1. Plying with Numbers

Exercise 1.1

1. Question

Write the following numbers in generalized form:

39, 52, 106, 359, 628, 3458, 9502, 7000.

Answer

(i) $39 = 3 \times 10 + 9 \times 10^0$

$= 3 \times 10^1 + 9 \times 1$

($\because 10^0 = 1$)

(ii) $52 = 5 \times 10 + 2\times 1$

$= 5 \times 10^1 + 2 \times 10^0$

(iii) $106 = 1 \times 100 + 0 \times 10 + 6 \times 1$

$= 1 \times 10^2 + 0 \times 10^1 + 6 \times 10^0$

(iv) $359 = 3 \times 100 + 5 \times 10 + 9 \times 1$

$= 3 \times 10^2 + 5 \times 10^1 + 9 \times 10^0$

(v) $628 = 6 \times 100 + 2 \times 10 + 8 \times 1$

$= 6 \times 10^2 + 2 \times 10^1+ 8 \times 10^0$

(vi) $3458 = 3 \times 1000 + 4 \times 100+ 5 \times 10 + 8 \times 1$

$= 3 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 8 \times 10^0$

(vii) $9502 = 9 \times 1000 + 5 \times 100 + 0 \times 10 + 2 \times 1$

$= 9 \times 10^3 + 5 \times 10^2 + 0 \times 10^1 + 2 \times 10^0$

(vii) $7000 = 7 \times 1000 + 0 \times 100 + 0\times 10 + 0 \times 1$

$= 7 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 +0 \times 10^0$

2. Question

Write the following in the decimal form:

(i) (5 × 10) + (6 × 1);

(ii) (7 × 100) + (5 × 10) + (8 × 1);

(iii) (6 × 1000) + (5 × 10) + (8 × 1);

(iv) (7 × 1000) + (6 × 1);

(v) (1 × 1000) + (1 × 10).

Answer

(i) (5 × 10) + (6 × 1)

= 50 + 6

= 56

(i) (7 × 100) + (5 × 10) + (8 × 1);

= 700 + 50 + 8

= 758

(ii) (6 × 1000) + (5 × 10) + (8 × 1);

= 6000+ 50 + 8

= 6058

(iii) (7 × 1000) + (6 × 1);

= 7000+ 6

= 7006

(iv) (1 × 1000) + (1 × 10).

= 1000+ 10

= 1010

Exercise 1.2

1. Question

In the following, find the digits represented by the letters:

(i) $\begin{array}{r} 3 \\ +\ B \\ \hline 7 \\ \hline \end{array}$ (ii) $\begin{array}{r} 1\ 6 \\ +\ 2\ A \\ \hline B\ 1 \\ \hline \end{array}$

(iii) $\begin{array}{r} 2\ A \\ \times\ \ A \\ \hline 12A \\ \hline \end{array}$ (iv) $\begin{array}{r} 1\,A\,A \\ +\ 1\,A\,A \\ \hline 2\,A\,A \\ \hline \end{array}$

(v) $\begin{array}{r} 1\,A \\ \times\ 1\,A \\ \hline 1\,B\,A \\ \hline \end{array}$ (vi) $\begin{array}{r} 3\,A \\ \times\ \ A \\ \hline 2\,B\,A \end{array}$

Answer

(i) Here B = 4 as when we subtract 3 from 7 that is 7-3 = 4 and 3 + 4 = 7

(ii) here A = 5 as only 6 + 5 = 11

From 11 the one`s place 1 has been written down, and another ten`s place 1 has been carried over to the ten`s place of given question

So the value of B = 4

(iii) Here A is occurring at all the three places (in the question and answer)

⇒ A could take values1, 5 and 6

If we take A = 1 then the answer would be 21 × 1 = 21 which is not in accordance the given solution

If A = 6 then the sum becomes 26 × 6 = 156 which is not the solution

So A = 5

As 25 × 5 = 125

(iv) Here all the A are the same in the question as well as answer

So A can take the only value as 0 as 0 + 0 = 0

Which is the only solution?

(v) here A is coming in question and answer both so

Values A can have are 0 and 1

With A = 0 the sum becomes 10 × 10 = 100

So if the value of A = 0 then B = 0

And if we take value of A = 1, the sum is 11 × 11 = 121

So if A = 1 and then B = 2

(iv) Here A can take value of 1, 5 or 6

Now if A = 1 then the multiplication would be 31 × 1 = 31 which is not the solution

If A = 5 the multiplication would be 35 × 5 = 175 which is not the solution again

If A = 6 then the multiplication becomes 36 × 6 = 216

which satisfies the condition of the solution

⇒ A = 6 and B = 1

2. Question

In the adjacent sum, A, B, C are consecutive digits. In the third row, A, B, C appear in some order. Find A, B, C.

$$\begin{array}{cccc} & A & B & C \\ + & C & B & A \\ + & - & - & - \\ \hline 1 & 2 & 4 & 2 \\ \hline \end{array}$$

Answer

Given A, B and C are consecutive numbers

Let A = x, then B = x + 1 and C = x + 2

C + A = x + x + 2 = 2x +2

B + B = x + 1 + x + 1 = 2x +2

A + C = 2x+2

Considering A, B and C at one`s place respectively

(a) C + A + A = 2x + 2 + x = 3x + 2 = 12 or 22

$\therefore x = \frac{10}{3}$ or $\frac{20}{3}$ which are not possible solutions

(b) C + A + B = 2x +2 + x + 1 = 3x +3 = 12 or 22

$\Rightarrow x = 3$ or $\frac{19}{3}$

But $\frac{19}{3}$ is not a possible solution

So x = 3

⇒ A = 3 and B = 4 and C = 5

We can verify the solution by putting the values in the question

$$\begin{array}{r} A\ B\ C \\ +\ C\ B\ A \\ +\ -\ -\ - \\ \hline 1\ 2\ 4\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3\ 4\ 5 \\ +\ 5\ 4\ 3 \\ +\ 3\ 5\ 4 \\ \hline 1\ 2\ 4\ 2 \\ \hline \end{array}$$ (Here order is A C B)

Exercise 1.3

1. Question

Find the quotient and the remainder when each of the following numbers is divided by 13.

8, 31, 44, 85, 1220.

Answer

We know If s → (q, r) denotes the quotient q and r denotes the remainder r

Then s = (b × q) + r where s and b > 0 and 0 ≤ r < b

When s is divided by 13 then

(i) s = 8

⇒ 8 = 13 × 0 + 8

Here quotient = 0 and r = 8

(ii) s = 31

⇒ 31 = 13 × 2 + 5

Here quotient = 2 and r = 5

(iii) s = 44

⇒ 44 = 13 × 3 + 5

Here quotient = 3 and r = 5

(iv) s= 85

⇒ 85 = 13 × 6 + 7

quotient = 6 and r = 7

(v) s = 1220

⇒ 1220 = 13 × 93 + 11

13) 1220 (93

117

50

39

11

Here quotient = 93 and remainder = 11

2. Question

Find the quotient and the remainder when each of the following numbers is divided by 304.

128, 636, 785, 1038, 2236, 8858.

