Chapter - 4
Electrical Capacitance

In chapter third we made ourselves familiar with the electrostatic potential energy of a system of charges. In this chapter our basic aim is to study about "capacitor" a device in which electrical energy can be stored. The electrical energy so stored can be recovered in other forms of energy. For example the flash attachment in a camera uses electrical energy stored by a capacitor. Once the capaciter is charged it can supply energy at a much greater rate by discharing through the associated circuit to provide a sudden bright flash of light, in cameras. Capacitors have many uses as a circuit element in various electric and electronic circuits.

4.1 Conductors and Insulators

On the basis of their ability to conduct electricity materials found in nature can be classified into two broad categories.

(a) Conductors and (b) Insulators

Conductors - The materials in which electric charges and electric current can flow easily for exanple silver, copper, aluminium, iron, mercury, common salt solution, human body and earth etc. are conductors of electricity. Among these silver is the best conductor of electricity.

Insulators - Ideally in such materials electrical current can not flow. Such materials are called insulators or dielectrics. For example rubber, glass, plastic, ebonite, dry wood etc are all insulators.

In addition to these, solid insulators materials can also be semiconductors. We will learn more about semiconductors in chapter 16. In that chapter we will also learn why different solid materials have different behaviour towards electric conduction.

4.2 Free and Bound Charges

Every material is composed of atoms. Each atom contains a positively charged nucleus and several electrons revoling around it. The electrons belonging to inner orbits are subjected to a greater attractive force from the nucleus and are tightly bound to their respective parent atoms.

In metallic solids outer electrons (valence electrons) of each atom are only weakly bound to nucleus. These electrons are almost free to move with in the bulk of the material and are called free electrons. However such free electrons can not escape through the surface of metal. Atoms after loosing valence electron(s) called as positive ions are rigidly bound at their respective locations in the solid. In such materials when an external electric field is applied, these free electrons move in direction opposite to field, consitituting an electric current. Inner electrons are bound to nucleus and can not contribute to electric current. The same is true for positive ions of metals, both these are bound charges. There is no contribution from bound charges in current flow in solids. In electrolytic solutions current flow is due to positive and negative ions.

4.3 Dielectric materials and Polarization

Dielectric materials do not conduct electricity but exhibit electrical effect when subjected to external electric field. In dielectric materials, effectively there are no free electrons hence no electrical conduction. The effect of external field results in a slight rearrangement of charges in atoms or molecules of the dielectric material, however, this rearrangement is enough to modify the electric field inside the material. The dielectric materials can be divided into two categories (a) polar dielectrics and (b) non olar dielectrics.

(a) Polar dielectric : For polar dielectric material, centre of negative charge distribution do not coincide with centre of positive charge distribution. The separation between centres of negative and positive charge distribution causes molecules to have a permanent dipole moment. Such molecules are known as polar molecules. An ionic molecule like HCl (Fig 4.1a) or a molecule of water (H_2O) (Fig 4.1 b) are examples of polar molecule.

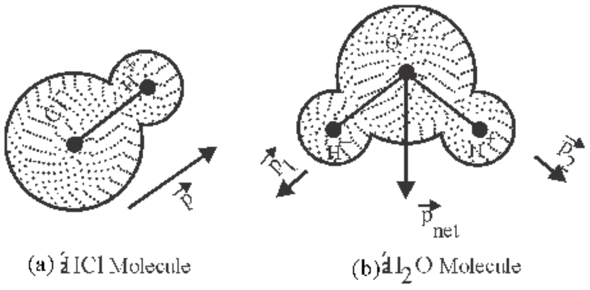


Fig 4.1 : Polar molecules

In the absence of external electric field, the different permanent dipole moments (molecules) are randomly oriented due to their random thermal motion. Thus in any volume containing a large number of atoms the net dipole moment is zero (Fig 4.2 a). When such a dielectric material is subjected to an external electric field, the individual dipoles experience torque due to the electric field and tend to align with the field as shown in Fig 4.2 (b) on increasing the magnitude of the applied field the alignment becomes more complete and net dipole moment is developed in the material.

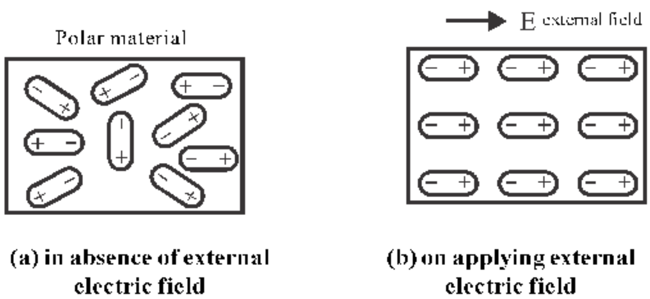


Fig 4.2 : Effect of external electric field (a) external electric field on polar molecules (b) on applying external field

(b) Non Polar dielectrics : Non polar dielectrics are composed of such atoms or molecules whose centre of the negative charges coincides with centre of distribution of positive charges. Such molecules have no permanent dipole moment and are called non polar molecules. Examples of non polar molecules are H_2, CO_2, N_2 and O_2. In Fig 4.2 H_2 and CO_2 molecules are shown. In the absence of external electric field the net dipole moment of a non polar dielectric material is zero [Fig 4.4a]

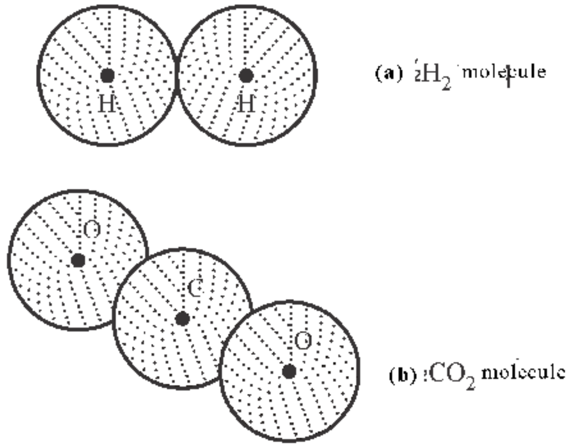


Fig 4.3 : Non polar molecules

If a non polar dielectric material is placed in an external electric field, the centre of negative charge distribution is slightly shifted opposite to electric field while that of positive charge in direction electic field. [Fig 4.4 (b)]. Thus atoms or molecules acquire a dipole moment by induction. The dipole moments of different molecules now tend to align along the electric field and we get a net dipole moment in the material.

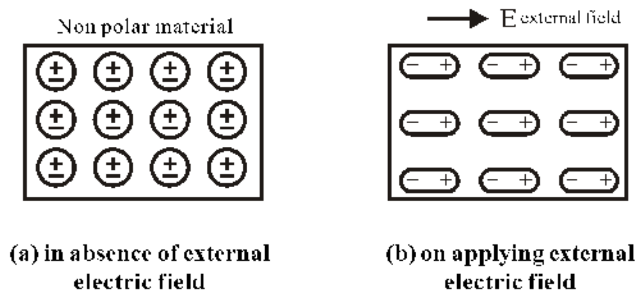


Fig 4.4 : Effect of external electric field on non polar material

Thus, in either case, whether polar or non polar, a dielectric acquires a net dipome moment in electric field. This phenomena is called dielectric polarization.

The materials in which the dipole moment is proportional to the external electric field and is along the field are called linear isotropic dielectrics.

The dipole moment per unit volume of a substance is called polarisation vector and is denoted by $\vec{P}$. For linear isotropic dielectrics the polarisation vector $\vec{P}$ is proportional to applied electric field $(\vec{E})$ i.e $\vec{P} \propto \vec{E}$

or $\quad \vec{P} = \chi_e \vec{E} \quad \quad ...(4.1)$

Where χ_e is a constant, characteristic of the dielectric and is known as electric susceptibility of dielectric. It is a measure of polarisation of the material and is a dimension less quantity.

4.4 Capacitance of a Conductor

The charge on a body is due to transfer of electrons i.e the body either gains or looses electrons. When some charge is given to a body, its potential rises. This is expected, to explain this in simple terms we may assume that a body is given some charge Q in a number of steps involving a transfer of charge dq in each step. When first such installment dq is given to the body no work is involved, but once this charge has been transferred a small potential develops on the body. Therefore the work must be done to move next incremental charge dq through this potential difference, Equivalently we may say that work must be done in putting this additional charge against the repulsion due to charge already present on the body whereby increasing its potential energy or electric potential. Thus if we go on giving charge to a conductor its potential rises in the same ratio.

If on giving a charge Q to an isolated conductor its potential rises by V then

$$Q \propto V \quad \text{or} \quad Q = CV \qquad \ldots (4.2)$$

Here C is a constant of proportionality and is called capacitance of the conductor. Further if we let V = 1 then from equation (4.2) we have C = Q

i.e capacitance of a conductor is numerically equal to the charge required to raise its potential through unity. The capacitance of a capacitor determines its ability to store electric charge. A conductor can store charge upto a certain maximum value of electric potential. The graph between the charge given to a conductor and rise in its potential is a straight line as shown in Fig 4.3 and the slope of this line represents the capacitance of the conductor.

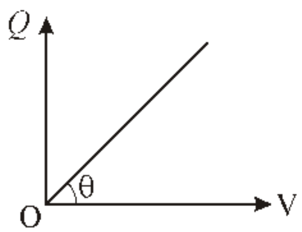


Fig 4.5 : Graph between Q and V for a conductor

The capacitance of a conductor depends on the shape and size of conductor, the nature of medium surrounding it and on presence of other conductor in its neighbourhood. However, it does not depend on the charge given to it or its potenial.

The SI unit of capacitance is coulomb/volt and is called farad (symbol F) Thus

$$1F = \frac{1C}{1V}$$

Capacitance of a conductor is thus said to be one farad if its potential rises through one volt when a charge of one coulomb is given to it.

The farad is a rather large unit so its submultiples like mili farad (mF) = 10^{-3} F, micro farad (μF) = 10^{-6} F, nano farad (nF) = 10^{-9} F and picofarad (pF) = 10^{-12} F are more commonly employed.

As C = Q/V the dimesional formula for C is

$$= \frac{[TA]}{[ML^2T^{-2}/TA]}$$

$$= [M^{-1}L^{-2}T^4A^2]$$

4.5 Capacitance of an Isolated Spherical conductor

Consider an isolated spherical conductor of radius R placed in free space. If a charge Q is given to it the electric potential V at its surface is given by

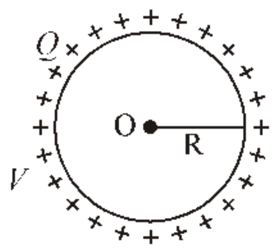


Fig 4.6 : Charged spherical conductor

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

So its capacitance $C = \frac{Q}{V} = 4\pi\epsilon_0 R \qquad \ldots (4.3a)$

Thus the capacitance of an isolated spherical conductor is directly proportional to its radius (i.e

$C \propto R$)

If the sphere is placed in a medium of dielectric constant $\in_r$ then its capacitance

$$C_m = \frac{Q}{V} = 4\pi \in_0 \in_r R \qquad \ldots (4.3b)$$

or $\quad C_m = C \in_r$

or $\quad \frac{C_m}{C} = \in_r$

Thus the dielectric constant of the medium is equal to the ratio of the capactiance of conductor in the medium and its capactiance in free space.

