EXERCISE. 22 (A)

Question 1:

Find the volume of a cone whose slant height is 17 cm and radius of base is 8 cm.

Solution 1:

Slant height (ℓ) = 17 cm

Radius (r) = 8 cm

But,

$l^2 = r^2 + h^2$

$\Rightarrow h^2 = l^2 - r^2$

$\Rightarrow h^2 = 17^2 - 8^2$

$\Rightarrow h^2 = 289 - 64 = 225 = (15)^2$

$\therefore h = 15$

Now, volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 15 \text{ cm}^3$

$= \frac{7040}{7} \text{ cm}^3$

$= 1005.71 \text{ cm}^3$

Question 2:

The curved surface area of a cone is 12320 cm^2. If the radius of its base is 56 cm, find its height.

Solution 2:

Curved surface area = 12320 cm^2

Radius of base (r) = 56 cm

Let slant height = ℓ

$\therefore \pi r \ell = 12320$

$\Rightarrow \frac{22}{7} \times 56 \times \ell = 12320$

$\Rightarrow \ell = \frac{12320 \times 7}{56 \times 22}$

$\Rightarrow \ell = 70$ cm

Height of the cone =
$= \sqrt{\ell^2 - r^2}$
$= \sqrt{(70)^2 - (56)^2}$
$= \sqrt{4900 - 3136}$
$= \sqrt{1764}$
= 42 cm

Question 3:

The circumference of the base of a 12 m high conical tent is 66 m. Find the volume of the air contained in it.

Solution 3:

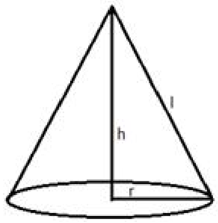
Circumference of the conical tent = 66 m
and height (h) = 12 m
$\therefore$ Radius $= \frac{c}{2\pi} = \frac{66 \times 7}{2 \times 22} = 10.5 \text{ m}$
Therefore, volume of air contained in it = $\frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 12 \text{ m}^3$
$= 1386 \text{ m}^3$

Question 4:

The radius and the height of a right circular cone are in the ratio 5 : 12 and its volume is 2512 cubic cm. Find the radius and height of the cone (Take π = 3.14)

Solution 4:

The ratio between radius and height = 5:12
Volume = 5212 cubic cm
Let radius (r) = 5x, height (h) = 12x and slant height = ℓ



$\ell^2 = r^2 + h^2$

$\Rightarrow \ell^2 = (5x)^2 + (12x)^2$

$\Rightarrow \ell^2 = 25x^2 + 144x^2$

$\Rightarrow \ell^2 = 169\ x^2$

$\Rightarrow \ell = 13x$

Now volume = $\frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3}\pi r^2 h = 2512$

$\Rightarrow \frac{1}{3}(3.14)(5x)^2(12x) = 2512$

$\Rightarrow \frac{1}{3}(3.14)(300x^3) = 2512$

$\therefore\ x^3 = \frac{2512 \times 3}{3.14 \times 300} = \frac{2512 \times 3 \times 100}{314 \times 300} = 8$

$\Rightarrow x = 2$

$\therefore$ Radius = 5x = 5 × 2 = 10 cm

Height = 12x = 12 × 2 = 24 cm

Slant height = 13x = 13 × 2 = 26 cm

Question 5:

Two right circular cone x and y are made x having three times the radius of y and y having half the volume of x. Calculate the ratio between the heights of x and y.

Solution 5:

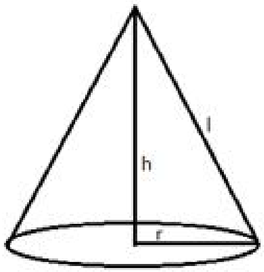
Let radius of cone y = r

Therefore, radius of cone x = 3r

Let volume of cone y = V

then volume of cone x = 2V

Let h_1 be the height of x and h2 be the height of y.



Therefore, Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of cone x = $\frac{1}{3}\pi(3r)^2 h_1 = \frac{1}{3}\pi 9r^2 h_1 = 3\pi r^2 h_1$

Volume of cone y = $\frac{1}{3}\pi r^2 h_2$

$\therefore \frac{2V}{v} = \frac{3\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2}$

$\Rightarrow \frac{2}{1} = \frac{3h_1 \times 3}{h_2} = \frac{9h_1}{h_2}$

$\Rightarrow \frac{h_1}{h_2} = \frac{2}{1} \times \frac{1}{9} = \frac{2}{9}$

$\therefore h_1 : h_2 = 2 : 9$

Question 6:

The diameter of two cones are equal. If their slant heights are in the ratio 5 : 4, find the ratio of their curved surface areas.

Solution 6:

Let radius of each cone = r

Ratio between their slant heights = 5:4

Let slant height of the first cone = 5x

and slant height of second cone = 4x

Therefore, curved surface area of the first cone =

$\pi r\ell = \pi r \times (5x) = 5\pi rx$

curved surface area of the second cone = $\pi r\ell = \pi r \times (4x) = 4\pi rx$

Hence, ratio between them = $5\pi rx : 4\pi rx = 5 : 4$

Question 7:
There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former find the ratio of their radii.

Solution 7:
Let slant height of the first cone = ℓ
then slant height of the second cone = 2ℓ
Radius of the first cone = r_1
Radius of the second cone = r_2
Then, curved surface area of first cone = $\pi r_1 \ell$
curved surface area of second cone = $\pi r_2 (2\ell) = 2\pi r_2 \ell$
According to given condition:

$\pi r_1 l = 2(2\pi r_2 l)$

$\pi \; \pi r_1 l = 4\pi r_2 l$

$r_1 = 4r_2$

$\frac{r_1}{r_2} = \frac{4}{1}$

$\therefore r_1 : r_2 = 4 : 1$

Question 8:
A heap of wheat is in the form of a cone of diameter 16.8 m and height 3.5 m. Find its volume. How much cloth is required to just cover the heap?

Solution 8:
Diameter of the cone = 16.8 m
Therefore, radius (r) = 8.4 m
Height (h) = 3.5 m

(i) Volume of heap of wheat = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 8.4 \times 8.4 \times 3.5$

= 258.72 m^3

(ii) Slant height (ℓ) = $\sqrt{r^2 + h^2}$

$= \sqrt{(8.4)^2 + (3.5)^2}$

$= \sqrt{70.56 + 12.25}$

$= \sqrt{82.81}$

$= 9.1\text{m}$

Therefore, cloth required or curved surface area = $\pi r \ell$

$= \frac{22}{7} \times 8.4 \times 9.1$

$= 240.24 \text{ m}^2$

Question 9:

Find what length of canvas, 1.5 m in width, is required to make a conical tent 48 m in diameter and 7 m in height. Given that 10% of the canvas is used in folds and stitching's. Also. Find the cost of the canvas at the rate of Rs. 24 per metre.

Solution 9:

Diameter of the tent = 48 m

Therefore, radius (r) = 24 m

Height (h) = 7 m

Slant height (ℓ) = $\sqrt{r^2 + h^2}$

$= \sqrt{(24)^2 + (7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

$= 25 \text{ m}$

Curved surface area = $\pi r \ell$

$= \frac{22}{7} \times 24 \times 25$

$= \frac{13200}{7} \text{ m}^2$

Canvas required for stitching and folding

$= \frac{13200}{7} \times \frac{10}{100}$

$= \frac{1320}{7} \text{ m}^2$

Total canvas required (area)

$= \frac{13200}{7} + \frac{1320}{7}$

$= \frac{14520}{7}\ m^2$

Length of canvas

$= \frac{\frac{14520}{7}}{\frac{3}{2}}$

$= \frac{14520}{7} \times \frac{2}{3}$

$= \frac{9680}{7}$

$= 1382.86\ m$

Rate = Rs 24 per meter

Total cost $= \frac{9680}{7} \times$ Rs 24 = Rs 33,188.64

Question 10:

A solid cone of height 8 cm and base radius 6 cm is melted and recast into identical cones each of height 2 cm and diameter 1 cm. Find the number of cones formed.

Solution 10:

Height of solid cone (h) = 8 cm

Radius (r) = 6 cm

Volume of solid cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8$

$= 96\pi\ cm^3$

Height of smaller cone = 2 cm

and radius = $\frac{1}{2}$ cm

Volume of smaller cone

$= \frac{1}{3} \times \pi \times \frac{1}{2} \times \frac{1}{2} \times 2$

$= \frac{1}{6}\pi cm^3$

Number of cones so formed

$= \frac{96\pi}{\frac{1}{6}\pi}$

$= 96\pi \times \frac{6}{\pi}$

$= 576$

Question 11:

The total surface area of a right circular cone of slant height 13 cm is 90π cm^2. Calculate:

(i) its radius in cm

(ii) its volume in cm^3 [Take π = 3.14]

Solution 11:

Total surface area of cone =90π cm^2

slant height (l) = 13 cm

(i) Let r be its radius, then

Total surface area = $\pi r\ell + \pi r^2 = \pi r(\ell + r)$

$\therefore \pi r(\ell + r) = 90\pi$

$\Rightarrow r(13 + r) = 90$

$\Rightarrow r^2 + 13r - 90 = 0$

$\Rightarrow r^2 + 18r - 5r - 90 = 0$

$\Rightarrow r(r + 18) - 5(r + 18) = 0$

$\Rightarrow (r + 18)(r - 5) = 0$

Either r+18 = 0, then r = -18 which is not possible

or r – 5 = 0, then r = 5

Therefore, radius = 5 cm

(ii) Now

$h = \sqrt{\ell^2 - r^2}$

$= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

h = 12 cm

Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 5 \times 5 \times 12$

$= 314 \text{ cm}^3$

Question 12:

The area of the base of a conical solid is 38.5 cm^2 and its volume is 154 cm^3. Find curved surface area of the solid.