Answer

We know If s → (q, r) denotes the quotient q and r denotes the remainder r

Then s = (b × q) + r where s and b > 0 and 0 ≤ r < b

When s is divided by 304 then

(i) s = 128

here 128 = 304 × 0 + 128

quotient = 0 and r = 128

(ii) s = 636

636 = 304 × 2 + 28

Here the quotient = 2 and r = 28

(iii) s = 785

785 = 304 × 2 + 177

Here quotient = 2 and r = 177

(iv) s = 1038

1038 = 304 × 3 +126

Here quotient = 3 and r = 126

(v) s = 2236

2236 = 304 × 7 + 108

Here the quotient = 7 and r = 108

(vi) s = 8858

8858 = 304 × 29 + 42

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304) 8858 (29
      608
       2778
        2736
          42
```

3. Question

Find the least natural number larger than 100 which leave the remainder 12 when divided by 19.

Answer

Since we have to find a number greater than 100

So first we will find the quotient and remainder for 100 when divided by 19

Here s = 100

We know If s → (q, r) denotes the quotient q and r denotes the remainder r

Then s = (b × q) + r where s and b > 0 and 0 ≤ r < b

When s is divided by 19 then

100 = 19 × 5 + 5

Here the quotient = 5 and r = 5

But the remainder has to be 12 so the 5+ 7 = 12

So the least natural number 107 (100 +7)

4. Question

What is the least natural number you have to add to 1024 to get a multiple of 181?

Answer

We know If s → (q, r) denotes the quotient q and r denotes the remainder r

Then s = (b × q) + r where s and b > 0 and 0 ≤ r < b

When s is divided by 181 then

s = 1024

1024 = 181 × 5 + 119

Here the quotient = 5 and r = 119

If we subtract the reminder from the divisor we will get the required number which should be added to get the multiple of 181

⇒ 181 – 119 = 62

The required number is 62 which when added to 1024 gives the multiple of 181

Exercise 1.4

1. Question

Without actual division, using divisibility rules, classify the following numbers as divisible by 3, 4, 5, 11.

803, 845, 474, 583, 1067, 350, 657, 684, 2187, 4334, 1905, 2548.

Answer

An integer ‘a’ is divisible by 3 if and only if the sum of the digits of ‘a’ are divisible by 3.

A number ‘a’ (having more than one digit) is divisible by 4 if and only if the 2-digit number formed by the last two digits of ‘a’ are divisible by 4.

An integer ‘a’ is divisible by 5 if and only if it ends with 0 or 5.

The number is divisible by 11 if and only if the difference between the sum of the digits in the odd place and the sum of the digits in the even place is divisible by 11.

Using these divisibility test we can find out which number is divisible by 3, 4, 5 and 11 without actually dividing

1) 803 – the sum of the digit at an odd place that is 8+3 = 11 and the sum of the digit at even place that is 0 and their difference = 11- 0 = 11 is divisible by 11 so the number is divisible by 11

2) 845- since the last digit is 5, the number is divisible by 5

3) 474 – since the sum of the digits 4 +7 +4 = 15 is divisible by 3, the number is divisible by 3

4) 583 - The sum of the digit at an odd place that is 5+3 = 8 and the sum of the digit at even place that is 8 and their difference = 8 – 8 = 0 is divisible by 11 so the number is divisible by 11

5) 1067 - The sum of the digit at an odd place that is 7+0 = 7 and the sum of the digit at even place that is 1 +6 = 7 and their difference = 7 - 7 = 0 is divisible by 11 so the number is divisible by 11

6) 350 - Since the last digit is 0, the number is divisible by 5

7) 657 - Since the sum of the digits 6 +5 +7 = 18 is divisible by 3, the number is divisible by 3

8) 684- since the last two digit that is 84 of the number is divisible by 4, the number is divisible by 4

9) 2187 - since the sum of the digits 2+1 +8 +7 = 18 is divisible by 3, the number is divisible by 3

10) 4334 - the sum of the digit at an odd place that is 4+3 = 7 and the sum of the digit at even place that is 3+4 = 7 and their difference = 7 - 7 = 0 is divisible by 11 so the number is divisible by 11

11) 1905 - since the last digit of the number is 5, the number is divisible by 5

12) 2548 - since the last two digit that is 48 of the number is divisible by 4, the number is divisible by 4

2. Question

How many numbers from 1001 to 2000 are divisible by 4?

Answer

The numbers between 1001 - 2000 would be divisible by 4 if and only if the last two digits of the number are divisible or are multiple of 4

So the number would be 1004, 1008, 1012, 1016, 1020 ……… 2000

Thus the numbers are

$$\frac{2000 - 1004}{4} + 1 = 250$$

So there would be 250 numbers between 1001- 2000 which would be divisible by 4

3. Question

Suppose a 3-digit number $\overline{abc}$ is divisible by 3. Prove that $\overline{abc} + \overline{bca} + \overline{cab}$ is divisible by 9.

Answer

Given that the $\overline{abc}$ is divisible by 3

Now,

the expanded form of
$\overline{abc} + \overline{bca} + \overline{cab} = a \times 100 + b \times 10 + c + b \times 100 + c \times 10 + a + c \times 100 + a \times 10 + b$

= 111a + 111b + 111c

= 111(a +b +c)

Since $\overline{abc}$ is divisible by 3

⇒ a +b +c is divisible by 3

Also, 111 (a +b +c) as 1+1+1 = 3 is also divisible by 3

Hence the number $\overline{abc} + \overline{bca} + \overline{cab}$ is divisible by 9 as the divisibility test of 9 states the number is divisible by 9 if and only if the sum of the digits is divisible by 9

4. Question

If $\overline{4a3b}$ is divisible by 11, find all possible values of a + b.

Answer

For the number $\overline{4a3b}$ to be divisible by 11

the sum of digit at odd place that is a +b and the sum of digit at even place that is 4 +3= 7 and their difference = (a + b)- 7 should divisible by 11

⇒ (a+ b) -7 = 0

or (a+ b) -7 = 11

⇒ Thus the possible values of a + b = 7 or 18

Exercise 1.5

1. Question

Using the numbers from 5 to 13, construct a 3 × 3 magic square. What is the magic sum here? What relation is there between the magic sum and the number in the central cell?

Answer

First, let us understand what magic square is.

A magic square is an arrangement of numbers in a square in such a way that the sum of each row, column, and diagonal is one constant number, the so-called “magic sum” or sometimes “magic constant”.

Now, let us try to create one using the numbers from 5 to 13 stepwise.

Step 1: Place 5 (the smallest number is given) in the centre box in the top row.

This is where you always begin when your magic square has odd-numbered sides, regardless of how large or small that number is.

For example, in 3×3 square, you place number 5 in box 2. Similarly, if you had 11×11 square, you would have placed your number in box 6.

	5	

Step 2: Fill the next number, that is, number 6 to the one-upper row and to the one-right column.

	5	
		6

The question arises, as to why did we place number 6 in the bottom-most row?

Notice, there is no row above the row where number 5 is placed.

This is a case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" above the magic square's top row, remain in the one right box's column, but place the number in the bottom row of the column.

Step 3: Similarly, fill the next number 7 in the one-upper row and one-right column.

	5	
7		
		6

Since there is a row above the row where number 6 is placed, but no column to number 6 column's right. We have placed the number 7 to the left-most column of the middle row.

This is another case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" to the right of the magic square's right column, remain in the one above row, but place the number in the furthest left column of that row.

Step 4: Similarly, place the next number 8 in the one-upper row and one-right column.

	5	
7		
8		6

Here, we have placed 8 below 7, because there was no space for the decided movement. It was already occupied by number 5.

This is the third case while placing numbers in the magic square having odd-numbered sides:

If the movement takes you to a box that is already occupied, go back to the last box that has been filled in, and place the next number directly below it.

We have covered all the possible cases while filling odd-numbered side magic square. Now, repeat the process.

We get,

12	5	10
7	9	11
8	13	6

So, magic sum can be calculated as:

Sum of first row = 12 + 5 + 10 = 27

Sum of second row = 7 + 9 + 11 = 27

Sum of third row = 8 + 13 + 6 = 27

Sum of first column = 12 + 7 + 8 = 27

Sum of second column = 5 + 9 + 13 = 27

Sum of third column = 10 + 11 + 6 = 27

Sum of diagonal = 12 + 9 + 6 = 10 + 11 + 6 = 27

⇒ **Magic sum = 27**

Now, let us find the relationship between the magic number and the number in the central cell.

Here, we have

Magic number = 27

The number in the central cell = 9

We can say that,

Magic number = 3 × (Number in the central cell)

⇒ 27 = 3 × 9

2. Question

Using the numbers from 9 to 17, construct a 3 × 3 magic square. What is the magic sum here? What relation is there between the magic sum and the number in the central cell?

Answer

First, let us understand what magic square is.

A magic square is an arrangement of numbers in a square in such a way that the sum of each row, column, and diagonal is one constant number, the so-called "magic sum" or sometimes "magic constant".

Now, let us try to create one using the numbers from 9 to 17 stepwise.

Step 1: Place 9 (the smallest number is given) in the centre box in the top row.

This is where you always begin when your magic square has odd-numbered sides, regardless of how large or small that number is.

For example, in 3×3 square, you place number 9 in box 2. Similarly, if you had 11×11 square, you would have placed your number in box 6.

	9	

Step 2: Fill the next number, that is, number 10 to the one-upper row and to the one-right column.

	9	
		10

The question arises, as to why did we place number 10 in the bottom-most row?