Example 4.1 : Considering earth to be a spherical conductor calculate its capacitance (Radius of earth $= 6.4 \times 10^6\ m$)

Solution : Capacitance $C = 4\pi \in_0 R = \frac{R}{1/4\pi \in_0}$

$$= \frac{6.4 \times 10^6}{9 \times 10^9} = 0.711 \times 10^{-3} F$$

$$C = 0.711 mF = 711 \mu F$$

It is obvious that the capacitance of a conductor is not very large.

Example 4.2 : What is the radius of a spherical capacitance of 1 F capacitance? Can you put it your cupboard?

Solution : As $C = 4\pi \in_0 R$

So $\quad R = \frac{C}{4\pi \in_0} = \frac{1}{4\pi \in_0} = 9 \times 10^9\ m$

This radius is nearly 1400 times larges than the radius of earth so it is not possible to put this capcitor in a cupboard. (The example also illustrate that Farad is a rather large unit of capacitance).

Example 4.3 : If the capacitances of some spherical capacitor in air and some medium are 2pF and 12pF respectively, what is the value of dielectric constant of medium.

Solution : Dielectric constant of medium

$$C_m = 4\pi \in_0 \in_r R = \in_r (4\pi \in_0 R) = \in_r C_0$$

Here $C_0 = 2pF$,

$$C_m = 12pF$$

or $\quad 12 = \in_r \times 2 \quad \in_r = 6$

Example 4.4 : On giving same amount of charge to two spheres of different radii the ratio of potential at their surfaces is 1 : 2. What is the ratio of their capacitances?

Solution : $C = \frac{Q}{V}$ or $\frac{C_1}{C_2} = \frac{Q_1 V_2}{Q_2 V_1}$

Here $\quad Q_1 = Q_2$ and $V_1 : V_2 = 1 : 2 \quad \frac{V_2}{V_1} = \frac{2}{1}$

therefore $\quad \frac{C_1}{C_2} = \frac{2}{1}$ or $C_1 : C_2 = 2 : 1$

4.6 Capacitor

The capacitance of a conductor can be increased by increasing its size but from practical considertion this is not convinient. Thus capcitance of a conductor is small and limited.

A capacitor is a device whose function is to increase the ability of a conductor to take up charge without increasing its size. This is done by reducing the electric potential of the given conductor for a given amount of charge.

A capacitor is a combination of two conductors called plates placed close to each other. One of the two plates is given a positive cahrge and other on equal amount of negative charge. For this these plates may be connected to the terminals of a battery (Fig 4.7). After charging if the battery is removed the plates retain their charge. So capaciator is a device to store charge.

Note that the net charge on a capacitor is $Q + (-Q) = 0$. Thus the term charge on a capacitor does not mean the net charge on capacitor. In our discussion of capacitor we let Q represent the magnitude of charge on either plate. Likewise the potential difference between plates is called the potential of capacitor.

For a given capacitor, the magintude of charge Q stored on either plate is directly proportional to the potential difference between plates.

$$Q \propto V$$

or $$Q = CV \qquad \ldots (4.4)$$

Where the constant of proportionality is called capacitance of the capacitor.

The shape of capacitor plates may be rectangular cylinderical, spherical or of any arbitrary shape. (No matter what their geometry, the two conductors farming a capacitor are called plates). The capacitance of a capacitor depends on the shape size, relative positions and medium between plates.

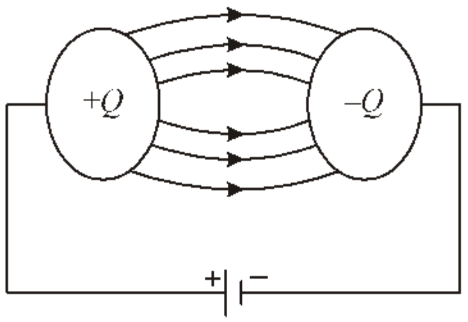


Fig 4.7 : Charging a capacitor

In Fig 4.8 the circuit symbols for capacitor used in electric circuits are shown

(a) Capacitor with fixed capacitance

(b) Capacitor with variable capacitnce

Fig 4.8 : Circuit symbols for a capcitor

4.6.1 Principle of Capacitor

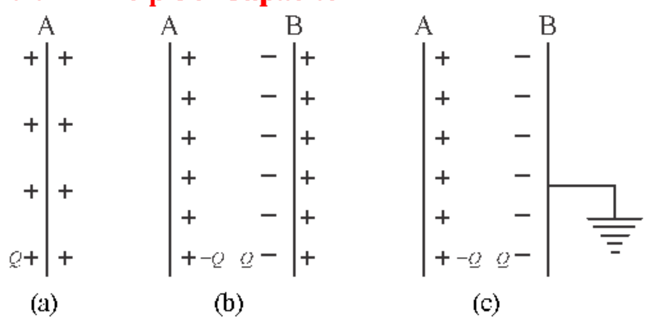


Fig 4.9 : Illustration of principle of capacitor

To understand the principle of a capacitor we consider various situations shown in fig 4.9 in sequence. In fig 4.9 (a) an insulated metallic plate A is shown to which a positive charge +Q has been given. Now if an identical uncharged metallic plate B is placed near plate A, by induction a charge -Q develops on the inner side of B and A positive charge + Q develops on the outer side of B (Fig 4.9 b). The negative charge on B tends to reduce the potential of A whereas the positive charge on it ends to enchance the potential of A. However, as negative charge on B is relatively closer to A than positive charge on B, the potential of plate A is slightly reduced. To make the potential of A at its original value we can give additional charge to plate A. Thus capacitance of A has increased. If we now connect the plate to ground the flow of electrons from earth neutralize the positive charge on B and the negative charge remains on plate B as it is bound (Fig 4.9 c). Under these condition there is nothing to increase potential of A. Thus the potential of the plate is considerably reduced due to induced negative charge on inner side of the plate B. Therefore capacitance of system increases considerably.

Thus "the capacitance of an insulated conductor is increased appreciably by bringing a grounded (earth connected) uncharged conductor near it".

Depending upon the shape of conductors we come across mainly three types of capacitors (a) parallel plate capacitor (b) shperical capacitors and (c) cylindrical capacitors. In this chapter our study is limited to parallel plate and spherical capacitors.

4.7 Parallel Plate Capacitance

A parallel plate cpapcitor consists of two equal plane parallel conductors separated by a small distance see fig 4.10 (a) here we are assuming that the space between plates contain vaccum or air.

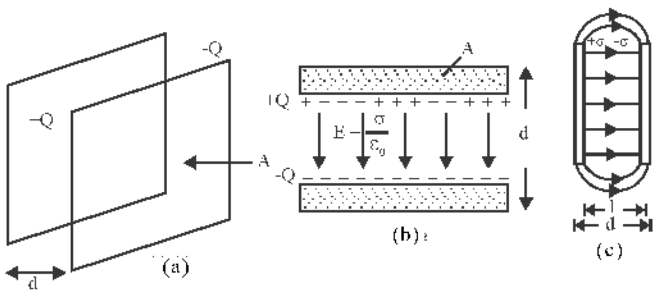


Fig 4.10 : Parallel plate capacitor

To charge a capacitor its plates are connected to the terminals of a battery. The plate connected to the positive terminal of the battery looses electron and the plate connected to the negative terminals recieves as many electrons. Thus equal and opposite charges + Q and - Q appears on the respective plates. As the plates are of equal area the magnitude of surface charge density for both the plates is same. Let $+\sigma$ and $-\sigma$ be the charge densities on the positive and negative plates.

Since the separation between plates is much smaller than the linear dimension of the plates, the electric field is uniform everywhere between the plates (fig 4.10 (b). The electric field in this region due to each plate is $\sigma / 2 \in_0$ and in same direction, perpendicular to the plates. Thus the two fields add up, giving

$$E = \frac{\sigma}{2\in_0} + \frac{\sigma}{2\in_0} = \frac{\sigma}{\in_0}$$

$$\because \quad \sigma = \frac{Q}{A}$$

where A = area of either plate

So $$E = \frac{Q}{A\in_0} \qquad \ldots(4.5)$$

Fringing - The electric field due to charged plates is uniform in the central region between the plates as indicated by parallel field lines in Fig 4.10 (b) and Fig 4.10 (c). As at the edges the surface charge density is relatively large field lines repel each other and becomes curved near the edges as in fig 4.10 (c). Thus near the edges field is non uniform. This effect is called fringing. For sufficiently large plates fringing can be ignored and electric field between the plates can be assumed uniform throughout this region.

If the separation between capacitor plates is d then the potential difference between the plates is

$$V = Ed = \frac{Qd}{A\in_0} \qquad \ldots(4.6)$$

therefore, the capacity of parallel plate capacitor

$$C = \frac{Q}{V} = \frac{Q}{Qd / A\in_0}$$

or $$C = \frac{A\in_0}{d} \Rightarrow C = \frac{\in_0 A}{d} \qquad \ldots(4.7)$$

Thus, the capacitance of a parallel plate capacitor is proportional to the area of plates and inversily proportional to the separation between them, i.e

$$C \propto A \text{ and } C \propto 1/d$$

Example 4.5 : A capacitor of capacity 20μF is charged to a potential difference of 10 kV. What is the magnitude of charge on its each plate?

Solution :

$$\because \quad Q = CV = 20\times10^{-6}\times10\times10^{3}$$

$$= 20\times10^{-2} = 0.2C$$

Example 4.6 : A parallel plate capacitor has plates of area A and the separation between the plates d. If the area of plates is doubled while the separation is halved, the new capacitance is how many times of its orginal value.

Solution : Initial capacitance $C = \frac{A\in_0}{d}$

and new capacitance

$$C' = \frac{(2A)\in_0}{d/2} = \frac{4A\in_0}{d} = 4C$$

Hence new capacity is four times the initial capacity

Example 4.7 : On connecting a capacitor of capacitance C to a battery of potential difference V, charges on its plates are ± 360 μC. On decreasing the potential difference by 120V charges become ± 120 μC. Find

(a) potential difference V across plates

(b) capacitance of capacitor

(c) magnitude of charge if the applied potential difference is increased by 120 V.

