Sample Paper 11

Class IX 2022-23

Mathematics

Time: 3 Hours **Max. Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 3 Qs of 5 marks, 3 Qs of 3 marks and 2 Questions of 2 marks has been provided.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A

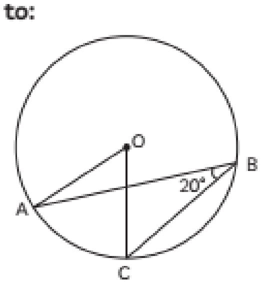
(Section A consists of 20 questions of 1 mark each).

1. Which of the following is not equal to $\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{-\frac{1}{6}}$?

(a) $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}}$ (b) $\frac{1}{\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{\frac{1}{6}}}$

(c) $\left(\frac{6}{5}\right)^{\left(\frac{1}{30}\right)}$ (d) $\left(\frac{5}{6}\right)^{\left(-\frac{1}{30}\right)}$ 1

2. In the given figure, if $\angle ABC = 20°$, then $\angle AOC$ is equal to:



(a) 20° (b) 40°

(c) 60° (d) 10° 1

3. If a straight line falling on two straight lines makes the interior angles on the same side of it, whose sum is 120°, then the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is:

(a) less than 120° (b) greater than 120°

(c) is equal to 120° (d) greater than 180° 1

4. The value of $x^3 + y^3$, if $x + y = 15$ and $xy = 25$ is:

(a) 2250 (b) 1250

(c) 1750 (d) 3250 1

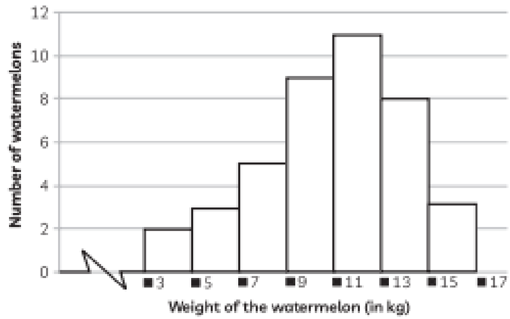
5. The table below shows the weights of number of watermelons at a store.

Weight of Watermelon (in kg)	Number of Watermelons
3 – 5	2
5 – 7	3

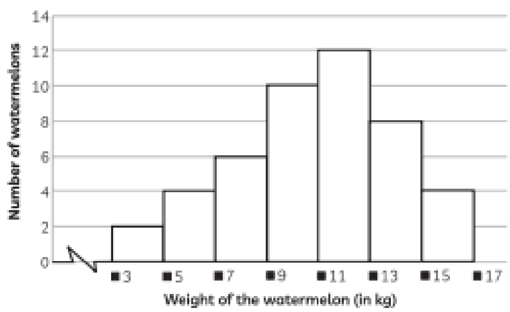
Weight of Watermelon (in kg)	Number of Watermelons
7 – 9	5
9 – 11	9
11 – 13	11
13 – 15	8
15 – 17	3

Which of the following histogram represents the given data correctly?

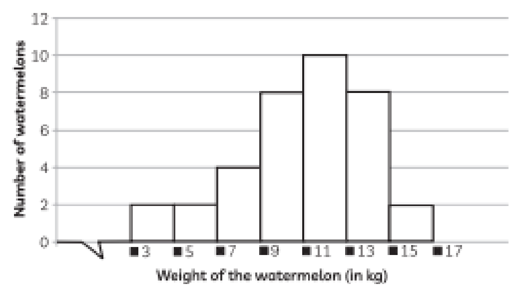
(a)



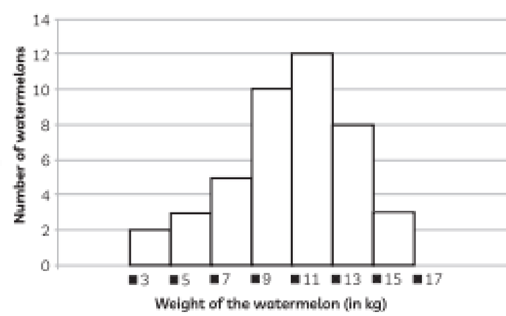
(b)



(c)



(d)



1

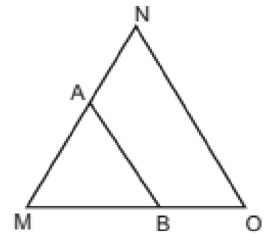
6. The line with the coordinates (4, 1), (2, 1) and (1, 1) is parallel to:

(a) x-axis (b) y-axis

(c) cannot be determined

(d) None of these 1

7. In the given figure ΔMNO is an isosceles triangle in which MN = MO and AB is parallel to NO. If $\angle N = 75°$, then the value of $\angle BAN$.



(a) 105° (b) 75°

(c) 65° (d) 85° 1

8. Is –10 a rational number? Why or why not?

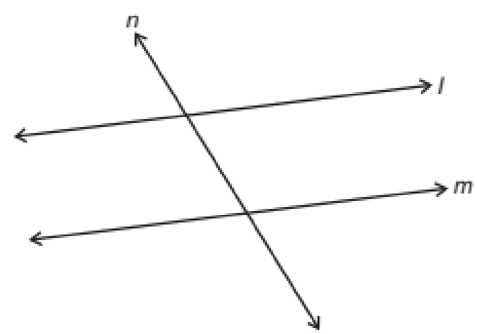
(a) No, as $-10 = \frac{-10}{0}$ and rational numbers are ratios of integers m and n, where $n \neq 0$.

(b) Yes, as $-10 = \frac{-10}{1}$ and rational numbers are the ratios of integers m and n, where $n \neq 0$.

(c) No, as $-10 = -\frac{-10}{0}$ and rational numbers are ratios of integers m and n, where $n \neq 1$.

(d) Yes, as $-10 = \frac{-10}{1}$ and rational numbers are ratios of integers m and n. 1

9. In the given figure, m and l are two parallel lines intersecting by a transversal n.



If another line is drawn parallel to the line m, what would be the increase in the pairs of alternate interior angles that will be formed?