Notice, there is no row above the row where number 9 is placed.

This is a case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" above the magic square's top row, remain in the one right box's column, but place the number in the bottom row of the column.

Step 3: Similarly, fill the next number 11 in the one-upper row and one-right column.

	9	
11		
		10

Since there is a row above the row where number 10 is placed, but no column to number 10 column's right. We have placed the number 11 on the left-most column of the middle row.

This is another case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" to the right of the magic square's right column, remain in the one above row, but place the number in the furthest left column of that row.

Step 4: Similarly, place the next number 12 in the one-upper row and one-right column.

	9	
11		
12		10

Here, we have placed 12 below 11, because there was no space for the decided movement. It was already occupied by number 9.

This is the third case while placing numbers in the magic square having odd-numbered sides:

If the movement takes you to a box that is already occupied, go back to the last box that has been filled in, and place the next number directly below it.

We have covered all the possible cases while filling odd-numbered side magic square. Now, repeat the process.

We get,

16	9	14
11	13	15
12	17	10

So, magic sum can be calculated as:

Sum of first row = 16 + 9 + 14 = 39

Sum of second row = 11 + 13 + 15 = 39

Sum of third row = 12 + 17 + 10 = 39

Sum of first column = 16 + 11 + 12 = 39

Sum of second column = 9 + 13 + 17 = 39

Sum of third column = 14 + 15 + 10 = 39

Sum of diagonal = 16 + 13 + 10 = 14 + 13 + 12 = 39

⇒ **Magic sum = 39**

Now, let us find the relationship between the magic number and the number in the central cell.

Here, we have

Magic number = 39

The number in the central cell = 13

We can say that,

Magic number = 3 × (Number in the central cell)

⇒ **39 = 3 × 13**

3. Question

Starting with the middle cell in the bottom row of the square and using numbers from 1 to 9, construct a 3 × 3 magic square.

Answer

Here, we are supposed to start with the middle cell in the bottom row of the square using numbers from 1 to 9 in the 3×3 magic square. This is another way of filling magic square of odd-numbered sides.

First, let us understand what a magic square is.

A magic square is an arrangement of numbers in a square in such a way that the sum of each row, column, and diagonal is one constant number, the so-called “magic sum” or sometimes “magic constant”.

Now, let us try to create one using the numbers from 1 to 9 stepwise.

Step 1: Place 1 (the smallest number is given) in the centre box in the bottom row.

This is the step according to the question.

	1	

Step 2: Fill the next number, that is, number 2 to the one-lower row and to the one-left column.

2		
	1	

The question arises, as to why did we place number 2 in the uppermost row?

Notice, there is no row below the row where number 1 is placed.

This is a case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" below the magic square's bottom-most row, remain in the one left box's column, but place the number in the upper row of the column.

Step 3: Similarly, fill the next number 3 in the one-lower row and one-left column.

2		
		3
	1	

Since there is a row below the row where number 2 is placed, but no column to number 2 column's left. We have placed the number 3 to the right-most column of the middle row.

This is another case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" to the left of the magic square's left column, remain in the one below row, but place the number in the furthest right column of that row.

Step 4: Similarly, place the next number 4in the one-lower row and one-left column.

2		4
		3
	1	

Here, we have placed 4 above 3, because there was no space for the decided movement. It was already occupied by number 1.

This is the third case while placing numbers in the magic square having odd-numbered sides:

If the movement takes you to a box that is already occupied, go back to the last box that has been filled in, and place the next number directly above it.

We have covered all the possible cases while filling odd-numbered side magic square. Now, repeat the process.

We get,

2	9	4
7	5	3
6	1	8

Check:

If magic sum or magic constant can be calculated using the constructed magic square, then the magic square is correct.

So, let us calculate the magic sum.

Sum of first row = 2 + 9 + 4 = 15

Sum of second row = 7 + 5 + 3 = 15

Sum of third row = 6 + 1 + 8 = 15

Sum of first column = 2 + 7 + 6 = 15

Sum of second column = 9 + 5 + 1 = 15

Sum of third column = 4 + 3 + 8 = 15

Sum of diagonal = 2 + 5 + 8 = 4 + 5 + 6 = 15

⇒ Magic sum = 15

Thus, the magic square so constructed is correct.

4. Question

Construct a 3 × 3 magic square using all odd numbers from 1 to 17.

Answer

First, let us understand what magic square is.

A magic square is an arrangement of numbers in a square in such a way that the sum of each row, column, and diagonal is one constant number, the so-called "magic sum" or sometimes "magic constant".

Collect all odd numbers from 1 to 17.

They are: 1, 3, 5, 7, 9, 11, 13, 15, 17

Now, let us try to create a magic square using these odd numbers from 1 to 17 stepwise.

Step 1: Place 1 (the smallest number is given) in the centre box in the top row.

This is where you always begin when your magic square has odd-numbered sides, regardless of how large or small that number is.

For example, in 3×3 square, you place number 1 in box 2. Similarly, if you had 11×11 square, you would have placed your number in box 6.

	1	

Step 2: Fill the next number, that is, number 3 to the one-upper row and to the one-right column.

	1	
		3

The question arises, as to why did we place number 3 in the bottom-most row?

Notice, there is no row above the row where number 1 is placed.

This is a case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" above the magic square's top row, remain in the one right box's column, but place the number in the bottom row of the column.

Step 3: Similarly, fill the next number 5 in the one-upper row and one-right column.

	1	
5		
		3

Since there is a row above the row where number 3 is placed, but no column to number 3 column's right. We have placed the number 5 to the left-most column of the middle row.

This is another case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a "box" to the right of the magic square's right column, remain in the one above row, but place the number in the furthest left column of that row.

Step 4: Similarly, place the next number 7 in the one-upper row and one-right column.

	1	
5		
7		3

Here, we have placed 7 below 5, because there was no space for the decided movement. It was already occupied by number 1.

This is the third case while placing numbers in the magic square having odd-numbered sides:

If the movement takes you to a box that is already occupied, go back to the last box that has been filled in, and place the next number directly below it.

We have covered all the possible cases while filling odd-numbered side magic square. Now, repeat the process.

We get,

15	1	11
5	9	13
7	17	3

Check:

If magic sum or magic constant can be calculated using the constructed magic square, then the magic square is correct.

So, let us calculate the magic sum.

Sum of first row = 15 + 1 + 11 = 27

Sum of second row = 5 + 9 + 13 = 27

Sum of third row = 7 + 17 + 3 = 27

Sum of first column = 15 + 5 + 7 = 27

Sum of second column = 1 + 9 + 17 = 27

Sum of third column = 11 + 13 + 3 = 27

Sum of diagonal = 15 + 9 + 3 = 11 + 9 + 7 = 27

$\Rightarrow$ Magic sum = 27

Thus, the magic square so constructed is correct.

5. Question

Construct a 5 × 5 magic square using all even numbers from 1 to 50.

Answer

First, let us understand what magic square is.

A magic square is an arrangement of numbers in a square in such a way that the sum of each row, column, and diagonal is one constant number, the so-called “magic sum” or sometimes “magic constant”.

Collect all even numbers from 1 to 50.

They are: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50.

Now, let us try to create a magic square using these even numbers from 1 to 50 stepwise.

Step 1: Place 2 (the smallest number is given) in the centre box in the top row.

This is where you always begin when your magic square has odd-numbered sides, regardless of how large or small that number is.

For example, in 5×5 square, you place number 2 in box 3. Similarly, if you had 11×11 square, you would have placed your number in box 6.

		2		

Step 2: Fill the next number, that is, number 3 to the one-upper row and to the one-right column.

		2		
			4	

The question arises, as to why did we place number 4 in the bottom-most row?

Notice, there is no row above the row where number 2 is placed.

This is a case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a “box” above the magic square’s top row, remain in the one right box’s column, but place the number in the bottom row of the column.

Step 3: Similarly, fill the next number 6 in the one-upper row and one-right column.

		2		
				6
			4	

Since there is a row above the row where number 4 is placed, and a column to number 4 column’s right. We have placed the number 6 to the one-upper and one-right cell.

Step 4: Similarly, place the next number 8 in the one-upper row and one-right column.

		2		
8				
				6
			4	

Since there is a row above the row where number 6 is placed, but no column to number 6 column’s right. We have placed the number 8 to the left-most column of the row, above the row of number 6.

This is another case while placing numbers in a magic square having odd-numbered sides:

