Chapter – 11

Work and Energy

Work

Term 'Work' in our daily life:

Suppose a person pushes the rock, but the rock does not move.

Is he doing some work?

If we are considering our daily life term work, then yes, he did.

But in science, he doesn't do any work as there is no resultant motion results because of the force he applied on the rock.

Work done (Scientific approach): Work is said to be done when a force produces motion in a body.

If the force 'F' is applied on an object and the object displace a distance 'S' in the direction of the force.

Then, work done W is defined as the product of the force and displacement.

$$W = F \times S$$

The SI unit of work is Joule. 1 J = 1 Nm

Tip: Remember the work is a scaler quantity although force and displacement are the vector quantity itself.

Example: Manan pushes a wooden table across the floor. A net force of 150 N is applied on the table and the work done by this force is 600 J. Calculate the distance that the table moved.

Solution: The net force applied on the table, F = 150 N

Work done on the table, W = 600 J

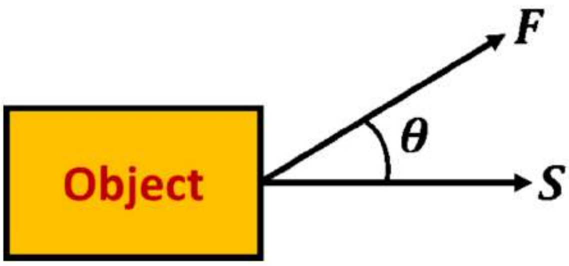
Here, we have to find the distance of table displaces from its initial position, S = ?

We know that,

W = F.S

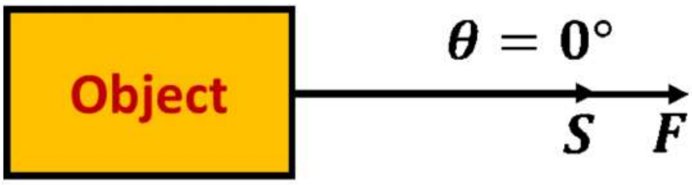
Therefore, $S = \frac{W}{F} = \frac{600}{150} = 4$ m

Work done when force is applied at some angle on the object: The force applied F and the displacement S covered by the car having an angle θ between them.



$W = FScos\theta$

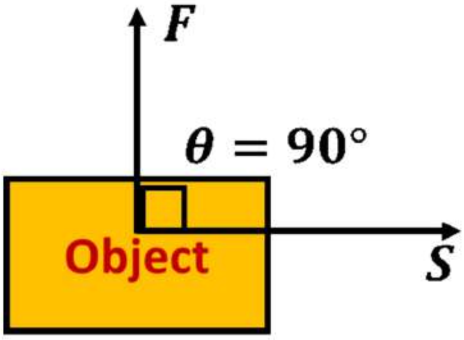
Positive Work: When the applied force and the displacement are in the same direction, the angle $\theta = 0^{o}$ and the value of $\cos 0^{o} = 1$.



So, work done $W = FS \times 1 = FS$

Hence, the work done is positive $W = FS$

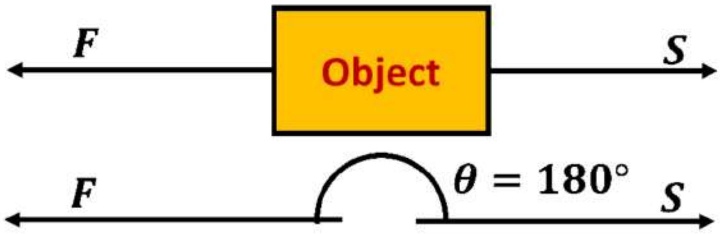
Zero Work: When the applied force and the displacement are perpendicular to each other, the angle $\theta = 90^{o}$ and the value of $\cos 90^{o} = 0$.



So, work done $W = FS \times 0 = 0$

Hence, the work done is zero $W = 0$

Negative Work: When the applied force acts opposite to the direction of displacement, the angle $\theta = 180^{\circ}$ and the value of $\cos 180^{\circ} = -1$.