Solution :

(a) $\because q = CV = 360\times10^{-6}C$ coulomb

Given $q' = C(V-120) = 120\times10^{-6}C$ coulomb

So, $$\frac{CV}{C(V-120)} = \frac{360\times10^{-6}}{120\times10^{-6}} = 3$$

or $3V - 360 = V$ or $V = 180$ V

(b) As $\because CV = 360\times10^{-6}$

$$C = \frac{360\times10^{-6}}{V} = \frac{360\times10^{-6}}{180}$$

$$= 2\times10^{-6} F = 2\mu F$$

(c) On increasing the potential differnce by 120 V, new charge

$$q'' = C(V+120) = 2\times10^{-6}(180+120)$$

$$q'' = 2\times300\times10^{-6} = 600\times10^{-6} C = 600\mu C$$

Example 4.8 Calculate the capacitance of the capacitor formed by two circular discs of radius 5 cm each at a separation of 1 mm.

Solution : Here, plate area

$$A = \pi r^2 = 3.14\times\left(5\times10^{-2}\right)^2$$

$$= 78.5\times10^{-4} m^2$$

So, $$C = \frac{A\in_0}{d} = \frac{78.5\times10^{-4}\times8.85\times10^{-12}}{1\times10^{-3}}$$

$$C = 69.47\times10^{-12} F = 69.5\, pF$$

4.6 Effect of Dielectric medium filled between plates of Capacitor

When the space between the capacitor plates is filled with some dielectric material (say wax, paper, mica etc) the molecular dipoles (polar or induced) tend to align along the direction of electric field $\left(E = \frac{\sigma}{\in_0}\right)$ due to plates (Fig 4.11). In this process of polarzation, in the interior of dielectric, charges on near by molecular dipole tends to neutralize each other, however, this is not so for dipoles near the edges of dielectric medium. As a result a layer of induced negative charge density $-\sigma_p$ is formed on one edge (near positive plate) and an equal magnitude positive charge $+\sigma_p$ on other edge (near negative plate).

Because of this an induced electric field $E_p = \frac{\sigma_p}{\in_0}$ is established within the dielectric medium. The direction of this induced electric field is opposite to electric field due to capacitor plates.

So the net electric field in the dielectric medium

$$E_1 = E - E_P$$

or $$E_1 = \frac{\sigma}{\in_0} - \frac{\sigma_p}{\in_0} = \frac{\sigma - \sigma_p}{\in_0} \qquad \ldots (4.8)$$

By definition of dielectric constant

$$E_1 = \frac{E}{\in} = \frac{\sigma}{\in_0\in_r}$$

hence equation (4.8) can be rewritten as

$$E_1 = \frac{\sigma - \sigma_p}{\in_0} = \frac{\sigma}{\in_r\in_0} \qquad \ldots (4.9)$$

Therefore the electric field between capacitor plates in presence of a dielectric is now smaller than the field in absence of dielectric. Consequently the potential difference between plates decrease whereby capacitance increases.

From equation (4.9)

$$\sigma - \sigma_p = \frac{\sigma}{\in_r}$$

So, induced charge density

$$\sigma_p = \sigma\left(1 - \frac{1}{\in_r}\right) \qquad \ldots (4.10)$$

Magnitude of Polarsation vector is equal to the magnitude of induced surface charge density, so

$$\left|\vec{P}\right| = \chi_e E_1 = \chi_e \frac{\sigma}{\in_0\in_r} = \sigma_p \qquad \ldots (4.11)$$

From equations (4.10) and (4.11) following relation is obtained between electric succesptiblity and dielectric constant of the material.

$$\chi_e = \in_0 (\in_r - 1) \qquad \ldots (4.12)$$

4.8.1 Capacitance of a parallel plate capacitor completly filled with a dielectric

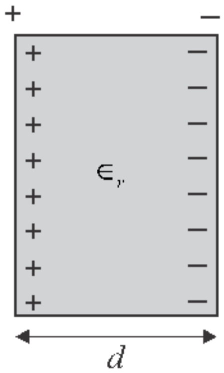


Fig 4.12 : A parallel plate capacitor filled with dielectric

As shown in fig 4.12, consider a parallel plate capacitor with space between plates completely filled with a dielectric of dielectric constant $\in_r$ let A and d are respectively the plate area and separation between plates. In presence of dielectric the net electric field between the plates

$$E = \frac{\sigma}{\in_0 \in_r} = \frac{Q}{A \in_0 \in_r} \qquad \ldots (4.13)$$

So the potential difference between plates

$$V = Ed = \frac{Qd}{A \in_0 \in_r}$$

and capacitance $C = \frac{Q}{V} = \frac{Q}{Qd / A \in_0 \in_r}$

$$C = \frac{A \in_0 \in_r}{d} = \in_r C_0 \qquad \ldots (4.14)$$

where $C_0 = \frac{\in_0 A}{d}$ is capacitance with free space or air as medium between the capacitor plates. Thus the capacitance of a capacitor is increased by a factor of $\in_r$ when the space between the plates is filled with a dielectric of dielectric constant $\in_r$. Although the result has been derived for a parallel capacitor but is valid for any capacitor.

4.8.2 Capacitance of a parallel plate capacitor partilly filled with a dielectric

This situation is shown in fig 4.13 where a dielectric slab of thickness t (< d) is placed in space between the capacitor plates. In this situation the thickness of region between plates where free space or air is present is d-t.

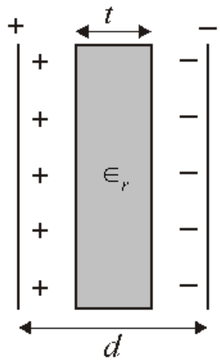


Fig 4.13 : Capacitor partially filled with dielectric

If the electric field in free space or air portion of the space is $E_0 = \frac{\sigma}{\in_0} = \frac{Q}{A \in_0}$ the electric field in the dielectric portion will be $E_0 / \in_r$ and the effective potential difference between the plates is then given by

V = potential difference across portion containing air + potential difference across portion containing dielectric

$$V = E_0 (d - t) + \frac{E_0}{\in_r} t$$

or $V = E_0 \left[d - t + \frac{t}{\in_r} \right] = \frac{Q}{A \in_0} \left[d - t + \frac{t}{\in_r} \right]$

So capacitance $C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A \in_0} \left[d - t + \frac{t}{\in_r} \right]}$

or $C = \frac{A \in_0}{(d - t + t / \in_r)} = \frac{A \in_0}{[d - t(1 - 1/\in_r)]} \qquad \ldots (4.15)$

If t = d i.e the dielectric fills the space between plates completely then we obtain $C = \frac{A \in_0 \in_r}{d} = \in_r C_0$ which is same is in equation (4.14). Also if t = 0 we obtain $C = \frac{A \in_0}{d}$ as expected in the absence of dielectric medium.

4.8.3 Capaitance of a parallel plate capaitor filled with different dielectric materials of different thicknesses

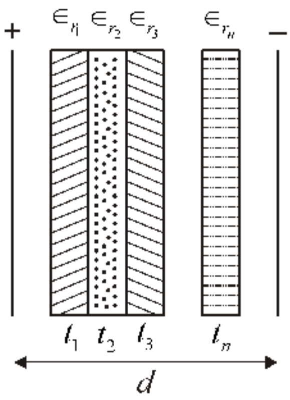


Fig 4.14 : Capacitor filled with different dielectrics

As shown in fig 4.14 if a number of dielectric media of dielectric constants $\in_{r_1}, \in_{r_2}, \in_{r_3}, ... \in_{r_n}$ having thicknesses $t_1, t_2, t_3, ... t_n$ respectively are present in region between capacitor plates then the thickness of the portion having free sapce or air as medium will be $\left[d - (t_1 + t_2 + t_3 + ... + t_n)\right]$. In this case the potential difference across the capacitor is given by

V = potential difference across air portion + sum of potential across different media

$$V = E_0 \left[d - (t_1 + t_2 + t_3 + ... + t_n)\right] + \frac{E_0}{\in_{r_1}} t_1 + \frac{E_0}{\in_{r_2}} t_2 + \frac{E_0}{\in_{r_3}} t_3 + ... + \frac{E_0}{\in_{r_n}} t_n$$

where $E_0 = \frac{\sigma}{\in_0} = \frac{Q}{A \in_0}$ is electric field in region of capacitor with air or free space.

or $V = E_0 \left[d - (t_1 + t_2 + ... + t_n) + \frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}} + ... + \frac{t_n}{\in_{r_n}}\right]$

$$V = \frac{Q}{A \in_0}\left[d - (t_1 + t_2 + ... + t_n) + \frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}} + ... + \frac{t_n}{\in_{r_n}}\right]$$

So, capacitance

$$C = \frac{Q}{V} = \frac{A \in_0}{\left[d - (t_1 + t_2 + t_3 + ... + t_n) + \frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}} + ... + \frac{t_n}{\in_{r_n}}\right]} \quad ...(4.16)$$

If $(t_1 + t_2 + t_3 + ... + t_n) = d$ i.e there is no part of the space between capacitor plates where air or free space is present, then

$$C = \frac{A \in_0}{\frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}} + ... + \frac{t_n}{\in_{r_n}}} \quad ...(4.17)$$

Example 4.9 : The plate area of a parallel plate capacitor is 100 cm^2 and separation between plates is 1 mm. On connecting this capacitor across a 120 volt battery a charge of 0.12 μC accumulates on the plates. Find the dielectric constant of material present between the capacitor plates.

Solution : Capacitance with dielectric

$$C = \frac{Q}{V} = \frac{A \in_0 \in_r}{d}$$

So $\quad \in_r = \frac{Qd}{A \in_0 V} = \frac{0.12 \times 10^{-6} \times 1 \times 10^{-3}}{100 \times 10^{-4} \times 8.85 \times 10^{-12} \times 120}$

$$= 0.001129 \times 10^4$$

or $\quad \in_r = 11.3$

Example 4.10 : A slab of ebonite (dielectric constant = 3) of 6 mm thickness is placed in space between capacitor plates of a parallel plate capacitor having plate area $2\times10^{-2}\,m^2$ and plate separation 0.01 m. What is the capacitance of this capacitor.

Solution : Use $C = \dfrac{A\in_0}{d-t+\dfrac{t}{\in_r}}$

$$= \frac{2\times10^{2}\times8.85\times10^{12}}{0.01-6\times10^{3}+\dfrac{6\times10^{3}}{3}}$$

$$C = \frac{17.70\times10^{14}}{0.01-4\times10^{-3}} = \frac{17.70\times10^{14}}{0.01-0.004} = \frac{17.70\times10^{14}}{0.006}$$

$$C = 2.95\times10^{11}F = 29.5\times10^{12}F = 29.5\,pF$$

Example 4.11 The capacitance of a capacitor is C. The separation between the plates of this capacitor is d. If the space between the plates is filled with a dielectric material of dielectric constant $\in_r$ upto a distance of 3d/4 then calculate the new capacitance.