If the movement takes you to a “box” to the right of the magic square’s right column, remain in the one above row, but place the number in the furthest left column of that row.

Step 5: Again, place the next number 10 in the one-upper row and one-right column.

		2		
	10			
8				
				6
			4	

Since there is a row above the row where number 8 is placed, and a column to number 8 column's right. We have placed the number 10 to the one-upper and one-right cell.

Step 6: Again, place the next number 12 in the one-upper row and one-right column.

		2		
	10			
8	12			
				6
			4	

Here, we have placed 12 below 10, because there was no space for the decided movement. It was already occupied by number 2.

This is the third case while placing numbers in the magic square having odd-numbered sides:

If the movement takes you to a box that is already occupied, go back to the last box that has been filled in, and place the next number directly below it.

We have covered all the possible cases while filling odd-numbered side magic square. Now, repeat the process.

We get,

34	48	2	16	30
46	10	14	28	32
8	12	26	40	44
20	24	38	42	6
22	36	50	4	18

Check:

If magic sum or magic constant can be calculated using the constructed magic square, then the magic square is correct.

So, let us calculate the magic sum.

Sum of first row = 34 + 48 + 2 + 16 + 30 = 130

Sum of second row = 46 + 10 + 14 + 28 + 32 = 130

Sum of third row = 8 + 12 + 26 + 40 + 44 = 130

Sum of fourth row = 20 + 24 + 38 + 42 + 6 = 130

Sum of fifth row = 22 + 36 + 50 + 4 + 18 = 130

Sum of first column = 34 + 46 + 8 + 20 + 22 = 130

Sum of second column = 48 + 10 + 12 + 24 + 36 = 130

Sum of third column = 2 + 14 + 26 + 38 + 50 = 130

Sum of fourth column = 16 + 28 + 40 + 42 + 4 = 130

Sum of fifth column = 30 + 32 + 44 + 6 + 18 = 130

Sum of diagonal = 34 + 10 + 26 + 42 + 18 = 30 + 28 + 26 + 24 + 22 = 130

⇒ Magic sum = 130

Thus, the magic square so constructed is correct.

Additional Problems 1

1 A. Question

The general form of 456 is

A. (4×100) + (5×10) + (6×1)

B. (4×100) + (6×10) + (5×1)

C. (5×100) + (4×10) + (6×1)

D. (6×100) + (5×10) + (4×1)

Answer

For Option A,

The general or generalised form of a number is represented by base 10 of the given number.

We have been given the number, 456.

456 can be split as,

456 = 400 + 50 + 6

Or can be represented by base 10,

$456 = (4 \times 100) + (5 \times 10) + (6 \times 10^0)$

$\Rightarrow 456 = (4 \times 100) + (5 \times 10) + (6 \times 1)$

[$\because$ $400 = 4 \times 100$,

$50 = 5 \times 10$,

$6 = 6 \times 1$]

For Option B,

Solve: $(4 \times 100) + (6 \times 10) + (5 \times 1)$.

We get

$(4 \times 100) + (6 \times 10) + (5 \times 1) = 400 + 60 + 5$

$\Rightarrow (4 \times 100) + (6 \times 10) + (5 \times 1) = 465 \neq 456$

For Option C,

We need to solve $(5 \times 100) + (4 \times 10) + (6 \times 1)$.

$(5 \times 100) + (4 \times 10) + (6 \times 1) = 500 + 40 + 6$

$\Rightarrow (5 \times 100) + (4 \times 10) + (6 \times 1) = 546 \neq 456$

For Option D,

We need to solve $(6 \times 100) + (5 \times 10) + (4 \times 1)$.

$(6 \times 100) + (5 \times 10) + (4 \times 1) = 600 + 50 + 4$

$\Rightarrow (6 \times 100) + (5 \times 10) + (4 \times 1) = 654 \neq 456$

1 B. Question

Computers use

A. decimal system

B. binary system

C. base 5 system

D. base 6 system

Answer

Binary and hexadecimal number systems are widely used in computer science. Binary numbers can be considered the very basic representation of a number in an electronic device. Binary is also known as base 2 system. It has 2 numerals (0 and 1), and it used to represent the value of bits – the type of information stored in computer’s memory.

They represent the ON state (1) and OFF state (0), many of these ON and OFF states represent a decimal number.

1 C. Question

If $\overline{abc}$ is a 3-digit number, then the number

$$n = \overline{abc} + \overline{acb} + \overline{bac} + \overline{bca} + \overline{cab} + \overline{cba}$$

is always divisible by

A. 8

B. 7

C. 6

D. 5

Answer

We have been given a 3-digit number, $\overline{abc}$ and another number,
$n = \overline{abc} + \overline{acb} + \overline{bac} + \overline{bca} + \overline{cab} + \overline{cba}$

Here, take one of the numbers from the given additions of n, say, $\overline{abc}$.

The cyclic permutation of this number can be written as,

$$\overline{abc} + \overline{bca} + \overline{cab}$$

And this we know, can be written as

$$\overline{abc} = (a \times 100) + (b \times 10) + (c \times 1)$$

$$\overline{bca} = (b \times 100) + (c \times 10) + (a \times 1)$$

$$\overline{cab} = (c \times 100) + (a \times 10) + (b \times 1)$$

So,

$$\overline{abc} + \overline{bca} + \overline{cab} = (a \times 100) + (b \times 10) + (c \times 1) + (b \times 100) + (c \times 10) + (a \times 1) + (c \times 100) + (a \times 10) + (b \times 1)$$

$$\Rightarrow \overline{abc} + \overline{bca} + \overline{cab} = 100a + 10b + c + 100b + 10c + a + 100c + 10a + b$$

$$\Rightarrow \overline{abc} + \overline{bca} + \overline{cab} = 111a + 111b + 111c$$

$$\Rightarrow \overline{abc} + \overline{bca} + \overline{cab} = 111(a + b + c) \quad ...(i)$$

Now take another set of numbers, $\overline{acb}$, $\overline{cba}$ and $\overline{bac}$.

These are also cyclic permutation of $\overline{acb}$.

Add these cyclic permutation, we get

$\overline{acb} + \overline{cba} + \overline{bac}$

And this in turn can be written as,

$\overline{acb} = (a \times 100) + (c \times 10) + (b \times 1)$

$\overline{cba} = (c \times 100) + (b \times 10) + (a \times 1)$

$\overline{bac} = (b \times 100) + (a \times 10) + (c \times 1)$

So,

$$\overline{acb} + \overline{cba} + \overline{bac} = (a \times 100) + (c \times 10) + (b \times 1) + (c \times 100) + (b \times 10) + (a \times 1) + (b \times 100) + (a \times 10) + (c \times 1)$$

$$\Rightarrow \overline{acb} + \overline{cba} + \overline{bac} = 100a + 10c + b + 100c + 10b + a + 100b + 10a + c$$

$$\Rightarrow \overline{acb} + \overline{cba} + \overline{bac} = 111a + 111b + 111c$$

$$\Rightarrow \overline{acb} + \overline{cba} + \overline{bac} = 111(a + b + c) \ldots(ii)$$

Adding equations (i) and (ii), we get

$$\overline{abc} + \overline{bca} + \overline{cab} + \overline{acb} + \overline{cba} + \overline{bac} = 111(a + b + c) + 111(a + b + c)$$

$\Rightarrow n = 222\ (a + b + c)$

$\Rightarrow n = \mathbf{6} \times 37 \times (a + b + c)$

Notice that, the number is always divisible by 6.

1 D. Question

If $\overline{abc}$ is a 3-digit number, then

$$n = \overline{abc} - \overline{acb} + \overline{bac} - \overline{bca} + \overline{cab} - \overline{cba}$$

is always divisible by

A. 12

B. 15

C. 18

D. 21

Answer

Let us collect numbers and their reversal.

Take for instance, $\overline{abc}$, then its reversal is $\overline{cba}$.

We know, $\overline{abc}$ in its general form can be written as,

$$\overline{abc} = (a \times 100) + (b \times 10) + (c \times 1)$$

And $\overline{cba}$ in its general form can be written as,

$$\overline{cba} = (c \times 100) + (b \times 10) + (a \times 1)$$

Taking their difference, we get

$$\overline{abc} - \overline{cba} = [(a \times 100) + (b \times 10) + (c \times 1)] - [(c \times 100) + (b \times 10) + (a \times 1)]$$

$$\Rightarrow \overline{abc} - \overline{cba} = [100a + 10b + c] - [100c + 10b + a]$$

$$\Rightarrow \overline{abc} - \overline{cba} = 100a + 10b + c - 100c - 10b - a$$

$$\Rightarrow \overline{abc} - \overline{cba} = 99a - 99c$$

$$\Rightarrow \overline{abc} - \overline{cba} = 99(a - c) \text{ ...(i)}$$

Now, take $\overline{bac} + \overline{cab} - \overline{acb} - \overline{bca}$. Expanding each in general form, we get

$$\overline{bac} = (b \times 100) + (a \times 10) + (c \times 1)$$

$$\overline{cab} = (c \times 100) + (a \times 10) + (b \times 1)$$

$$\overline{acb} = (a \times 100) + (c \times 10) + (b \times 1)$$

$$\overline{bca} = (b \times 100) + (c \times 10) + (a \times 1)$$

Then, we get

$$\overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = [(b \times 100) + (a \times 10) + (c \times 1)] + [(c \times 100) + (a \times 10) + (b \times 1)] - [(a \times 100) + (c \times 10) + (b \times 1)] - [(b \times 100) + (c \times 10) + (a \times 1)]$$

$$\Rightarrow \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca}$$
$$= [100b + 10a + c] + [100c + 10a + b] - [100a + 10c + b] - [100b + 10c + a]$$

$$\Rightarrow \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca}$$
$$= 100b + 10a + c + 100c + 10a + b - 100a - 10c - b - 100b - 10c - a$$

$$\Rightarrow \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca}$$
$$= (100b + b - b - 100b) + (10a + 10a - 100a - a) + (c + 100c - 10c - 10c)$$

$$\Rightarrow \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = 0 + (20a - 101a) + (101c - 20c)$$

$$\Rightarrow \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = -81a + 81c$$

$$\Rightarrow \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = 81(c - a) \text{ ...(ii)}$$

Adding equations (i) and (ii), we get

$$\overline{abc} - \overline{cba} + \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = 99(a - c) + 81(c - a)$$

$$\Rightarrow \overline{abc} - \overline{cba} + \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = 99a - 99c + 81c - 81a$$

$$\Rightarrow \overline{abc} - \overline{cba} + \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = 18a - 18c$$

$$\Rightarrow \overline{abc} - \overline{cba} + \overline{bac} + \overline{cab} - \overline{acb} - \overline{bca} = 18(a - c)$$

$\Rightarrow n = 18 (a - c)$

We can easily say that, n is divisible by 18.

1 E. Question

If 1K × K1 = K2K, the letter K stands for the digit.

A. 1

B. 2

C. 3

D. 4

Answer

We have,

$$\begin{array}{r} 1\,K \\ \times\, K\,1 \\ \hline K\,2\,K \\ \hline \end{array}$$

Let us check for option (A), if K = 1,

