

MIND MAP : LEARNING MADE SIMPLE

CHAPTER - 8

Application of the Integrals

Area between two curves

Area under simple curves

The area of the region enclosed between two curves $y = f(x), y = g(x)$

and the lines $x=a, x=b$ is given by

$$A = \int_a^b [f(x) - g(x)] dx, \text{ where } f(x) \geq g(x) \text{ in } [a, b]$$

For eg: To find the area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$

$(0,0)$ and $(1,1)$ are points of intersection of $y = x^2$ and

$y^2 = x$ and $y^2 = x \Rightarrow y = \sqrt{x} = f(x)$, and $y = x^2 = g(x)$, where $f(x) \geq g(x)$ in $[0, 1]$.

$$\begin{aligned} \text{Area, } A &= \int_0^1 [f(x) - g(x)] dx \\ &= \int_0^1 [\sqrt{x} - x^2] dx \\ &= \left[\frac{2}{3} x^{3/2} - \frac{x^3}{3} \right]_0^1 \\ &= \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ Sq. units.} \end{aligned}$$

The area of the region bounded by the curve $y = f(x)$, x -axis and the lines $x=a$ and $x=b$ ($b > a$) is given by

$$A = \int_a^b y dx \text{ or } \int_a^b f(x) dx.$$

For eg : the area bounded by $y = x^2$, x -axis in I quadrant and the lines $x=2$ and $x=3$ is -

$$A = \int_2^3 y dx = \int_2^3 x^2 dx = \left[\frac{x^3}{3} \right]_2^3 = \frac{1}{3} (27 - 8) = \frac{19}{3} \text{ Sq. units.}$$

if $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b], a < c < b$, then the area is

$$A = \int_a^c [f(x) - g(x)] dx + \int_c^b [g(x) - f(x)] dx$$

The area of the region bounded by the curve $x = f(y)$, y -axis and the lines $y=c$ and $y=d$ ($d > c$)

is given by $A = \int_c^d x dy$ or $\int_c^d f(y) dy$.

For eg : the area bounded by $x = y^3$, y -axis in the I quadrant and the lines $y=1$ and $y=2$ is

$$A = \int_1^2 x dy = \int_1^2 y^3 dy = \left[\frac{1}{4} y^4 \right]_1^2 = \frac{1}{4} (2^4 - 1^4) = \frac{15}{4} \text{ Sq. units}$$