Solution 12:

Area of the base, $\pi r^2 = 38.5\ cm^2$

Volume of the solid, v = 154 cm^3

Curved surface area of the solid = $\pi r^2 h$

Volume, v = $\frac{1}{3}\pi r^2 h$

$\Rightarrow 154 = \frac{1}{3}\pi r^2 h$

$\Rightarrow h = \frac{154 \times 3}{\pi r^2}$

$\Rightarrow h = \frac{154 \times 3}{38.5} = 12\,cm$

Area = 38.5

$\pi r^2 = 38.5$

$\Rightarrow r^2 = \frac{38.5}{3.14}$

$\Rightarrow r = \sqrt{\frac{38.5}{3.14}} = 3.5$

Curved surface area of solid = πrl

$= \pi r\sqrt{r^2 + h^2}$

$= \pi \times 3.5 \times \sqrt{3.5^2 + 12^2}$

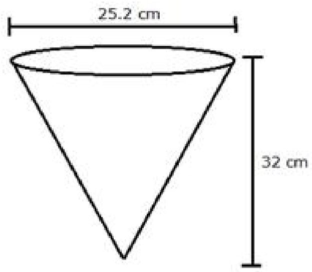
$= \pi \times 3.5 \times 12.5$

$= 137.44\ cm^2$

Question 13:

A vessel, in the form of an inverted cone, is filled with water to brim. Its height is 32 cm and diameter of the base is 25.2 cm. Six equal solid cones are dropped in it, so that they are fully submerged. As a result one fourth of water is volume of each of the solid cone submerged?

Solution 13:



Volume of vessel = volume of water = $\frac{1}{3}\pi r^2 h$

diameter = 25.2 cm, therefore radius = 12.6 cm

height = 32 cm

Volume of water in the vessel = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 32$

$= 5322.24 \text{ cm}^3$

On submerging six equal solid cones into it, one-fourth of the water overflows.

Therefore, volume of the equal solid cones submerged

= Volume of water that overflows

$= \frac{1}{4} \times 5322.24$

$= 1330.56 \text{ cm}^3$

Now, volume of each cone submerged

$= \frac{1330.56}{6} = 221.76 \text{ cm}^3$

Question 14:

The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the:

(i) radius of the floor.

(ii) height of the tent

(iii) length of the canvas required to cover this conical tent if its width is 2 m.

Solution 14:

(i) Let r be the radius of the base of the conical tent, then area of the base floor = $\pi r^2 m^2$

$\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{22} = 49$

$\Rightarrow r = 7$

Hence, radius of the base of the conical tent i.e. the floor = 7m

(ii) Let h be the height of the conical tent, then the volume =

$\frac{1}{3}\pi r^2 hm^3$

$\therefore \frac{1}{3}\pi r^2 h = 1232$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h = 1232$

$\Rightarrow h = \frac{1232 \times 3}{22 \times 7} = 24$

Hence, radius of the base of the conical tent i.e. the floor = 7 m

(iii) Let l be the slant height of the conical tent, then $= \ell = \sqrt{h^2 + r^2}$ m

$\therefore \ell = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25$ m

The area of the canvas required to make the tent = $\pi r \ell m^2$

$\therefore \pi r \ell = \frac{22}{7} \times 7 \times 25 \text{ m}^2 = 550 \text{ m}^2$

Length of the canvas required to cover the conical tent of its width 2 m = $\frac{550}{2} = 275$ m

EXERCISE. 22 (B)

Question 1:

The surface area of a sphere is 2464 cm^2, find its volume.

Solution 1:

Surface area of the sphere = 2464 cm^2

Let radius = r, then

$4\pi r^2 = 2464$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 2464$

$\Rightarrow r^2 = \frac{2462 \times 7}{4 \times 22} = 196$

$\Rightarrow r = 14 \text{cm}$

Volume = $\frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 = 11498.67 \text{ cm}^3$

Question 2:

The volume of a sphere is 38808 cm^3; find its diameter and the surface area.

Solution 2:

Volume of the sphere = 38808 cm^3

Let radius of sphere = r

$\therefore \frac{4}{3}\pi r^3 = 38808$

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 38808$

$\Rightarrow r^3 = \frac{38808 \times 7 \times 3}{4 \times 22} = 9261$

$\Rightarrow r = 21 \text{ cm}$

$\therefore$ diameter = 2r = 21 × 2 cm = 42 cm

Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^3 = 5544 \text{ cm}^3$

Question 3:

A spherical ball of lead has been melted and made into identical smaller balls with radius equal to half the radius of the original one. How many such balls can be made?

Solution 3:

Let the radius of spherical ball = r

$\therefore$ Volume = $\frac{4}{3}\pi r^3$

Radius of smaller ball = $\frac{r}{2}$

$\therefore$ Volume of smaller ball = $\frac{4}{3}\pi \left(\frac{r}{2}\right)^3 = \frac{4}{3}\pi \frac{r^3}{8} = \frac{\pi r^3}{6}$

Therefore, number of smaller balls made out of the given ball =

$$\frac{\frac{4}{3}\pi r^3}{\frac{\pi r^3}{6}} = \frac{4}{3} \times 6 = 8$$

Question 4:

How many balls each of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm.

Solution 4:

Diameter of bigger ball = 8 cm

Therefore, Radius of bigger ball = 4 cm

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 4 \times 4 \times 4 = \frac{265\pi}{3}$ cm^3

Radius of small ball = 1 cm

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 1 \times 1 \times 1 = \frac{4\pi}{3}$ cm^3

Number of balls = $\frac{\frac{256\pi}{3}}{\frac{4\pi}{3}} = \frac{256\pi}{3} \times \frac{3}{4\pi} = 64$

Question 5:

Eight metallic spheres; each of radius 2 mm, are melted and cast into a single sphere. Calculate the radius of the new sphere.

Solution 5:

Radius of metallic sphere = 2 mm $= \frac{1}{5}$ cm

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{88}{21 \times 125}$ cm^3

Volume of 8 spheres = $\frac{88 \times 8}{21 \times 125} = \frac{704}{21 \times 125}$ cm^3 …… (i)

Let radius of new sphere = R

$\therefore$ Volume $= \frac{4}{3}\pi R^3 = \frac{4}{3} \times \frac{22}{7} R^3 = \frac{88}{21} R^3$ ……….. (ii)

From (i) and (ii)

$$\frac{88}{21}R^3 = \frac{704}{21\times125}$$

$$\Rightarrow R^3 = \frac{704}{21\times125}\times\frac{21}{88} = \frac{8}{125}$$

$$\Rightarrow R = \frac{2}{5} = 0.4 \text{ cm} = 4 \text{ mm}$$

Question 6:

The volume of one sphere is 27 times that of another sphere. Calculate the ratio of their:

(i) radii (ii) surface areas.

Solution 6:

Volume of first sphere = 27 × volume of second sphere

Let radius of first sphere = r_1

and radius of second sphere = r_2

Therefore, volume of first sphere = $\frac{4}{3}\pi r_1^3$

and volume of second sphere = $\frac{4}{3}\pi r_2^3$

(i) Now, according to the question

$$= \frac{4}{3}\pi r_1^3 = 27\times\frac{4}{3}\pi r_2^3$$

$$r_1^3 = 27r_2^3 = (3r_2)^3$$

$$\Rightarrow r_1 = 3r_2$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{3}{1}$$

$$\therefore\ r_1 : r_2 = 3:1$$

(ii) Surface area of first sphere = $4\pi r_1^2$

and surface area of second sphere = $4\pi r_2^2$

Ratio in surface area = $\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{3^2}{1^2} = \frac{9}{1} = 9:1$

Question 7:

If the number of square centimeters on the surface of a sphere is equal to the number of cubic centimeters in its volume, what is the diameter of the sphere?

Solution 7:

Let r be the radius of the sphere.

Surface area = $4\pi r^2$ and volume = $\frac{4}{3}\pi r^3$

According to the condition:

$$4\pi r^2 = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{r^3}{r^2} = 4\pi \times \frac{3}{4\pi}$$

$$\Rightarrow r = 3 \text{ cm}$$

Diameter of sphere = 2 × 3 cm = 6 cm

Question 8:

A solid metal sphere is cut through its center into 2 equal parts. If the diameter of the sphere is $3\frac{1}{2}$ cm, find the total surface area of each part correct to two decimal places.

Solution 8:

Diameter of sphere = $3\frac{1}{2}$ cm = $\frac{7}{2}$ cm

Therefore, radius of sphere = $\frac{7}{4}$ cm

Total curved surface area of each hemispheres =

$$2\pi r^2 + \pi r^2 = 3\pi r^2$$

$$= 3 \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}$$

$$= 28.88 \text{ cm}^2$$

Question 9:

The internal and external diameter of a hollow hemispherical vessel are 21 cm and 28 cm respectively Find:

(i) internal curved surface area

(ii) external curved surface area

(iii) total surface area

(iv) volume of material of the vessel

Solution 9:

External radius (R) = 14 cm

Internal radius (r) = $\frac{21}{2}$ cm

(i) Internal curved surface area =

$2\pi r^2$

$= 2\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$

$= 693\ cm^3$

(ii) External curved surface area =

$2\pi R^2$

$= 2\times\frac{22}{7}\times 14\times 14$

$= 1232\ cm^2$

(iii) Total surface area =

$2\pi R^2 + 2\pi r^2 + \pi\left(R^2 - r^2\right)$

$= 693 + 1232 + \frac{22}{7}\left((14)^2 - \left(\frac{21}{2}\right)^2\right)$

$= 1925 + \frac{22}{7}\left(196 - \frac{441}{4}\right)$

$= 1925 + \frac{22}{7}\times\frac{343}{4}$

$= 1925 + 269.5$

$= 2194.5\ cm^3$

(iv) Volume of material used =

$\frac{2}{3}\pi\left(R^3 - r^3\right)$

$= \frac{2}{3}\times\frac{22}{7}\left((14)^3 - \left(\frac{21}{2}\right)^3\right)$

$= \frac{44}{21}(2744 - 1157.625)$

$= \frac{44}{21}\times 1586.375$

$= 3323.83\ cm^3$

Question 10:

A solid sphere and a solid hemi-sphere have the same total surface area. Find the ratio between their volumes.