```
   1 1
 × 1 1
 -----
   1 1
 1 1 ×
 -----
 1 2 1
```

We can see that the answer matches in the end.

So, we can conclude that K = 1 is the suitable answer.

Now, check for other options,

For (B), when K = 2,

```
   1 2
 × 2 1
 -----
   1 2
 2 4 ×
 -----
 2 5 2
```

We can see that it clearly doesn’t matches the final answer.

Option (B) doesn’t fit the answer.

For (C), when K = 3,

```
   1 3
 × 3 1
 -----
   1 3
 3 9 ×
 -----
 4 0 3
```

Clearly, the answer doesn’t match.

Option (C) is not the answer,

For (D), when K = 4,

```
   1 4
 × 4 1
 -----
   1 4
 5 6 ×
 -----
 5 7 4
```

Clearly, the answer doesn’t match.

Option (C) is not the answer,

1 F. Question

The numbers 345111 is divisible by

A. 15

B. 12

C. 9

D. 3

Answer

Let us separately check for each option.

For Option (A),

If 345111 is divisible by 15.

Then, 345111 is also divisible by (5 × 3).

⇒ 345111 is divisible by 5; 345111 is divisible by 3.

Add all digits of the number 345111, we get

3 + 4 + 5 + 1 + 1 + 1 = 15, which is divisible by 3.

⇒ 345111 is divisible by 3.

But clearly, 345111 is not divisible by 5, since the last digit is neither 5 nor 0.

∴ 345111 is not divisible by 15. [∵ it is not divisible by 5]

Option (A) is incorrect.

For Option (B),

If 345111 is divisible by 12.

Then, 345111 is also divisible by (4 × 3).

⇒ 345111 is divisible by 4; 345111 is divisible by 3.

Add all digits of the number 345111, we get

3 + 4 + 5 + 1 + 1 + 1 = 15, which is divisible by 3.

⇒ 345111 is divisible by 3.

But clearly, 345111 is not divisible by 4, since the number ought to be even to get divided by another even number.

∴ 345111 is not divisible by 12. [∵ it is not divisible by 4]

Option (B) is incorrect.

For Option (C),

If 345111 is divisible by 9 then let us check its feasibility.

A number is divisible by 9 if and only if the sum of digits of the number is also divisible by 9.

Sum of digits of 345111 = 3 + 4 + 5 + 1 + 1 + 1

⇒ Sum of digits of 345111 = 15

15 is not divisible by 9.

∴ 345111 is not divisible by 9.

Option (C) is incorrect.

For Option (D),

If 345111 is divisible by 3 then let us check its feasibility.

A number is divisible by 3 if and only if the sum of digits of the number is also divisible by 3.

Sum of digits of 345111 = 3 + 4 + 5 + 1 + 1 + 1

⇒ Sum of digits of 345111 = 15

15 is divisible by 3.

∴ 345111 is divisible by 3.

Option (D) is correct.

1 G. Question

The number of integers of the form 3AB4, where A, B denote some digits, which are divisible by 11 is

A. 0

B. 4

C. 7

D. 9

Answer

We are given that AB is divisible by 11.

Now, AB can be divisible by 11, if and only if, A = B.

A and B can take numbers from 0 to 9. (It won't take negative integers)

The number AB can be,

0, 11, 22, 33, 44, 55, 66, 77, 88, 99

⇒ There are 9 such combination of numbers which when put in AB will make it divisible by 11.

2. Question

What is the smallest 5-digit number divisible by 11 and containing each of the digits 2, 3, 4, 5, 6?

Answer

We have the digits 2, 3, 4, 5, 6. Using these digits, we need to form the smallest 5-digit number divisible by 11.

If we had had the (smallest) number as abcde, this would have satisfied the criteria:

$(a + c + e) = (b + d) \pmod{11}$

Adding these digits, $2 + 3 + 4 + 5 + 6 = 20$, so this means that $(a + c + e)$ and $(b + d)$ must be either equal or the difference must come out to be 11 or a multiple of 11. Since, the numbers are small, we can say that the difference won't come out to be greater multiples of 11.

Observe that from the given digits, $(a + c + e) = (b + d) = 10$

With the given digits, $b + d = 10$ is possible.

So, if $(b + d) = 10$, then $(a + c + e) = 10$

Such that, $(a + c + e) = (b + d) \pmod{11}$

$\Rightarrow 10 = 10 \pmod{11}$

From the given digits, $\{b, d\} = \{4, 6\}$

$\Rightarrow \{a, c, e\} = \{2, 3, 5\}$

$\Rightarrow$ The number is abcde, that is, 24365, which is the smallest possible 5-digit number containing each of the digit 2, 3, 4, 5, 6.

Thus, the number is 24365.

3. Question

How many 5-digit numbers divisible by 11 are there containing each of the digits 2, 3, 4, 5, 6?

Answer

We know the rule of divisibility by 11. A number abcde is divisible by 11, if the difference of the sum of digits at odd places and even places is a multiple of 11.

That is, if $(a + c + e) - (b + d) = 11z$, where z = integers

We have the digits 2, 3, 4, 5, 6. If using these we can estimate a possible number divisible by 11, then our work would become easier.

The 5-digit number abcde would obviously follow the criteria to be divisible by 11.

(a + c + e) = (b + d) (mod 11)

Adding digits 2, 3, 4, 5, 6, we get

2 + 3 + 4 + 5 + 6 = 20, which means (a + c + e) and (b + d) must be either equal or the difference must come out to be equal to 11 or a multiple of 11. Noticing the numbers, it is not possible to obtain a result equal to 11 or multiple of 11.

So, we can say that (a + c + e) = (b + d) = 10

From the digits, {b, d} = {4, 6}

⇒ {a, c, e} = {2, 3, 5}

Using this result, we can take a number of combinations which will satisfy the question.

For instance, if we take a = 2, b = 4, c = 3, d = 6, e = 5,

We get the number, 24365.

Now, if we take a = 3, b = 4, c = 2, d = 6, e = 5,

Then, we get 34265.

Like that, using permutation and combination, there are 2 orders possible for 4 and 6.

∵ There are two numbers in the set {b, d}, so arrangement can be done in 2! ways.

⇒ Arrangement can be done in (2 × 1) ways.

⇒ Arrangement can be done in 2 ways. …(i)

Similarly, there are 6 ways possible for 2, 3 and 5.

∵ There are 3 numbers in the set {a, c, e}, so arrangement can be done in 3! ways.

⇒ Arrangement can be done in (3 × 2 × 1) ways.

⇒ Arrangement can be done in 6 ways. …(ii)

Using result (i) and (ii), we get

2 × 6 = 12 ways to form a number using digits 2, 3, 4, 5, 6, which is divisible by 11.

Hence, the answer is 12.

4. Question

If 49A and A49, where A > 0, have a common factor, find all possible values of A.

Answer

Given that, A > 0.

We need to find the possible values of A, such that 49A and A49 have a common factor.

A can take values from 1, 2, 3, 4, 5, 6, 7, 8, 9.

If A = 1,

We have 491 and 149.

Split 491, we get 491 = 491 × 1 [∵ 491 is a prime number]

Split 149, we get 149 = 149 × 1 [∵ 149 is a prime number]

So, 491 and 149 have no common factors.

If A = 2,

We have 492 and 249.

Split 492, we get 492 = 2 × 2 × **3** × 41

Split 249, we get 249 = **3** × 83

So, 492 and 249 have a common factor, 3.

If A = 3,

We have 493 and 349.

Split 493, we get 493 = 17 × 29

Spit 349, we get 349 = 349 × 1 [∵ 349 is a prime number]

So, 493 and 349 have no common factors.

If A = 4,

We have 494 and 449.

Split 494, we get 494 = 2 × 13 × 19

Split 449, we get 449 = 449 × 1 [∵ 449 is a prime number]