(a) 2 (b) 8

(c) 4 (d) 6 1

10. A tent of conical shape of base radius 28 m and slanted height 20 m is to be made from a certain material. What is the total cost of clothing material if the rate of material is ₹80/m^2? (Take π = 227)

(a) ₹140800 (b) ₹150000

(c) ₹130500 (d) ₹140000 1

11. Name the quadrilateral that doesn't have the diagonals intersecting at right-angles.

(a) Parallelogram, Kite, Rhombus

(b) Square, Rectangle, Trapezium

(c) Parallelogram, Rectangle, Trapezium

(d) Rectangle, Kite, Trapezium 1

12. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is:

(a) $5 + x$ (b) $5 - x$

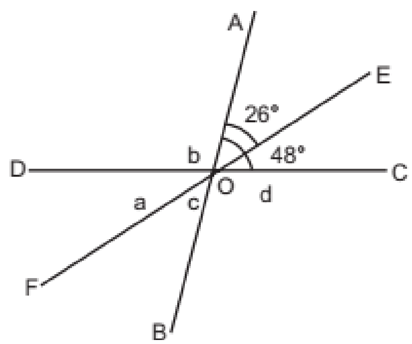
(c) $5x - 1$ (d) $10x$ 1

13. A rhombus and an equilateral triangle have equal perimeters. If the perimeter of the rhombus is 60 cm. The altitude of the equilateral triangle is:

(a) $10\sqrt{3}$ cm (b) $15\sqrt{3}$ cm

(c) $20\sqrt{3}$ cm (d) $14\sqrt{3}$ cm 1

14. In the adjoining figure, if $\angle AOC = 48°$, then the value of a is:



(a) 26° (b) 22°

(c) 42° (d) 24° 1

15. The class marks of a frequency distribution are given as follows: 15, 20, 25, ...

The class corresponding to the class-mark 20 is:

(A) 12.5–17.5 (b) 17.5–22.5

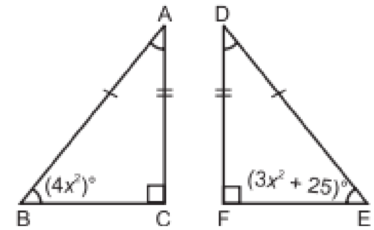
(c) 18.5–21.5 (d) 19.5–20.5 1

16. The value of $399^2 - 398^2$ is:

(a) 779 (b) 979

(c) 879 (d) 797 1

17. In the given figure $\triangle ABC \equiv \triangle DEF$ by AAA congruence rule. The value of $\angle x$ is:



(a) 75° (b) 105°

(c) 125° (d) 5° 1

18. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The upper limit of the class is:

(a) 10 (b) 7

(c) 8 (d) 13 1

Direction: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).

Choose the correct option.

(a) Both assertion (A) and reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both assertion (A) and reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Statement A (Assertion): If $x = 1 + \sqrt{2} + \sqrt{3}$ and $y = 1 + \sqrt{2} - \sqrt{3}$, then $xy = 6 + 2\sqrt{2}$

Statement R (Reason): $(a + b)(a - b) = a^2 - b^2$. 1

20. Statement A (Assertion): The bisectors of the angles of a linear pair are at right angle.

Statement R (Reason): If the sum of two adjacent angles is 180°, then the non-common arms of the angles are in a straight line. 1

SECTION - B

(Section B consists of 5 questions of 2 marks each.)

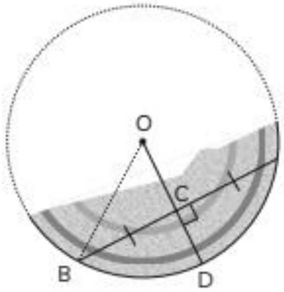
21. A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. Find the radius of the sphere.

OR

The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and volume of a cone. 2

22. Rationalize the denominator of $\frac{1}{\sqrt{3}-\sqrt{2}}$ and evaluate by taking $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$. 2

23. The diagram shows a fragment of a circular plate. AB = 8 cm and CD = 2 cm. What is the diameter of a plate?



2

24. If a and b are rational numbers and $(a - 3)\sqrt{6} + 7 = b\sqrt{6} + a$, then find $a^2 + b^2$. Also, find the value of x, if $a^2 + b^2$ when divided by 13 gives quotient x.

OR

Yogita being a teacher, taught students about irrational numbers and then asked them to find three irrational numbers between $\sqrt{2}$ and $\sqrt{5}$. 2

25. If Shintu told Rinu that $x = -k$ and $y = k + 2$, is the solution of a linear equation $x + 7y = a$. Find the value of a , if $y = 8$. 2

SECTION - C

(Section C consists of 6 questions of 3 marks each.)

26. For the polynomial $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$, write:

(A) the degree of the polynomial

(B) the coeffcient of x^3

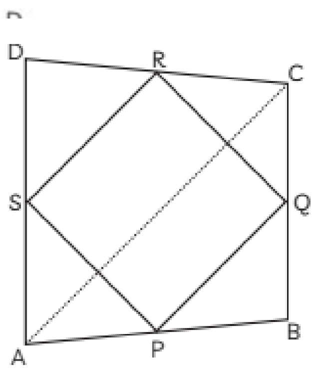
(C) the coeffcient of x^6 3

27. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (as shown in the figure below). AC is a diagonal, Show that:

(A) SR ∥ AC and SR = $\frac{1}{2}$ AC

(B) PQ = SR

(C) PQRS is a parallelogram.



3

28. Find the value of x.

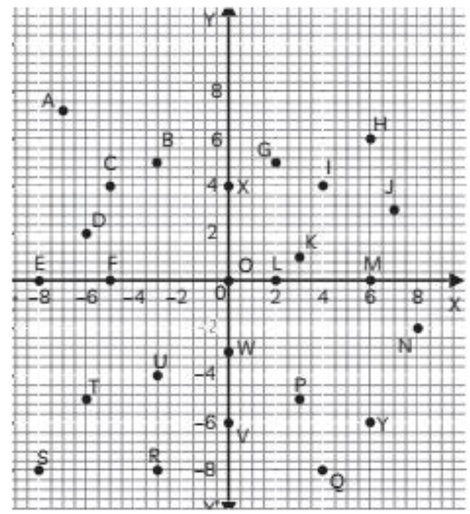
$$\left(\frac{1}{2}\right)^{-4} - 3 \times \left(\frac{64}{8}\right)^{\frac{2}{3}} \times 71^0 + \left(\frac{36}{64}\right)^{\frac{-256}{512}} = \frac{x}{3}$$ 3

29. Based on the given graph, answer the following questions:

(A) Identify all the points with a negative abscissa and a negative ordinate.

