IIT Foundation Material

SECTION - I

Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c), (d), out of which ONLY ONE is correct. Choose the correct option.

1. The equation $\frac{x-2}{x-1}=\frac{1-2}{x-1}$ has

(a) no root (b) one root
(c) two equal roots (d) infinitely many roots

2. The number of real solutions of the equation $|x^2|-3|x|+2=0$ is

(a) 4 (b) 1 (c) 3 (d) 2

3. The roots of $(x-a)(x-b)=abx^2$ are always

(a) real (b) depends upon a
(c) depends upon b (d) depends upon a and b

4. The equation $(b-c)x^2+(c-a)x+(a-b)=0$ has

(a) equal roots (b) irrational numbers
(c) rational roots (d) none of these

5. The number of roots of the equation $\frac{(x+2)(x-5)}{(x-3)(x+6)}=\frac{x-2}{x+4}$ is

(a) 3 (b) 2 (c) 1 (d) 0

6. The roots of the equation $7(q-r)x^2+(r-p)x+(p-q)=0$ are

(a) $\frac{r-p}{q-r},1$ (b) $\frac{p-q}{q-r},1$ (c) $\frac{q-r}{p-q},1$ (d) $\frac{r-p}{p-q},1$

7. The expression $ax^2+bx+c, a>0$ is positive for all real x only if

(a) $b^2-4ac=0$ (b) $b^2-4ac\neq 0$
(c) $b^2-4ac<0$ (d) $b^2-4ac>0$

8. Both the roots of the equation $(x-b)(x-c)+(x-c)(x-a)+(x-a)(x-b)=0$ are always

(a) positive (b) negative (c) real (d) None of these

9. If the sum of the roots of the equation $(M+1)x^2+2mx+3=0$ is 1, then the value of M is

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

10. *The roots of the equation* $2^{2x}-10.2^{x}+16=0$ *are*

(a) 2, 8 (b) 1, 3 (c) 1, 8 (d) 2, 3

11. *If* x^2+Px+1 *is a factor of* ax^3+bx+c *then*

(a) $a^2+c^2=-ab$ (b) $a^2-c^2=-ab$

(c) $a^2-c^2=ab$ (d) None of these

12. *If the roots of the equation* $\frac{a}{x-a}+\frac{b}{x-b}=1$ *are equal in magnitude and opposite in sign then*

(a) $a-b=0$ (b) $a+b=0$

(c) $a-b=1$ (d) $a+b=0$

13. *The* $a<b$ *then solution* $x^2+(a+b)x+ab<0$ *is given by*

(a) $a<x<b$ (b) $x<a \text{ or } x>b$

(c) $-b<x<-a$ (d) $x<-b, x<-a$

14. *If* $x=\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+----\alpha}}}}$

(a) x is an irrational number (b) $2<x<3$

(c) $x=3$ (d) None of these

15. *If* $2.x^{1/3}+2x^{-1/3}=5$ *then x is equal to*

(a) 1 or – 1 (b) 2 or $\frac{1}{2}$ (c) 8 or $\frac{1}{8}$ (d) 4 or $\frac{1}{4}$

16. *If* α,β *are roots of* $ax^2+bx+b=0$. *The* $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{b}{a}}$ *is*

(a) 0 (b) 1 (c) 2 (d) $2\sqrt{\frac{b}{a}}$

17. *If x satisfies* $|x-1|+|x-2|+|x-3|\geq 6$. *Then*

(a) $0\leq x\leq 4$ (b) $x\leq -2 or x\geq 4$

(c) $x\leq -or\, x\geq 4$ (d) None of these

18. *If* $x^2+6x-27>0$ *and* $x^2-3x-4<0$ *then*

(a) $x > 3$ (b) $x < 4$ (c) $3 < x < 4$ (d) $x = 3\frac{1}{2}$

19. *The value of x which satisfy both the in equations* $x^2 - 1 < 0$ *and* $x^2 - x - 2 \geq 0$ *lies on*

