Chapter 6
Lines and Angles

Exercise 6.1

Question:1 In Fig. 6.13, lines AB and CD intersect at O. If$\angle$ AOC + $\angle$ BOE = 70° and $\angle$ BOD = 40°, find$\angle$ BOE and reflex $\angle$ COE.

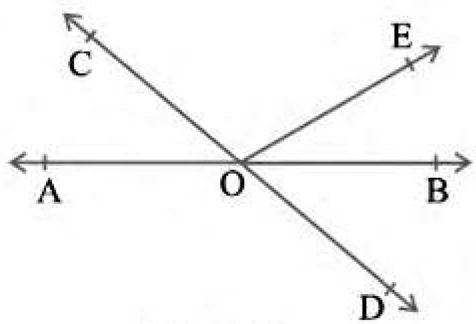


Fig. 6.13

Answer:

It is given: $\angle AOC + \angle BOE = 70^o$

And, $\angle BOD = 40^o$

Now, according to the question,

$\angle AOC + \angle BOE + \angle COE = 180^o$ (Sum of linear pair is always 180^o)

$\Rightarrow 70^o + \angle COE = 1^{80o}$

$\Rightarrow \angle COE = 110o$ ....(1)

And,

$\angle COE + \angle BOD + \angle BOE = 180^o$ (Sum of linear pair is always 180^o)

putting the value of $\angle COE$, we get,

$110^o + 40^o + \angle BOE = 180^o$

$150^o + \angle BOE = 180^o$

$\therefore \angle BOE = 30^o$

Now, reflex $\angle COE = 360^o - \angle COE$

$\Rightarrow$ reflex $\angle COE = 360^o - 110^o$

$\therefore$ reflex $\angle COE = 250^o$

Note: Reflex here means the reflex angles which are greater than 180° but less than 360°.

Question:2 In Fig. 6.14, lines XY and MN intersect at O. If $\angle$ POY = 90° and a: b = 2 : 3, find c.

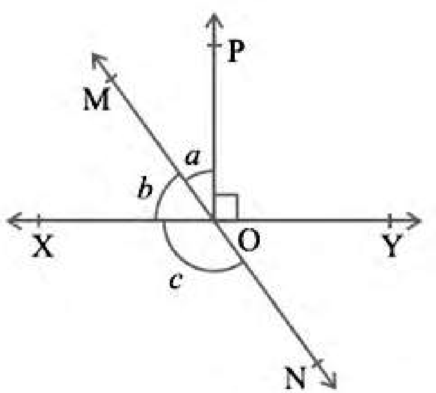


Fig. 6.14

Ans.: Given: $\angle POY = 90^O$

a: b = 2 : 3

Now, according to the question,

$\angle POY + a + b = 180^o$ (Angles made on a straight line are supplementary or sum of angles equals to 180°)

$90^o + a + b = 180^o$

$a + b = 90^o$ ....eq(1)

Now a:b = 2:3 ....eq(2)

Let a be 2 x and b be 3 x

So putting this value in eq(1),

$2x + 3x = 90^o$

$5x = 90^o$

$x = 18^o$

Hence,

$a = 2 \times 18^o = 36^o$

And,

$b = 3 \times 18^o = 54^o$

And,

$b + c = 180^o$ (Linear pair)

$54^o + c = 180^o$

$c = 126^o$

Question: 3 In Fig. 6.15, ∠ PQR = ∠ PRQ, then prove that ∠ PQS = ∠ PRT.