(B) Identify any 4 coordinates that will make a square.

(C) Identify points with their abscissa as zero.



OR

Find the coordinates of the point:

(A) which lies on x-axis and y-axis both.

(B) whose ordinate is –4 and which lies on y-axis.

(C) whose abscissa is 5 and which lies on x-axis. 3

30. Ravi is a salesperson and earns a fixed salary per month and earns a commission on his monthly sales. Ravi's monthly earning is given by the equation $10{,}000 + 0.05x$. Ravi gets a hike of 10% on his fixed salary and will now be earning ₹140 on every 2000 rupees worth of sales. Write the equation which shows Ravi's monthly earnings after the hike? 3

31. Prepare a continuous grouped frequency distribution table from the following data:

Midpoint	Frequency
5	4
15	8
25	13
35	12
45	6

Also, find the size of the class interval.

OR

The following table shows the number of scooters sold by a dealer during six consecutive years.

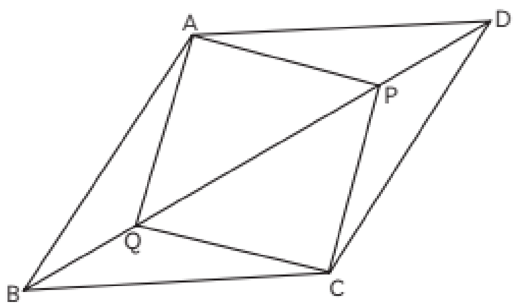
Year	Number of Scooters Sold (In Thousands)
2011	16
2012	20
2013	32
2014	36
2015	40
2016	48

Draw a bar graph to represent this data. 3

SECTION - D

(Section D consists of 4 questions of 5 marks each.)

32. In a parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (as shown in the figure below).



Show that:

(A) $\Delta APD \cong \Delta CQB$

(B) AP = CQ

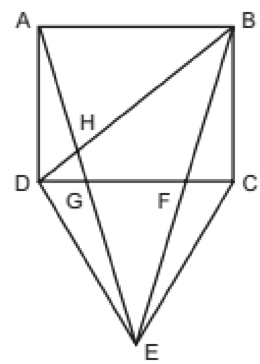
(C) $\Delta AQB \cong \Delta CPD$

(D) AQ = CP

(E) APCQ is a parallelogram 5

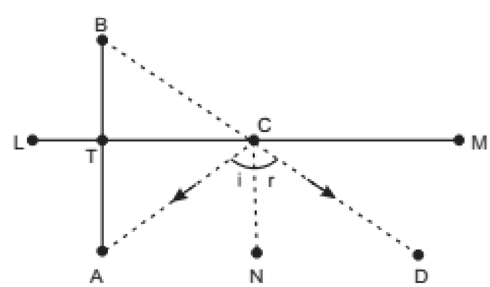
33. A toy is in form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of the base of the cone is 21 cm and its volume is $\frac{2}{3}$ of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy. 5

34. Ram has a wooden box on which he exactly fitted a triangle and joined with a straight line to prove that AE = BE and also, find $\angle BDE$ and $\angle AHD$.



OR

The image of an object placed at a point A before a plane mirror, LM is seen at the point B by an observer at D as shown in figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



[Hint: CN is normal to the mirror, also, angle of incidence = angle of reflection]. 5

35. For which values of k, do the linear equations $kx + y - k^2 = 0$ and $x + ky - 1$ have:

(A) no solution?

(B) a unique solution?

(C) infinitely many solutions?

(D) if $x = 1$ and $y = 2$, then find the value of k in linear equation $kx + y - k^2 = 0$

OR

The linear equation that converts Fahrenheit

(F) to Celsius (C) is given by the relation $C = \frac{5F - 160}{9}$.

(A) If the temperature is 86°F, what is the temperature in Celsius?

(B) If the temperature is 35°C, what is the temperature in Fahrenheit?

(C) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(D) What is the numerical value of the temperature which is the same in both scales? 5

SECTION - E

(Case study based questions are compulsory.)

36. *Junk food is food that contains high levels of salt, sugar, fats and lack of nutrients such as vitamins, fibre and minerals, consuming them can lead to short and long-term health complications, including weight gain. If α be the number of children who take junk food and β be the number of children who take healthy food such that $\alpha > \beta$ where α and β are the zeros of the quadratic polynomial*

$$f(y) = 2y^2 - 18y + 40$$

(A) Find the number of students who take healthy food. 1

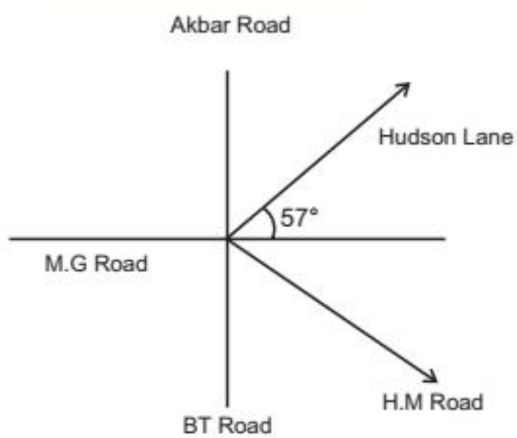
(B) How many students take junk food?

OR

Find the value of k. If $p(0) + p(1) = k.\ p(2)$. 2

(C) Find the value of $p(-1)$. 1

37. *Raju and Priya are cousins and both went to visit Mughal Garden. Before going, they searched the location of their destination on a map. During searching, they found on map that Akbar Road and M.G. Road form a Right angle at their intersection point and Hudson lane form 57° angle with M.G. Road.*



(A) What is the measure of acute angle between Akbar Road and Hudson lane? 1

(B) If a Hudson lane makes 57° angle with M.G. road and H.M. road making 37°, with M.G. road, then what type angle does form between Hudson lane and H.M road?

OR

If Raju is standing on M.G Road in the west direction and Priya is on H.M road, what is the shortest angle they can cover in order to meet? 2

(C) Find the measure of reflex angle formed between M.G. Road [in east direction] with Hudson lane. 1

38. *Interior decorator Natasha designed a floral carpet that was made up of 32 well-designed triangular pieces, the measurements of the triangular pieces are 18 cm, 56 cm, 70 cm. The rate of stitching the carpet is 70 paise per cm^2.*

(A) Find the perimeter of one triangular piece. **1**

(B) Find the semi-perimeter of one triangular piece. **1**

(C) Find the area of one of the triangular pieces.