Solution : Let new capacity be C[1] then

$$C' = \frac{A\in_0}{d-t+\dfrac{t}{\in_r}}$$

Given $t = \dfrac{3}{4}d$

$$\therefore \quad C' = \frac{A\in_0}{d-\dfrac{3d}{4}+\dfrac{3d}{4\in_r}} = \frac{A\in_0}{\dfrac{d}{4}+\dfrac{3d}{4\in_r}} = \frac{A\in_0}{\dfrac{d}{4\in_r}(\in_r+3)}$$

So $$C' = \frac{4\in_r A\in_0}{d(\in_r+3)} = \frac{4\in_r}{\in_r+3}C$$

$$\because \quad C = \frac{A\in_0}{d}$$

Hence capacity of capacitor will be $\dfrac{4\in_r}{(\in_r+3)}$ times.

4.9 Capacitance of a spherical capacitors

A spherical capacitor consists of two concentric shperical conductors of nearly equal size, one of the conductor is given a positive charge Q and other an equal and opposite charge -Q.

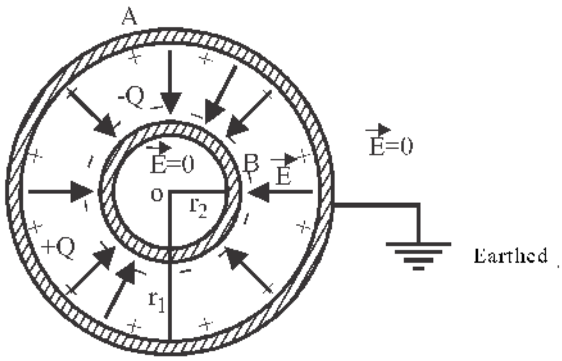


Fig 4.15 : A spherical capacitor

Fig. 4.15 shows a spherical capacitor. Inner conducting sphere can be solid or hollow and is surrounded by a concentric conducting shell A. The radii of A and B are r_1 and r_2 respectively. Note that the outer sphere B is grounded (earhted). Let a negative charge -Q is given to the inner sphere, induced charges +Q and -Q appears on the inner and outer surfaces of shell A. Since shell A is grounded the charge - Q on its outersurface flows to earth.

Potential on sphere B due to its own charge - Q is

$$V_B = \frac{1}{4\pi\in_0}\cdot\frac{(-Q)}{r_2} \qquad \ldots(4.18)$$

However, sphere B is surrounded by shell A which has a positive charge Q and as for all internal points of a charged shell the potential is same as that on its surface, so the potential at surface of B due to charge on sphere A

$$V'_B = \frac{1}{4\pi\in_0}\cdot\frac{(+Q)}{r_1} \qquad \ldots(4.19)$$

So the net potential on sphere B is

$$V = V_B + V'_B$$

$$V = \frac{Q}{4\pi \in_0 r_1} - \frac{Q}{4\pi \in_0 r_2}$$

$$V = \frac{Q}{4\pi \in_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$V = -\frac{Q}{4\pi \in_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right] \qquad \ldots (4.20)$$

As the outer sphere A is earthed its potential is zero. Thus the potential difference between A and B

$$V_{AB} = 0 - V$$

$$V_{AB} = 0 - \left[\frac{-Q}{4\pi \in_0}\frac{(r_1 - r_2)}{r_1 r_2}\right]$$

or $$V_{AB} = \frac{Q}{4\pi \in_0}\frac{(r_1 - r_2)}{r_1 r_2} \qquad \ldots (4.21)$$

So if for free space between A and B if the capacitance of this capacitor is C_0 then

$$C_0 = \frac{Q}{V_{AB}} = \frac{Q}{\frac{Q}{4\pi \in_0}\left(\frac{r_1 - r_2}{r_1 r_2}\right)}$$

or $$C_0 = 4\pi \in_0 \left(\frac{r_1 r_2}{r_1 - r_2}\right) \qquad \ldots (4.22)$$

Thus, the capacitance of a spherical capciator depends on the size of spheres and (r_1-r_2). To have a large capacity, two spheres of sufficient size with nearly equal radii must be taken.

If a dielectric medium of dielectric constant $\in_r$ is filled between the spheres, then capcitance is given by

$$C_m = 4\pi \in \frac{r_1 r_2}{r_1 - r_2}$$

$$C_m = \in_r 4\pi \in_0 \left(\frac{r_1 r_2}{r_1 - r_2}\right) \Rightarrow C_m = \in_r C_0$$

From equation 4.22 it can be seen that]

(i) It $r_1 = R$ and $r_2 = \infty$ than $C_0 = 4\pi \in_0 R$

Which is the capacitance of an isolated spherical conductor i.e a spherical conductor can be treated as a spherical capacitor with outer sphere at infinity.

(ii) If both r_1 and r_2 are made large but $r_1 - r_2 = d$ is kept fixed we can write $r_1 r_2 \approx 4\pi r^2 = A$ where r is approximatelythe radius of each sphere and A is its area. Equation 4.22 then becomes which is same as the equation for a parallel plate capacitor.

Example 4.12 : The radii of inner and outer spheres of a spherical capacitors are 2 m and 1 m and a dielectric mediumof dielectric constant $\in_r = 8$ fills the space between them. Calculate the capacitance of the capacitor.

Solution : For a spherical capacitor filled with dielectric

$$C_m = \frac{4\pi \in_0 \in_r r_1 r_2}{(r_1 - r_2)}$$

Here $r_1 = 2\,\text{m},\ r_2 = 1\,\text{m},\ \in_r = 8$

$$C_m = \frac{1}{9\times 10^9}\frac{8\times 2\times 1}{(2-1)}$$

$$C_m = \frac{16}{9}\times 10^{-9}\ \text{F}$$

$$C_m = 1.78\times 10^{-9}\ F = 1.78\,\text{nF}$$

4.10 Combination of Capacitors

There are many situations in electric circuit where two or more capacitors are used. Two special methods of combinations frequently used are series and parallel combinations. Any combination should have two points which may be connected to a battery to apply a potential difference.

4.10.1 Series Combination

In series combination capacitors are wired serially one after the other and a potential difference may be applied across the two ends of the series. In other words in such a combination second plate of first capacitor is connected to the first plate of second capacitor the second plate of the second capacitor is connected to the first plate of the third and so on. The charge delivered to each capacitor of the series combination has the same value.

Figure 4.16 shows the series combination of three capacitors having capacitance C_1, C_2 and C_3 respectively. The points P and N are end points of this series combination and serve as the points through which a potential difference may be applied. In figure point P is connected to the positive terminal and N to the negative terminal of the battery. Second plate of capacitor C_1 is connected to the first plate of capacitor C_2 and second plate of C_2 is connected the first plate of capacitor C_3. In this manner any number of capacitors can be connected in series.

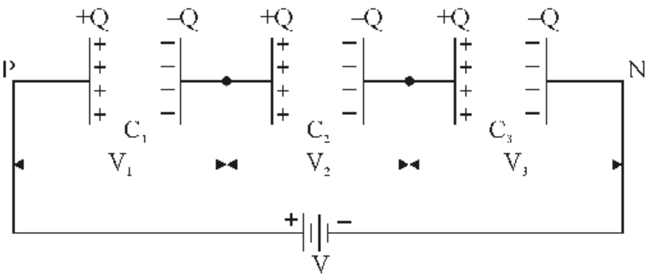


Fig 4.16 : Series combination of capacitors

We can understand how the capacitors end up with identical charge by following reasoning. Let the battery supply a charge + Q to the first plate of the first capacitor (C_1). By electrostatic induction a charge - Q appears on the inner face of second plate of C_1 and a charge + Q an its outer face. This + Q charge on the second plate of C_1 flows to the first plate of second capacitor C_2 inducing charge + Q on its outer face and charge - Q on inner face. The process is repeated for the third capacitor and the charge + Q on outer plate of C_3 flows to the negaive terminal of the battery. Thus we have a positive charge + Q on the inner side of the first plate of capacitor C_1 and a negative charge - Q on the inner side of the second plate of capacitor C_3. This compeletes the charge distribution with each capacitor ending with same charge. Since their capacitances are different the potential differences across the plates of each capacitor are different. Let the potential differences across C_1, C_2 and C_3 are V_1, V_2 and V_3 respectively then

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

If the potential difference across P and N is V then

$$V = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad \ldots (4.23)$$

If the total (equivalent) capacity of the combination is C_s, then

$$V = \frac{Q}{C_S} \qquad \ldots (4.24)$$

So $$\frac{Q}{C_S} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \qquad \ldots (4.24)$$

or $$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad \ldots (4.25)$$

Like wise the equivalent capacitance of n capacitors $C_1, C_2 \ldots\ldots\ldots C_n$ connected in series is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n} \qquad \ldots (4.26)$$

If the combination is replaced by a single capacitor of this capacitance (C_s) the single capacitor will store the same amount of charge for a given potential difference as the combination does.

It is obvious that

(i) The reciprocal of the equivalent capacitance of the series combination is equal to the sum of the reciprocals of individual capacitances of capacitors in series combination.

(ii) The equivalent capacitance is smaller then the smallest capcitance present in the series combination.

Some points worth noting regarding series combination are

(i) Series combination of the capacitors is used in situations where the applied voltage is high and a single capacitor can not sustain it or when we require a capacitor of smaller capacitance than that of capacitors available.

(ii) In series combination amount of charge is same for all capacitors irrespective of their capacitances

$Q_1 : Q_2 : Q_3 : ... = 1 : 1 : 1 : ...$

(iii) If n capacitors of equal capacitance C are connected in series then their equivalnt capacitance is $C_S = \frac{C}{n}$ and potential difference across each is V/n.

(iv) If a number of dielectric slabs having dielectric constants $\in_{r_1}, \in_{r_2}, \in_{r_3}$ ... are placed between the plates of a capacitor parallel to plates as shown in Fig 4.17, then this capacitor can be regraded as a series combination of as many capacitors, however, the distance between plates of each such capacitor is equal to the thickness of corresponding dielectric slab.

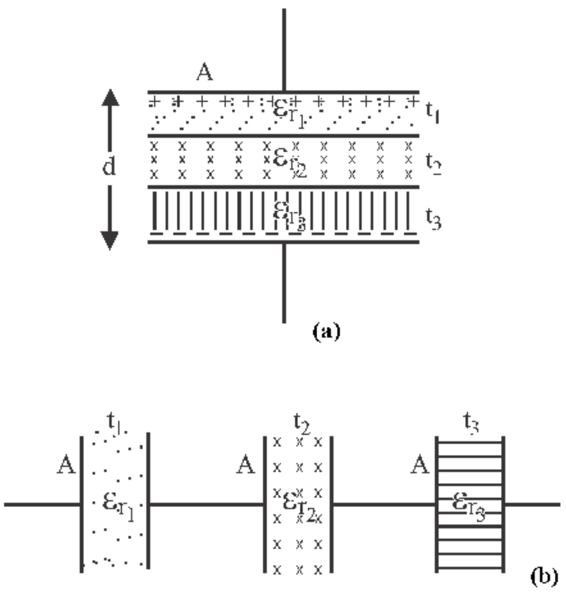


Fig 4.17 : Capacitor containing different dielectric slab parallel to plates

For series combination 4.17(b)

$$C_1 = \frac{\in_{r_1} \in_0 A}{t_1}, \quad C_2 = \frac{\in_{r_2} \in_0 A}{t_2}$$

$$C_3 = \frac{\in_{r_3} \in_0 A}{t_3}$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C_S} = \frac{t_1}{\in_{r_1} \in_0 A} + \frac{t_2}{\in_{r_2} \in_0 A} + \frac{t_3}{\in_{r_3} \in_0 A}$$

$$\frac{1}{C_S} = \frac{1}{\in_0 A}\left[\frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}}\right]$$

$$C_S = \frac{\in_0 A}{\frac{t_1}{\in_{r_1}} + \frac{t_2}{\in_{r_2}} + \frac{t_3}{\in_{r_3}}}$$

4.10.2 Parallel Combination of Capacitors

A number of capacitors are said to be connected in parallel if potential difference across each is the same (and is equal to the applied voltage). In such a combinations capacitors are arranged (wired) in such a manner that all right hand (first) plates of all capacitor are connected at one common point and left hand (second) plates are connected at another common point. One of these common points is at a higher potential than other when a battery is connected between these points.