(a) $(-1, 2)$ (b) (-1, 1) (c) (1, 3) (d) { - 1}

20. *In a quadratic equation with leading coefficient 1, a student reads the coefficient 16 of x wrongly by 19 and obtain the roots as – 15 and – 4. The correct roots are*

(a) 6, 10 (b) – 6, - 10 (c) 8, 8 (d) – 8, - 8

21. *The equation* $x^2 - ax + b = 0$ *and* $x^2 + bx - a = 0$ *have common root, then*

(a) a = b (b) a + b = 0 (c) a – b = 1 (d) a – b = 2

22. *If the quadratic equation* $ax^2 + 2cx + b = 0$ *and* $ax^2 + 2bx + c = 0$ *have a common root. Then is equal to*

(a) – 1 (b) 3 (c) 0 (d) 2

23. *If the roots of* $ax^2 + bx + c = 0 (a > 0)$ *be greater then unity then*

(a) a + b + c =0 (b) a + b + c > 0

(c) a + b + c < 0 (d) None

24. *The number of real roots of equation* $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ *is*

(a) 2 (b) 1 (c) 0 (d) 3

25. *If the ratio of the roots of the equation* $x^2 + bx + c = 0$ *is the same as that of* $x^2 + qx + r = 0$ *then*

(a) $r^2 b = qc^2$ (b) $r^2 b = qb^2$

(c) $c^2 r = q^2 b$ (d) $b^2 r = q^2 c$

26. *One root of the equation* $(x+1)(x+3)(x+2)(x+4) = 120$

(a) – 1 (b) + 2 (c) 1 (d) = 0

27. *The solution of equation* $\left|3 + \frac{1}{x}\right| = 2$ *are*

(a) $0, -1$ (b) $2, -1$ (c) $-1, \frac{-1}{5}$ (d) None

28. *If* $f(x) = 2x^3 + mx^2 - 13x + n$ *and 2, 3 are roots of the equation* $f(x) = 0$. *Then the value of m and n are*

(a) – 5, –30 *(b) – 5, 30* *(c) 5, 30* *(d) None*

29. *If the equation* $x^2 - (2+m)x + (m^2 - 4m + 4)$ *has coincident roots, then*

(a) $m = 0, m = 1$ *(b)* $m = 0, m = 2$

(c) $m = \frac{2}{3}, m = 6$ *(d)* $m = \frac{2}{3}, m = 1$

30. *If one root of the equation* $(x-1)(7-x) = m$ *is three times the other, then m equals*

(a) – 5 *(b) 0* *(c) 2* *(d) 5*

SECTION – II

Assertion - Reason Questions

This section contains certain number of questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT - 2 (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct. Choose the correct option.

31. *Statement – 1 : The roots an equation* $4x^2 - 8x + 3 = 0$ *are* $\frac{1}{2}, \frac{3}{2}$

because

Statement – 2 : The roots of an equation $ax^2 + bx + c = 0$ *are* $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1

(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False

(d) Statement–1 is False, Statement–2 is True

32. *Statement – 1 : The product of the roots of* $9x^2+4x-11=0$ *is* $\frac{-4}{9}$

because

Statement – 2 : The product of the roots $ax^2+bx+c=0$ *is* $\frac{c}{a}$

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1
(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1
(c) Statement–1 is True, Statement–2 is False
(d) Statement–1 is False, Statement–2 is True

33. *Statement – 1 : If one root* $x^2-5x+k=0$ *is 2 then the value of k is 6.*

because

Statement – 2 : If $\Delta=0$ *then* $ax^2+bx+c=0$ *has equal roots.*
(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1
(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1
(c) Statement–1 is True, Statement–2 is False
(d) Statement–1 is False, Statement–2 is True

34. *Statement – 1 : A root of* $px^2+qx+r=0$ *is thrice the other root. Then* $13q^2=16pr$

because

Statement – 2 : The quadratic equation with roots α,β *is* $x^2-(\alpha+\beta)x+\alpha\beta$

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1
(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1
(c) Statement–1 is True, Statement–2 is False
(d) Statement–1 is False, Statement–2 is True