OR

Find the total area of carpet made up of 32 triangular pieces. **2**

SOLUTION

SAMPLE PAPER - 1

SECTION - A

1. *(a)* $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}}$

Explanation: Given $\left[\left(\frac{5}{6}\right)\right]^{-\frac{1}{6}}$

$$\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{-\frac{1}{6}} = \left[\frac{5}{6}\right]^{\frac{1}{5}\times-\frac{1}{6}} \qquad [\text{Since, } (a^x)^y = a^{xy}]$$

$$= \left[\left(\frac{5}{6}\right)\right]^{-\frac{1}{30}}$$

$$= \left(\frac{6}{5}\right)^{\frac{1}{30}} \qquad \left[\because \left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^{x}\right]$$

And, $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}} = \frac{5}{6}^{\left(\frac{6-5}{30}\right)} = \left(\frac{5}{6}\right)^{\frac{1}{30}} \neq \left(\frac{6}{5}\right)^{\frac{1}{30}}$

2. *(b)* $40°$

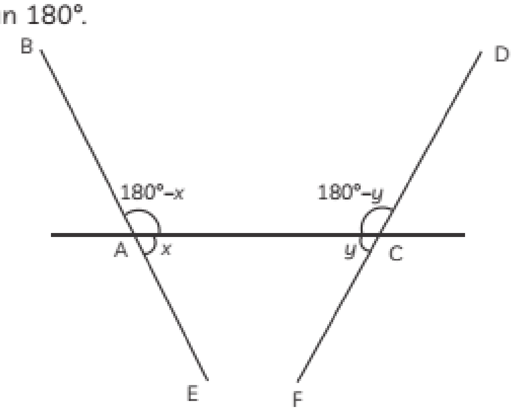
Explanation: Here, arc AC of the circle making an angle of $20°$ on the circumference of a circle.

Hence, arc$\angle AOC = 2\angle ABC$

$\angle AOC = 2 \times 20° = 40°$

3. *(c) Is equal to 120°*

Explanation: We have to Form a triangle to meet the line BE and DF. if the lines AB and CD is considered it won't meet as the sum of base angle is greater than 180°. It is given that $x + y = 120°$. It will only intersect on AE and CF side as the base angles $x + y = 120°$ is less than 180°.



Therefore, the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is equal to 120°.

4. *(a) 2250*

Explanation: We know,

$$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$$

And $$(x + y)^2 = x^2 + y^2 + 2xy$$

Now, $$x + y = 15$$

Squaring both sides

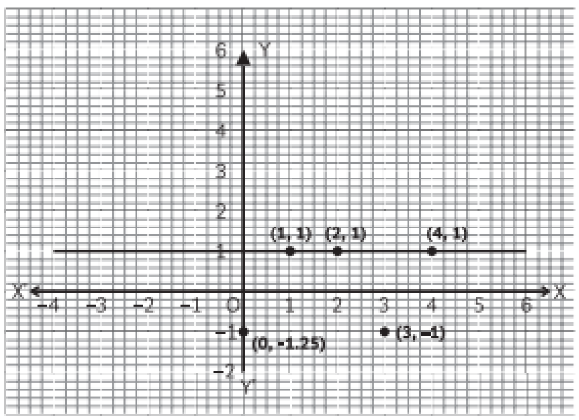
$$x^2 + y^2 + 2xy = 225$$
$$x^2 + y^2 = 225 - 2xy$$
$$= 225 - 2(25) = 175$$
$$\therefore \quad x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$$
$$= 15(175 - 25) = 15(150)$$
$$x^3 + y^3 = 2250.$$

5. *(a).*

Explanation: Histogram given in option (a) is the correct representation of the given table.

6. *(a) x-axis*

Explanation: Plot the given points (4, 1), (2, 1) and (1, 1) on the Cartesian plane.



It is clear from the graph, the line with the coordinate is parallel to x-axis.

7. *(a) 105°.*

Explanation: In ΔMNO,

$$MN = MO \qquad \text{[Given]}$$
$$\therefore \quad \angle N = \angle O$$
$$\angle O = 75^\circ$$
$$\angle NAB + \angle ANO = 180^\circ \qquad \text{[Co-interior angles]}$$
$$\angle NAB + 75^\circ = 180^\circ$$
$$\angle NAB = 180^\circ - 75^\circ$$
$$\angle NAB = 105^\circ$$

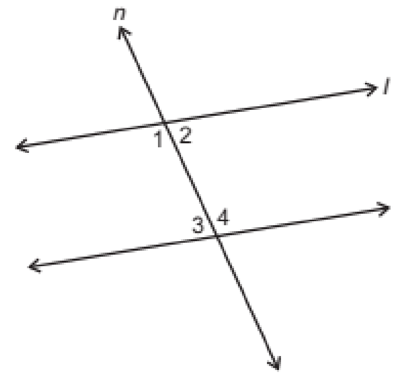
8. *(b). Yes, as $-10 = -\dfrac{10}{1}$ and rational numbers are ratios of integers m and n, where $n \neq 0$.*

Explanation: 10 can be written as $-10 = \dfrac{-10}{1}$ and it is in $\dfrac{p}{q}$ form, where p and q both are integers and $q \neq 0$.

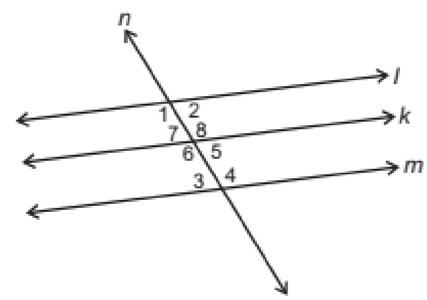
Hence, –10 is a rational number.

9. *(c) 4.*

Explanation: We can see in the figure below, 4 alternate interior angles were formed or, 2 pairs of alternates interior angles were formed which are [1, 4] and [2, 3].



Now, after drawing one more parallel line, let's name it k as shown in the figure below.



The alternate interior angles formed are 8 in number or, 6 pairs of alternate interior angles are formed. Which are [1, 8], [2, 7], [6, 4], [5, 3], [1, 4] and [2, 3].