Solution 10:

Let the radius of the sphere be 'r_1'.
Let the radius of the hemisphere be 'r_2'
TSA of sphere = $4\pi r_1^2$
TSA of hemisphere = $3\pi r_2^2$
TSA of sphere = TSA of hemi-sphere

$$4\pi r_1^2 = 3\pi r_2^2$$

$$\Rightarrow r_2^2 = \frac{4}{3} r_1^2$$

$$\Rightarrow r_2 = \frac{2}{\sqrt{3}} r_1$$

Volume of sphere, $v_1 = \frac{4}{3}\pi r_1^3$

Volume of hemisphere, $V_2 = \frac{2}{3}\pi r_2^3$

$$v_2 = \frac{2}{3}\pi r_2^3$$

$$\Rightarrow v_2 = \frac{2}{3}\pi\left(\frac{r_1 2}{\sqrt{3}}\right)^3$$

$$\Rightarrow v_2 = \frac{2}{3}\pi\frac{r_2^3 8}{3\sqrt{3}}$$

Dividing v_1 by v_2

$$\frac{v_1}{v_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{2}{3}\pi\frac{8}{3\sqrt{3}} r_1^3}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\frac{4}{3}}{\frac{2}{3}\frac{8}{3\sqrt{3}}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{4}{3} \times \frac{9\sqrt{3}}{16}$$

$\Rightarrow \frac{v_1}{v_2} = \frac{3\sqrt{3}}{4}$

Question 11:

Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted and re casted into a single solid sphere. Taking π = 3.1, find the surface area of solid sphere formed.

Solution 11:

Let radius of the larger sphere be 'R'

Volume of single sphere

= Vol. of sphere 1 + Vol. of sphere 2 + Vol. of sphere 3

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r6^3 + \frac{4}{3}\pi r8^3 + \frac{4}{3}\pi 10^3$

$\Rightarrow R^3 = \left[6^3 + 8^3 + 10^3\right]$

$\Rightarrow R^3 = 1728$

$\Rightarrow R = 12$

Surface area of the sphere

$= 4\pi R^2$

$= 4\pi 12^2$

$= 1785.6 \text{ cm}^2$

Question 12:

The surface area of a solid sphere is increased by 12% without changing its shape. Find the percentage increase in its:

(i) radius (ii) volume

Solution 12:

Let the radius of the sphere be 'r'.

Total surface area the sphere, S = $4\pi r^2$

New surface area of the sphere, S’

$= 4\pi r^2 + \frac{21}{100} \times 4\pi r^2$

$= \frac{121}{100} 4\pi r^2$

(i) let the new radius be r_1

$S^I = 4p\, r_1^2$

$S^I = \frac{121}{100} 4\pi r^2$

$\Rightarrow 4\pi r_1^2 = \frac{121}{100} 4\pi r^2$

$\Rightarrow r_1^2 = \frac{121}{100} r^2$

$\Rightarrow r_1 = \frac{11}{10} r$

$\Rightarrow r_1 = r + \frac{r}{10}$

$\Rightarrow r_1 - r = \frac{r}{10}$

$\Rightarrow$ change in radius = $\frac{r}{10}$

Percentage change in radius = $\frac{change\ in\ radius}{original\ radius} \times 100$

$= \frac{r/10}{r} \times 100$

= 10

Percentage change in radius = 10%

(ii) Let the volume of the sphere be V

Let the new volume of the sphere be V'.

$v = \frac{4}{3}\pi r^3$

$v^I = \frac{4}{3}\pi r_1^3$

$\Rightarrow v^I = \frac{4}{3}\pi \left(\frac{11r}{10}\right)^3$

$\Rightarrow v^I = \frac{4}{3}\pi \frac{1331}{1000} r^3$

$\Rightarrow v^I = \frac{4}{3}\pi r^3 \frac{1331}{1000}$

$\Rightarrow v^I = \frac{1331}{1000} v$

$\Rightarrow v^{I} = v + \frac{1331}{1000}v$

$\Rightarrow v^{I} - v = \frac{331}{1000}v$

$\therefore$ Change in volume = $\frac{331}{1000}v$

Percentage change in volume = $\frac{change\ in\ volume}{original\ volume} \times 100$

$= \frac{\frac{331}{1000}v}{v} \times 100$

$= \frac{331}{10}$

$= 33.1$

Percentage change in volume = 33.1 %

EXERCISE. 22 (C)

Question 1:

A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast.

Solution 1:

Radius of sphere = 8 cm

$\therefore$ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 8 \times 8 \times 8 = \frac{45056}{21}$ cm^3 …….(i)

Height of the cone = 32 cm

Let radius = r

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32 = \frac{704}{21} r^2 cm^3$ ……...(ii)

From (i) and (ii)

$\frac{704}{21} r^2 = \frac{45056}{21}$

$\Rightarrow r^2 = \frac{45056}{21} \times \frac{21}{704}$

$\Rightarrow r^2 = 64$

$\Rightarrow r = 8$ cm

Question 2:

A hollow sphere of internal and external diameter 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.

Solution 2:

External diameter = 8 cm

Therefore, radius (R) = 4 cm

Internal diameter = 4 cm

Therefore, radius (r) = 2 cm

Volume of metal used in hollow sphere =

$$\frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3}\times\frac{22}{7}\times(4^3 - 2^3) = \frac{88}{21}(64-8) = \frac{88}{21}\times 56 \text{ cm}^3 \text{ ...... (i)}$$

Diameter of cone = 8 cm

Therefore, radius = 4 cm

Let height of cone = h

$$\therefore \text{Volume} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 4\times 4\times h = \frac{352}{21}h \text{ ..... (ii)}$$

From (i) and (ii)

$$\frac{352}{21}h = \frac{88}{21}\times 56$$

$$\Rightarrow h = \frac{88\times 56\times 21}{21\times 352} = 14 \text{ cm}$$

Height of the cone = 14 cm.

Question 3:

The radii of the internal and external surface of a metallic spherical. It is melted and recast into a solid right circular cone of height 32 cm. Find the diameter of the base of the cone.

Solution 3:

Internal radius = 3cm

External radius = 5 cm

Volume of spherical shell

$$= \frac{4}{3}\pi\left(5^3 - 3^3\right)$$

$$= \frac{4}{3}\times\frac{22}{7}(125-27)$$

$$= \frac{4}{3}\times\frac{22}{7}\times 98$$

Volume of solid circular cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32$

Vol. of Cone = Vol. of sphere

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32 = \frac{4}{3} \times \frac{22}{7} \times 98$

$\Rightarrow r^2 = \frac{4 \times 98}{32}$

$\therefore\ r = \frac{7}{2} = 3.5\ \text{cm}$

Question 4:

Total volume of three identical cones is the same as that of a bigger cone whose height is 9 cm and diameter 40 cm. Find the radius of the base of each smaller cone, if height of each is 108 cm.

Solution 4:

Let the radius of the smaller cone be 'r' cm.

Volume of larger cone

$= \frac{1}{3}\pi \times 20^2 \times 9$

Volume of smaller cone

$= \frac{1}{3}\pi \times r^2 \times 108$

Volume of larger cone = 3 × Volume of smaller cone

$\frac{1}{3}\pi \times 20^2 \times 9 = \frac{1}{3}\pi \times r^2 \times 108 \times 3$

$\Rightarrow r^2 = \frac{20^2 \times 9}{108 \times 3}$

$\Rightarrow r = \frac{20}{6} = \frac{10}{3}$

Question 5:

A solid rectangular block of metal 49 cm by 44 cm by 18 cm is melted and formed into a solid sphere. Calculate the radius of the sphere.

Solution 5:

Volume of rectangular block = 49 × 44 × 18 cm^3 = 38808 cm^3 ……… (i)

Let r be the radius of sphere

$\therefore$ Volume = $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times r^3 = \frac{88}{21}r^3$ …… (ii)

From (i) and (ii)

$$\frac{88}{21}r^3 = 38808$$

$$\Rightarrow r^3 = 38808\times\frac{21}{88} = 441\times 21$$

$$\Rightarrow r^3 = 9261$$

$$\Rightarrow r = 21 \text{ cm}$$

Radius of sphere = 21 cm

Question 6:

A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into conical shaped small container each of diameter 3 cm and height 4 cm. How many container are necessary to empty the bowl?

Solution 6:

Radius of hemispherical bowl = 9 cm

Volume $= \frac{1}{2}\times\frac{4}{3}\pi r^3 = \frac{2}{3}\pi 9^3 \times \frac{2}{3}\pi\times 729 = 486\pi \text{ cm}^3$

Diameter each of cylindrical bottle = 3 cm

Radius = $\frac{3}{2}$ cm, and height = 4 cm

$\therefore$ volume of bottle = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\times\left(\frac{3}{2}\right)^2\times 4 = 3\pi$

$\therefore$ No of bottles = $\frac{486\pi}{3\pi} = 162$

Question 7:

A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into a inverted cone of radius 4.8 cm. Find the height of the cone is it is completely filled.

Solution 7:

Diameter of the hemispherical bowl = 7.2 cm

Therefore, radius = 3.6 cm

Volume of sauce in hemispherical bowl = $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (3.6)^3$

Radius of the cone = 4.8 cm

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4.8)^2 \times h$

Now, volume of sauce in hemispherical bowl = volume of cone

$\Rightarrow \frac{2}{3}\pi \times (3.6)^3 = \frac{1}{3}\pi \times (4.8)^2 \times h$

$\Rightarrow h = \frac{2 \times 3.6 \times 3.6 \times 3.6}{4.8 \times 4.8}$

$\Rightarrow h = 4.05$ cm

Height of the cone = 4.05 cm

Question 8:

A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of sphere formed.

Solution 8:

Radius of a solid cone (r) = 5 cm

Height of the cone = 8 cm

$\Rightarrow$ Volume of a cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 5 \times 5 \times 8 \text{ cm}^3$

$= \frac{200\pi}{3} \text{cm}^3$

Radius of each sphere = 0.5 cm

$\therefore$ Volume of one sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \Pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \text{cm}^3$

$= \frac{\Pi}{6} \text{ cm}^3$

Number of spheres = $\frac{Total\ volume}{Volume\ of\ one\ sphere}$

$= \frac{\frac{200\pi}{3}}{\frac{\pi}{6}} \times \frac{6}{\pi}$

$= \frac{200\pi}{3} \times \frac{6}{\pi}$

$= 400$

Question 9:

The total area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate:

(i) the radius of the solid sphere

(ii) the number of cones recast

Solution 9:

Total area of solid metallic sphere = 1256 cm^2

(i)Let radius of the sphere is r then

$4\pi r^2 = 1256$

$4 \times \frac{22}{7} r^2 = 1256$

$\Rightarrow r^2 = \frac{1256 \times 7}{4 \times 22}$

$\Rightarrow r^2 = \frac{157 \times 7}{11}$

$\Rightarrow r^2 = \frac{1099}{11}$

$\Rightarrow r = \sqrt{99.909} = 9.995 \text{ cm}$

$\Rightarrow r = 10 \text{ cm}$

(ii) Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 = \frac{88000}{21} \text{ cm}^3$

volume of right circular cone =

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times 8 = \frac{1100}{21} \text{ cm}^3$$

Number of cones

$$= \frac{88000}{21} \div \frac{1100}{21}$$

$$= \frac{88000}{21} \times \frac{21}{1100}$$

$$= 80$$

Question 10:

A solid metallic cone, with radius 6 cm and height 10cm, is made of some heavy metal A. In order to reduce its weight, a conical hole is made in the cone as shown and it is completely filled with a lighter metal B. The conical hole has a diameter of cm and depth 4 cm calculate the ratio of the volume of metal A to the volume of the metal B in the solid.

