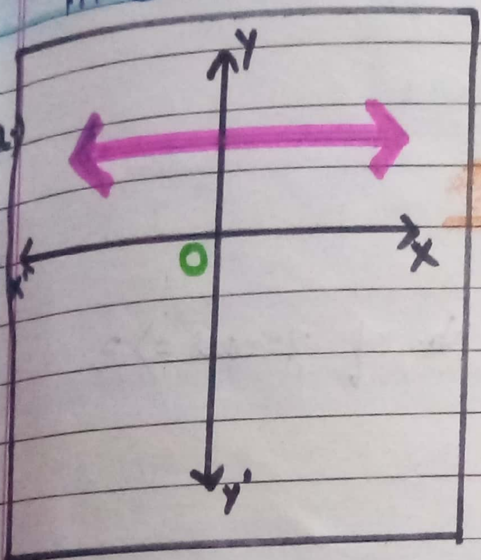
Chapter :- 2

Polynomial

Ex :- 2.1

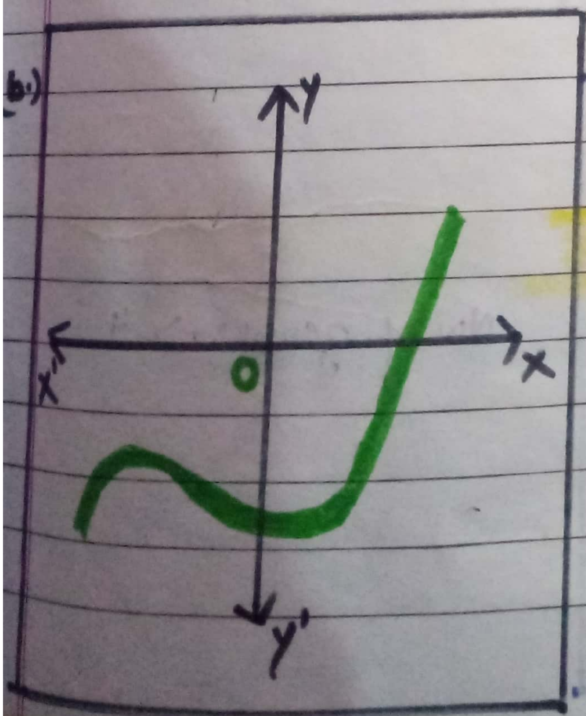
Ques1.) The graphs of $y = p(x)$ are given below for some polynomials $p(x)$. Find the No. of zeroes of $p(x)$ in each case.

(a.)



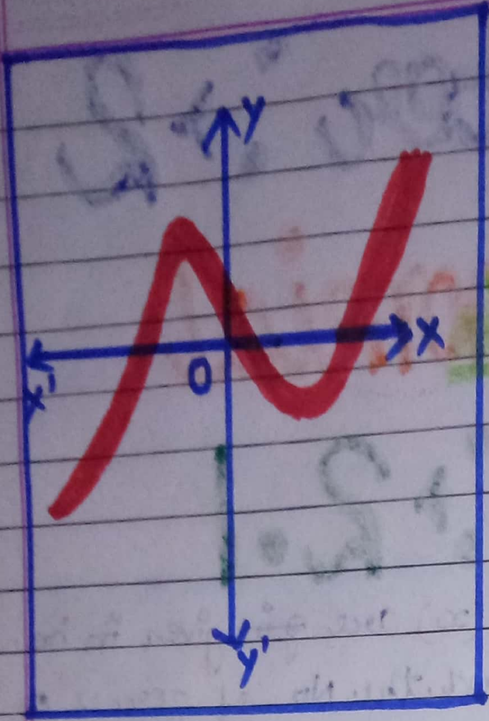
Ans!- No. of zeroes => 0

(b.)



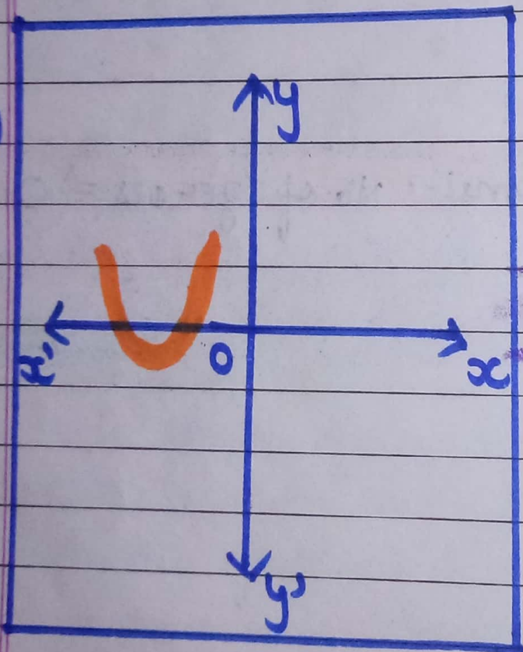
Ans!- No. of zeroes => 1

(c.)