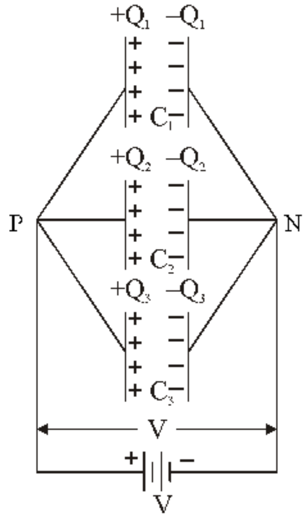


Fig 4.18 : Parallel combination of capacitors

Figure 4.18 shows parallel combination of three capacitors of capacitances C_1, C_2 and C_3 respectively. Note that first plates of each capacitor is connected to higher potential point P (positive terminal of battery) while negative plates are connected to lower potential point N (negative terminal of battery). Thus potential difference across each capacitor is same (equal to the potential difference across the terminals of battery).

Let the battery supplies a charge + Q which is distributed on the first plates of the three capacitors according to the cpapacitance of each capacitor. Let the charges on these plates be Q_1, Q_2 and Q_3. Due to electrostatie induction the charges $-Q_1$, $-Q_2$ and $-Q_3$ appears on inner faces of second plates of respective capacitors. Positive induced charges appearing on outer faces of these plates flows to the negative terminal of battery. So

$$Q_1 = C_1V,\ Q_2 = C_2V \text{ and } Q_3 = C_3V$$

and total charge $Q = Q_1 + Q_2 + Q_3$

or $Q = C_1V + C_2V + C_3V$

or $Q = (C_1 + C_2 + C_3)V$ ... (4.27)

If the equivalent capacitance of the parallel combination is denoted by C_P then

$$Q = C_PV \quad \text{... (4.28)}$$

From equation (4.27)m and (4.28)

$$C_PV = (C_1 + C_2 + C_3)V$$

or $$C_P = C_1 + C_2 + C_3 \quad \text{... (4.29)}$$

Like wise for a parallel combination of n different capacitors

$$C_P = C_1 + C_2 + C_3 + ... + C_n \quad \text{... (4.30)}$$

It is obvious that

(a) For parallel combination of capacitors, equivalent capacitance is equal to the sum of he capacitances of individual capacitors.

(b) In a parallel combination the equivalent capacitance is always greater then the largest capacitance present in the combination.

Following points are worth noting regarding the parallel combination of capacitors.

(i) Such a combination is used when a large capacitance required for a given working voltage or a capacitor of large capacitance is to be obtained from given capacitors having smaller capacitances.

(ii) In parallel combination potential difference across each capacitor is some

$$V_1 : V_2 : V_3 : ... = 1 : 1 : 1 : ...$$

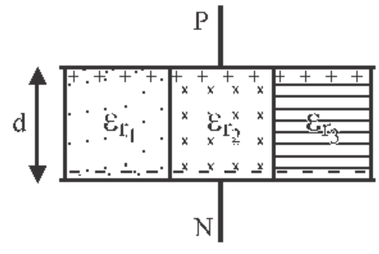
but the charge on each capacitor is proportional to its capacitance

$$Q_1 : Q_2 : Q_3 \text{ .... } = C_1 : C_2 : C_3$$

(iii) If n identical capacitors each of capacitance C are connected in parallel the equivalent capacitance of the combination is n times the individual capacitance

$$C_P = nC$$

(iv) If n a number of dielectric slabs each of some thickness d having dielectric constants $\in_{r_1}, \in_{r_2}, \in_{r_3}$ ... are placed in the space between a capacitor (of plate area A) as shown in fig 4.19 then these can be considered equivalent to a parallel combination of as may capators for the capacitor with three dielectric placed as in fig 4.19 (a), the equivlalent parallel combination is shown in fig 4.19 (B)



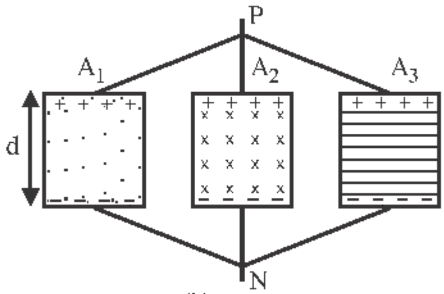


Fig. 4.19 : Parallel combination for different media

$$C_1 = \frac{\in_{r_1} \in_0 A_1}{d},\ C_2 = \frac{\in_{r_2} \in_0 A_2}{d}$$

$$C_3 = \frac{\in_{r_3} \in_0 A_3}{d}$$

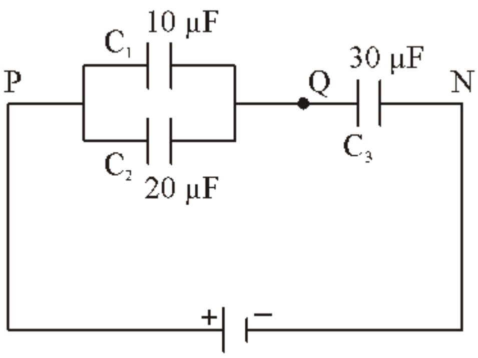
here A_1, A_2 and A_3 are areas corresponding dielectric slabs respectively.

Capacitance of parallel combination

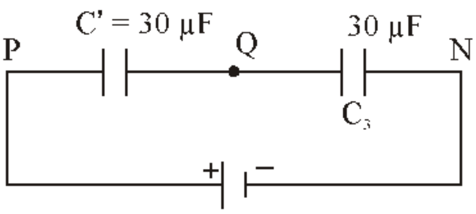
$$C_P = C_1 + C_2 + C_3$$

or $$C_P = \frac{\in_0}{d}\left[A_1 \in_{r_1} + A_2 \in_{r_2} + A_3 \in_{r_3}\right]$$

Example 4.13 : For the capacitor combination shown in Fig find the equivalent capacitance between points P and N



Solution : In the circuit shown capacitors of 10 μF and 20 μF are connected in parallel and their equivalent capacitance $C' = 10 + 20 = 30\,\mu F$ so these can be replaced by a 30 μF capacitor, which is now in series with another 30 μF (C_3) capacitor already present in the circuit so the capacitance of the given combination.

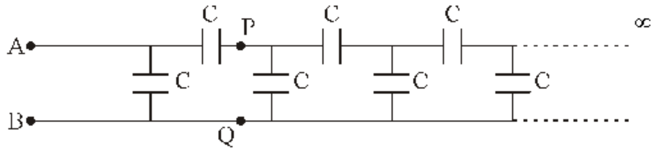


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_3}$$

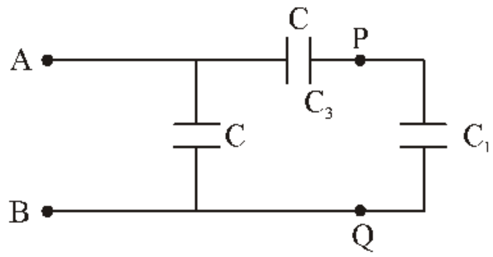
$$\frac{1}{C} = \frac{1}{30} + \frac{1}{30} = \frac{2}{30}$$

$$C = 15\,\mu F$$

Example 4.14 An infinite circuit is formed by the repetition of same link consisting of two identical capacitors each of capacitance C. Find the effective capacitance between points A and B.



Solution : As the ladder shown in fig of question is infinity long the capacitance of ladder to the right of points P and Q (say C) is same as that between A and B. The ladder shown above can then be replaced by the circuit shown in Fig below



Fom this figure equivalent capacitance between A and B

$$C_1 = C + \frac{CC_1}{C + C_1}$$

$$C_1 = \frac{C(C + C_1) + CC_1}{(C + C_1)}$$

$$C_1^2 - CC_1 - C^2 = 0$$

$$C_1 = \frac{C \pm \sqrt{C^2 + 4C^2}}{2} = \frac{C \pm \sqrt{5C^2}}{2}$$

$$C_1 = \frac{C\left(1 \pm \sqrt{5}\right)}{2}$$

$$C_1 = \frac{\left(1 + \sqrt{5}\right)C}{2} \text{ or } C_1 = \frac{\left(1 - \sqrt{5}\right)C}{2}$$

but as C_1 can not negative we have to choose

$$C_1 = \frac{\left(1+\sqrt{5}\right)C}{2}$$

as correct value out of the two values calculated above.

4.11 Energy stored in a capacitor

As has been mentioned earlier a capacitor is a device to store electrical energy. In this section we discuss about this aspect.

We have seen in previous chapter that electrostatic potential energy is associated with any charge configuration which is equal to the work done by extenral force in assembling this configuration by bringing the constituent charges from infinity (where charges are assumed at rest) to their respective locations in configuration under condsideration. As work must be done by an external agent in charging a capacitor which is stored in the form of electrostatic potnetial energy in the electric field between capacitor plates. To understand the process of charging we imagine that some external agent is transfering electrons from one plate to other plate of a capacitor. The plate from which electrons are removed is becoming positively cahrged while the plate recieving electrons is becoming negative, thus a charge separation takes plate in the process. As the charge accumlates on the capacitor plates this external agent has to do increasingly large amount of work to transfer more electrons. In practice this work is done by a battery at the cost of chemical energy stored in it.