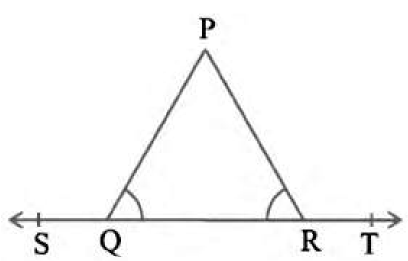


Fig. 6.15

Ans.:

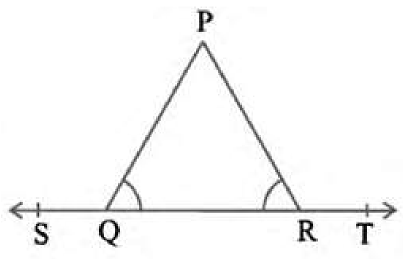


Fig. 6.15

Given: ∠PQR = ∠PRQ

To prove: ∠PQS = ∠PRT

Proof:

Now,

∠PQR + ∠PQS = 180^o (Angles on a straight line are supplementary)

⇒ ∠PQR = 180^o - ∠PQS ......(i)

Similarly,

∠PRQ + ∠PRT = 180^o (Angles on a straight line are supplementary)

⇒ ∠PRQ = 180^o - ∠PRT .....(ii)

Now, ∠PQR = ∠PRQ (Given)

Therefore, (i) and (ii) will be equal 180^o - ∠PQS = 180^o - ∠PRT

∠PQS = ∠PRT

Hence, Proved.

Question: 4 In Fig. 6.16, if x + y = w + z, then prove that AOB is a line.

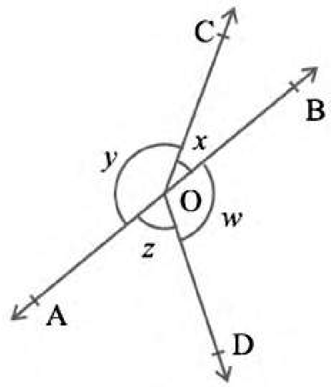


Fig. 6.16

Ans.: Given: x + y = w + z

To prove: AOB is a line or x + y = 180° (Linear pair)

Proof:

Now, according to the question,

$x + y + w + z = 360^{o}$ (Angles at a point)

$(x + y) + (w + z) = 360°$

(Given, x + y = w + z)

$2 (x + y) = 360^{o}$

$(x + y) = 180^{o}$

Therefore, x + y makes a linear pair.

And, AOB is a straight line

Hence, Proved.

Question: 5 In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ ROS $= \frac{1}{2}$(∠ QOS –∠ POS).

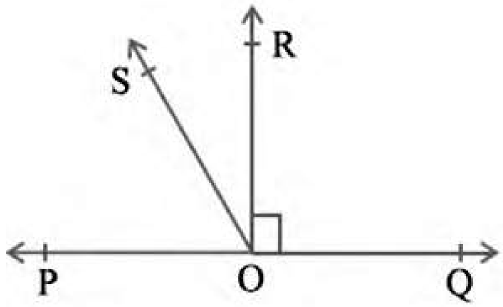


Fig. 6.17

Ans.:

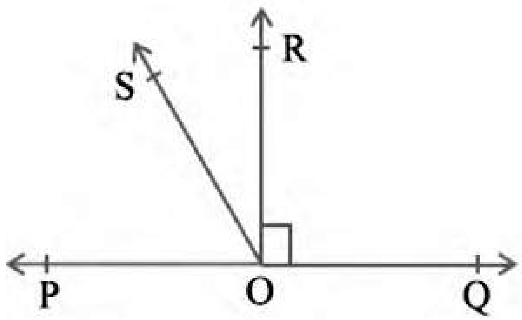


Fig. 6.17

Given: OR is perpendicular to line PQ

To prove: ∠ROS = (∠QOS - ∠POS)

Proof:

Now, according to the question,

∠POR = ∠ROQ = 90° (∵ OR is perpendicular to line PQ)

∠QOS = ∠ROQ + ∠ROS = 90° + ∠ROS ....eq(i)

We can write,

∠POS = ∠POR - ∠ROS = 90° - ∠ROS ....eq(ii)

Subtracting (ii) from (i), we get

∠QOS - ∠POS = 90° + ∠ROS – (90° - ∠ROS)

$\angle QOS - \angle POS = 90^o + \angle ROS - 90° + \angle ROS$

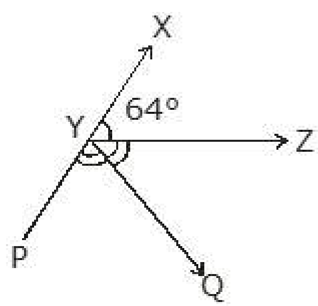
$\angle QOS - \angle POS = 2\angle ROS$

$\angle ROS = \frac{1}{2}(\angle QOS) - (\angle POS)$

Hence, proved

Question: 6 It is given that ∠ XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ ZYP, find ∠ XYQ and reflex ∠ QYP.