Solution 10:

Volume of the whole cone of metal A

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 6^2 \times 10$$

$$= 120\pi$$

Volume of the cone with metal B

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 3^2 \times 4$$

$$= 12\pi$$

Final Volume of cone with metal A=120π − 12π =108π

$$\frac{\textit{Volume of cone with metal A}}{\textit{Volume of cone with metal B}}$$

$$= \frac{108\pi}{12\pi} = \frac{9}{1}$$

Question 11:

A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and react into small cones of base radius 2 cm and height 8 cm. Find the number of cones.

Solution 11:

Let the number of small cones be 'n'

Volume of sphere

$= \frac{4}{3}\pi\left(8^3 - 6^3\right)$

$= \frac{4}{3} \times \pi \times 2 \times 148$

Volume of small spheres

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3^2 \times 4$

Volume of sphere = n × Volume of small sphere

$\Rightarrow \frac{4}{3} \times \pi \times 2 \times 148 = n \times \frac{1}{3} \times \pi \times 2^2 \times 8$

$\Rightarrow n = \frac{4 \times 2 \times 148 \times 3}{4 \times 8 \times 3}$

$\Rightarrow n = 37$

The number of cones = 37 cm.

EXERCISE. 22 (D)

Question 1:

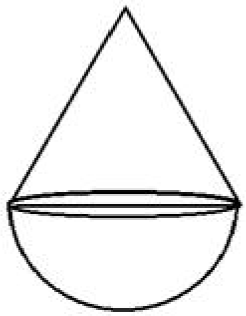
A cone of height 15 cm and diameter 7 cm is mounted on a hemisphere of same diameter. Determine the volume of the solid thus formed.

Solution 1:

Height of cone = 15 cm

and radius of the base = $\frac{7}{2}$ cm

Therefore, volume of the solid = volume of the conical part + volume of hemispherical part.



$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(15 + 2 \times \frac{7}{2}\right)$

$= \frac{847}{3}$

$= 282.33 \text{ cm}^3$

Question 2:

A buoy is made in the form of hemisphere surmounted by a right cone whose circular base coincides with the plane surface of hemisphere. The radius of the base of the cone is 3.5 metres and its volume is two third of the hemisphere. Calculate the height of the cone and the surface area of the buoy, correct to two places of decimal

Solution 2:

Radius of hemispherical part (r) = 3.5 m = $\frac{7}{2}$ m

Therefore, Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{539}{6} \text{ m}^3$

Volume of conical part = $\frac{2}{3} \times \frac{539}{6} \text{ m}^3$ (2/3 of hemisphere)

Let height of the cone = h

Then,

$\frac{1}{3}\pi r^2 h = \frac{2\times 539}{3\times 6}$

$\Rightarrow \frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times h = \frac{2\times 539}{3\times 6}$

$\Rightarrow h = \frac{2\times 539\times 2\times 2\times 7\times 3}{3\times 6\times 22\times 7\times 7}$

$\Rightarrow h = \frac{14}{3} m = 4\frac{2}{3} m = 4.67\ m$

Height of the cone = 4.67 m

Surface area of buoy = $2\pi r^2 + \pi r\ell$

But $\ell = \sqrt{r^2 + h^2}$

$\ell = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{14}{3}\right)^2}$

$= \sqrt{\frac{49}{4} + \frac{196}{9}} = \sqrt{\frac{1225}{36}} = \frac{35}{6} m$

Therefore, Surface area =

$= \left(2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right) + \left(\frac{22}{7}\times\frac{7}{2}\times\frac{35}{6}\right) m^2$

$= \frac{77}{1} + \frac{385}{6} = \frac{847}{6}$

$= 141.17\ m^2$

Surface Area = 141.17 m^2

Question 3:

From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a conical cavity of diameter 14 cm and depth 24 cm is drilled out. Find:

(i) the surface area of remaining solid,

(ii) the volume of remaining solid

(iii) the weight of the material drilled out if it weighs 7 gm per cm^3.

Solution 3:

(i) Total surface area of cuboid = $2(\ell b + bh + \ell h)$

= 2 (42 × 30 + 30 × 20 + 20 × 42)

= 2 (1260 + 600 + 840)

= 2 × 2700

= 5400 cm^2

Diameter of the cone = 14 cm

$\Rightarrow$ Radius of the cone = $\frac{14}{2} = 7\,\text{cm}$

Area of circular base = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ \text{cm}^2$

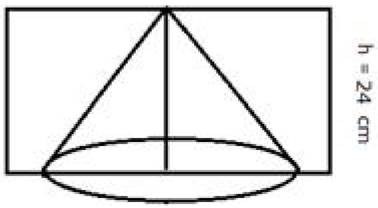
Area of curved surface area of cone =

$\pi r \ell = \frac{22}{7} \times 7 \times \sqrt{7^2 + 24^2} = 22\sqrt{49 + 576} = 22 \times 25 = 550\ \text{cm}^2$

Surface area of remaining part = 5400 + 550 – 154 = 5796 cm^2

(ii) Dimensions of rectangular solids = (42 × 30 × 20) cm

volume = (42 × 30 × 20) = 25200 cm^3



Radius of conical cavity (r) =7 cm

height (h) = 24 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

$= 1232\ \text{cm}^3$

Volume of remaining solid = (25200 - 1232) = 23968 cm^3

(iii) Weight of material drilled out

=1232 × 7 g = 8624g = 8.624 kg

Question 4:

A cubical block of side 7 cm is surmounted by a hemisphere of the largest size. Find the surface area of the resulting solid.

Solution 4:

The diameter of the largest hemisphere that can be placed on a face of a cube of side 7 cm will be 7 cm.

Therefore, radius = $r = \frac{7}{2}$ cm

Its curved surface area = $2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 77\ \text{cm}^2$ ....(i)

Surface area of the top of the resulting solid = Surface area of the top face of the cube − Area of the base of the hemisphere

$= (7 \times 7) - \left(\frac{22}{7} \times \frac{49}{4}\right)$

$= 49 - \frac{77}{2}$

$= \frac{98 - 77}{2}$

$= \frac{21}{2}$

$= 10.5\ \text{cm}^2$ …….. (ii)

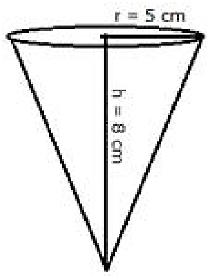
Surface area of the cube = 5 × (side)^2 = 5 × 49 = 245 cm^2 ………. (iii)

Total area of resulting solid = 245 + 10.5 + 77 = 332.5 cm^2

Question 5:

A vessel is in the form of an inverted cone its height is 8 cm and the radius of its top, which is open, is 5 cm. If is filled with water up to the rim. When lead shots each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots sopped in the vessel.

Solution 5:



Height of cone = 8 cm

Radius = 5 cm

Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8\ \text{cm}^3$

$= \frac{4400}{21} \text{cm}^3$

Therefore, volume of water that flowed out =

$= \frac{1}{4} \times \frac{4400}{21} \text{ cm}^3$

$= \frac{1100}{21} \text{ cm}^3$

Radius of each ball = 0.5 cm = $\frac{1}{2}$ cm

Volume of a ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \text{ cm}^3$

$= \frac{11}{21} \text{ cm}^3$

Therefore, No. of balls = $= \frac{1100}{21} \div \frac{11}{21} = 100$

Hence, number of lead balls = 100

Question 6:

A hemi-spherical bowl has negligible thickness and the length of its circumference is 198 cm. Find the capacity of the bowl.

Solution 6:

Let r be the radius of the bowl.

$\therefore\ 2\pi r = 198$

$\Rightarrow r = \frac{198 \times 7}{2 \times 22}$

$\Rightarrow r = 31.5 \text{ cm}$

Capacity of the bowl =

$\frac{2}{3}\pi r^3$

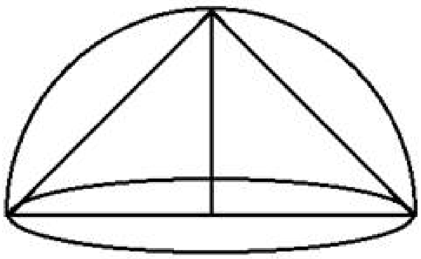
$= \frac{2}{3} \times \frac{22}{7} \times (31.5)^3$

$= 65488.5 \text{ cm}^3$

Question 7:

Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r cm.

Solution 7:



For the volume of cone to be largest, h = r cm

Volume of the cone

$\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times r$

$= \frac{1}{3}\pi r^3$

Question 8:

The radii of the bases of two solid right circular cones of same height are r_1 and r_2 respectively. The cones area melted and recast into a solid sphere of radius R. Find the height of each cone in terms r_1, r_2 and R.

Solution 8:

Let the height of the solid cones be 'h'

Volume of solid circular cones

$v_1 = \frac{1}{3}\pi {r_1}^2 h$

$v_2 = \frac{1}{3}\pi {r_2}^2 h$

Volume of sphere

$= \frac{4}{3}\pi R^3$

Volume of sphere = Volume of cone 1 + volume of cone 2

$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi {r_1}^2 h + \frac{1}{3}\pi {r_2}^2 h$

$\Rightarrow 4R^3 = {r_1}^2 h + {r_2}^2 h$

$$\Rightarrow h\left(r_1^2 + r_2^2\right) = 4R^3$$

$$\Rightarrow h = \frac{R^3}{\left(r_1^2 + r_2^2\right)}$$

Question 9:

A solid metallic hemisphere of diameter 28 cm is melted and recast into a number of identical solid cones, each of diameter 14 cm and height 8 cm. find the number of cones so formed.