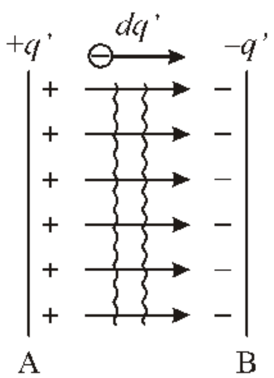


Fig. 4.20 : Imagination of charge transfer from one plate of a capacitor to other

Suppose that a certain instant a charge q^1 is already present one plate of capacitor during the charging process. At that instant the potential difference V between capacitor plates is q^1/C if now an increment of charge dq is to be transferred to this plate the increment of work needed is given by

$$dW = V'\,dq \qquad \ldots (4.31)$$

the total work required to charge the capcitor from

$$W = \int_0^Q V'\,dq$$

$$W = \int_0^Q \frac{q'}{C}\,dq$$

$$W = \frac{1}{C}\int_0^Q q'\,dq$$

$$\Rightarrow \quad W = \frac{1}{C}\left[\frac{q'^2}{2}\right]_0^Q$$

$$W = \frac{1}{2}\frac{Q^2}{C}$$

This work is stored as potential energy U in the capcitor

$$\text{So } U = \frac{1}{2}\frac{Q^2}{C} \qquad \ldots (4.32)$$

On substituting Q = CV in equation (4.32) we obtain

$$U = \frac{1}{2}CV^2 \qquad \ldots (4.33)$$

and if we substitute V = Q / C in equation (4.32)

$$U = \frac{1}{2}QV \qquad \ldots (4.34)$$

thus we can express the energy stored in a charged capacitor as

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This result applies to any capcitor regardless of its geometry. These equations, however, do not tell us where this energy is stored. By following argument we can explain that this energy is stored in capacitor in electric field between the plates. Consider two parallel plate capacitors 1 and 2 having same plate area A but the plate separation for capacitor 1 is double of that for capacitor 2. Thus the capacitance of capacitor 1 is half the capacitance of capacitor 2. If both the capacitors are given same amount of charge q then electric field between the plates given by $E = \frac{q}{\in_0 A}$ is same for both the capacitors. Because of the (above mentioned) difference in capacitance, from equation 4.33 we note that the energy stored in capacitor 1 is twice that in capacitor 2. Also note that for the volume between capacitor plates capacitor 1 has twice the volume than capacitor 2. So for same value of q for both capacitors, one with twice volume has twice the stored energy. However as the electric field is present in the entire volume between the capacitor plates it is logical to associate electrostatic potential energy with electric field present in this region.

4.11.1 Energy Density of Electric Field between Plates for a parallel Capacitor

For a parallel plate capaciotor having plate area A and charge Q the electric field between capacitor plates is given by

$$E = \frac{\sigma}{\in_0} = \frac{Q}{\in_0 A}$$

$$Q = \in_0 EA$$

The electrostatic energy of a charged capacitor

$$U = \frac{1}{2}\frac{Q^2}{C}$$

but $C = \frac{\in_0 d}{d}$

So $$U = \frac{1}{2}\frac{(\in_0 EA)^2}{\left(\frac{\in_0 A}{d}\right)}$$

$$U = \frac{1}{2}\in_0 E^2 Ad$$

but Ad = V (Volume between capacitor plates)

not to confuse with potential difference.

So energy density, u, i.e energy per unit volume

$$u = \frac{U}{V} = \frac{1}{2}\hat{I}_0 E^2 \qquad \ldots (4.35)$$

Although the equation 4.35 is derived for the special case of a parallel plate capacitor, it has a general validity. Thus wherever electric field E is present the energy per unit volume u is given by 1/2 $\hat{I}_0$ E^2. In general E varies with position, so u is a function of coordinates. For the special case of the parallel plate capacitor E and u do not vary with position in the region between capacitor plates.

Example 4.15 : The area of each plate for a parallel plate capacitor is 90 cm^2 and plate separation is 2.5 mm. It is charged by connecting across a 400 V supply. Find the electrostatic energy stored in the capacitor.

Solution : Here $A = 90\,\text{cm}^2 = 90\times10^{-4}\,\text{m}^2$

$$d = 2.5\,\text{mm} = 2.5\times10^{-3}\,\text{m}$$

$$V = 400\,\text{V}$$

Use $$U = \frac{1}{2}CV^2$$

Here $$C = \frac{\in_0 A}{d} = \frac{8.85\times10^{12}\times90\times10^{4}}{2.5\times10^{-3}}$$

$$C = 3.18\times10^{-11}F = 31.8\,pE$$

$\therefore$

$$U = \frac{1}{2}\times3.18\times10^{11}\times(400)^2 = 2.54\times10^{6}J$$

4.11.2 Energy stored in combination of capacitors

(a) Series Combination : Consider n capacitances $C_1, C_2, C_3, \ldots, C_n$ connected in series, the equivalent capacitance of the combination is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

Here charge Q on each capacitor is same, Energy stored in equivalent capacitor $U = \frac{1}{2}\frac{Q^2}{C_S}$

$$U = \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}\right]$$

or $U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + ... + \frac{Q^2}{2C_n}$

or $U = U_1 + U_2 + U_3 + ... + U_n$

(b) Parallel combination : For n capacitors having capacitances $C_1, C_2, C_3, ..., C_n$ connected in parallel the equivalent capacitance is given by

$$C_P = C_1 + C_2 + C_3 + ... + C_n$$

Here potential difference V across each capacitor is same energy stored in equivalent capacitor

$$U = \frac{1}{2}C_P V^2$$

or $U = \frac{1}{2}[C_1 + C_2 + C_3 + ... + C_n]V^2$

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + ... + \frac{1}{2}C_nV^2$$

or $U = U_1 + U_2 + U_3 + ... + U_n$

Therefore, for both series and parallel combination of capacitor the total energy stored in the combination is equal to the sum of energies stored in individual capacitors.

4.12 Redistribution of Charge and Loss of energy on sharing of Charges

When two conductors charged to different potential are connected through, a conducting wire or brought in contact with each other, charge flows from the conductor at higher potential to the conductor at lower potential till both acquire a common potential. There is a loss in energy in this process of redistribution of charges.

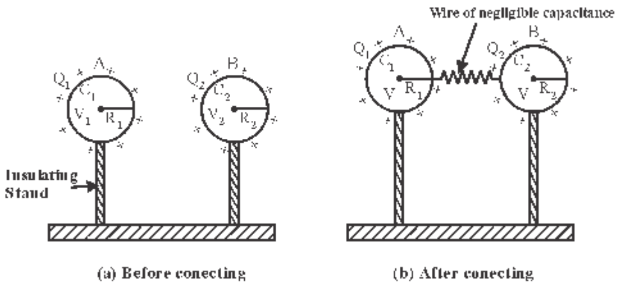


Fig 4.21 : (a) Before conecting (b) After conecting

Consider two isolated spherical conductors A and B of radii R_1 and R_2 charged to potentials V_1 and V_2 [Fig 4.21 a]. If capacitances of the conductors are C_1 and C_2 and charges on them are Q_1 and Q_2 respectively.

Then $Q_1 = C_1V_1$ ...(4.36)

and $Q_2 = C_2V_2$ ...(4.37)

So total charge on the system before the two conductor are brought into contact

$$Q = Q_1 + Q_2$$

$Q = C_1V_1 + C_2V_2$ ...(4.38)

If the two conductors are now connected through a conducting wire of negligble capacitance then the charge will flow from the conductor at higher potential to other at lower potential till both acquire same potential V(say). In this case treating both condcutors as one the capacity of the combined system is $C_1 + C_2$ as the two conductors are at same potential can be treated in a parallel combination.

After the redistribution of charges if Q_1 and Q_2 are the cahrges on C_1 and C_2

$Q_1' = C_1V$ ...(4.39)

and $Q_2' = C_2V$ ...(4.40)

From conservation of charge, total charge before redistribution must equal the total charge after redistribution i.e

$\therefore \quad Q = Q_1' + Q_2' = (C_1 + C_2)V$

or $C_1V_1 + C_2V_2 = (C_1 + C_2)V$

therefore the common potential is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \qquad \ldots (4.41)$$

and ratio of charges after redistribution

$$\frac{Q_1'}{Q_3'} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2} \qquad \ldots (4.42)$$

thus after redistribution charge is shared in proportion to capacity

If $V_1 > V_2$

then $V_1 > V > V_2$

the change in potential of first conductor is

$$\Delta V_1 = V_1 - V$$

$$\Delta V_1 = V_1 - \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$$

or $$\Delta V_1 = \frac{C_2(V_1 - V_2)}{C_1 + C_2} \qquad \ldots (4.43)$$

The change in potential of second conductor

$$\Delta V_2 = V - V_2$$

or $$\Delta V_2 = \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right) - V_2$$

or $$\Delta V_2 = \frac{C_1(V_1 - V_2)}{C_1 + C_2} \qquad \ldots (4.44)$$

so $$\frac{\Delta V_1}{\Delta V_2} = \frac{C_2}{C_1}$$

Thus after redistribution the magnitudes of change in potential of the two conductors are in inverse ratio of their capacitances. The above discussion is also true for sharing of charge between two charged capacitors initially at different potentials.

4.13 Energy loss

Initially before contact the total electrostaic potential energy of the system is

$$U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \qquad \ldots (4.45)$$

After the conductors are joined through a conducting wire, total electrostatic potential energy of the system is

$$U' = \frac{1}{2}(C_1 + C_2)V^2$$

$$U' = \frac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2$$

$\because$ $$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

So $$U^1 = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2} \qquad \ldots (4.46)$$

So energy loss $\Delta U = U - U'$

or $$\Delta U = \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

or $$\Delta U = \frac{1}{2}\left[\frac{C_1V_1^2(C_1 + C_2) + C_2V_2^2(C_1 + C_2) - (C_1V_1 + C_2V_2)^2}{C_1 + C_2}\right]$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2 \qquad \ldots (4.47)$$

Since $(V_1 - V_2)^2$ is always positive so $\Delta U = 0$

thus there is always a loss of electrostatic potential energy, when two conductors or two capacitors charged at different potentials are connected in parallel. This loss of energy is mainly in the form of heat when charge flows from one conductor to other through the conducting wire having some finite resistance and also in form of light and sound if sparking takes place. In sharing there is no loss of energy

If $V_1 = V_2$ so that $\Delta U = 0$

In process of charging a capacitor by a battery of emf V half of the energy QV supplied by battery gets stored in capacitor ($U = \frac{QV}{2}$) and remaining half (QV/2) is dissipated in form of heat in connecting wires.

Example 4.16 A 600 pF capacitor is charged to 200V using a battery. Now the battery is disconnected and the plates of this capacitor are connected in parallel to an initially uncharged capacitor of 600 pF. Find the loss of electric potential energy in the process.

Solution : Here $C_1 = C_2 = 600\ pF$

$$= 600 \times 10^{12} F = 6 \times 10^{10} F$$

$$V_1 = 200V,\ V_2 = 0$$

loss in energy $= \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$

$$= \frac{1}{2} \times \frac{6 \times 10^{10} \times 6 \times 10^{16}}{12 \times 10^{-10}}(200 - 0)^2$$

$$= 6 \times 10^{6}\ \mathrm{J}$$

Example 4.17 A 900 pF capacitor is charged to 200 V (a) Find the electrostatic potential energy stored in the capacitor (b) It now the capacitor is disconnected from battery and joined in parallel with another uncharged 900 pF capacitor then calculate the electrostatic potential energy stored in the combination.