Ans.:



Given: ∠XYZ = 64o

YQ bisects ∠ZYP, therefore ∠QYP = ∠QYZ

$\angle XYZ + \angle ZYP = 180^o$ (Sum of angles made on a straight line = 180º)

$64^o + \angle ZYP = 180^o$

$\angle ZYP = 116^o$

And,

$\angle ZYP = \angle ZYQ + \angle QYP$

$\angle ZYQ = \angle QYP$ (YQ bisects ∠ZYP)

$\angle ZYP = 2\angle ZYQ$

$2\angle ZYQ = 116^o$

$\angle ZYQ = 58^o = \angle QYP$

Now,

$\angle XYQ = \angle XYZ + \angle ZYQ$

$\angle XYQ = 64^o + 58^o$

$\angle XYQ = 122^o$

And,

Reflex of $\angle QYP = 180^o + \angle XYQ$

$\angle QYP = 180^o + 122^o$

$\angle QYP = 302^o$

Exercise 6.2

Question: 1 In Fig. 6.28, find the values of x and y and then show that AB || CD.

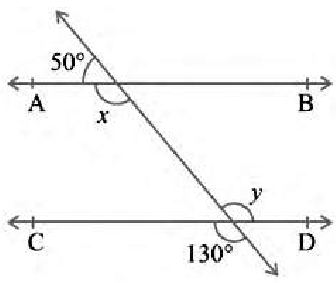


Fig. 6.28

Answer:

From the figure:

$x + 50^o = 180^o$ (Linear pair)

$x = 180^o - 50^o$

$x = 130^o$

And,

$y = 130^o$ (Vertically opposite angles)

Now,

$x = y = 130^o$ (Alternate interior angles)

Hence,

The theorem says that when the lines are parallel, that the alternate interior angles are equal. And thus the lines must be parallel.

AB || CD

Question: 2 In Fig. 6.29, if AB || CD, CD || EF and y: z = 3: 7, Find x.

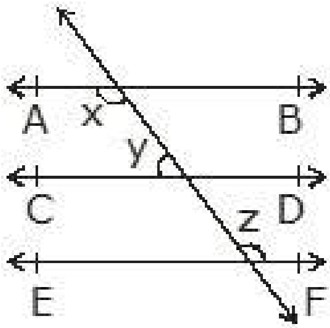
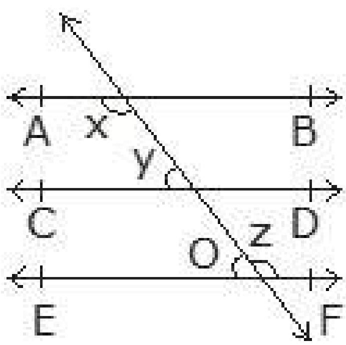


Fig. 6.29

Ans.



It is given in the question that:

AB || CD,

Thus, ∠x + ∠y = 180°

(Interior angles on same side of transversal)

(1) Also, AB || CD and CD || EF

Thus, AB || EF

⇒ ∠x = ∠z (Alternate interior angles)

(2)From (1) and (2) we can say that,

∠z + ∠y = 180°

(3) [Angles will be supplementary] It is given that, ∠y : ∠z = 3 : 7 (4)Let ∠y = 3 a and ∠z = 7

a Putting these values in (2)

3a + 7a = 180°

10a = 180°a

= 18°∠z = 7a

z = 7 x 18°

∠z = 126°

As ∠z = ∠x ∠x = 126°

Question: 3 In Fig. 6.30, if AB || CD, EF ⊥ CD and∠ GED = 126°, find ∠ AGE, ∠ GEF and ∠ FGE.