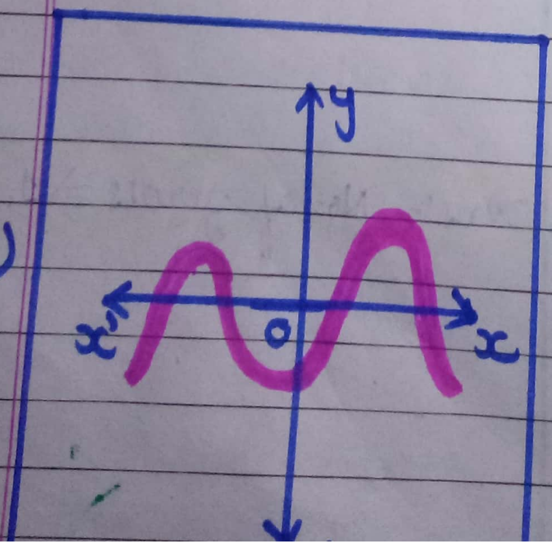
No. of zeroes => 3

(d.)



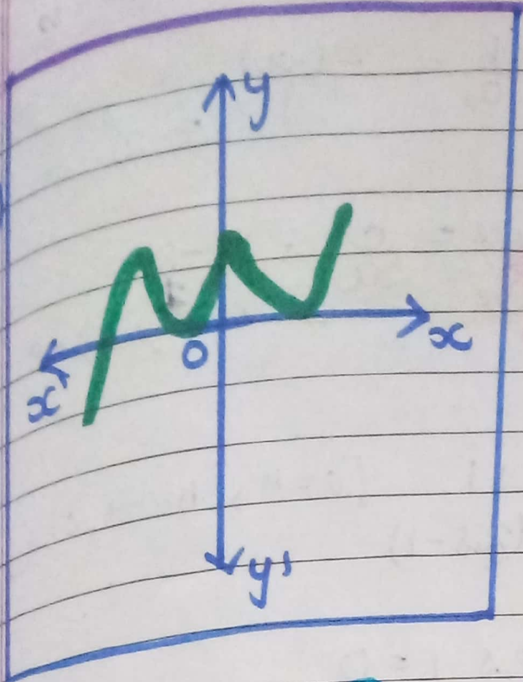
No. of zeroes => 2

(e.)



No. of zeroes => 4

(f.)



No. of zeroes => 3

Ex :- 2.2

Ques.) Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the cofficients :-

(i.) $x^2 - 2x - 8$

Ans:- $P(x) = x^2 - 4x + \cancel{2}2x - 8$

$x(x-4) + 2(x-4)$

$(x-4)\ (x+2)$

$x - 4 = 0$ | $x + 2 = 0$

$x = 4$ | $x = -2$

Verification :-

Sum of zeroes $= 4 + (-2)$

$= 4 - 2 = 2$

Product of zeroes $= 4 \times (-2)$

$= -8$

Sum of zeroes = $\frac{-b}{a} = \frac{-(-2)}{1} = 2$

Product of zeroes = $\frac{c}{a} = \frac{-8}{1} \Rightarrow -8$

(b.) $4s^2-4s+1$

Ans:- $P(x) = 4s^2-2s-2s+1$ $[a=4, b=-4, c=1]$

$2s(2s-1)-1(2s-1)$

$(2s-1)(2s-1)$

$2s-1=0$	$2s-1=0$
$2s=0+1$	$2s=0+1$
$2s=+1$	$2s=1$
$s=\frac{+1}{2}$	$s=\frac{1}{2}$