So, 494 and 449 have no common factors.

If A = 5,

We have 495 and 549.

Split 495, we get 495 = **3** × **3** × 5 × 11

Split 549, we get 549 = **3** × **3** × 61

So, 495 and 549 have 2 common factors, 3 and 3.

If A = 6,

We have 496 and 649.

Split 496, we get 496 = 2 × 2 × 2 × 2 × 31

Split 649, we get 649 = 11 × 59

So, 496 and 649 have no common factors.

If A = 7,

We have 497 and 749.

Split 497, we get 497 = **7** × 71

Split 749, we get 749 = **7** × 107

So, 497 and 749 has a common factor, 7.

If A = 8,

We have 498 and 849.

Split 498, we get 498 = 2 × **3** × 83

Split 849, we get 849 = **3** × 283

So, 498 and 849 has a common factor, 3.

If A = 9,

We have 499 and 949.

Split 499, we get 499 = 499 × 1 [∵ 499 is a prime number]

Split 949, we get 949 = 13 × 73

So, 499 and 949 have no common factors.

Hence, A = 2, 5, 7 and 8.

5. Question

Write 1 to 10 using 3 and 5, each at least once, and using addition and subtraction. (For example, 7 = 5 + 5 − 3.)

Answer

We need to write numbers 1 to 10, using 3 and 5 only, irrespective of how many times we use 3 and 5, using addition and subtraction.

Take number 1,

We know (3 + 3) = 6, and (6 – 5) = 1

⇒ 1 = 3 + 3 – 5

Take number 2,

Simply subtract 5 and 3.

⇒ 2 = 5 – 3

Take number 3,

We know, (8 – 5) = 3.

And 8 can be written as (5 + 3), then subtract 5 from the addition.

We get, 3 = (5 + 3) – 5

⇒ 3 = 8 – 5

Take number 4,

We know, (9 – 5) = 4.

And 9 can be written as (3 + 3 + 3), then subtract 5 from the addition.

We get, (3 + 3 + 3) – 5 = 9 – 5 = 4

⇒ 4 = 3 + 3 + 3 – 5

Take number 5,

Just cancel 3 from 3, then add 5 to it.

⇒ 5 = (3 – 3) + 5

Take number 6,

We know, (3 + 3) = 6 but we need to include 5 also. So, just add (5 – 5) to it, it would not mean anything and won't disturb the equation, while satisfy the question.

⇒ 6 = 3 + 3 + 5 – 5

Take number 7,

We know, (10 – 3) = 7.

And 10 can be written as (5 + 5), then subtract 3 from it.

We get, (5 + 5) – 3 = 10 – 3 = 7

⇒ 7 = 5 + 5 – 3

Take number 8,

We know, (5 + 3) = 8

⇒ 8 = 5 + 3

Take number 9,

Although 9 can be represented by several ways using 5 and 3.

One of the easiest way is:

We know, (3 + 3 + 3) = 9, then add (5 – 5) to it.

⇒ 9 = 3 + 3 + 3 + 5 – 5

Also, we know 9 = 15 – 6

Where 15 = 5 + 5 + 5

And 6 = 3 + 3

So, 9 = (5 + 5 + 5) – (3 + 3)

⇒ 9 = 5 + 5 + 5 – 3 – 3

Take number 10,

We know, (5 + 5) = 10.

Just add (3 – 3) to it, it won't affect the equation.

⇒ 10 = 5 + 5 + 3 – 3

Hence, we have found the answers.

6. Question

Find all 2-digit numbers each of which is divisible by the sum of its digits.

Answer

Let us start by collecting all 2-digit numbers.

They are from 10 – 99.

From these numbers, we have prime numbers also which are not divisible by any number except 1.

They are 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

For this, we need to check every 2-digit number for its divisibility by the sum of its digits.

For 10,

The digits are 1 and 0.

Sum of digits = 1 + 0 = 1

And 10 is divisible by 1 obviously.

⇒ 10 is divisible by the sum of its digits.

For 12,

The digits are 1 and 2.

Sum of digits = 1 + 2 = 3

And 12 is divisible by 3.

⇒ 12 is divisible by the sum of its digits.

For 14,

The digits are 1 and 4.

Sum of digits = 1 + 4 = 5

And 14 is not divisible by 5.

⇒ 14 is not divisible by the sum of its digits.

For 15,

The digits are 1 and 5.

Sum of digits = 1 + 5 = 6

And 15 is not divisible by 6.

⇒ 15 is not divisible by the sum of its digits.

For 16,

The digits are 1 and 6.

Sum of digits = 1 + 6 = 7

And 16 is not divisible by 7.

⇒ 16 is not divisible by the sum of its digits.

For 18,

The digits are 1 and 8.

Sum of digits = 1 + 8 = 9

And 18 is divisible by 9.

⇒ 18 is divisible by the sum of its digits.

For 20,

The digits are 2 and 0.

Sum of digits = 2 + 0 = 2

And 20 is divisible by 2.

⇒ 20 is divisible by the sum of its digits.

For 21,

The digits are 2 and 1.

Sum of digits = 2 + 1 = 3

And 21 is divisible by 3.

⇒ 21 is divisible by the sum of its digits.

For 22,

The digits are 2 and 2.

Sum of digits = 2 + 2 = 4

And 22 is not divisible by 4.

⇒ 22 is not divisible by the sum of its digits.

For 24,

The digits are 2 and 4.

Sum of digits = 2 + 4 = 6

And 24 is divisible by 6.

⇒ 24 is divisible by the sum of its digits.

For 25,

The digits are 2 and 5.

Sum of digits = 2 + 5 = 7

And 25 is not divisible by 7.

⇒ 25 is not divisible by the sum of its digits.

For 26,

The digits are 2 and 6.

Sum of digits = 2 + 6 = 8

And 26 is not divisible by 8.

⇒ 26 is not divisible by the sum of its digits.

For 27,

The digits are 2 and 7.

Sum of digits = 2 + 7 = 9

And 27 is divisible by 9.

⇒ 27 is divisible by the sum of its digits.

For 28,

The digits are 2 and 8.

Sum of digits = 2 + 8 = 10

And 28 is not divisible by 10.

⇒ 28 is not divisible by the sum of its digits.

For 30,

The digits are 3 and 0.

Sum of digits = 3 + 0 = 3

And 30 is divisible by 3.

⇒ 30 is divisible by the sum of its digits.

For 32,

The digits are 3 and 2.

Sum of digits = 3 + 2 = 5

And 32 is not divisible by 5.

⇒ 32 is not divisible by the sum of its digits.

For 33,

The digits are 3 and 3.

Sum of digits = 3 + 3 = 6

And 33 is not divisible by 6.

⇒ 33 is not divisible by the sum of its digits.

For 34,

The digits are 3 and 4.

Sum of digits = 3 + 4 = 7

And 34 is not divisible by 7.

⇒ 34 is not divisible by the sum of its digits.

For 35,

The digits are 3 and 5.

Sum of digits = 3 + 5 = 8

And 35 is not divisible by 8.

⇒ 35 is not divisible by the sum of its digits.

For 36,

The digits are 3 and 6.

Sum of digits = 3 + 6 = 9

And 36 is divisible by 9.

⇒ 36 is divisible by the sum of its digits.

For 38,

The digits are 3 and 8.

Sum of digits = 3 + 8 = 11

And 38 is not divisible by 11.

⇒ 38 is not divisible by the sum of its digits.

For 39,

The digits are 3 and 9.

Sum of digits = 3 + 9 = 12

And 39 is not divisible by 12.

⇒ 39 is not divisible by the sum of its digits.

For 40,

The digits are 4 and 0.

Sum of digits = 4 + 0 = 4

And 40 is divisible by 4.

⇒ 40 is divisible by the sum of its digits.

For 42,

The digits are 4 and 2.

Sum of digits = 4 + 2 = 6

And 42 is divisible by 6.

⇒ 42 is divisible by the sum of its digits.

For 44,

The digits are 4 and 4.

Sum of digits = 4 + 4 = 8

And 44 is not divisible by 8.

⇒ 44 is not divisible by the sum of its digits.

For 45,

The digits are 4 and 5.

Sum of digits = 4 + 5 = 9

And 45 is divisible by 9.

⇒ 45 is divisible by the sum of its digits.

For 46,

The digits are 4 and 6.

Sum of digits = 4 + 6 = 10

And 46 is not divisible by 10.

⇒ 46 is not divisible by the sum of its digits.

For 48,

The digits are 4 and 8.

Sum of digits = 4 + 8 = 12

And 48 is divisible by 12.