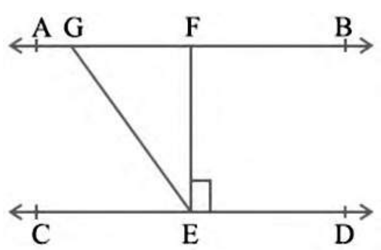


Fig. 6.30

Ans.:

It is given in the question that:

AB || CD

EF perpendicular CD

∠GED = 126°

Now, according to the question,

∠FED = 90° (EF perpendicular CD)

Now,

∠AGE = ∠GED (Since, AB parallel CD and GE is transversal, hence alternate interior angles)

Therefore,

$\angle AGE = 126^o$

And,

∠GEF = ∠GED - ∠FED

$\angle GEF = 126^o - 90^o$

$\angle GEF = 36^o$

Now,

$\angle FGE + \angle AGE = 180^o$ (Linear pair)

$\angle FGE = 180^o - 126^o$

$\angle FGE = 54^o$

Question: 4 In Fig. 6.31, if PQ || ST, ∠ PQR = 110° and∠ RST = 130°, find ∠ QRS.

[Hint: Draw a line parallel to ST through point R.]

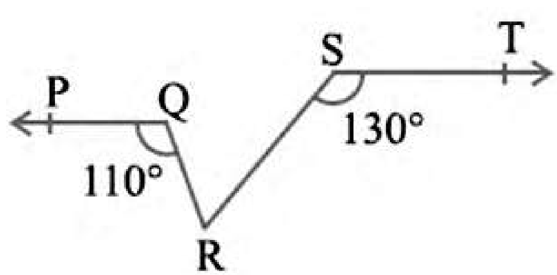


Fig. 6.31

Ans.:

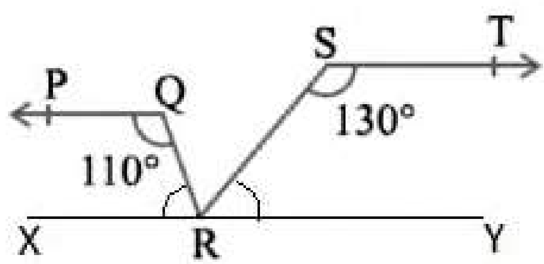
It is given in the question that:

PQ parallel ST,

$\angle PQR = 110^o$ and,

$\angle RST = 130^o$

Construction: Draw a line XY parallel to PQ and ST



$\angle PQR + \angle QRX = 180^o$ (Angles on the same side of transversal)

$110^o + \angle QRX = 180^o$

$\angle QRX = 70^o$

And,

$\angle RST + \angle SRY = 180^o$ (Angles on the same side of transversal)

$130^o + \angle SRY = 180^o$

$\angle SRY = 50^o$

Now,

$\angle QRX + \angle SRY + \angle QRS = 180^o$ (Angle made on a straight line)

$70^o + 50^o + \angle QRS = 180^o$

$\angle QRS = 60^o$

Question: 5 In Fig. 6.32, if AB || CD, ∠ APQ = 50° and∠ PRD = 127°, find x and y.

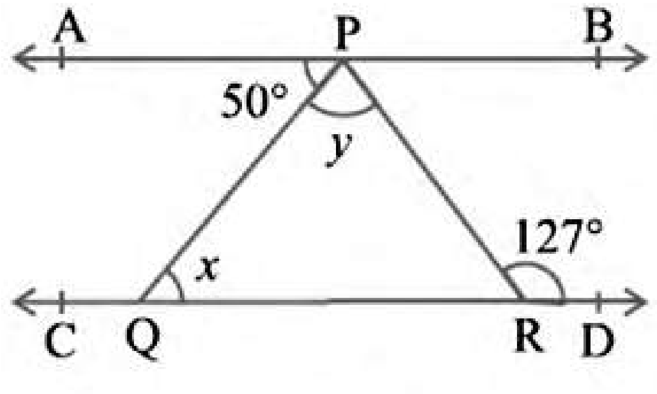


Fig. 6.32

Ans.;