So, (6 – 2) = 4 Pairs of alternate interior angles increased.

10. *(a) ₹140800*

Explanation: The curved surface area of a conical tent

$$= \pi r l$$
$$= \frac{22}{7} \times 28 \times 20$$
$$= 1760 \text{ m}^2$$

Cost of 1 m^2 of material = ₹80

Hence, the total cost of 1760 m^2

$$= 1760 \times 80$$
$$= ₹140800.$$

11. *(c) Parallelogram, Rectangle, Trapezium*

Explanation: Square, Rhombus and Kite have the diagonals bisect each other at 90°.

Hence, option *a*, *b* and *d* have been omitted.

Caution

➥ *Diagonals of rectangle does not intersect each other at 90°.*

12. *(d) 10x*

Explanation: Let

$$p(x) = (25x^2 - 1) + (1 + 5x)^2$$

On simplifying,

$$p(x) = 25x^2 - 1 + (1 + 5x)^2$$
$$p(x) = 25x^2 - 1 + (1 + 10x + 25x^2)$$
$$p(x) = 25x^2 - 1 + 1 + 10x + 25x^2$$
$$p(x) = 50x^2 + 10x$$
$$p(x) = 10x\,(5x + 1)$$

Here, $10x$ and $(5x + 1)$ are two factors of $p(x)$.

Hence, $10x$ is one of the factor of $(25x^2 - 1) + (1 + 5x)^2$.

13. *(a) $10\sqrt{3}$ cm*

Explanation: Given,

Perimeter of the rhombus

= Perimeter of equilateral triangle

60 = 3 × side of triangle

[Since, perimeter of equilateral triangle = 3 × side]

Side of triangle

$$= \frac{60}{3} = 20 \text{ cm}$$

Now, altitude (height) of the equilateral triangle

$$h = \frac{\sqrt{3}}{2} \times \text{side}$$
$$h = \frac{\sqrt{3}}{2} \times 20$$
$$= 10\sqrt{3} \text{ cm.}$$

14. *(b) 22°*

Explanation:

$$\angle AOC = \angle AOE + \angle EOC$$
$$48° = 26° + \angle EOC$$
$$\angle EOC = 48° - 26°$$
$$\angle EOC = 22°$$
$$a = \angle EOC$$

[Vertically opposite angle]

$$a = 22°.$$

15. *(b) 17.5 – 22.5*

Explanation: The class marks of a frequency distribution are given as: 15, 20, 25...

Class size for the given frequency distribution is:

20 – 15 = 5

$$\text{Upper limit} = \text{Class mark} + \frac{\text{Class Size}}{2}$$
$$= 20 + \frac{5}{2} = \frac{40+5}{2} = \frac{45}{2}$$
$$= 22.5$$
$$\text{Lower limit} = \text{Class mark} + \frac{\text{Class Size}}{2}$$
$$= 20 - \frac{5}{2} = \frac{35}{2} = 17.5$$

Hence, 17.5 – 22.5 is the corresponding class to the class-mark 20.

16. *(d) 797*

Explanation: We know,

$$a^2 - b^2 = (a + b)(a - b)$$
$$399^2 - 398^2 = (399 + 398)(399 - 398)$$
$$= 797 \times 1$$
$$= 797$$

17. Ans. *(d) 5°*

Explanation: In ΔABC and ΔDEF

$$\angle C = \angle F$$
$$AB = DE$$
$$\therefore \quad AC = DF$$
$$\Delta ABC \cong \Delta DEF \quad \text{[By RHS rule]}$$
$$\angle B = \angle E \quad \text{[By CPCT]}$$
$$(4x^2)° = (3x^2 + 25)°$$
$$x^2 = 25$$
$$x = \sqrt{25}$$
$$x = 5$$

18. *(d) 13*

Explanation: Mid-value = 10

Class width = 6

$$\text{Lower limit} = \text{mid-value} - \frac{\textit{Class width}}{2}$$
$$= 10 - \frac{6}{2}$$
$$= 10 - 3$$

Lower limit = 7

Class width = upper limit – lower limit

6 = upper limit – 7

Upper limit = 6 + 7

= 13

19. *(d) Assertion (A) is false but reason (R) is true.*

Explanation:

$$x = 1 + \sqrt{2} + \sqrt{3},\ y = 1 + \sqrt{2} - \sqrt{3}$$

Let $x = a + b$

$y = a - b$

where $a = 1 + \sqrt{2}$ and $b = \sqrt{3}$

$$xy = a^2 - b^2$$
$$\Rightarrow \quad = (1 + \sqrt{2})^2 - (\sqrt{3})^2$$
$$\Rightarrow \quad = 3 + 2\sqrt{2} - 3$$
$$\Rightarrow \quad = 2\sqrt{2}$$

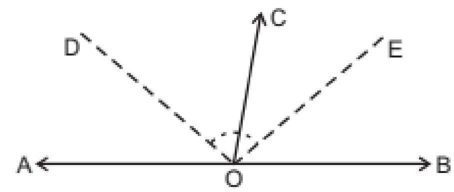
20. *(b). Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).*

Explanation: $\angle AOC + \angle BOC = 180°$

[Linear Pair]

$$\frac{1}{2}[\angle AOC + \angle BOC] = \frac{180°}{2}$$

$$\frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90°$$



$$\angle DOC + \angle EOC = 90°$$

The bisectors of the angles of a linear pair are at right angle.

SECTION - B

21. Volume of the cone

$$= \frac{1}{3}\pi r_c^2 h$$

Where, r_c is radius of cone

$$r_c = 2.1 \text{ cm}$$
$$h = 8.4 \text{ cm}$$

Volume of the sphere

$$= \frac{4}{3}\pi r_s^3$$

Where, r_s is radius of the sphere.

Volume of the cone = volume of the sphere

$$\frac{1}{3}\pi r_c^2 h = \frac{4}{3}\pi r_s^3$$

$$2.1 \times 2.1 \times 8.4 = 4 \times r_s^3$$

$$r_s^3 = (2.1)^3$$

$$r_s = 2.1 \text{ cm}$$

Hence, the radius of the sphere is 2.1 cm.

OR

Curved surface area $= \pi rl$

where, r is the radius of the cone and l is slant height.