Solution : (a) Given $C = 900\ pF = 9 \times 10^{-10} F$

$$V = 100V$$

energy stored

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}(9 \times 10^{10})(100)^2$$

$$U_1 = 4.5 \times 10^{6} J$$

(b) When given charged capacitor is connected in parallel to an identical uncharged capacitor the charge on first is now divided equally on the two. Let the final charge on each capacitor is Q[1] and common potential be V.

$$Q = \frac{Q}{2} \text{ and } V = \frac{V}{2}$$

In this case both capacitors have same value of stored energy so the total energy of system

$$U_2 = 2\left(\frac{1}{2}QV\right)$$

$$U_2 = 2 \times \frac{1}{2} \times \frac{Q}{2} \times \frac{V}{2} = \frac{1}{4}QV$$

$$U_2 = \frac{1}{2} \times 4.5 \times 10^{-6} = 2.25 \times 10^{-6}\ \mathrm{J}$$

and rest half of the energy is spent is the form of heat and EM radiations.

Important Points

1. **Dielectric Materials :** These materials are non conductor of electricity they get polarised when placed in external electric field.
2. **Capacitane** The electric potential V of a conductor is proportional to charge Q given to it i.e Q = CV here the constant of proportionality C is called capacitance of conductor. Numberically the capacitance of a conductor is the amount of charge given to a conductor to raise its electric potential by unity.
3. A capacitor is a device that consists of two closely spaced conductors (plates) with charges + Q and - Q. Its capacitance is also given by Q = CV where V is the potential difference between the plates.
4. The capacitance of a capacitor depends on area of plates, plate separation and dielectric constant of medium present between plates.
5. For an isolated spherical conductor radius R[1] placed in air or vacuum the capacitance is given by $C_0 = 4\pi \in_0 R$.

6. The radius of a spherical conductor of 1 F is greater even than the radius of earth.

7. On filling the space between the plates of a capacitor completely with a of medium of dielectric constant $\in_r$ its capacitance increases $\in_r$ times.

8. Capacitance for a parallel plate capacitor $C = \in_r \frac{\in_0 A}{d}$ having a medium between the plates.

with free space as medium $C_0 = \frac{\in_0 A}{d}$

with medium of dielectric constance $\in_r$

9. For a capacitor partially filled with a dielectric (dielectric constant $\in_r$, and thickness t)

$$C = \frac{\in_0 A}{d - t + \frac{t}{\in_r}}$$

10. Capacitance for a spherical capacitor with free space medium $C_0 = \frac{4\pi \in_0 r_1 r_2}{(r_1 - r_2)}$; with medium of dielectric constant $\in_r$, $C_m = \in_r \frac{4\pi \in_0 r_1 r_2}{(r_1 - r_2)}$

11. In series combination of capacitor charge on each capacitor is same and equivalent capacitance of the combination is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

12. For parallel combination of capacitor potential difference across each capacitor is same and equivalent capacitance is given by $C_P = C_1 + C_2 + C_3 + \ldots + C_n$

13. In charging a capacitor some work has to be done. This work is stored in capacitor in electric field between the plates in the form of electrostatic potential energy . This energy is given by

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

14. When two conductors charged to different potentials are connected, the redistribution of charge takes place in ratio of their capacitance i.e $\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$ and their common potential (after charge redistribution) is given by

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

15. The energy loss in above redistribution of charges is given by

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2 \, J$$

Questions for Practice

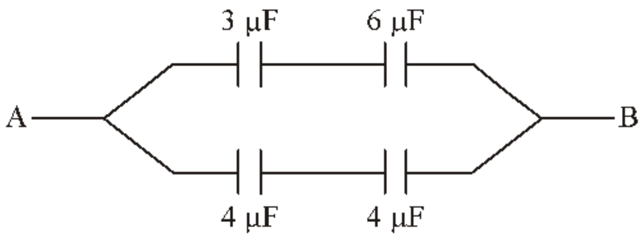
Multiple Choice Questions

1. The correct relation for the capacitance of a parallel plate capacitor is

(a) $C \propto R$ (b) $C \propto R^2$

(c) $C \propto R^{-2}$ (d) $C \propto R^{-1}$

2. For the figure shown equivalent capaitance between points A and B is



(a) 2 μF (b) 4 μF

(c) 25 μF (d) 3 μF

3. On connecting the two plates of a charged cpacitor by a conducting wire

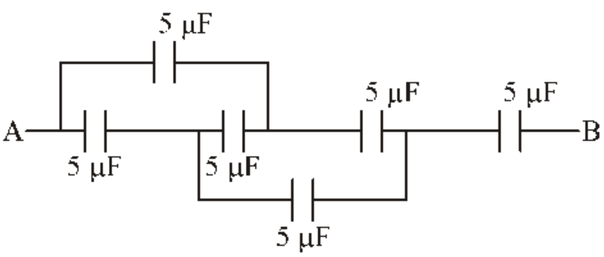
(a) potential difference across the plates will become infinite

(b) charge on capacitor will become infinite

(c) charge on capacitor will become double of its initial value

(d) Capacitor will be discharged

4. For the figure shown the equivalent capacitance between position A and B



(a) 5 μF (b) 2.5 μF

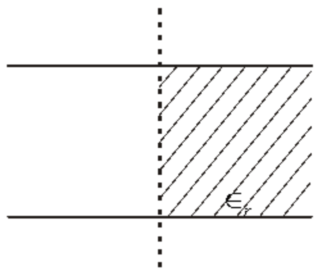
(c) 10 μF (d) 20 μF

5. The radii of two spherical conductors are in ratio 1 : 2 the ratio of their capacitanes is

(a) 4 : 1 (b) 1 : 4

(c) 1 : 2 (d) 2 : 1

6. As shown in Fig. a dielectric slab of dielectric constant $\in_r$ is slided in half the space between capacitor plates. If the initial capacitance of the capacitor is C its new value will be



(a) $\frac{C}{2}(\in_r + 1)$ (b) $\frac{1}{2}\cdot\frac{C}{(\in_r + 1)}$

(c) $\frac{(1+\in_r)}{2C}$ (d) $C(1+\in_r)$

7. Eight drops of mercury of equal radii and each possessing the same charge combine to form a big drop. The capacitance of this big drop as compared to that of each smaller drop is

(a) 2 times (b) 8 times

(c) 4 times (d) 16 times

8. A capacitor has capacitance C. It is charged to a potential difference V. It now be is connected across a resitor R then amount of heat dissipated will be

(a) CV^2 (b) $\frac{1}{2}CV^2$

(c) $\frac{1}{3}CV^2$ (d) $\frac{1}{2}QV^2$

9. On giving a charge Q to a capacitor its stored energy is W. On doubling the charge stored energy will be

(a) 2 W (b) 4 W

(c) 8 W (d) W/2

10. Two spherical conductors of 3 μF and 5 μF are charged to 300 V and 500 V and then connected together. The common potential will be

(a) 400 V (b) 375 V

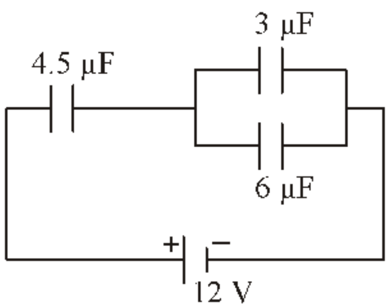
(c) 425 V (d) 350 V

11. The potential energy stored in the region between plates of a capacitor is U_0. If a dielectric slab of $\in_r$ now fills the space between the plates completely then new potential energy will be

(a) $\frac{U_0}{\in_r}$ (b) $U_0 \in_r^2$

(c) $\frac{U_0}{\in_r^2}$ (d) U_0

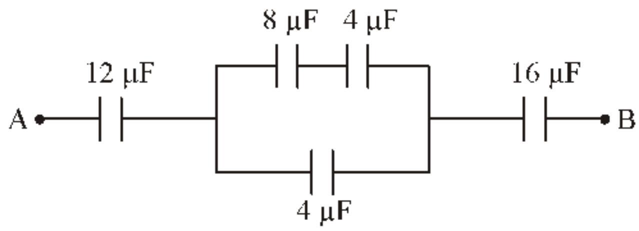
12. For the circuit shown in fig the potential difference across 4.5 μF capcitor is



(a) $\frac{8}{3}V$ (b) 4 V

(c) 6 V (d) 8 V

13. For the circuit shown in fig the equivalent capacitance between A and B will be



(a) 1μF (b) 9 μF

(c) 1.5 μF (d) $1/3\mu F$

Very Short Answer Questions

1. If area of one plate of a parallel plate capacitor is halved then can the device be called a capacitor.

2. What will be the maximum and minimum capacitance obtainable with three capacitors each of 6 μF capacitance.

3. Mention the factors affecting capacitance of a conductor.

4. On assuming earth as a spherical conductor what will be its capacitance?

5. What is the electric field between the plates of a charged parallel plate capacitor with surface charge density σ_on plates.

6. If n capacitors of equal capacitance C are concepted in series what will be the equivalent capacitance.

7. Write expression for energy density for electric field between plates of a parallel plate capacitor.

8. Write unit of energy density.

9. Two capacitors of capacitance C_1 and C_2 are given equal charge write the ratio of electrostatic potential energy stored in them.

10. Mention a conductor which can be given nearly infinite amount of charge.

11. Where and in what form does the energy is stored in a capacitor.

12. What is the net charge on a charged capcitor?

13. On filling the space between the plates of a parallel

plate capacitor with some dielectric its capacitance increases five times. What is th dielectric constant of this materials?

14. What is the basic use of a capacitor?

15. The plate separation of a parallel plate capacitor is d. If a metalic plate of thickness d/2 is placed in the space between plates not touching any plate, then what will be effect on its capacitance?

16. How much work has to be done in charging a 25 μF capacitor if potential differece across it is 500 V.

17. How will you obtain a capacitance of higher value if you have been provided with capacitors of relaively low capacitance.

18. What is the equivalent capacitance of two 2 μF capacitors connected in series.

19. On immersing a parllel plate capacitor in oil what will be the effect on its capactiance. The dielectric constant of oil is 2.

20. The radius of a circular plate for a circular parallel plate capactor is r. If its capacitance is equal to that of a spherical conductor of radius R then find the plate separation.

Short Answer Questions

1. Explain terms conductor and insulator with examples.

2. Distinguish between polar and non polar dielectrics.

3. Derive expression for capacitance of a spherical conductor.

4. What will be the effect on the potential difference across the plates of a capacitor if its plates are brought nearer keeping the charge constant? Explain.