35. *Statement – 1 :* $x^2+x+1=0$ *has no real roots*

because

Statement – 2 : $ax^2+bx+c=0$ *has two real roots and distance roots if*

$\Delta > 0$

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1

(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False

(d) Statement–1 is False, Statement–2 is True

36. *Statement – 1 : If* $3x^2 - 7x + 6 = a(x-2)^2 + b(x-2) + c$ *is true of all values of x then* $a + b + c = 12$

because

Statement – 2 : The roots of $x^4 + kx^3 + kx^2 + lx + m = 0$ *are a, b, c, d. The minimum value of* $a^2 + b^2 + c^2 + d^2$ *is -1*

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1

(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False

(d) Statement–1 is False, Statement–2 is True

37. *Statement – 1 : If* $|x-2| + |x-3| = 7$ *then* $x = -1, 6$

because

Statement – 2 : $x^2 + ax + b = 0$ *and* $x^2 + bx + a = 0 (a \neq b)$ *have a common root then* $a + b = -1$

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1

(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False

(d) Statement–1 is False, Statement–2 is True

38. *Statement – 1 :* $e^{\sin x} - e^{-\sin x} - 4 = 0$ *has no real roots*

because

Statement – 2 : If $\Delta < 0$ *then* $ax^2 + bx + c = 0$ *has no real roots.*

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1

(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False
(d) Statement–1 is False, Statement–2 is True

39. *Statement – 1 : The roots of one equation* $x^2+5|x|+4=0$ *are not real.*

because

Statement – 2 : $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$ *has no solutions.*

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1
(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1
(c) Statement–1 is True, Statement–2 is False
(d) Statement–1 is False, Statement–2 is True

40. *Statement – 1 :* $x^2-7x+2m=0$ *and* $x^2-11x+4m=0$ *will have a common root then m = 6*

because

Statement – 2 : The number of real solution of the equation $(x+4)^3+(x+3)^3+(x+2)^3+(x+1)^2+(x-5)^3+180=0$ *is one.*

(a) Statement–1 is True, Statement–2 is True, Statement–2 is a correct explanation for Statement–1
(b) Statement–1 is True, Statement–2 is True, Statement–2 is NOT a correct explanation for Statement–1
(c) Statement–1 is True, Statement–2 is False
(d) Statement–1 is False, Statement–2 is True

SECTION - III

Linked Comprehension Type

This section contains paragraphs. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. Choose the correct option.

Paragraph for Question Nos. 41 to 43

If the roots of the equation $ax^2+bx+c=0$ *are* α,β *then*

(i) When both the roots are positive, $\alpha+\beta$ *and* $\alpha\beta$ *are positive.*

(ii) When both the roots are negative, $\alpha+\beta<0$ *and* $\alpha\beta>0$

(iii) When both the roots are equal but of opposite signs then $\alpha+\beta=0$

(iv) When the roots are reciprocal then $\alpha=\frac{1}{\beta}$ *i.e,* $\alpha\beta=1$

41. *If one root of the equation* $(l-m)x^2+lx+1=0$ *is double of the other and l is then greatest value of m is*

(a) $\frac{1}{2}$ *(b)* $\frac{9}{8}$ *(c)* $\frac{3}{4}$ *(d) 1*

42. *The equations* $ax^2+bx+c=0$ *and* $x^2+x+1=0$ *have a common root then*

(a) $a=b=c$ *(b)* $a+b+c=0$

(c) $a=b\neq c$ *(d)* $a+b+c=1$

43. $x^2-11x+k=0$ *and* $x^2-14x+2k=0$ *may have a common root then the value of k is*

(a) 0 (b) 12 (c) 24 (d) 5

Paragraph for Question Nos. 44 to 46

If both the roots α,β *of the given equation* $f(x)=ax^2+bx+c=0$ *are greater then a fixed quantity P. Then the conditions to be satisfied are :*

(i) roots must be real $\Delta\geq 0$

(ii) $f(p)>0$

(iii) $a+b>2p\Rightarrow\frac{-b}{a}>2p$

If both roots are less then, P then

(i) $\Delta\geq 0$ *(ii)* $f(p)>0$ *(iii)* $\frac{-b}{a}<2p$

If on root is lens then P and other is greater than P. the conditions to be satisfied are