It is given in the question that:

AB parallel CD,

∠APQ = 50° and,

∠PRD = 127°

According to question,

x = 50° (Alternate interior angle)

∠PRD + ∠PRQ = 180° (Angles on the straight line are supplementary)

127° + ∠PRQ = 18^{0o}

∠PRQ = 53°

Now,

In Δ PQR,

x + y + ∠PRQ = 180° (Sum of interior angles of a triangle)

y + 50° + ∠ PRQ = 180°

$y + 50^o + 53^o = 180^o$

$y + 103^o = 180^o$

$y = 77^o$

Question: 6 In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.

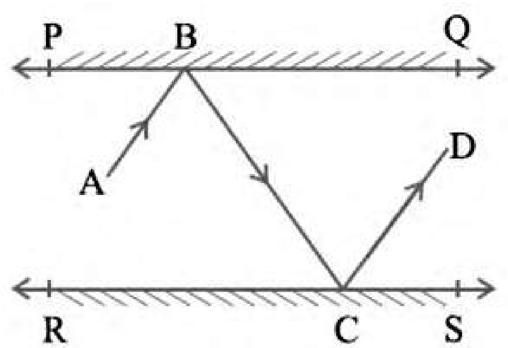


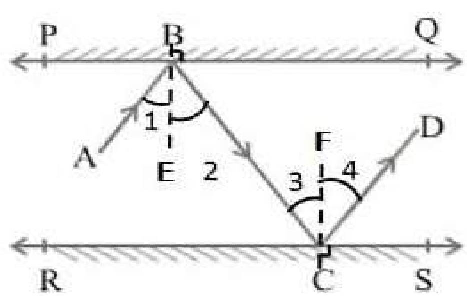
Fig. 6.33

Ans.:

Given: Two mirrors are parallel to each other, PQ||RS

To Prove: AB || CD

Proof:



Take two perpendiculars BE and CF, As the mirrors are parallel to each other their perpendiculars will also be parallel thus BE || CF

According to laws of reflection, we know that:

Angle of incidence = Angle of reflection

$\angle 1 = \angle 2$ and,

$\angle 3 = \angle 4$ (i)

And,

$\angle 2 = \angle 3$ (Alternate interior angles, since BE, is parallel to CF and a transversal line BC cuts them at B and C respectively) (ii)

We need to prove that $\angle ABC = \angle DCB \angle ABC = \angle 1 + \angle 2$ and $\angle DCB = \angle 3 + \angle 4$

From (i) and (ii), we get

$\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \angle ABC = \angle DCB$

$\Rightarrow$ AB parallel CD (Alternate interior angles)

Hence, Proved.

Exercise 6.3

Question: 1 In Fig. 6.39, sides QP and RQ of Δ PQR are produced to points S and T respectively. If ∠ SPR = 135° and ∠ PQT = 110°, find ∠ PRQ.

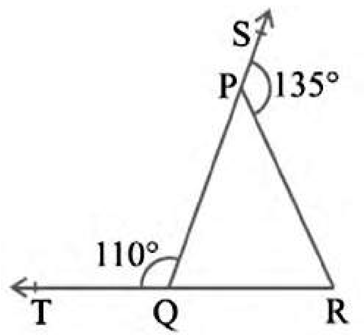


Fig. 6.39

Ans.:

Method 1:

It is given in the question that:

$\angle SPR = 135^{O}$

And,

$\angle PQT = 110^{o}$

Now, according to the question,

$\angle SPR + \angle QPR = 180°$ (SQ is a straight line)

$135° + \angle QPR = 180°$

$\angle QPR = 45°$

And,

$\angle PQT + \angle PQR = 180°$ (TR is a straight line)