5. A parallel plate capacitor with air as medium is charged to a potential difference V using a source (battery). Without disconnecting the battery air medium is replaced by a dielectric medium of dielectric constant $\in_r$. Explain with reasons the changes observed in the following -

   (i) potential difference

   (ii) electric field between plates

   (iii) capacitance

   (iv) charge

   (v) energy

6. A parallel plate capacitor with air as medium is charged to a potential difference V_0 using a voltage supply. Ater it is disconnected from the voltage supply the space between plates is completely filled by a dielectric. Explain with reasons the changes observed in the following

   (i) charge (ii) potential difference

   (iii) capacitance (iv) electric field

   (v) energy

7. Derive expression for energy stored in a charged capacitor.

8. Three capacitors of capcitance C each are connected first in series and then in parallel. Calculate the ratio of equivalent capacitances in these two situations.

9. n capactors of capaitance C each when connected in series yields an equivalent capacitance C_S and when in parallel yields equivalent capacitance C_p.

   Find the value of $\frac{C_p}{C_s}$.

10. Define electric capacitance and mentioned its SI unit.

11. What will happen to the capacitance of a spherical conductor on trippling charge on it? Give reason.

12. On filling the space between a 2 μF air capacitor by mica its capacitance becomes 5μF. Calculate the dielectric constant of mica.

13. Two spherical conductors of radii R_1 and R_2 have capacitance C_1 and C_2 and charges Q_1 and Q_2 and potential difference V_1 and V_2 respectively $(V_1 > V_2)$. These conductors are now connected by a conducting wire of negligilbe capacitance then show that the ratio of chages in their potential is

$$\frac{\Delta V_1}{\Delta V_2}=\frac{C_2}{C_1}$$

14. What is a capacitor? Explain.
15. Three capacitor having capacitance C_1, C_2 and C_3 are connected in series. Derive expression for equivalent capacitance.
16. Three capacitors having capacitances C_1, C_2 and C_3 are connected in parallel. Derive expression for equivalent capacitance.

Essay Type Questions

1. Discuss the principal of a capacitor and derive the expression for the capacitance of a parallel plate capacitor.
2. Obtain an expression for the capacitance of a partially filled capacitor.
3. Derive an expression for energy density of electric field between the plates of a parallel plate capacitor.
4. What is a spherical capacitor? Derive an expression for the capcitance of spherical capacitor.
5. Explain the charge redistribution when two charged conductors are connected together. Determine the ratio of charges after redistribution and expression for energy loss.

Answer (Multiple Choice Questions)

1. (A) 2. (B) 3. (D) 4. (B) 5. (C) 6. (A)
7. (A) 8. (B) 9. (B) 10. (C) 11. (A)
12. (D) 13. (A)

Short Answer Question

1. No
2. $C_{max}=18\,\mu F$ and $C_{min}=2\,\mu F$
3. The capacitance of a conductor depends on its geometry and nature of surrounding medium.
4. $C_0=4\pi\in_0 R$ here

   $R=6400km=6400\times10^3 m$, $C_0=711\mu F$
5. $E=\dfrac{\sigma}{\in_0}$
6. , $C_s=\dfrac{C}{n}$
7. $\dfrac{U}{V}$; or $u=\dfrac{1}{2}\in_0 E^2$
8. J/m^3
9. $$\frac{U_1}{U_2}=\frac{\frac{1}{2}\frac{Q^2}{C_1}}{\frac{1}{2}\frac{Q^2}{C_2}}=\frac{C_2}{C_1}$$
10. Earth, because its capacitance is very large.
11. In form of electrostatic potential energy in electric field between capacitor plates.
12. Zero as plates have equal and opposite charges.
13. $\in_r=\dfrac{C}{C_0}=\dfrac{5C_0}{C_0}=5$
14. To store electric charge and electric energy.
15. $$C'=\frac{\in_0 A}{(d-t)}=\frac{\in_0 A}{(d-d/2)}=2\left(\frac{\in_0 A}{d}\right)=2C$$

    i.e capacitance is doubled
16. $W=QV=(CV)V=CV^2$

    $=24\times10^{-6}\times500\times500=6\,J$
17. By joining given capacitors is parallel.
18. From $\dfrac{1}{C_s}=\dfrac{1}{C_1}+\dfrac{1}{C_2}$ this gives $C_s=1\mu F$
19. $C=\in_r C_0=2C_0$ i.e doubled
20. $C=4\pi\in_0 R\Rightarrow\dfrac{\in_0\pi r^2}{d}=4\pi\in_0 R\Rightarrow d=\dfrac{r^2}{4R}$

Numberical Problems

1. Calculate the radius of a spherical conductor of 1 pF capacitance. [Ans : 9 mm]

2. The area of each plate of a parallel capacitor is 100 cm^2 and the intensity of electric field in between the plates is 100 N/C. What is the charge on each plate.

   [Ans :$+8.85 \times 10^{-12}$ C, -8.85×10^{-12} C]

3. A parallel plate capacitor is kept at a certain potential difference. Keeping this potential difference constant to put a dielectric slab of 3 mm thickness between the plates the plate separation is to be increased by 2.4 mm. Calculate the dielectric constant of slab. [Ans : $\in_r = 5$]

4. Two capacitors are of 2μF and 4μF capacitance respectively. Compare the ratio of equivalent capacitances of their series and parallel combinations.

   [Ans : 2 : 9]

5. Consider two metallic spheres of radii 0.05 m and 0.10 m, each having a 75 μF charge. On connecting these spheres by a conducting wire find (i) common potential (ii) amount of charge flown.

   [Ans : 9×10^6 V, 25 μC]

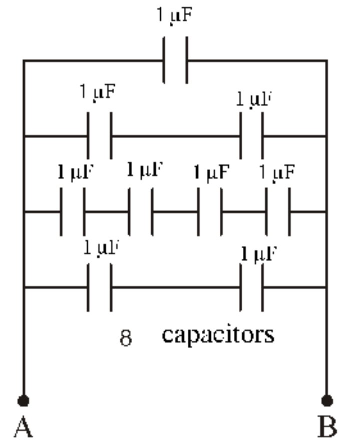
6. A spherical conductor of capacitance 2μF, charged at 150 volt is connected to an 1μF uncharged and conducting sphere. Find common potential, charge on each capacitor after joining.

   [Ans. : V = 100 V, $Q_1' = 200\mu C$, $Q_2' = 100\mu C$]

7. 125 droplets are charged to 200 V potential. These droplets are combined to make a single drop. Calculate the potential of big drop and change in potential energy.

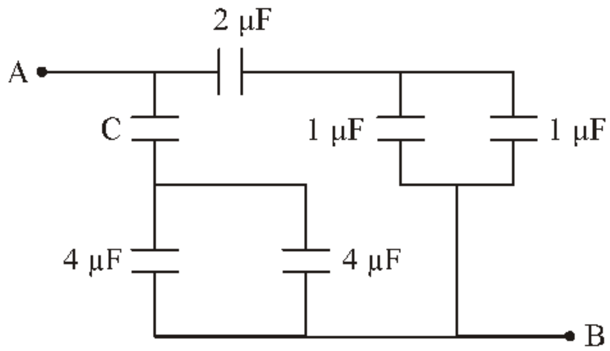
   [Ans : $V_b = 500$ V, $U_b = 25\ U_s$]

8. In fig shown each capacitor is of 1 μF capacitance. Find equivalent capacitance between points A and B.



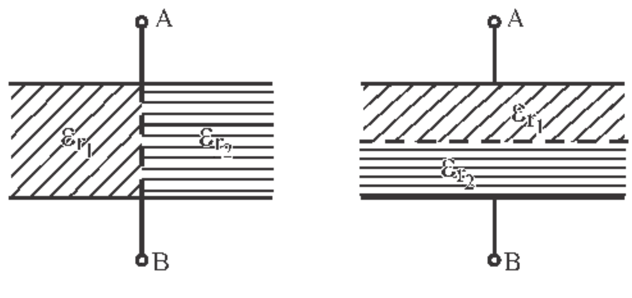
[Ans : 2 μF]

9. In fig shown the equivalent capacitance between points A and B is 5. Calculate the capacitance of capacitor C.



[Ans : C = 8 μF]

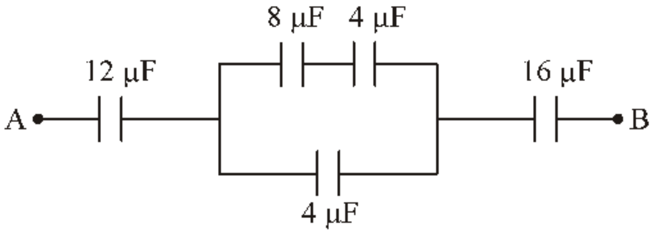
10. For capacitors shown in fig (a) and (b) find capacitances Area of each plate is A and the plate separation is d



(a) (b)

[Ans : (a) $\left[\frac{\in_0 A}{2d}\left(\in_{r_1} + \in_{r_2}\right)\right]$ (b) $\left[\frac{2 \in_{r_1} \in_{r_2} \in_0 A}{d\left(\in_{r_1} + \in_{r_2}\right)}\right]$]

11. Determine equivalent capacitance between points A and B for the Fig shown



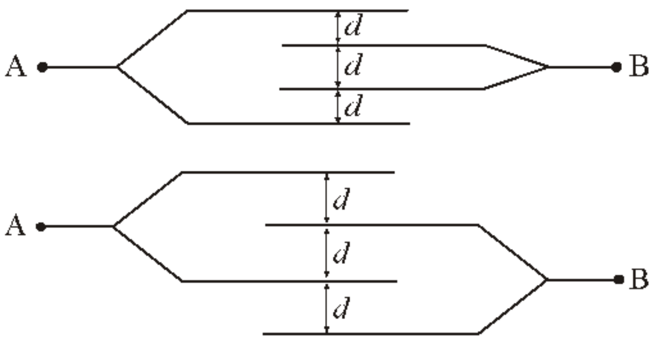
[Ans : $C = \frac{1}{31}\mu F$]

12. An isolated spherical conductor is surrounded by another concentric conducting sphere whose outer surface is earthed. the ratio of the radii of these two spheres is $\frac{n}{n-1}$. Prove that due to this arrangment the capacitance of spherical conductor increases n times.

13. The energy density of a parallel plate condensor is $4.43 \times 10^{-10}\ J/m^3$. Find electric field intensity between the plates. $\epsilon_0 = 8.86 \times 10^{-12}\ F/m$

[Ans : E = 10 N/C]

14. Determine the equivalent capacitance of the systems shown in fig (a) and (b) between A and B. Area of each plate is A and the separation between adjancent plates is d.



[Ans : (a) $\frac{2\epsilon_0 A}{d}$ (b) $\frac{3\epsilon_0 A}{d}$]

15. Let equivalent capacitances for n indentical capacitors each of capacitance C in series and parallel arrangements are Cs and Cp . Prove that

$$C_p - C_s = \frac{(n^2-1)}{n}C$$

16. The radius of plate used in a parallel plate capaitor is 10 cm. If the plate separation is 10 cm, determine the capacitance of this capacitor for air medium.

[Ans : C = 2.78 pF]