⇒ 48 is divisible by the sum of its digits.

For 49,

The digits are 4 and 9.

Sum of digits = 4 + 9 = 13

And 49 is not divisible by 13.

⇒ 49 is not divisible by the sum of its digits.

For 50,

The digits are 5 and 0.

Sum of digits = 5 + 0 = 5

And 50 is divisible by 5.

⇒ 50 is divisible by the sum of its digits.

For 51,

The digits are 5 and 1.

Sum of digits = 5 + 1 = 6

And 51 is not divisible by 6.

⇒ 51 is not divisible by the sum of its digits.

For 52,

The digits are 5 and 1.

Sum of digits = 5 + 1 = 6

And 51 is not divisible by 6.

⇒ 51 is not divisible by the sum of its digits.

For 54,

The digits are 5 and 4.

Sum of digits = 5 + 4 = 9

And 54 is divisible by 9.

⇒ 54 is divisible by the sum of its digits.

For 55,

The digits are 5 and 5.

Sum of digits = 5 + 5 = 10

And 55 is not divisible by 10.

⇒ 55 is not divisible by the sum of its digits.

For 56,

The digits are 5 and 6.

Sum of digits = 5 + 6 = 11

And 56 is not divisible by 11.

⇒ 56 is not divisible by the sum of its digits.

For 57,

The digits are 5 and 7.

Sum of digits = 5 + 7 = 12

And 57 is not divisible by 12.

⇒ 57 is not divisible by the sum of its digits.

For 58,

The digits are 5 and 8.

Sum of digits = 5 + 8 = 13

And 58 is not divisible by 13.

⇒ 58 is not divisible by the sum of its digits.

For 60,

The digits are 6 and 0.

Sum of digits = 6 + 0 = 6

And 60 is divisible by 6.

⇒ 60 is divisible by the sum of its digits.

For 62,

The digits are 6 and 2.

Sum of digits = 6 + 2 = 8

And 62 is not divisible by 8.

⇒ 62 is not divisible by the sum of its digits.

For 63,

The digits are 6 and 3.

Sum of digits = 6 + 3 = 9

And 63 is divisible by 9.

⇒ 63 is divisible by the sum of its digits.

For 64,

The digits are 6 and 4.

Sum of digits = 6 + 4 = 10

And 64 is not divisible by 10.

⇒ 64 is not divisible by the sum of its digits.

For 65,

The digits are 6 and 5.

Sum of digits = 6 + 5 = 11

And 65 is not divisible by 11.

⇒ 65 is not divisible by the sum of its digits.

For 66,

The digits are 6 and 6.

Sum of digits = 6 + 6 = 12

And 66 is not divisible by 12.

⇒ 66 is not divisible by the sum of its digits.

For 70,

The digits are 7 and 0.

Sum of digits = 7 + 0 = 7

And 70 is divisible by 7.

⇒ 70 is divisible by the sum of its digits.

For 72,

The digits are 7 and 2.

Sum of digits = 7 + 2 = 9

And 72 is divisible by 9.

⇒ 72 is divisible by the sum of its digits.

For 74,

The digits are 7 and 4.

Sum of digits = 7 + 4 = 11

And 74 is not divisible by 11.

⇒ 74 is not divisible by the sum of its digits.

For 75,

The digits are 7 and 5.

Sum of digits = 7 + 5 = 12

And 75 is not divisible by 12.

⇒ 75 is not divisible by the sum of its digits.

For 76,

The digits are 7 and 6.

Sum of digits = 7 + 6 = 13

And 76 is not divisible by 13.

⇒ 76 is not divisible by the sum of its digits.

For 77,

The digits are 7 and 7.

Sum of digits = 7 + 7 = 14

And 77 is not divisible by 14.

⇒ 77 is not divisible by the sum of its digits.

For 78,

The digits are 7 and 8.

Sum of digits = 7 + 8 = 15

And 78 is not divisible by 15.

⇒ 78 is not divisible by the sum of its digits.

For 80,

The digits are 8 and 0.

Sum of digits = 8 + 0 = 8

And 80 is divisible by 8.

⇒ 80 is divisible by the sum of its digits.

For 81,

The digits are 8 and 1.

Sum of digits = 8 + 1 = 9

And 81 is divisible by 9.

⇒ 81 is divisible by the sum of its digits.

For 82,

The digits are 8 and 2.

Sum of digits = 8 + 2 = 10

And 82 is not divisible by 10.

⇒ 82 is not divisible by the sum of its digits.

For 84,

The digits are 8 and 4.

Sum of digits = 8 + 4 = 12

And 84 is divisible by 12.

⇒ 84 is divisible by the sum of its digits.

For 85,

The digits are 8 and 5.

Sum of digits = 8 + 5 = 13

And 85 is not divisible by 13.

⇒ 85 is not divisible by the sum of its digits.

For 86,

The digits are 8 and 6.

Sum of digits = 8 + 6 = 14

And 86 is not divisible by 14.

⇒ 86 is not divisible by the sum of its digits.

For 87,

The digits are 8 and 7.

Sum of digits = 8 + 7 = 15

And 87 is not divisible by 15.

⇒ 87 is not divisible by the sum of its digits.

For 88,

The digits are 8 and 8.

Sum of digits = 8 + 8 = 16

And 88 is not divisible by 16.

⇒ 88 is not divisible by the sum of its digits.

For 90,

The digits are 9 and 0.

Sum of digits = 9 + 0 = 9

And 90 is divisible by 9.

⇒ 90 is divisible by the sum of its digits.

For 91,

The digits are 9 and 1.

Sum of digits = 9 + 1 = 10

And 91 is not divisible by 10.

⇒ 91 is not divisible by the sum of its digits.

For 92,

The digits are 9 and 2.

Sum of digits = 9 + 2 = 11

And 92 is not divisible by 11.

⇒ 92 is not divisible by the sum of its digits.

For 93,

The digits are 9 and 3.

Sum of digits = 9 + 3 = 12

And 93 is not divisible by 12.

⇒ 93 is not divisible by the sum of its digits.

For 94,

The digits are 9 and 4.

Sum of digits = 9 + 4 = 13

And 94 is not divisible by 13.

⇒ 94 is not divisible by the sum of its digits.

For 95,

The digits are 9 and 5.

Sum of digits = 9 + 5 = 14

And 95 is not divisible by 14.

⇒ 95 is not divisible by the sum of its digits.

For 96,

The digits are 9 and 6.

Sum of digits = 9 + 6 = 15

And 96 is not divisible by 15.

⇒ 96 is not divisible by the sum of its digits.

For 98,

The digits are 9 and 8.

Sum of digits = 9 + 8 = 17

And 98 is not divisible by 17.

⇒ 98 is not divisible by the sum of its digits.

For 99,

The digits are 9 and 2.

Sum of digits = 9 + 2 = 11

And 92 is not divisible by 11.

⇒ 92 is not divisible by the sum of its digits.

Thus, the answer is 10, 12, 18, 20, 21, 24, 27, 30, 36, 40, 42, 45, 48, 50, 54, 60, 63, 70, 72, 80, 81, 84, 90.

7. Question

The page numbers of a book written in a row gives a 216 digit number. How many pages are there in the book?

Answer

Let us start from 1-digit numbers of the pages.

Page numbers always start from 1, never 0.

So,

From Page No. 1 to 9, there are 9 digits.

From Page No. 10 to 99, there are (100 – 10) = 90 pages.

We know 2-digit numbers have 2 digits each, so each number has 2 digits.

For instance, 10 has 2-digits.

From Page No. 10 to 11, we have (2 × 2) = 4 digits.

From Page No. 10 to 12, we have (3 × 2) = 6 digits.

…

…

From Page No. 10 to 99, we have (90 × 2) = 180 digits.

So, from Page No. 1 to 99, we have (9 + 180) = 189 digits.

Now, 216 – 189 = 27

We need to find the number of pages, whose digits add up to 27.

We know 3-digit numbers have 3 digits each, so each number has 3 digits.

For instance, 100 has 3-digits.

From Page No. 100 to 101, we have (2 × 3) = 6 digits.

From Page No. 100 to 102, we have (3 × 3) = 9 digits.

…

…

From Page No. 100 to 108. We have (9 × 3) = 27 digits.

So, from Page No. 1 to 108, we have

9 + 180 + 27 = 216

Hence, there are 108 pages in the book.

8. Question

Look at the following pattern:

