{\frac{T}{32\times10^{-3}}}$$

$$\sqrt{300} = \sqrt{\frac{T}{16}}$$

$$300 = \frac{T}{16}$$

$$T = 4800K$$

(Pressure)

$$V_{RMS} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} \text{ (S.I unit)}$$

Derivation:

K.E of 1 mole of Gas $= \frac{1}{2} M V_{rms}^2$

$$\frac{3}{2} RT = \frac{1}{2} M V_{rms}^2$$

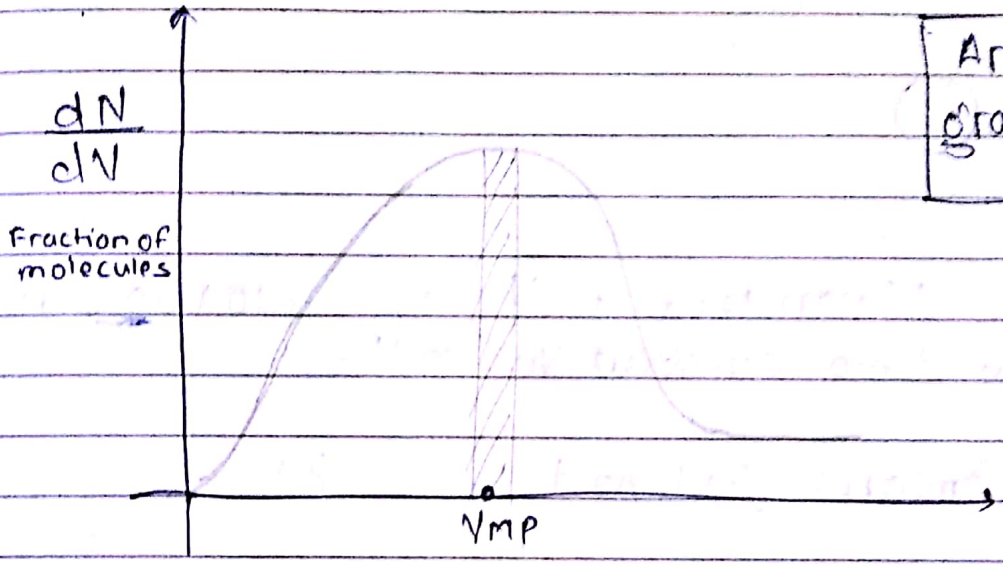
$$\frac{3}{2} PV = \frac{1}{2} M V_{rms}^2$$

$$V_{rms}^2 = \frac{3PV}{M} = \sqrt{\frac{3P}{(M/V)}}$$

$$= \sqrt{\frac{3P}{\rho}}$$

At constant temp
PT V_{rms} = constant.

Maxwell's distribution of velocities



Area of graph = Total no of molecules

$$V_{mp} = \sqrt{\frac{2RT}{M}}$$

$$V_{RMS} = \sqrt{\frac{3RT}{M}}$$

(Correct) $$V_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

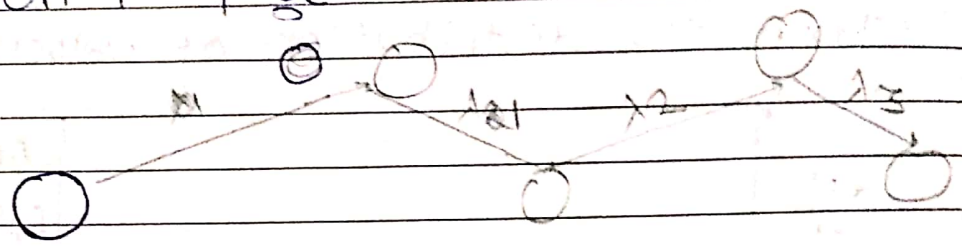
(Incorrect) $$V_{avg} = 0$$

$$R > A > M$$

$$V_{RMS} > V_{avg} > V_{mp}$$

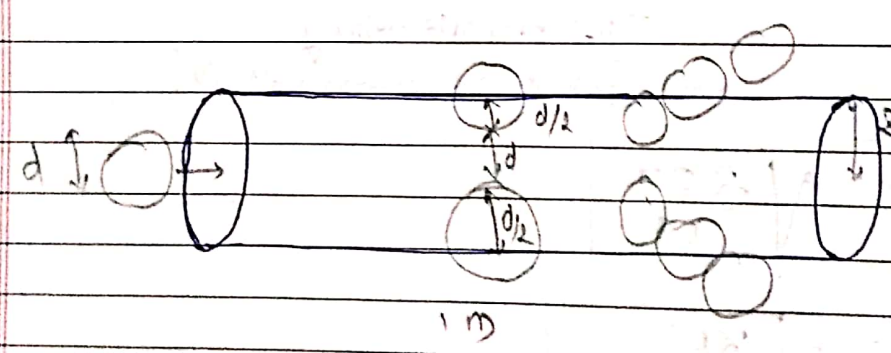
Mean Free Path:

on 1st page.



Mean free path is average distance b/w two consecutive coll^{n}s.

$$\lambda_{mean} = \frac{\lambda_1 + \lambda_2 + \cdots \lambda_n}{n}$$



no of molecules per unit vol^m = x

$1m^3 = x$

$2m^3 = 2x$

$Vm^3 = Vx$

no of molecules inside cylinder = $\pi(d)^2 \times 1 \times x$

no of coll^{n}s in 1m travelling distance = $\pi(d)^2 x$

$\pi d^2 x$ coll^{n}s $\longrightarrow$ 1 m distance

1 coll^n $\longrightarrow$ $\frac{1}{\pi d^2 x}$ distance.

$$\lambda = \frac{1}{\pi d^2 x}$$

$$\lambda = \frac{RT}{\pi d^2 P N_A}$$

N_A → Avogadro no

P → pressure in SI unit

d → diameter of molecule

$x \rightarrow$ no of molecules per unit vol^m

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{RT}$$

$$x = \frac{n}{V} \times N_A$$

$$x = \frac{PN_A}{RT}$$