The surface area of a sphere

$$= 4\pi r^2$$

where r is the radius of the sphere

$$4\pi r^2 = 5(\pi rl)$$

Substitute values

$$4 \times (5)^2 = 5 \times 4 \times l$$

$$l = 5 \text{ cm}$$

We know, $l^2 = r^2 + h^2$

$$h^2 = l^2 - r^2$$
$$= 5^2 - 4^2$$
$$h^2 = 25 - 16$$
$$h^2 = 9$$
$$h = 3$$

Hence, the height a cone is 3 cm.

Now the volume of a cone

$$= \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3$$

$$V = 50.29 \text{ cm}^3$$

22. Let

$$\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

[By rationalizing the denominator]

So, $$\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}+\sqrt{2}}{3-2} = \frac{\sqrt{3}+\sqrt{2}}{1}$$

or, $$\sqrt{3}+\sqrt{2} = 1.732 + 1.414$$
$$= 3.146.$$

23. As per question $OC \perp AB$

Perpendicular is drawn from the centre bisects the chord

$$BC = AC = \frac{AB}{2} = \frac{8}{2} = 4 \text{ cm}$$

Radius, $OB = OD = r$

$$OD = OC + CD$$
$$OC = OD - CD$$
$$OC = r - 2$$

In ΔOBC,

$$OB^2 = OC^2 + BC^2$$
$$r^2 = (r-2)^2 + 4^2$$
$$r^2 = r^2 + 4 - 4r + 16$$
$$4r = 20$$
$$r = 5$$

Hence, the diameter of the plate is (5×2) cm

$$= 10 \text{ cm}$$

24. $(a-3)\sqrt{6} + 7 = b\sqrt{6} + a$

On comparing both sides, we get

$$a - 3 = b \text{ and } a = 7$$
$$7 - 3 = b \quad (a = 7)$$
$$b = 4$$

Now, $a^2 + b^2 = (7)^2 + (4)^2$
$= 49 + 16$
$= 65$

Now, $x = \dfrac{a^2+b^2}{13} = \dfrac{65}{13} = 5$

Hence, $x = 5$.

OR

We know, if x and y are two distinct positive irrational numbers, then $\sqrt{xy}$ is an irrational number lying between x and y.

$\therefore$ An irrational number between $\sqrt{2}$ and $\sqrt{5}$

$$= \sqrt{\sqrt{2} \times \sqrt{5}}$$
$$= \sqrt{\sqrt{10}} = 10^{\frac{1}{4}}$$

Now, irrational number between $\sqrt{2}$ and $10^{\frac{1}{4}}$

$$= \sqrt{\sqrt{2} \times 10^{\frac{1}{4}}}$$
$$= 2^{\frac{1}{4}} \times 10^{\frac{1}{8}}$$

Irrational Number between $10^{\frac{1}{4}}$ and $\sqrt{5}$

$$= \sqrt{10^{\frac{1}{4}} \times \sqrt{5}}$$
$$= 10^{\frac{1}{8}} \times 5^{\frac{1}{4}}$$

Hence, the required irrational numbers are

$10^{\frac{1}{4}}, 2^{\frac{1}{4}} \times 10^{\frac{1}{8}}$, and $10^{\frac{1}{8}} \times 5^{\frac{1}{4}}$

25. Given linear equation is

$$x + 7y = a$$

and, $x = -k$

and $y = k + 2$

Since, $y = 8$

$\Rightarrow k + 2 = 8$

$\Rightarrow k = 6$

Therefore, $x = -6$

and $y = 8$

Now, substitute these in the given equation, we get

$$-6 + 7 \times 8 = a$$
$$-6 + 56 = a$$
$$a = 50$$

SECTION - C

26. Here, $\dfrac{x^3+2x+1}{5} - \dfrac{7}{2}x^2 - x^6$

$\Rightarrow -x^6 + \dfrac{1}{5}(x^3 + 2x + 1) - \dfrac{7}{2}x^2$

$\Rightarrow -x^6 + \dfrac{1}{5}x^3 - \dfrac{7}{2}x^2 + \dfrac{2}{5}x + \dfrac{1}{5}$

(A) The degree of the polynomial

$$\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$$

is 6 because the maximum power of the x is 6.

(B) The coefficient of x^3 in

$-x^6 + \dfrac{1}{5}x^3 - \dfrac{7}{2}x^2 + \dfrac{2}{5}x + \dfrac{1}{5}$ is $\dfrac{1}{5}$

(C) The coefficient of x^6 in

$-x^6 + \dfrac{1}{5}x^3 - \dfrac{7}{2}x^2 + \dfrac{2}{5}x + \dfrac{1}{5}$ is -1

27. (A) In ΔADC,

S and R are the mid-points of sides AD and CD, respectively.

Therefore, SR $\parallel$ AC and SR = $\dfrac{1}{2}$ AC ...(i)

[Mid-point Theorem]

(B) In ΔABC,

P and R are the mid-points of AB and BC respectively.

Therefore, PQ $\parallel$ AC and PQ = $\dfrac{1}{2}$ AC ...(ii)

[Mid-point Theorem]

From (i) and (ii)

PQ = SR and PQ $\parallel$ SR

Hence proved.

(C) Here, PQ $\parallel$ SR and PQ = SR

As, opposite sides are equal and parallel

Hence, PQRS is a parallelogram.

28. Given, $\left(\dfrac{1}{2}\right)^{-4} - 3 \times \left(\dfrac{64}{8}\right)^{\frac{2}{3}} \times 71^0 + \left(\dfrac{36}{64}\right)^{\frac{-256}{512}} = \dfrac{x}{3}$

$\Rightarrow (2)^4 - 3 \times (8)^{\frac{2}{3}} \times 71^0 + \left(\dfrac{9}{16}\right)^{\frac{-1}{2}} = \dfrac{x}{3}$

$\Rightarrow 16 - 3 \times 2^{3\times\frac{2}{3}} \times 1 + \left(\dfrac{3}{4}\right)^{2\times\frac{-1}{2}} = \dfrac{x}{3}$

$\Rightarrow 16 - 3 \times 2^2 \times 1 + \left(\dfrac{3}{4}\right)^{-1} = \dfrac{x}{3}$

$$\Rightarrow \qquad 16 - 12 \times 1 + \left(\frac{4}{3}\right) = \frac{x}{3}$$

$$\frac{x}{3} = \frac{16}{3}$$

$$\Rightarrow \qquad x = 16$$

29. (A) Negative abscissa means negative x-axis and similarly negative ordinate means negative y-axis.