Solution 9:

Volume of the solid hemisphere

$$= \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi 14^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

Volume of 1 cone

$$\frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 8$$

No of cones formed

$$= \frac{Volume\, of\ sphere}{volume\, of\ 1 cone}$$

$$= \frac{\frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14}{\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 8}$$

$$= 28$$

Question 10:

A cone and hemisphere have the same base and the same height. Find the ratio between their volumes.

Solution 10:

Let the radius of base be 'r' and the height be 'h'

Volume of cone, V_c

$= \frac{1}{3}\pi r^2 h$

Volume of hemisphere, V_h

$= \frac{2}{3}\pi r^3$

$$\frac{v_c}{v_h} = \frac{\frac{1}{3}\pi r^2 r}{\frac{2}{3}\pi r^3}$$

$$\Rightarrow \frac{v_c}{v_h} = \frac{1}{2}$$

EXERCISE. 22 (E)

Question 1:

From a solid right circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed. Find the volume of the remaining solid.

Solution 1:

Height of the cylinder (h) = 10 cm

and radius of the base (r) = 6 cm

Volume of the cylinder = $\pi r^2 h$

Height of the cone = 10 cm

Radius of the base of cone = 6 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

Volume of the remaining part

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 10$

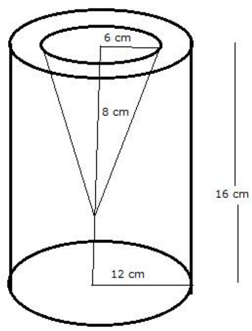
$= \frac{5280}{7}$

$= 754\frac{2}{7}\text{ cm}^3$

Question 2:

From a solid cylinder whose height is 16 cm and radius is 12 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume and total surface area of the remaining solid.

Solution 2:



Radius of solid cylinder (R) = 12 cm
and Height (H) = 16 cm

$\therefore$ Volume = $\pi R^2 H$

$= \frac{22}{7} \times 12 \times 12 \times 16$

$= \frac{50688}{7}\text{ cm}^3$

Radius of cone (r) = 6 cm, and height (h) = 8 cm.

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$

$= \frac{2112}{7}\text{ cm}^3$

(i) Volume of remaining solid

$= \frac{50688}{7} - \frac{2112}{7}$

$= \frac{48567}{7}$

$= 6939.43^{2}\text{ cm}^3$

(ii) Slant height of cone $\ell = \sqrt{h^2 + r^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$ cm

Therefore, total surface area of remaining solid = curved surface area of cylinder + curved surface area of cone + base area of cylinder + area of circular ring on upper side of cylinder

$= 2\pi Rh + \pi r\ell + \pi R^2 + \pi\left(R^2 - r^2\right)$

$= \left(2 \times \frac{22}{7} \times 12 \times 16\right) + \left(\frac{22}{7} \times 6 \times 10\right) + \left(\frac{22}{7} \times 12 \times 12\right) + \left(\frac{22}{7}\left(12^2 - 6^2\right)\right)$

$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{22}{7}(144 - 36)$

$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{2376}{7}$

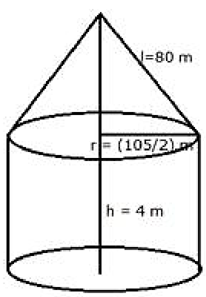
$= \frac{15312}{7}$

$= 2187.43 \text{ cm}^3$

Question 3:

A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 80 m, Calculate the total area of canvas required. Also, find the total cost of canvas used at Rs. 15 per metre if the width is 1.5 m

Solution 3:



Radius of the cylindrical part of the tent (r) = $\frac{105}{2}$ m

Slant height (ℓ) = 80 m

Therefore, total curved surface area of the tent = $2\pi rh + \pi r\ell$

$= \left(2 \times \frac{22}{7} \times \frac{105}{2} \times 4\right) + \left(\frac{22}{7} \times \frac{105}{2} \times 80\right)$

= 1320 + 13200

= 14520 m^2

Width of canvas used = 1.5 m

Length of canvas = $\frac{14520}{1.5} = 9680$ m

Total cost of canvas at the rate of Rs 15 per meter

= 9680 × 15 = Rs. 145200

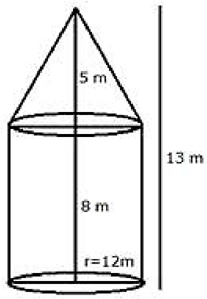
Question 4:

A circus tent is cylindrical to a height of 8m surmounted by a conical part. If total height of the tent is 13m and the diameter of its base is 24m; calculate:

(i) total surface area of the tent,

(ii) area of canvas, required to make this tent allowing 10% of the canvas used for folds and stitching.

Solution 4:



Height of the cylindrical part = H = 8 m

Height of the conical part = h = (13- 8)m = 5 m

Diameter = 24 m ⟶ radius = r = 12 m

Slant height of the cone = *l*

$\ell = \sqrt{r^2 + h^2}$

$\ell = \sqrt{12^2 + 5^2}$

$\ell = \sqrt{169}$ = 13 m

Slant height of cone = 13 m

(i) Total surface are of the tent = 2πrh + πr ℓ = πr(2h + ℓ)

$= \frac{22}{7} \times 12 \times (2 \times 8 + 13)$

$= \frac{264}{7}(16+13)$

$= \frac{264}{7} \times 29$

$= \frac{7656}{7} m^2$

$= 1093.71\ m^2$

(ii) Area of canvas used in stitching = total area

Total area of canvas = $\frac{7656}{7} + \frac{\text{Total area of canvas}}{10}$

$\Rightarrow$ Total area of canvas $- \frac{\text{Total area of canvas}}{10} = \frac{7656}{7}$

$\Rightarrow$ Total area of canvas $\left(1 - \frac{1}{10}\right) = \frac{7656}{7}$

$\Rightarrow$ Total area of canvas $\times \frac{9}{10} = \frac{7656}{7}$

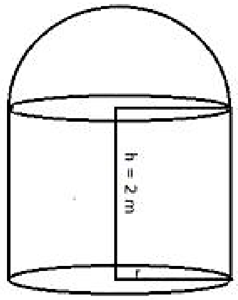
$\Rightarrow$ Total area of canvas $= \frac{7656}{7} \times \frac{10}{9}$

$\Rightarrow$ Total area of canvas $= \frac{76560}{63} = 1215.23\ m^2$

Question 5:

A cylindrical boiler, 2 m high, is 3.5 m in diameter. It has a hemispherical lid. Find the volume of its interior, including the part covered by the lid.

Solution 5:



Diameter of cylindrical boiler = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2} = \frac{35}{20} = \frac{7}{4}$ m

Height (h) = 2 m

Radius of hemisphere (R) = $\frac{7}{4}$ m

Total volume of the boiler = $\pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2\left(h + \frac{2}{3}r\right)$

$= \frac{22}{7}\times\frac{7}{4}\times\frac{7}{4}\left(2 + \frac{2}{3}\times\frac{7}{4}\right)$

$= \frac{77}{8}\left(2 + \frac{7}{6}\right)$

$= \frac{77}{8}\times\frac{19}{6}$

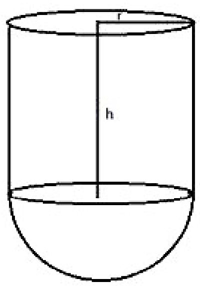
$= \frac{1463}{48}$

$= 30.48 \text{ m}^3$

Question 6:

A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylindrical part is $4\frac{2}{3}$ m and the diameter of hemisphere is 3.5 m. Calculate the capacity and the internal surface area of the vessel.

Solution 6:



Diameter of the base = 3.5 m

Therefore, radius = $\frac{3.5}{2}\text{m} = 1.75\text{m} = \frac{7}{4}\text{m}$

Height of cylindrical part = $4\frac{2}{3} = \frac{14}{3}$ m

(i) Capacity (volume) of the vessel = $\pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{2}{3}r\right)$

$=\frac{22}{7}\times\frac{7}{4}\times\frac{7}{4}\left(\frac{14}{3}+\frac{2}{3}\times\frac{7}{4}\right)$

$=\frac{77}{8}\left(\frac{14}{3}+\frac{7}{6}\right)$

$=\frac{77}{8}\left(\frac{28+7}{6}\right)$

$=\frac{77}{8}\times\frac{35}{6}$

$=\frac{2695}{48}$

$=56.15\ \text{m}^3$

(ii) Internal curved surface area = $2\pi rh+2\pi r^2=2\pi r(h+r)$

$=2\times\frac{22}{7}\times\frac{7}{4}\left(\frac{14}{3}+\frac{7}{4}\right)$

$=11\left(\frac{56+21}{12}\right)$

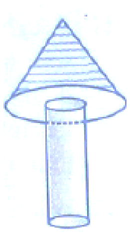
$=11\times\frac{77}{12}$

$=\frac{847}{12}$

$=70.58\ \text{m}^2$

Question 7:

A wooden toy is in the shape of a cone mounted on a cylinder as shown alongside.
If the height of the cone is 24 cm, the total height of the toy is 60 cm and the radius of the base of the cone = twice the radius of the base of the cylinder = 10 cm; find the total surface area of the toy.



Solution 7:

Height of the cone = 24 cm
Height of the cylinder = 36 cm

Radius of the cone = twice the radius of the cylinder = 10 cm
Radius of the cylinder = 5 cm
Slant height of the cone = $\sqrt{r^2 + h^2}$
$= \sqrt{10^2 + 24^2}$
$= \sqrt{100 + 576}$
$= \sqrt{676}$
$= 26$ cm
Now, the surface area of the toy = curved area of the conical point + curved area of the cylinder
$= \pi r\ell + \pi r^2 + 2\pi RH$
$= \pi\left[r\ell + r^2 + 2RH\right]$
$= 3.14\left[10 \times 26 + (10)^2 + 2 \times 5 \times 36\right]$
$= 3.14[260 + 100 + 360]$
$= 3.14[720]$
$= 2260.8 \ \ cm^2$

Question 8:
A cylindrical container with diameter of base 42 cm contains sufficient water to submerge a rectangular solid of iron with dimensions 22 cm × 14 cm × 10.5 cm. Find the rise in level of the water when the solid is submerged.