```
      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
 • • • • • •
• • • • • • •
```

This is called Pascal's triangle. What is the middle number in the 9^{th} row?

Answer

Let us try to draw out a formula by which this Pascal's triangle is formed.

Notice, corners always contain 1.

Also, the number appearing in the next row in the middle of two numbers from the preceding row is the sum of the numbers of the preceding row.

Following this, we have

```
  1
 1 1
1 (1 + 1) 1
```

Understand the pattern,

In the first row, the corner is occupied by 1.

In the second row, the corners are occupied by 1.

In the third row, the corners are occupied by 1, and the middle number is found by adding the two numbers on its either sides in the preceding row.

We have,

```
  1
 1 1
1 2 1
1 (1 + 2) (2 + 1) 1
```

In the fourth row, the corners are occupied by 1, and the other middle numbers are found by adding the two numbers on its either sides in the preceding row.

We have,

```
  1
 1 1
1 2 1
1 3 3 1
```

1 (1 + 3) (3 + 3) (3 + 1) 1

In the fifth row, the corners are occupied by 1, and the middle row numbers are computed by adding the numbers on its either sides in the preceding row.

We have,

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

1 (1 + 4) (4 + 6) (6 + 4) (4 + 1) 1

In the sixth row, the corners are occupied by 1, and the middle row numbers are computed by adding the numbers on its either sides in the preceding row.

We have,

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

1 (1 + 5) (5 + 10) (10 + 10) (10 + 5) (5 + 1) 1

In the seventh row, the corners are occupied by 1, and the middle row numbers are just derived by adding the numbers on its either sides in the preceding row.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

1 (1 + 6) (6 + 15) (15 + 20) (20 + 15) (15 + 6) (6 + 1) 1

In the eighth row, the corners are occupied by 1, and the middle row numbers are just derived by adding the numbers on its either sides in the preceding row.

We get

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

1 (1 + 7) (7 + 21) (21 + 35) (35 + 35) (35 + 21) (21 + 7) (7 + 1) 1

⇒

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 **70** ← 56 28 8 1

Thus, the middle number in the ninth row is 70.

9. Question

Complete the adjoining magic square. (Hint: in a 3 × 3 magic square, the magic sum is three times the central number.)

8		
3	7	

Answer

First, let us understand what magic square is.

A magic square is an arrangement of numbers in a square in such a way that the sum of each row, column, and diagonal is one constant number, the so-called “magic sum” or sometimes “magic constant”.

The magic number is given as,

Magic number = 3 × (central number)

⇒ Magic number = 3 × 7

⇒ Magic number = 21

This means, sum of rows = 21.

Take row 2,

Sum of row 2 = 3 + 7 + ?

⇒ 21 = 3 + 7 + ?

⇒ ? = 21 – 10

⇒ ? = 11

8		
3	7	11
?		

Also, in column 1,

Sum of column 1 = 8 + 3 + ?

⇒ 21 = 8 + 3 + ? [∵ Magic sum = 21]

⇒ 21 = 11 + ?

⇒ ? = 21 – 11

⇒ ? = 10

We get,

8		
3	7	11
10		?

Also, along the diagonal, we can say that

Sum of diagonal = 8 + 7 + ?

⇒ 21 = 15 + ?

⇒ ? = 21 – 15

⇒ ? = 6

We get,

8		?
3	7	11
10		6

Here, Sum of third column is the magic number.

⇒ Sum of third column = ? + 11 + 6

⇒ 21 = ? + 17

⇒ ? = 21 – 17

⇒ ? = 4

We get,

8	X	4
3	7	11
10	Y	6

For X and Y,

We can say that,

Sum of first row = 21

⇒ 8 + X + 4 = 21

⇒ 12 + X = 21

⇒ X = 21 – 12

⇒ X = 9

Sum of third row = 21

⇒ 10 + Y + 6 = 21

⇒ 16 + Y = 21

⇒ Y = 21 – 16

⇒ Y = 5

We get,

8	9	4
3	7	11
10	5	6

Thus, this is the magic square.

10. Question

Final all 3-digit natural numbers which are 12 times as large as the sum of their digits.

Answer

There is only one number that exist which is 12 times as large as the sum of their digits, that number is, 108.

Check: Let us take the smallest 3-digit number, say, 100.

Add the digits of 100, we get

$1 + 0 + 0 = 1$

Multiplying 12 by 1, we get

$12 \times 1 = 12 \neq 100$

Similarly, check numbers till 999. We will notice that there is only one number that satisfy such condition and that is, 108.

The digits in the number 108 are 1, 0 and 8.

Sum of these digits = 1 + 0 + 8

⇒ Sum of these digits = 9

Now, multiply 12 by 9,

$12 \times 9 = 108$

Hence, the 3-digit natural number which is 12 times as large as the sum of its digit is 108.

11. Question

Find all digits x, y such that $\overline{34x5y}$ is divisible by 36.

Answer

The number divisible by 36 is also divisible by 4 & 9.

Since, $36 = 4 \times 9$

For divisibility by 4,

The last two digits of the number should be divisible by 4 for the number to be divisible by 4.

The last two digits of the number 34x5y is 5y.

5y should be divisible by 4.

$\Rightarrow y = 2$ or $y = 6$

$(0 \leq y \leq 9)$

Because 52 is divisible by 4 and 56 is also divisible by 4.

For divisibility by 9,

The sum of digits of the number should be divisible by 9 for the number to be divisible by 9.

Sum of digits of 34x5y is,

Sum = 3 + 4 + x + 5 + y

⇒ Sum = 12 + x + y

If y = 2,

Sum = 12 + x + 2

⇒ Sum = 14 + x

Now if x = 4, $(0 \leq x \leq 9)$

Then sum = 18, which is divisible by 9.

And if x = 13,

Then sum = 27, which is divisible by 9 but $x \neq 13$ as 13 is a 2-digit number.

So, we have found one pair of solution. That is, **(x, y) = (4, 2).**

If y = 6,

Sum = 12 + x + 6

⇒ Sum = 18 + x

Now if x = 0, $(0 \leq x \leq 9)$

Then sum = 18, which is divisible by 9.

Also, if x = 9, $(0 \leq x \leq 9)$

Then sum = 27, which is divisible by 9.

So, we have found two pair of solutions. That is, **(x, y) = (0, 6) and (x, y) = (9, 6).**

Thus, we have x = 4, y = 2 or x = 0, y = 6 or x = 9, y = 6.

12. Question

Can you divide the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 into two groups such that the product of numbers in one group divides the product of numbers in the other group and the quotient is minimum?

Answer

Note that, multiply least numbers together:

We have 1, 2, 3, 4, 5, 6, 7

Product = 1 × 2 × 3 × 4 × 5 × 6 × 7

⇒ Product = 5040

And the other numbers are 8, 9, 10.

Product = 8 × 9 × 10

⇒ Product = 720

Divide them,

$$\frac{5040}{720} = 7$$

This quotient is minimum.

Thus, Group 1 has numbers, 1, 2, 3, 4, 5, 6, 7

Group 2 has numbers, 8, 9, 10

13. Question

Find all 8-digit numbers 273A49B5 which are divisible by 11 as well as 25.

Answer

For the divisibility by 25,

A number is divisible by 25, if and only if, it ends in 00, 25, 50 and 75.

The number 273A49B5 ends in B5.

⇒ B can be either 2 or 7.

For the divisibility by 11,

A number is divisible by 11, if and only if, (Sum of digits at odd places) – (Sum of digits at even places) = Multiple of 11

So, (5 + 9 + A + 7) – (B + 4 + 3 + 2) = Multiple of 11

⇒ (21 + A) – (9 + B) = Multiple of 11

⇒ 21 + A – 9 – B = Multiple of 11

⇒ A – B + 12 = Multiple of 11 …(i)

Now, if B = 2,

We get from equation (i),

A – 2 + 12 = Multiple of 11

⇒ A + 10 = Multiple of 11

Put A = 1 ($0 \le A \le 9$) arbitrarily, we get

1 + 10 = Multiple of 11

⇒ 11 = Multiple of 11

So, one pair of solution is (A, B) = (1, 2).

If B = 7,

From equation (i), we get

A − 7 + 12 = Multiple of 11

⇒ A + 5 = Multiple of 11

Put A = 6 ($0 \le A \le 9$) arbitrarily, we get

6 + 5 = Multiple of 11

⇒ 11 = Mulitple of 11

And, another pair of solution is (A, B) = (6, 7).

So, the number from solution (1, 2) comes out to be 27314925.

And, the number from solution (6, 7) comes out to be 27364975.

Thus, the numbers are 27314925 and 27364975.

14. Question

Suppose a, b are integers such that 2 + a and 35 − b are divisible by 11. Prove that a + b is divisible by 11.

Answer

Given: (2 + a) and (35 − b) are divisible by 11, such that, a and b are integers.

To Prove: (a + b) is divisible by 11.

Proof: If (2 + a) and (35 − b) are divisible by 11, then their sum and difference will also be divisible by 11.

Let us take difference of these two numbers.

We can write as,

Difference = (2 + a) − (35 − b) = Divisible by 11

⇒ 2 + a − 35 + b = Divisible by 11

⇒ a + b − 33 = Divisible by 11

Note that, here -33 is absolutely divisible by 11.

⇒ a + b must also be divisible by 11, as a + b – 33 is divisible by 11.

Hence, proved.

15. Question

In the multiplication table A8 × 3B = 2730, A and B represent distinct digits different from 0. Find A + B.

Answer

We have been given, A8 × 3B = 2730

What should be multiplied to 8 that gives 0 in the end digit.

We know, 8 × 0 = 0 and 8 × 5 = 40.

But B ≠ 0, so B = 5.

Now, we have

A8 × 35 = 2730

$$\Rightarrow A8 = \frac{2730}{35}$$

⇒ A8 = 78

Comparing the left side and right side in the equation, we get

A = 7

Thus, A = 7 and B = 5.

16. Question

Find the least natural number which leaves the remainders 6 and 8 when divided by 7 and 9 respectively.

Answer

Take L.C.M of 7 and 9.

We get, L.C.M (7, 9) = 7 × 9 [∵ 7 and 9 are co-primes]

⇒ L.C.M (7, 9) = 63

Now, we need to subtract 1 from L.C.M of 7 and 9.

So, 63 – 1 = 62

Dividing 62 by 7,

62 = (7 × 8) + 6

Notice, by dividing 62 by 7, we have got remainder as 6.

Dividing 62 by 9,

62 = (9 × 6) + 8

Notice, by dividing 62 by 9, we have got remainder as 8.

Thus, we can conclude that the least natural number is 62, which leaves the remainder 6 and 8 when divided by 7 and 9 respectively.

17. Question

Prove that the sum of cubes of three consecutive natural numbers is always divisible by 3.

Answer

Given: We have information that, we can assume three consecutive natural numbers.

To Prove: Sum of the cubes of three consecutive natural numbers is always divisible by 3.

Proof: Let three consecutive numbers be x, (x + 1) and (x + 2).

Sum of cubes of these numbers = $x^3 + (x + 1)^3 + (x + 2)^3$

⇒ Sum of cubes of these numbers = $x^3 + x^3 + 1 + 3x (x + 1) + x^3 + 2^3 + 3x (x + 2)$

⇒ Sum of cubes of these numbers = $x^3 + x^3 + 1 + 3x^2 + 3x + x^3 + 8 + 6x^2 + 12x$

⇒ Sum of cubes of these numbers = $3x^3 + 9x^2 + 15x + 9$

⇒ Sum of cubes of these numbers = $3 (x^3 + 3x^2 + 5x + 3)$

This clearly shows that the expression, $3 (x^3 + 3x^2 + 5x + 3)$ is divisible by 3.

Thus, proved that the sum of cubes of three consecutive natural numbers is always divisible by 3.

18. Question

What is the smallest number you have to add to 100000 to get a multiple of 1234?

Answer

For the smallest number that needs to be added to 100000 to get multiple of 1234, we need to divide 100000 by 1234.

So, let us represent in division form,

100000 = (1234 × 81) + 46

By dividing 100000 by 1234, we get

Quotient = 81

Remainder = 46

Now, we need to find [Divider – Remainder] value,

Divider – Remainder = 1234 – 46

⇒ Divider – Remainder = 1188

Now, add 1188 to 100000, then divide by 1234.

100000 + 1188 = 101188

Now, $\frac{101188}{1234} = 82$

Thus, the smallest number need to be added to 100000 to get multiple of 1234 is 1188.

19. Question

Using the digits 4, 5, 6, 7, 8, each once, construct a 5-digit number which is divisible by 264.

Answer

First, factorize 264.

Factors of 264 = 2 × 2 × 2 × 3 × 11

Or factors of 264 = 2^3 × 3 × 11

So, if a number is to be divisible by 264, then the number must be divisible by 2, 3 and 11 necessarily.

For divisibility by 2,

The number should contain an even number in the unit digit.

So, using the given numbers 4, 5, 6, 7, 8, we can say that the number so formed will have either 4 or 6 or 8 in the end.

For divisibility by 3,

The sum of the digits of the number must be divisible by 3.

Add the numbers so given,

4 + 5 + 6 + 7 + 8 = 30

30 is divisible by 3.

For divisibility by 11,

The difference of the sum of digits at odd places and sum of the digits at even places must be equal to a multiple of 11.

Sum of all numbers = 4 + 5 + 6 + 7 + 8

⇒ Sum of all numbers = 30

Let us try to form a combination whose sum of digits at even places and sum of digits at odd places is equal to 15.

Such combination of numbers are {8, 7} and {4, 5, 6}.

Now, form a 5-digit number using these two groups, provided the unit digit be either 4, 6 or 8.

For unit digit of 4, we have 68574, 57684, 67584, 58674.

For unit digit of 6, we have 48576, 58476, 57486, 47586.

And the number 8 can't be used in the unit place.

So, we have 5-digit numbers namely, 68574, 57684, 67584, 58674, 48576, 58476, 57486 and 47586.