These all points will lie in III quadrant.

Hence, W, T, U, V, R, S are the points.

(B) According to question O, V, Y and M are the coordinates forming square.

(C) All points with abscissa zero with ultimately lies on y-axis.

Hence, points are V, W, O and X.

OR

(A) If a point lies on the x-axis, then its ordinate will be zero and abscissa can be any number. If a point lies on the y-axis, then its abscissa will be zero and ordinate can be any number. Therefore, the only point which lies on both axes is the origin that is (0, 0).

(B) If a point lies on the y-axis, then its abscissa will be zero and ordinate can be any number. The coordinates are (0, –4).

(C) If a point lies on the x-axis, then its Coordinate will be zero and abscissa can be any number. The coordinates are (5, 0).

30. Ravi's monthly earnings is given by the equation

$$= ₹10{,}000 + 0.05x$$

Ravi gets a hike of 10% of the fixed salary.

Salary's fixed part = ₹10000

[As this is constant]

Variable part = $0.05x$

[As this will change according to the value of x]

where x is total sales.

10% hike on fixed part

$$= \frac{10}{100} \times 10{,}000 = ₹1000$$

Hence, fixed part after hike

$$= ₹10{,}000 + ₹1000$$

$$= ₹11{,}000$$

According to the question,

Earning ₹140 on every 2000 Rupees worth of sales.

$$\therefore \qquad \frac{140x}{2000} = 0.07x$$

Ravi's Monthly earning after the hike

$$= ₹11{,}000 + ₹0.07x$$

Let y represent Ravi's monthly earnings after the hike. Therefore,

$$y = 11{,}000 + 0.07x$$

31. Here, we see that the difference between two midpoints is 15–5 *i.e.,* 10. It means the width of the class interval is 10. Let the lower limit of the first class interval be a. Then, its upper limit $= a + 10$.

Now, mid-value of the first class interval

$$= 5$$

$$\Rightarrow \qquad \text{Mid–value} = \frac{\textit{Lower limit + Upper limit}}{2}$$

$$\Rightarrow \qquad 5 = \frac{a+a+10}{2}$$

$$\Rightarrow \qquad 2a + 10 = 10$$

$$\Rightarrow \qquad 2a = 0$$

$$\Rightarrow \qquad a = 0$$

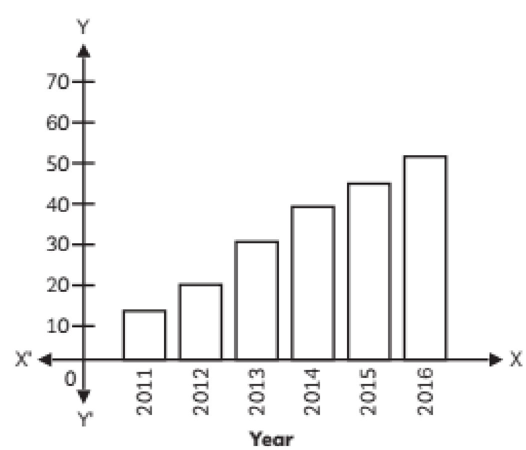
So, the first class interval is 0–10. Now, we prepare a continuous grouped frequency distribution table as follows:

Midpoint	Class Interval	Frequency
5	0 – 10	4
15	10 – 20	8
25	20 – 30	13
35	30 – 40	12
45	40 – 50	6

Hence, the size of the class interval is 10 *i.e.,* 10 – 0.

OR

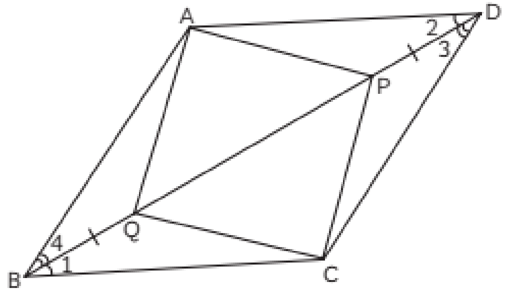
Please see the figure given below for answer.



SECTION - D

32. Given: ABCD is a parallelogram and DP = BQ

(A) In ΔAPD and ΔCQB,



$$AD = BC$$

[Opposite sides of parallelogram are equal]

$\angle 1 = \angle 2$ [Alternate angles]

$DP = BQ$ [Given]

Therefore, $\Delta APD \cong \Delta CQB$

[By SAS Congruency Rule]

(B) Since, $\Delta APD \cong \Delta CQB$

So, $AP = QC$ [BY CPCT]

(C) In ΔAQB and ΔCPD,

$$AB = CD$$

[Opposite sides of parallelogram are equal]

$\angle 4 = \angle 3$ [Alternate angles]

$DP = BQ$ [Given]

Therefore, $\Delta AQB \cong \Delta CPD$

[By SAS Congruency Rule]

(D) Since, $\Delta AQB \cong \Delta CPD$

So, $AQ = CP$ [BY CPCT]

(E) Now, $AP = CQ$

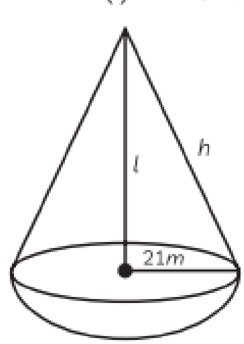
and $AQ = CP$

Since opposite sides of parallelogram are equal.

Hence, $\Delta APCQ$ is a parallelogram.

33. Radius of base of the conical part

$(r) = 21$ cm.



Volume of cone

$$= \frac{2}{3} \times \text{volume of hemisphere}$$

Let h be the height of the conical part.