Solution 8:
Diameter of cylindrical container = 42 cm
Therefore, radius (r) = 21 cm
Dimensions of rectangular solid = 22cm × 14cm × 10.5cm
Volume of solid = 22 × 14 × 10.5 cm^3 ……….(i)
Let height of water = h
Therefore, volume of water in the container = $\pi r^2 h$
$= \frac{22}{7} \times 21 \times 21 \times h\, cm^3 = 22 \times 63h \ \ cm^3$ ……. (ii)
From (i) and (ii)
$22 \times 63h = 22 \times 14 \times 10.5$
$\Rightarrow h = \frac{22 \times 14 \times 10.5}{22 \times 63}$

$\Rightarrow h = \frac{7}{3}$

$\Rightarrow h = 2\frac{1}{3}$ or 2.33 cm

Question 9:

Spherical marbles of diameter 1.4 cm are dropped into beaker containing some water and are fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5.6 cm.

Solution 9:

Diameter of spherical marble = 1.4 cm

Therefore, radius = 0.7 cm

Volume of one ball = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.7)^3 \text{ cm}^3$ ....(i)

Diameter of beaker = 7 cm

Therefore, radius = $\frac{7}{2}$ cm

Height of water = 5.6 cm

Volume of water = $\pi r^2 h = \pi \times \left(\frac{7}{2}\times\frac{7}{2}\times 5.6\right) \text{ cm}^3 = \pi \times \frac{49\times 56}{4\times 10} \text{ cm}^3$

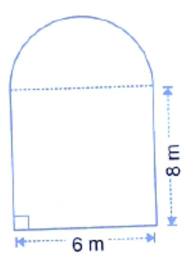
No. of balls dropped

$= \frac{\pi \times 49 \times 56 \times 3}{4\times 10\times 4\pi \times (0.7)^3}$

$= 150$

Question 10:

The cross-section of a railway tunnel is a rectangle 6 m broad and 8 m high surmounted by a semi-circle as shown in the figure. The tunnel is 35 m long. Find the cost of plastering the internal surface of the tunnel (excluding the floor) at the rate of Rs. 2.25 per m^2.



Solution 10:

Breadth of the tunnel = 6 m
Height of the tunnel = 8 m
Length of the tunnel = 35 m
Radius of the semi-circle = 3 m

Circumference of the semi-circle = $\pi r = \frac{22}{7} \times 3 = \frac{66}{7}$ m

Internal surface area of the tunnel

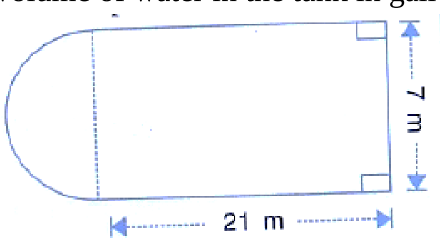
$$= 35\left(8 + 8 + \frac{66}{7}\right)$$
$$= 35\left(16 + \frac{66}{7}\right)$$
$$= 35\left(\frac{112 + 66}{7}\right)$$
$$= 35 \times \frac{178}{7}$$
$$= 890 \text{ m}^2$$

Rate of plastering the tunnel = Rs 2.25 per m^2

Therefore, total expenditure = $\text{Rs. } 890 \times \frac{225}{100} = \text{Rs } 890 \times \frac{9}{4} = \text{Rs } \frac{8010}{4} = \text{Rs. } 2002.50$

Question 11:

The horizontal cross-section of a water tank is in the shape of a rectangle with semi-circle at one end, as shown in the following figure. The water is 2.4 metres deep in the tank. Calculate the volume of water in the tank in gallons.



Solution 11:

Length = 21 m
Depth of water = 2.4 m
Breadth = 7 m

Therefore, radius of semicircle = $\frac{7}{2}$ m

Area of cross-section = area of rectangle + Area of semicircle

$= l \times b + \frac{1}{2}\pi r^2$

$= 21 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 147 + \frac{77}{4}$

$= \frac{588 + 77}{4}$

$= \frac{665}{4}\ \text{m}^2$

Therefore, volume of water filled in gallons

$= \frac{665}{4} \times 2.4\ \ \text{m}^3$

$= 665 \times 0.6$

$= 399\ \text{m}^3$

$= 399 \times 100^3\,\text{cm}^3$

$= \frac{399 \times 100 \times 100 \times 100}{1000}$ gallons

$= \frac{399 \times 100 \times 100 \times 100}{1000 \times 4.5}$

$= \frac{399 \times 100 \times 100 \times 100 \times 10}{1000 \times 45}$ gallons

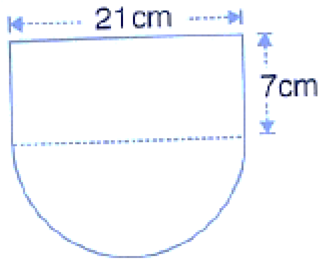
$= \frac{1330000}{15}$ gallons

$= \frac{266000}{3}$ gallons

$= 88666.67$ gallons

Question 12:

The given figure shows the cross section of a water channel consisting of a rectangle and a semi-circle. Assuming that the channel is always full, find the volume of water discharged through it in one minute if water is flowing at the rate of 20 cm per second. Give your answer in cubic metres correct to one place of decimal.



Solution 12:

Length = 21 cm, Breadth = 7 cm

Radius of semicircle $= \frac{21}{2}$ cm

Area of cross section of the water channel $= l \times b + \frac{1}{2}\pi r^2$

$= 21 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$= 147 + \frac{693}{4}$

$= \frac{588 + 693}{4}$

$= \frac{1281}{4}\ \text{cm}^2$

Flow of water in one minute at the rate of 20 cm per second

$\Rightarrow$ Length of the water column = 20 × 60 = 1200 cm

Therefore, volume of water =

$= \frac{1281}{4} \times 1200\ \text{cm}^3$

$= 384300\ \text{cm}^2$

$= \frac{384300}{100 \times 100 \times 100}\ \text{m}^3$

$= 0.3843\ \text{m}^3$

$= 0.4\ \text{m}^3$

Question 13:

An open cylindrical vessel of internal diameter 7 cm and height 8 cm stands on a horizontal table. Inside this is placed a solid metallic right circular cone, the diameter of whose base is $3\frac{1}{2}$ cm and height 8 cm. Find the volume of water required to fill the vessel.

If this cone is replaced by another cone, whose height is $1\frac{3}{4}$ cm and the radius of whose base is 2 cm, find the drop in the water level.

Solution 13:

Diameter of the base of the cylinder = 7 cm

Therefore, radius of the cylinder = $\frac{7}{2}$ cm

Volume of the cylinder $\pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 8 = 308\ cm^3$

Diameter of the base of the cone $= \frac{7}{2}$ cm

Therefore, radius of the cone $= \frac{7}{4}$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 8 = \frac{77}{3}\ cm^3$

On placing the cone into the cylindrical vessel, the volume of the remaining portion where the water is to be filled

$= 308 - \frac{77}{3}$

$= \frac{924 - 77}{3}$

$= \frac{847}{3}$

$= 282.33\ cm^3$

Height of new cone = $1\frac{3}{4} = \frac{7}{4}$ cm

Radius = 2 cm

Therefore, volume of new cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times \frac{7}{4} = \frac{22}{3}\ cm^3$

Volume of water which comes down = $\frac{77}{3} - \frac{22}{3} cm^3 = \frac{55}{3}\ cm^3$ …….(i)

Let h be the height of water which is dropped down.

Radius $= \frac{7}{2}$ cm

$\therefore$Volume $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{77}{2} h$ …… (ii)

From (i) and (ii)

$\frac{77}{2} h = \frac{55}{3}$

$\Rightarrow h = \frac{55}{3} \times \frac{22}{77}$

$\Rightarrow h = \frac{10}{21}$

Drop in water level $= \frac{10}{21}$ cm

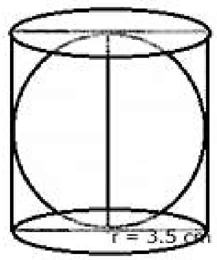
Question 14:

A cylindrical can, whose base is horizontal and of radius 3.5 cm, contains sufficient water so that when a sphere is placed in the can, the water just covers the sphere. Given that the sphere just fits into the can, calculate:

(i) the total surface area of the can in contact with water when the sphere is in it;

(ii) the depth of water in the can before the sphere was put into the can.

Solution 14:



Radius of the base of the cylindrical can = 3.5 cm

(i) When the sphere is in can, then total surface area of the can = Base area + curved surface area

$= \pi r^2 + 2\pi rh$

$= \left(\frac{22}{7} \times 3.5 \times 3.5\right) + \left(2 \times \frac{22}{7} \times 3.5 \times 7\right)$

$= \frac{77}{2} + 154$

$= 38.5 = 154$

$= 192.5 \text{ cm}^2$

(ii) Let depth of water = x cm

When sphere is not in the can, then volume of the can = volume of water + volume of sphere

$$\Rightarrow \pi r^2 h + \pi r^2 x \times + \frac{4}{3}\pi r^3$$

$$\Rightarrow \pi r^2 h + \pi r^2 \left(x + \frac{4}{3} r \right)$$

$$\Rightarrow h = x + \frac{4}{3} r$$

$$\Rightarrow x = h - \frac{4}{3} r$$

$$\Rightarrow x = 7 - \frac{4}{3} \times \frac{7}{2}$$

$$\Rightarrow x = 7 - \frac{14}{3}$$

$$\Rightarrow x = \frac{21 - 14}{3}$$

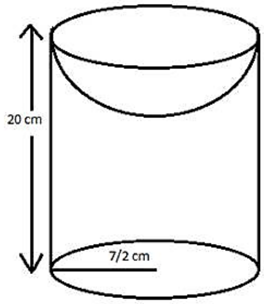
$$\Rightarrow x = \frac{7}{3}$$

$$\Rightarrow x = 2\frac{1}{3} \text{cm}$$

Question 15:

A hollow cylinder has solid hemisphere inward at one end and on the other end it is closed with a flat circular plate. The height of water is 10 cm when flat circular surface is downward. Find the level of water, when it is inverted upside down, common diameter is 7 cm and height of the cylinder is 20 cm.