So, work done $W = FS \times -1 = -FS$

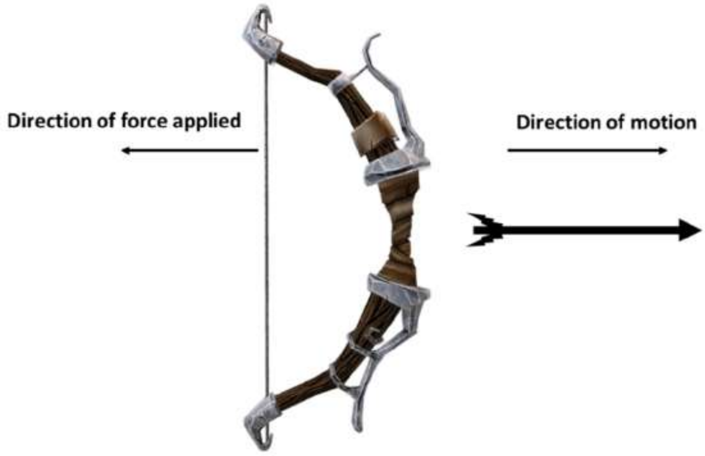
Hence, the work done is negative $W = -FS$

Example: Identify the type of work as positive, negative or zero in the following cases.

(i) The archer shooting an arrow.

(ii) A satellite revolving around the earth.

Solution: (i) To shoot the arrow, the archer pulls the string towards him and the arrow moves away from him.



In this case, the applied force acts opposite to the direction of motion. So, the angle between the applied force and the direction of displacement is 180°.

Therefore, the work done $W = -FS$

Hence, the work done is negative.

(ii) The satellite moves around the earth in a circular orbit. Here the force of gravitation act on the satellite at right angles to the direction of motion of the satellite. So, the angle between the direction force and the direction of displacement is 90°.

Therefore, the work done W = 0.

Hence, the work done is zero in this case.

Energy

Energy: The capacity of doing work is known as energy.

The SI unit of energy is also Joule.

Forms of Energy: Around us energy is present in many different forms, some main forms are:

- Electrical energy
- Mechanical energy
- Heat energy
- Chemical energy
- Light energy

- Nuclear energy
- Sound energy

Mechanical Energy: Mechanical energy is the energy possessed by a body during its state of motion, i.e energy possessed when the object is at rest or moving.

- **Potential Energy:** The type of energy possessed by an object at rest is called potential energy.
- **Kinetic Energy:** The type of energy possessed by a moving object is called kinetic energy.

The sum of kinetic energy and potential energy is always equal to the mechanical energy of the body.

Kinetic Energy: The energy possessed by a body due to its motion is called kinetic energy.

For example, wind moves the blade of a windmill because the wind is in motion, so it possesses kinetic energy which helps in doing the work of rotating the blades of a windmill.

Derivation for the formula of kinetic energy:

Consider an object having mass m, moving with a uniform velocity u. Assume it is displaced by a distance 'S' when a constant force F acts on it in the direction of its displacement. So, $\theta = 0^\circ$ and $cos 0^\circ = 1$.

Thus, $W = FS$

The work is done W on a body produces a uniform acceleration a and due to that, the object starts moving with a changed velocity v.

Now, $W = FS$

And $F = ma$

So, $W = ma \times S$ ------eq(1)

Using 3rd equation of motion,

$$v^2 - u^2 = 2as$$

$$S = \frac{v^2 - u^2}{2a}$$

By, substituting the value of in equation (1), We get

$$W = ma \times \frac{v^2 - u^2}{2a}$$

After simplification

$$W = \frac{m(v^2 - u^2)}{2}$$ --------eq(2)

If we take the object initially at rest, then u $= 0$

And, $W = \frac{1}{2}mv^2$

Here, this work is the kinetic energy possessed by a moving body with a uniform velocity v.

$$K.E = \frac{1}{2}mv^2$$

This is the expression for kinetic energy.

Tip: Remember the kinetic energy can be expressed in the form of momentum, P = mv as:

$$K.E = \frac{P^2}{2m}$$

Work-Energy Theorem:

In the above derivation using equation (2) $\Rightarrow W = \frac{m(v^2 - u^2)}{2}$ we can calculate the work done by the moving body by finding the change in kinetic energy.

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = K.E_i - K.E_f$$

According to the work-energy theorem, the net work done by the forces on the object is equal to the change in its kinetic energy.

Potential Energy: The energy of a body due to its position or change in shape is known as potential energy.

For example, when we release the compressed spring from the rest position it starts moving, this means at this rest position the spring possesses some energy to do work and that energy is the potential energy.