$110° + \angle PQR = 180°$

$\angle PQR = 70°$

Now,

$\angle PQR + \angle QPR + \angle PRQ = 180°$ (Sum of the interior angles of the triangle)

$70° + 45° + \angle PRQ = 180°$

$115° + \angle PRQ = 180°$

$\angle PRQ = 65°$

Method 2:

It is given in the question that:

$\angle SPR = 135^{\circ}$

And,

$\angle PQT = 110°$

$\angle PQT + \angle PQR = 180°$ (TR is a straight line)

$110° + \angle PQR = 180°$

$\angle PQR = 70°$

Now, we know that the exterior angle of the triangle equals the sum of interior opposite angles. Therefore, $\angle SPR = \angle PQR + \angle PRQ$

$135^{\circ} = 70° + \angle PRQ$

$\angle PRQ = 65^{\circ}$

Question: 2 In Fig. 6.40, ∠ X = 62°, ∠ XYZ = 54°. If YO and ZO are the bisectors of ∠ XYZ and∠ XZY respectively of Δ XYZ, find ∠ OZY and ∠ YOZ.

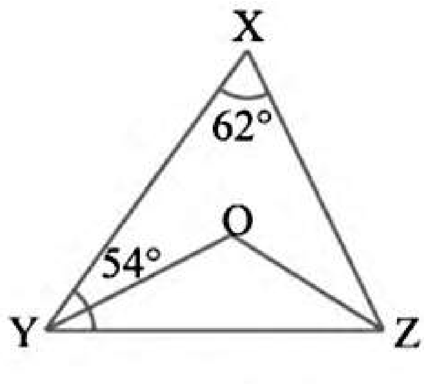


Fig. 6.40

Ans.:

Given: $\angle X = 62^o$, $\angle XYZ = 54^o$

YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively.

To Find: ∠ OZY and ∠ YOZ.

Now, according to the question,

$\angle X + \angle XYZ + \angle XZY = 180^o$ (Sum of the interior angles of a triangle = 180°)

$62^o + 54^o + \angle XZY = 180^o$

$116^o + \angle XZY = 180^o$

$\angle XZY = 64^o$

Now,

As ZO is the bisector of ∠ XZY

$\angle OZY = \frac{1}{2} \angle XZY$

$\angle OZY = 32^o$

And,

As YO is bisector of ∠XYZ

$\angle OYZ = \frac{1}{2}\angle XYZ$

∠OYZ = 27º

Now,

∠OZY + ∠OYZ + ∠O = 180º

(Sum of the interior angles of the triangle = 180°)

= 32º + 27º + ∠O = 180º

= 59º + ∠O = 180º

= ∠O = 121º

Question: 3 In Fig. 6.41, if AB || DE, ∠ BAC = 35° and ∠ CDE = 53°, find ∠ DCE.

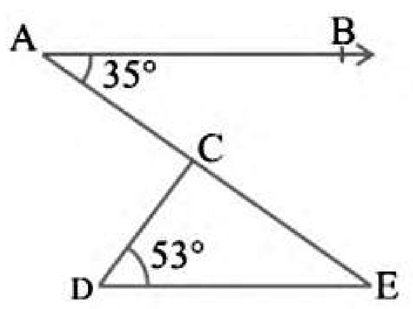


Fig. 6.41

Ans.: Given: AB parallel DE, ∠BAC = 35º, ∠CDE = 53º

To Find: ∠DCE

According to question,

∠BAC = ∠CED (Alternate interior angles)

Therefore,

∠CED = 35°

Now, In Δ DEC,

∠DCE + ∠CED + ∠CDE = 180°

(Sum of the interior angles of the triangle)

∠DCE + 35° + 53° = 180°

∠DCE + 88° = 180°

∠DCE = 92°

Question: 4 In Fig. 6.42, if lines PQ and RS intersect at point T, such that ∠ PRT = 40°, ∠ RPT = 95°and ∠ TSQ = 75°, find ∠ SQT.