Solution 15:



Let the height of the water level be 'h', after the solid is turned upside down.
Volume of water in the cylinder

$$= \pi\left(\frac{7}{2}\right)^2 10$$

Volume of the hemisphere

$$= \frac{2}{3}\pi\left(\frac{7}{2}\right)^3$$

Volume of water in the cylinder
= Volume of water level − Volume of the hemisphere

$$\pi\left(\frac{7}{2}\right)^2 10 = \pi\left(\frac{7}{2}\right)^2 h - \frac{2}{3}\pi\left(\frac{7}{2}\right)^2$$

$$\Rightarrow 10 = h - \frac{7}{3}$$

$$\Rightarrow h = 10 + \frac{7}{3}$$

$$\Rightarrow h = 12\frac{1}{3} \text{ cm}$$

The height of water when the hemisphere is facing downwards is $12\frac{1}{3}$ cm

EXERCISE. 22 (F)

Question 1:

What is the least number of solid metallic spheres, each of 6 cm diameter, that should be melted and recast to form a solid metal cone whose height is 45 cm and diameter 12 cm?

Solution 1:

Let the number of solid metallic spheres be 'n'
Volume of 1 sphere

$$= \frac{4}{3}\pi(3)^3$$

Volume of metallic cone

$$= \frac{1}{3}\pi 6^2 \times 45$$

$$n = \frac{\text{Volume of metal cone}}{\text{Volume of 1 sphere}}$$

$$\Rightarrow n = \frac{\frac{1}{3}\pi 6^2 \times 45}{\frac{4}{3}\pi (3)^3}$$

$$\Rightarrow n = \frac{6 \times 6 \times 45}{4 \times 3 \times 3 \times 3}$$

$$\Rightarrow n = 15$$

The least number of spheres needed to form the cone is 15

Question 2:

A largest sphere is to be carved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere.

Solution 2:

Radius of largest sphere that can be formed inside the cylinder should be equal to the radius of the cylinder.

Radius of the largest sphere = 7 cm

Volume of sphere

$$= \frac{4}{3}\pi 7^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{4312}{3}$$

$$= 1437 \text{ cm}^3$$

Question 3:

A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice cream is to be filled in identical cones of height 12 cm and diameter 6 cm having a hemi – spherical shape on the top. Find the number of cones required.

Solution 3:

Let the number of cones be 'n'.

Volume of the cylinder = $\pi \times 6^2 \times 15$

Volume of 1 cone = $\frac{1}{3}\pi \times 3^2 \times 12$

$n = \dfrac{\text{volume of cylinder}}{\text{Volume of 1 cone}}$

$= \dfrac{\pi \times 6^2 \times 15}{\frac{1}{3}\pi \times 3^2 \times 12}$

$= 15$

Number of cones required = 15

Question 4:

A solid is in the form of a cone standing one hemi-sphere with both their radii being equal to 8 cm and the height of cone is equal to its radius. Find, in terms of π, the volume of the solid.

Solution 4:

Volume of the solid

$= \frac{1}{3}\pi r^2 r + \frac{2}{3}\pi r^3$

$= \frac{1}{3} \times \pi \times 8^3 + \frac{2}{3} \times \pi \times 8^3$

$= \pi 8^3$

$= 512\pi \text{ cm}^3$

Question 5:

The diameter of a sphere is 6 cm, It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.

Solution 5:

Diameter of a sphere = 6 cm

Radius = 3 cm

$\therefore$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = \frac{792}{7} \text{ cm}^3$ ……(i)

Diameter of cylindrical wire = 0.2 cm

Therefore, radius of wire $= \frac{0.2}{2} = 0.1 = \frac{1}{10}$ cm

Let length of wire = h

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times h \text{ cm}^3 = \frac{22h}{700} \text{ cm}^3$ ……(ii)

From (i) and (ii)

$$\frac{22h}{700} = \frac{792}{7}$$

$$\Rightarrow h = \frac{792}{7} \times \frac{700}{22}$$

$$\Rightarrow h = 3600 \text{ cm} = 36 \text{ m}$$

Hence, length of the wire = 36 m

Question 6:

Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.

Solution 6:

Let edge of the cube = a

volume of the cube = a × a × a = a^3

The sphere, which exactly fits in the cube, has radius $= \frac{a}{2}$

Therefore, volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{a}{2}\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{a^3}{8} = \frac{11}{21}a^3$

Volume of cube : volume of sphere

$$= a^3 : \frac{11}{21}a^3$$

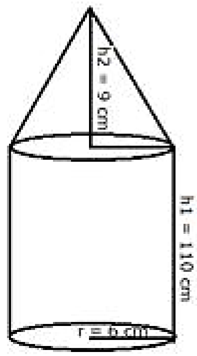
$$= 1 : \frac{11}{21}$$

$$= 21 : 11$$

Question 7:

An iron pole consisting of a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm^3 of iron has 8 gm of mass (approx).

Solution 7:



Radius of the base of poles (r) = 6 cm
Height of the cylindrical part (h_1) = 110 cm
Height of the conical part (h_2) = 9 cm

Total volume of the iron pole = $\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = \pi r^2\left(h_1 + \frac{1}{3}h_2\right)$

$= \frac{355}{113} \times 6 \times 6\left(110 + \frac{1}{3} \times 9\right)$

$= \frac{355}{113} \times 36 \times 113$

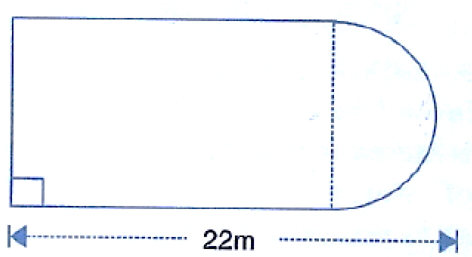
$= 12780 \text{ cm}^3$

Weight of 1 cm^3 = 8 gm

Therefore, total weight = 12780 × 8 = 102240 gm = 102.24 kg

Question 8:

In the following diagram a rectangular platform with a semi-circular end on one side is 22 metres long from one end to the other end. If the length of the half circumference is 11 metres. Find the cost of constructing the platform, 1.5 metres high at the rate of Rs. 4 per cubic metres.



Solution 8:

Length of the platform = 22 m
Circumference of semicircle = 11 m

$\therefore$ Radius = $\dfrac{c \times 2}{2 \times \pi} = \dfrac{11 \times 7}{22} = \dfrac{7}{2}$ m

Therefore, breadth of the rectangular part = $\dfrac{7}{2} \times 2 = 7$ m

And length $= 22 - \dfrac{7}{2} = \dfrac{37}{2} = 18.5\text{m}$

Now area of platform = $l \times b + \dfrac{1}{2}\pi r^2$

$= 18.5 \times 7 + \dfrac{1}{2} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} \text{m}^2$

$= 129.5 + \dfrac{77}{4} \text{m}^2$

$= 148.75 \text{ m}^2$

Height of the platform = 1.5 m

$\therefore$ Volume = 148.75 × 1.5 = 223.125 m^3

Rate of construction = Rs 4 per m^3

Total expenditure = Rs 4 × 223.125 = Rs 892.50

Question 9:

The cross-section of a tunnel is a square of side 7m surmounted by a semi circle as shown in the adjoining figure. The tunnel is 80 m long.

(i) its volume

(ii) the surface area of the tunnel (excluding the floor) and

(iii) its floor area.

Solution 9:

Side of square = 7 m

Radius of semicircle = $\dfrac{7}{2}$ m

Length of the tunnel = 80 m

Area of cross section of the front part = $a^2 + \dfrac{1}{2}\pi r^2$

$= 7 \times 7 + \dfrac{1}{2} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}$

$= 49 + \dfrac{77}{4} \text{m}^2$

$= \frac{196+77}{4}$

$= \frac{273}{4} \ \ m^2$

(i) therefore, volume of tunnel = area × length

$= \frac{273}{4} \times 80$

$= 5460 \ m^3$

(ii) Circumference of the front of tunnel

$= 2 \times 7 + \frac{1}{2} \times 2\pi r$

$= 14 + \frac{22}{7} \times \frac{7}{2}$

$= 14 + 11$

$= 25 \ m$

Therefore, surface area of the inner part of the tunnel

= 25 × 80

= 2000 m^2

(iii) Area of floor = l × b = 7 × 80 = 560 m^2

Question 10:

A cylindrical water tank of diameter 2.8 m and height 4.2 m is being fed by a pipe of diameter 7 cm through which water flows at the rater of 4 m s^{-1}. Calculate, in minutes, the time it takes to fill the tank.

Solution 10:

Diameter of cylindrical tank = 2.8 m

Therefore, radius = 1.4 m

Height = 4.2 m

Volume of water filled in it = $\pi r^2 h$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 4.2 \ \ m^3$

$= \frac{181.104}{7} \ \ m^3$

$= 25.872 \ \ m^3$ …….(i)

Diameter of pipe = 7 cm

Therefore, radius (r) = $\frac{7}{2}$

Let length of water in the pipe = h_1

$\therefore$ Volume = $\pi r^2 h_1$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h_1$

$= \frac{77}{2}\ h_1 cm^3$ ....(ii)

From (i) and (ii)

$\frac{77}{2}\ h_1 cm^3 = 25.872 \times 100^3\ cm^3$

$\Rightarrow h_1 = \frac{25.872 \times 100^3 \times 2}{77}$

$\Rightarrow h_1 = \frac{25.872 \times 100^3 \times 2}{77 \times 100}$

$\Rightarrow h_1 = 0.672 \times 100^2\ m$

$\Rightarrow h_1 = 6720\ m$

Therefore, time taken at the speed of 4 m per second

$= \frac{6720}{4 \times 60}$ minutes = 28 minutes

Question 11:

Water flows, at 9 km per hour, through a cylindrical pipe of cross-sectional area 25 cm^2. If this water is collected into a rectangular cistern of dimensions 7.5 m by 5 m by 4 m; calculate the rise in level in the cistern in 1 hour 15 minutes.

Solution 11:

Rate of flow of water = 9 km/hr

Water flow in 1 hour 15 minutes

i.e. in $\frac{5}{4}$ hr $= 9 \times \frac{5}{4} = \frac{45}{4}$ km $= \frac{45}{4} \times 1000 = 11250$ m

Area of cross-section = $25\ cm^2 = \frac{25}{10000} m^2 = \frac{1}{400}\ m^2$

Therefore, volume of water = $\frac{1}{400} \times 11250 = 28.125\, m^3$

Dimensions of water tank = 7.5m × 5m × 4m

Area of tank = $l \times b = 7.5 \times 5 = 37.5\ m^2$

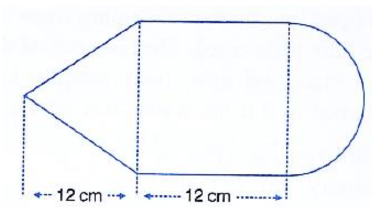
Let h be the height of water then,

$37.5 \times h = 28.125$

$h = \frac{28.125}{37.5} = 0.75 \text{ m} = 75\text{cm}$

Question 12:

The given figure shown the cross-section of a cone, a cylinder and a hemisphere all with the same diameter 10 cm, and the other dimensions are as shown.