Verification:-

Sum of zeroes $\Rightarrow \frac{1}{2}+\frac{1}{2}$

$= \frac{1+1}{2} = \frac{\cancel{2}}{\cancel{2}}$

$= 1$

Product of zeroes = $\frac{1}{2} \times \frac{1}{2} \Rightarrow \frac{1}{4}$

Sum of zeroes = $\frac{-b}{a} \Rightarrow \frac{-(-\cancel{4})}{\cancel{4}}$

$= 1$

Product of zeroes = $\frac{c}{a}$

$= \frac{1}{4}$

(c) $6x^2-3-7x$

Ans: $p(x) = 6x^2-7x-3$

$6x^2+2x-9x-3$

$2x(3x+1) \; -3(3x+1)$

$(3x+1)\,(2x-3)$

$3x+1=0$	$2x-3=0$
$3x=0-1$	$2x=0+3=3$
$x=\frac{-1}{3}$	$x=\frac{3}{2}$

Verification :-

Sum of zeroes $= \frac{-1}{3}+\frac{3}{2}$

$= \frac{-2+9}{6} = \frac{7}{6}$

Product of zeroes $= \frac{-1}{3}\times\left(\frac{3}{2}\right)$

$= \frac{-\cancel{3}}{\cancel{6}} = \frac{-1}{2}$

Sum of zeroes $= \frac{-b}{a} =$

$= -\left(\frac{-7}{6}\right)$

$= \frac{7}{6}$

Product of zeroes $= \frac{c}{a} = \frac{-\cancel{3}}{\cancel{6}}$

$= \frac{-1}{2}$

(d) $4u^2+8u$

Ans:- $a=4, b=8, c=0$

$4u(u+2)$

$4u=0$

$u=\frac{0}{4}$

$\boxed{u=0}$

$u+2=0$

$u=0-2$

$\boxed{u=-2}$

Verification :-

Sum of zeroes $= 0+(-2)$

$= 0-2 = -2$

Product of zeroes $= 0\times -2 = 0$

Sum of zeroes $= \frac{-b}{a} = \frac{-\cancel{8}}{\cancel{4}} = -2$

Product of zeroes $= \frac{c}{a} = \frac{0}{4}$

$=0$

(v.) t^2-15

Ans:- $a=1, b=0, c=15$

$(t)^2-(\sqrt{15})^2$ $[\because a^2-b^2=(a+b)(a-b)]$

$(t+\sqrt{15})$ $(t-\sqrt{15})$

$t+\sqrt{15}=0$

$t=-\sqrt{15}$

$t-\sqrt{15}=0$

$t=\sqrt{15}$

Verification:-

Sum of zeroes $= -\cancel{\sqrt{15}}+\cancel{\sqrt{15}}=0$

Product of zeroes $= -\sqrt{15} \times \sqrt{15}$

$= -\sqrt{15}$

Sum of zeroes $= \frac{-b}{a} = \frac{-0}{1}$

$= 0$

Product of zeroes $= \frac{c}{a} = \frac{-15}{1}$

$= -15$

(vi) $3x^2 - x - 4$

Ans:- $a = 3,\ b = 1,\ c = 4$

$3x^2 - 4x + 3x - 4$

$x(3x-4) + 1(3x-4)$

$(3x-4)\ (x+1)$

$3x - 4 = 0$	$x + 1 = 0$
$3x = 4$	$\boxed{x = -1}$
$\boxed{x = \frac{4}{3}}$	

Verification:-

Sum of zeroes $= \frac{4}{3} + (-1)$

$= \frac{4-3}{3} = \frac{1}{3}$

Product of zeroes $= \frac{4}{3} \times (-1) = \frac{-4}{3}$

Sum of zeroes $= \frac{-b}{a} = \frac{-(-1)}{3} = \frac{1}{3}$

Product of zeroes $= \frac{c}{a} = \frac{-4}{3}$

Ques (2) Find the quadratic polynomial each with the given No. as the sum and product of its zeroes respectively.

P = Product

S = Sum of roots

(i) $\frac{1}{4}, -1$

Ans:- $x^2 - Sx + P$

$x^2 - \frac{1}{4}x - 1$

$\left(\frac{4x^2 - 1x - 4}{4}\right)$

$\frac{k}{4}(4x^2 - 1x - 4)$

where k is real No.

$4x^2 - 1x - 4$ Ans

(ii) $\sqrt{2}, \frac{1}{3}$

Ans:- $x^2 - Sx + P$

$x^2 - \sqrt{2}x + 1$

$\left(\frac{3x^2 - 3\sqrt{2}x + 1}{3}\right)$

$\frac{k}{3}(3x^2 - 3\sqrt{2}x + 1)$

Where k is real No.

$3x^2 - 3\sqrt{2}x + 1$ Ans

(iii) $0, \sqrt{5}$

Ans. $x^2 - Sx + P$

$x^2 - 0 \times x + \sqrt{5}$

$x^2 - 0 + \sqrt{5}$

$x^2 + \sqrt{5}$ Ans

(iv) $1, 1$

Ans. $x^2 - Sx + P$

$x^2 - 1 \times x + 1$

$x^2 - 1x + 1$ Ans

(v) $\frac{-1}{4}, \frac{1}{4}$

Ans. $x^2 - Sx + P$

$x^2 - \left(\frac{-1}{4}\right)x + \frac{1}{4}$

$x^2 + \frac{1x}{4} + \frac{1}{4}$

$\frac{4x^2 + 1x + 1}{4}$

$\frac{K}{4}(4x^2 + 1x + 1)$

Where K is real No.