Potential Energy of an Object at a Height: When an object is raised to a height say (h), the work is done against gravity and this work is stored in the form of energy. This energy possessed by an object is called gravitational potential energy.

The **gravitational potential energy** is defined as the work done in raising an object from the ground to the height (h) against gravity.

Formula for Potential Energy:

Work done = Force X Displacement

$$W = ma \times s$$

Here, the displacement is h and a is the acceleration due to gravity g as the object is moving in the vertical direction.

So, $W = mgh$

The Work done is stored in the body as potential energy.

Therefore,

$$P.E = mgh$$

This is the expression for potential energy.

Tip: Remember the potential energy does not depend on the path of the motion.

Example: Rohan lifts a 10 kg box kept on the ground and keeps it on the top of a table of height 1.5 m. Calculate the energy possessed by the box on the table. [Take g = 9.8 ms^{-2}]

Solution: Mass of the box, m = 10 kg {Given}

Height of the table (the distance covered by the box), h = 1.5 m {Given}

Acceleration due to gravity, g = 9.8 ms^{-2}

The object is at a certain height, so it possesses potential energy.

$$P.E = mgh$$

$$P.E = 10 \times 9.8 \times 1.5$$

$$P.E = 147\ J$$

Law of conservation of Energy: The law of conservation of energy states that the energy can neither be created nor be destroyed. It is only transformed from one form to another.

For example, in the hydroelectric power house,

- Dams are used to block the water and it comes into rest and possesses **potential energy**.
- When the dams are opened the **potential energy** possessed by the collective water transform into **kinetic energy** and water starts flowing.
- The moving water fall on the turbine and the turbine comes into motion, here the **kinetic energy** possessed by the water is transformed into **mechanical energy**.
- The turbine is connected with the dynamo box that transforms **mechanical energy** into **electric energy**.

In this example, the new energy is not created. Energy is just transformed from one form to another.

Rate of doing work (Power)

Power: 'The rate of doing work' or 'the rate of energy consumption' is called power.

$$Power = \frac{Work}{Time}$$

The SI unit of power is Watt denoted by W.

1 Watt = 1 Js^{-1} and 1 kW = 1000 Js^{-1}

Tip: $Power = Force \times Velocity$

We can easily drive this expression for power or directly use it to solve the questions.

Example: An elevator carrying passengers having a total load of 500 kg is moving up with a constant speed of 3 m/s. The frictional force acting on the elevator is 2000 N. What is the minimum power required by the motor to drive the elevator? [Take g = 10 ms^{-2}]

Solution: We know, $P = \frac{W}{t}$

So, $P = \frac{F \times s}{t}\ [\because W = Fs]$

Now using, $Speed = \frac{Distance}{time}$

$$v = \frac{s}{t}$$

So, by substituting the value of $\frac{s}{t}$ in the formula of power, we get

$$P = F \times v$$

The speed of elevator, v = 3 m/s {Given}

Now, we calculate the minimum force required to move the elevator upward, which should be greater than or equal to the total downward force.

The downward force acting on the elevator will be the force of passengers and the frictional force which opposes the upward motion.

Therefore,

$f_{downward} = mg + f_{friction}$

$f_{downward} = 500 \times 10 + 2000$

$f_{downward} = 7000\ N$

To drive, this elevator motor must supply enough power to balance this downward force.

Hence, the minimum Power = F x v = 7000 x 3 = 21000 W

Commercial Unit of Energy: The commercial unit of energy is kWh.

The energy consumed in households and industries is measured in a commercial unit.

1 kWh = 1 unit = 3.6 x 10^6 J

Example: A bulb rated 25 W is used for 6 hours a day. What will be the expenses of using the bulb for 30 days in the month of January, if the cost of electricity is 3 rupees per unit?

Solution: Energy consumed by the bulb in one day = 25 x 6 = 150 Wh

So, the energy consumed in 30 days by the bulbs = 150 x 30 = 4500 Wh

Now, we convert this energy from Wh into kWh because the cost is given for each unit and we know 1 unit = 1 kWh.

Therefore, Energy consumed = $\frac{4500}{1000} = 4.5\ kWh = 4.5\ units$

Now, it is given that the cost of each unit is ₹ 3.

Therefore, the total cost of electricity consumption = 4.5 x 3 = ₹ 13.5