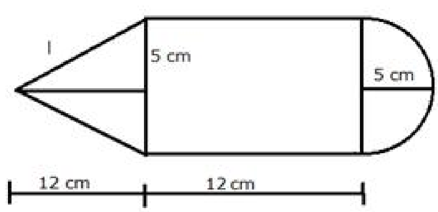
Calculate:

(a) the total surface area,

(b) the total volume of the solid and

(c) the density of the material if its total weight is 1.7 kg..

Solution 12:



Diameter = 10 cm

Therefore, radius (r) = 5 cm

Height of the cone (h) = 12 cm

Height of the cylinder = 12 cm

$\therefore \ell = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}$

(i) Total surface area of the solid

$= \pi r\ell + 2\pi rh + 2\pi r^2$

$= \pi r(\ell + 2h + 2r)$

$= \frac{22}{7} \times 5[13 + (2 \times 12) + (2 \times 5)]$

$= \frac{110}{7}[13 + 24 + 10]$

$= \frac{110}{7} \times 47$

$= \frac{5170}{7}$

$= 738.57 \ \text{cm}^2$

(ii) Total volume of the solid

$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2 \left[\frac{1}{3}h + h + \frac{2}{3}r\right]$

$= \frac{22}{7} \times 5 \times 5 \left[\frac{1}{3} \times 12 + 12 + \frac{2}{3} \times 5\right]$

$= \frac{550}{7}\left[4 + 12 + \frac{10}{3}\right]$

$= \frac{550}{7}\left[16 + \frac{10}{3}\right]$

$= \frac{550}{7} \times \frac{58}{3}$

$= \frac{31900}{21}$

$= 1519.0476 \ \text{cm}^3$

(iii) Total weight of the solid = 1.7 kg

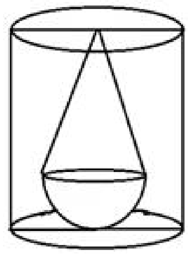
$\therefore$ Density = $\frac{1.7 \times 1000}{1519.0476} \ \text{gm / cm}^3 = 1.119 \ \text{gm / cm}^3$

$\Rightarrow$ Density = 1.12 gm / cm^3

Question 13:

A solid, consisting of a right circular cone standing one a hemisphere, is placed upright in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder, having given that the radius of the cylinder is 3 cm and its height is 6 cm; the radius of the hemisphere is 2 cm and the height of cone is 4 cm. Give your answer to the nearest cubic centimeter.

Solution 13:



Radius of cylinder = 3 cm
Height of cylinder = 6 cm
Radius of hemisphere = 2 cm
Height of cone = 4 cm
Volume of water in the cylinder when it is full =
$\pi r^2 h = \pi \times 3 \times 3 \times 6 = 54\pi\ cm^3$
Volume of water displaced = volume of cone + volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3}\pi \times 2 \times 2(4 + 2 \times 2)$$

$$= \frac{1}{3}\pi \times 4 \times 8$$

$$= \frac{32}{3}\pi cm^3$$

Therefore, volume of water which is left

$$= 54\pi - \frac{32}{3}\pi$$

$$= \frac{130}{3}\pi cm^3$$

$$= \frac{130}{3} \times \frac{22}{7} cm^3$$

$$= \frac{2860}{21} cm^3$$

$$= 136.19\ cm^3$$

$$= 136\ cm^3$$

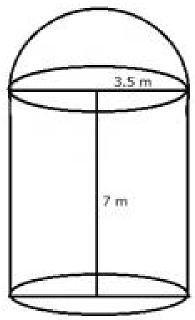
Question 14:

A metal container in the form of a cylinder is surmounted by a hemisphere of the same radius. The internal height of the cylinder is 7 m and the internal radius is 3.5 m. Calculate:

(i) the total area of the internal surface, excluding the base;

(ii) the internal volume of the container in m^3.

Solution 14:



Radius of the cylinder = 3.5 m

Height = 7 m

(i) Total surface area of container excluding the base = Curved surface area of the cylinder + area of hemisphere

$= 2\pi rh + 2\pi r^2$

$= \left(2\times\frac{22}{7}\times 3.5\times 7\right)+\left(2\times\frac{22}{7}\times 3.5\times 3.5\right)$

$= 154 + 77\ m^2$

$= 231\ m^2$

(ii) Volume of the container = $\pi r^2 h + \frac{2}{3}\pi r^3$

$= \left(\frac{22}{7}\times 3.5\times 3.5\times 7\right)+\left(\frac{2}{3}\times\frac{22}{7}\times 3.5\times 3.5\times 3.5\right)$

$= \frac{539}{2}+\frac{539}{6}$

$= \frac{1617+539}{6}$

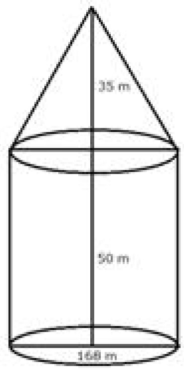
$= \frac{2156}{6}$

$= 359.33\ m^3$

Question 15:

An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and height of the cylindrical part of 50 m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for fold and for stitching. Give your answer to the nearest m^2.

Solution 15:



Total height of the tent = 85 m

Diameter of the base = 168 m

Therefore, radius (r) = 84 m

Height of the cylindrical part = 50 m

Then height of the conical part = (85 - 50) = 35 m

Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{84^2 + 35^2} = \sqrt{7056 + 1225} = \sqrt{8281} = 91$ cm

Total surface area of the tent = $2\pi rh + \pi r\ell = \pi(2h + \ell)$

$= \frac{22}{7} \times 84(2 \times 50 + 91)$

$= 264(100 + 91)$

$= 264 \times 191$

$= 50424\ m^2$

Since 20% extra is needed for folds and stitching,

total area of canvas needed

$= 50424 \times \frac{120}{100}$

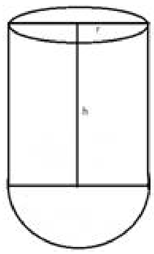
$= 60508.8$

$= 60509\ m^2$

Question 16:

A test tube consists of a hemisphere and a cylinder of the same radius. The volume of the water required to fill the whole tube is $\frac{5159}{6}$ cm^3 and $\frac{4235}{6}$ cm^3 of water is required to fill the tube to a level which is 4 cm below the top of the tune. Find the radius of the tube and the length of its cylindrical part.

Solution 16:



Volume of water filled in the test tube = $\frac{5159}{6}$ cm^3

Volume of water filled up to 4 cm = $\frac{4235}{6}$ cm^3

Let r be the radius and h be the height of test tube.

$$\therefore \frac{2}{3}\pi r^3 + \pi r^2 h = \frac{5159}{6}$$

$$\Rightarrow \pi r^2\left(\frac{2}{3}r + h\right) = \frac{5159}{6}$$

$$\Rightarrow \frac{\pi r^2}{3}(2r + 3h) = \frac{5159}{6}$$

$$\Rightarrow \pi r^2(2r + 3h) = \frac{5159}{2} \quad ......(i)$$

And

$$\frac{2}{3}\pi r^3 + \pi r^2(h - 4) = \frac{4235}{6}$$

$$\Rightarrow \pi r^2\left(\frac{2}{3}r + h - 4\right) = \frac{4235}{6}$$

$$\Rightarrow \frac{\pi r^2}{3}(2r + 3h - 12) = \frac{4235}{6}$$

$$\Rightarrow \pi r^2(2r + 3h - 12) = \frac{4235}{2} \quad ....(ii)$$

Dividing (i) by (ii)

$$\frac{2r+3h}{2r+3h-12}=\frac{5259}{4235} \quad ....(iii)$$

Subtracting (ii) from (i)

$$\pi r^2(12)=\frac{5159}{2}-\frac{4235}{2}=\frac{924}{2}$$

$$\Rightarrow 12\times\frac{22}{7}\times r^2=\frac{924}{2}$$

$$\Rightarrow r^2=\frac{924\times 7}{2\times 12\times 22}=\frac{7\times 7}{2\times 2}$$

$$\Rightarrow r^2=\frac{49}{4}$$

$$\Rightarrow r=\frac{7}{2}=3.5\,\text{cm}$$

Subtracting the value of r in (iii)

$$\frac{2\times\frac{7}{2}+3h}{2\times\frac{7}{2}+3h-12}=\frac{5159}{4235}$$

$$\Rightarrow \frac{7+3h}{7+3h-12}=\frac{5159}{4235}$$

$$\Rightarrow \frac{7+3h}{7+3h-12}=\frac{469}{385}$$

$$\Rightarrow 2695+1155h=1407h-2345$$

$$\Rightarrow 252h=5040$$

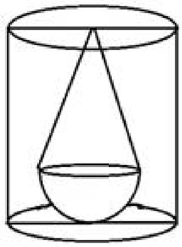
$$\Rightarrow h=20$$

Hence, height = 20 cm and radius = 3.5 cm

Question 17:

A solid is in the form of a right circular cone mounted on a hemisphere. The diameter of the base of the cone, which exactly coincides with hemisphere, is 7 cm and its height is 8 cm. the solid is placed in a cylindrical vessel of internal radius 7 cm and height 10 cm. How much water, in cm^3, will be required to fill the vessel completely.

Solution 17:



Diameter of hemisphere = 7 cm
Diameter of the base of the cone = 7 cm
Therefore, radius (r) = 3.5 cm
Height (h) = 8 cm
Volume of the solid =

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 (8 + 2 \times 3.5)$$

$$= \frac{77}{6}(8+7)$$

$$= \frac{385}{2}$$

$$= 192.5 \text{ cm}^3$$

Now, radius of cylindrical vessel (R) = 7 cm
Height (H) = 10 cm
∴ Volume = $\pi R^2 H$

$$= \frac{22}{7} \times 7 \times 7 \times 10$$

$$= 1540 \text{ cm}^3$$

Volume of water required to fill = 1540 − 192.5 = 1347.5 cm^3