(i) roots must be red and district $\Delta>0$

(ii) $f(P)<0$

44. *At what vale of K does the equation* $x^2+2(k-1)x+k+5=0$ *posses latest one positive root ?*

(a) $(-\alpha,-1]$ (b) $(\alpha,-1]$ (c) $(-\alpha,+\alpha]$ (d) $(-\alpha,2]$

45. If $a<b$ and $a<c<b$ then the roots of the equation $(a-b)^2x^2+2(a+b-2c)x+1=0$ are

(a) real and equal (b) red, not equal

(c) imaginary (d) none

46. If the equations $ax+by=1$ and $cx^2+dy^2=1$ have only one solution then the value of x is

(a) $\frac{a}{c}$ (b) $\frac{b}{d}$ (c) $\frac{a}{b}$ (d) $\frac{a}{d}$

Paragraph for Question Nos. 47 to 49

Let $f(x)=a(x-\alpha)(x-\beta)$

When $a>0, f(x)>0 \Rightarrow x<a$ or $x>\beta$

When $a<0, f(x)<0 \Rightarrow a<X<\beta$

47. If $(x-2)(x-5)>0$ then

(a) $x<2 or x>5$ (b) $2<x<5$

(c) $x<-2 or x>5$ (d) None

48. If $(x-2)(x-5)<0$ then

(a) $x<2 or x>5$ (b) $2<x<5$

(c) $x<-2 or x>5$ (d) None

49. If $(x+2)(x-5)>0$ then

(a) $x<2 or x>5$ (b) $2<x<5$

(c) $x<-2 or x>5$ (d) None

Paragraph for Question Nos. 50 to 52

If $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are two linear equations of first degree, then

(i) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow$ equations are dependent equations and number of solutions are infinite.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ *Number of solution of the equations are zero.*

(iii) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ *The number of solutions of the equations are finite.*

50. *Number of solution of the equations* $2x+3y+5=0$ *and* $6x+9y+15=0$ *are*

(a) finite (b) infinite (c) No solution (d) None

51. *Number of solutions of the equations* $2x+3y+5=0$ *and* $2x+3y=7=0$ *are*

(a) finite (b) infinite (c) No solution (d) None

52. *Number of solution of the equations* $2x+3y+5=0$ *and* $3x+2y+7=0$ *are*

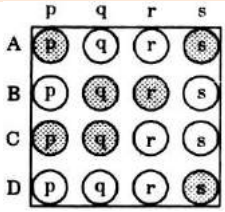
(a) finite (b) infinite (c) No solution (d) None

SECTION - IV

Matrix - Match Type

This section contains Matrix-Match type questions. Each question contains statements given in two columns which have to be matched. Statements (a, b, c, (d) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are a-p, a-s, b-q, b-r, c-p, c-q and d-s, then the correctly bubbled 4 x 4 matrix should be as follows:



53. *The quadratic equation whose roots are*

Column I	***Column II***
(a) 3, - 2	*(p)* $9x^2-15x+4=0$
(b) $\frac{4}{3},\frac{1}{3}$	*(q)* $x^2-x-6=0$
(c) $-2,-4$	*(r)* $x^2+6x+8=0$
(d) a, - b	*(s)* $x^2+(b-a)x-ab=0$

54. *The roots of the equation are*

Column I	***Column II***
(a) $x^2-4x-12=0$	*(p)* $\frac{4}{3},-2$
(b) $x^2+4-42=0$	*(q)* $6,-7$
(c) $x^2+16x+48=0$	*(r)* $6,-2$
(d) $3x^2+2x-8=0$	*(s)* $\frac{3}{2},\frac{-4}{5}$

55. *Nature of roots of the equations*

Column I	***Column II***
(a) $x^2-x+5=0$	*(p) real distinct and irrational*
(b) $5x^2-2x-7=0$	*(q) real equal*
(c) $3x^2+7x+20=0$	*(r) real, distinct and rational*
(d) $x^2+6x-5=0$	*(s) Complex*