$4x^2 + 1x + 1$

(vi.) (4, 1)

Ans:- $x^2 - Sx + P$

$x^2 - 4x + 1$ Ans

Ex :- 2.3

Ques(1) Divide the polynomial $p(x)$ by the polynomial g(x) and find the Quotient and remainder in each of the following :

(i) $x^2 - 2 \overline{) x^3 - 3x^2 + 5x - 3}$ ($x - 3 \rightarrow$ Quotient

$\cancel{x^3} \quad - 2x$

$-3x^2 + 7x - 3$

$\cancel{-3x^2} \quad + 6$

$7x - 9 \rightarrow$ Remainder

$x^2 - 2$
$\times\ x$
$x^3 - 2x$

$x^2 - 2$
$\times\ -3$
$-3x^2 + 6$

(ii) $x^2 + 1 - x \overline{) x^4 - 3x^2 + 4x + 5}$ ($x^2 + x - 3$ ~~quotient~~

$\cancel{x^4} + 1x^2 \quad - x^3$

$-4x^2 + 4x + 5 \quad \cancel{+x^3}$

$-1x^2 + 1x \quad \cancel{+x^3}$

$\cancel{-3x^2} + \cancel{3x} + 5$

$\cancel{-3x^2} + \cancel{3x} - 3$

$8 \rightarrow$ Remainder

$x^2 + 1 - x$
$\times\ x^2$
$x^4 + x^2 - x^3$

$x^2 + 1 - x$
$\times\ x$
$x^3 + x - x^2$

$x^2 + 1 - x$
$\times\ -3$
$-3x^2 - 3 + 3x$

$$-x^2-2$$

(iii) $2-x^2 \overline{\big)\, x^4 - 5x + 6}$

$\cancel{x^4} \quad -2x^2$

$-5x + 6 + \cancel{2x^2}$

$-4 + \cancel{2x^2}$

$-5x + 10$

$2-x^2$
$x - x^2$
$-2x^2 + x^4$
$2-x^2$
$x - 2$
$-4+2x^2$

Ques(3.) Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$ if two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans:- $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)$

$x^2-\frac{5}{3}$

Now $x^2-\frac{5}{3} \overline{\big)\, 3x^4+6x^3-2x^2-10x-5} \big(3x^2+6x+3$

$\cancel{3x^4} \quad -5x^2$

$\cancel{6x^3} + 3x^2 - \cancel{10x} - 5$

$\cancel{6x^3} \quad \cancel{-10x}$

$\cancel{3x^2} - \cancel{5}$

$\cancel{3x^2} - \cancel{5}$

0

$3x^2+6x+3$

$3x^2+3x+3x+3$

$3x(x+1)\ 3(x+1)$

$(x+1)(3x+3)$

$3(x+1)(x+1)$

$3(x+1)=0$

$x+1=\frac{0}{3}$

$x+1=0$

$\boxed{x=-1}$

$x+1=0$

$\boxed{x=-1}$

zeroes are $x=-1$ and $x=-1$

Ques(4) On dividing x^3-3x^2+x+2 by a polynomial $g(x)$ the quotient and remainder were $(x-2)$ $(-2x+4)$ respectively. Find $g(x)$

Ans:- Dividend = Quotient × Divisor + Remainder

$$x^3-3x^2+x+2-4+2x = (x-2)\times g(x)$$

$$x^3-3x^2+3x-2 = (x-2)\,g(x)$$

$$g(x) = \frac{x^3-3x^2+3x-2}{x-2}$$

$$x-2 \,\big)\, x^3-3x^2+3x-2 \,\big(\, x^2-x+1$$

$$\cancel{x^3} - 2x^2$$

$$(-) \quad (+)$$

$$-\cancel{x^2}+3x-2$$

$$-\cancel{x^2}+2x$$

$$(+) \quad (-)$$

$$\cancel{x}-\cancel{2}$$

$$x-2$$

$$(-) \quad (+)$$

$$0$$

Hence $g(x) = x^2-x+1$

Ques 5) Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

Ans:- Let $p(x) = x-1$
and $g(x) = 1$
when we divide $p(x)$ by $g(x)$, then we get the quotient $q(x) = x-1$ and remainder $r(x) = 0$
Hence, $\deg p(x) = \deg q(x)$

(ii) $\deg q(x) = \deg r(x)$

Ans:- Let $p(x) = x^3 - 3x^2 + 5x - 3$
and $g(x) = x^2 - 2$
When we divide $p(x)$ by $g(x)$ then we get the quotient $q(x) = x-3$ and remainder $r(x) = 7x-9$
Hence $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

Ans:- Let $p(x) = -x^3 + 3x^2 - 3x + 5$
and $g(x) = -x^2 + x - 1$
when we divide $p(x)$ by $g(x)$ then we get get the quotient $q(x) = x-2$ and remainder $r(x) = 3$

Hence, $\deg r(x) = 0$