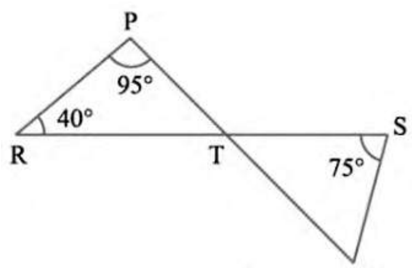


Fig. 6.42

Ans.:

Given:

∠PRT = 40°

∠RPT = 95° and,

∠TSQ = 75°

Now according to the question,

∠PRT + ∠RPT + ∠PTR = 180° (Sum of interior angles of the triangle)

40° + 95° + ∠PTR = 180°

$40^o + 95^o + \angle PTR = 180^o$

$135^o + \angle PTR = 180^o$

$\angle PTR = 45^o$

$\angle PTR = \angle STQ = 45^o$ (Vertically opposite angles)

Now,

$\angle TSQ + \angle PTR + \angle SQT = 180^o$ (Sum of the interior angles of the triangle)

$75^o + 45^o + \angle SQT = 180^o$

$120^o + \angle SQT = 180^o$

$\angle SQT = 60^o$

Question: 5 In Fig. 6.43, if PQ ⊥ PS, PQ || SR, ∠ SQR = 28° and ∠ QRT = 65°, then find the values of x and y.

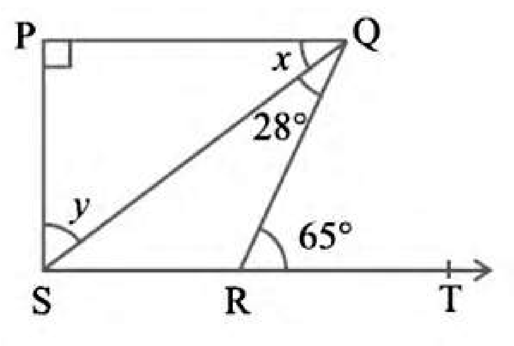


Fig. 6.43

Ans.:

To Find: Values of x and y

Given: PQ is perpendicular to PS, PQ parallel SR

$\angle SQR = 28^o$

And, $\angle QRT = 65^o$

Now according to the question,

x + ∠SQR = ∠QRT (Alternate angles are equal as QR is transversal)

x + 28° = 65°

x = 37°

Now, in Δ PQS, Sum of interior angles of a triangle = 180°

∠ PQS + ∠ PSQ + ∠ QPS = 180°Therefore,

y + 37° + 90° = 180°

y= 53°So x=37° and y= 53°

Question: 6 In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR =1/2 ∠QPR.

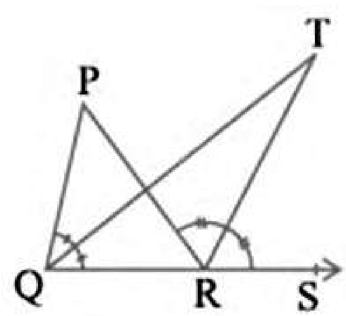


Fig. 6.44

Ans.: To Prove: ∠QTR = $\frac{1}{2}$ ∠QPR

Given: Bisectors of ∠PQR and ∠PRS meet at point T

Proof:

In ΔQTR,

∠TRS = ∠TQR + ∠QTR (Exterior angle of a triangle equals to the sum of the two opposite interior angles)

$\angle QTR = \angle TRS - \angle TQR$ -----------(i)

Similarly, in ΔQPR,

$\angle SRP = \angle QPR + \angle PQR$

$2\angle TRS = \angle QPR + 2\angle TQR$

($\because$ $\angle TRS$ and $\angle TQR$ are the bisectors of $\angle SRP$ and $\angle PQR$ respectively.)

$\angle QPR = 2\angle TRS – 2\angle TQR$

$\angle TRS - \angle TQR = \frac{1}{2}\angle QPR$ ....(ii)

From (i) and (ii), we get

$\angle QTR = \frac{1}{2}\angle QPR$

Hence, proved.