Now volume of hemispherical part

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 \text{ cm}^3$$

$$= 19404 \text{ cm}^3$$

$\therefore$ Volume of conical part

$$= \frac{2}{3} \times 19404$$

$$= 12936 \text{ cm}^3$$

$$\therefore \quad h = \frac{\text{Volume}}{\frac{1}{3}\pi r^2}$$

$$= \frac{12936 \times 3 \times 7}{22 \times 1 \times 21 \times 21} = 28 \text{ cm}$$

Now surface area of the toy

= curved surface area of conical part + surface area of hemispherical part

$$= \pi rl = 2\pi r^2 = \pi r\,(l + 2r)$$

$$= \pi r\left[\sqrt{h^2 + r^2} + 2r\right]$$

$$= \frac{22}{7} \times 21\left[\sqrt{(28)^2 + (21)^2} + 2 \times 21\right] \text{cm}^2$$

$$= 66\left[\sqrt{784 + 441} + 42\right] \text{cm}^2$$

$$= 66\left[\sqrt{1225} + 42\right]$$

$$= 66[35 + 42]$$

$$= 66 \times 77$$

$$= 5082 \text{ cm}^2$$

34. Since, ABCD is a square and ΔDCB is an equilateral triangle.

As $\angle BCD = 90^\circ$ and $\angle DCE = 60^\circ$

$\Rightarrow \quad \angle BCD + \angle DCE = 90^\circ + 60^\circ$

$$\angle BCE = 150^\circ$$

Similarly, we have

$$\angle ADE = 150^\circ$$

Thus, in triangle ΔADE, we have

$$AD = BC$$

$$\angle BCE = \angle ADE = 150^\circ$$

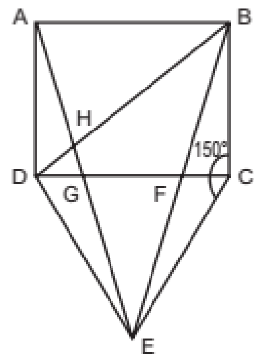
and $EC = DE$

So, by SAS Congruence Rule,

$$\Delta ECB \cong \Delta EDA$$

$\Rightarrow$ $$AE = BE$$

Now,



In ΔBCE,

$$BC = CE$$

[Angles opposite to equal sides are equal]

$\therefore$ $$\angle CBE = \angle CEB = x$$

In ΔBCE,

$$\angle EBC + \angle BEC + \angle BCE = 180^\circ$$

$$x + x + 150^\circ = 180^\circ$$

$$2x = 180^\circ - 150^\circ$$

$$2x = 30^\circ$$

$$x = 15^\circ$$

Also, $$\angle FEC = 15^\circ$$

$$\angle DEC = \angle DEF + \angle FEC$$

$$\angle DEF = \angle DEC - \angle FEC$$

$$= 60^\circ - 15^\circ$$

$$\angle DEF = 45^\circ$$

Now, $$\angle BDE = \angle BDC + \angle CDE$$

$$= 45^\circ + 60^\circ$$

$$\angle BDE = 105^\circ$$

$\therefore$ In ΔDBE,

$$\angle DEB + \angle BDE + \angle DBE = 180^\circ$$

[Angle sum property]

$$45^\circ + 105^\circ + \angle DBE = 180^\circ$$

$$\angle DBE = 180^\circ - 45^\circ - 105^\circ$$

$$= 180^\circ - 150^\circ$$

$$\angle DBE = 30^\circ$$

In ΔADH,

$$\angle A + \angle D + \angle H = 180^\circ$$

[Angle sum property]

$$15^\circ + 45^\circ + \angle H = 180^\circ$$

$$60^\circ + \angle H = 180^\circ$$

$$\angle H = 120^\circ$$

OR

According to the figure we need to prove that

$$AT = BT$$

According to the given question,

The angle of incidence

= Angle of reflection

$\therefore$ $$\angle ACN = \angle DCN \quad ...(i)$$

Since, AB || CN and AC is the transversal,

From the figure, we know that $\angle TAC$ and $\angle ACN$ are alternate angles.

$$\angle TAC = \angle ACN \quad ...(ii)$$

We know that AB || CN and BD are the transversals.

From the figure, we know that $\angle TBC$ and $\angle DCN$ are corresponding angles.

$$\angle TBC = \angle DCN \quad ...(iii)$$

By considering the equation (i), (ii) and (iii)

We get, $$\angle TAC = \angle TBC \quad ...(iv)$$

Now, In ΔACT and ΔBCT

$$\angle ATC = \angle BTC = 90^\circ$$

CT is common *i.e.*,

$$CT = CT$$

By AAS Congruence Rule,

$$\Delta ACT \cong \Delta BCT$$

$$AT = BT \quad \text{[CPCT]}$$

Therefore, it is proved that the image is as far behind the mirror as the object is in front of the mirror.

35. The given pair of linear equation is

$$kx + y - k^2 = 0$$

and $$x + ky - 1 = 0$$

Comparing with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we have,

$a_1 = k,\quad b_1 = 1,\quad c_1 = -k^2;$

$a_2 = 1,\quad b_2 = k,\quad c_2 = -1;$

$$\frac{a_1}{a_2} = \frac{k}{1};\quad \frac{b_1}{b_2} = \frac{1}{k};\quad \frac{c_1}{c_2} = \frac{-k^2}{-1} = \frac{k^2}{1}$$

(A) For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{k}{1} = \frac{1}{k} \neq \frac{k^2}{1}$$

$$\frac{k}{1} = \frac{1}{k} \text{ and } \frac{k}{1} = \frac{k^2}{1}$$

$$k^2 - 1 = 0 \text{ and } k \neq k^2$$

$$(k-1)(k+1)=0 \text{ and } (k^2-k)\neq 0$$
$$(k-1)(k+1)=0 \text{ and } k(k-1)\neq 0$$
$$k=1,-1 \text{ and } k\neq 0, 1$$

Here, we take only

$$k=-1.$$

Hence, for $k=-1$, the pair of linear equations for no solution.

(B) For a unique solution,

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$
$$\frac{k}{1}\neq\frac{1}{k}$$
$$k^2-1\neq 0$$
$$(k-1)(k+1)\neq 0$$
$$k\neq -1, 1$$

Hence, for real values of k except ± 1, the given pair of equations has a unique solution.

(C) For infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$
$$\frac{k}{1}=\frac{1}{k}=\frac{k^2}{1}$$
$$\frac{k}{1}=\frac{1}{k} \text{ and } \frac{k}{1}=\frac{k^2}{1}$$
$$k^2-1=0 \text{ and } k^2= k$$
$$(k-1)(k+1)=0 \text{ and } k^2-k=0$$
$$(k-1)(k+1)=0 \text{ and } k(k-1)=0$$
$$k=1,-1 \text{ and } k=0, 1$$

$k=1$ satisfies both the equations.

Hence, for $k=1$, the linear equations has infinitely many solutions.

(D) Given that $x=1$ and $y=2$

Given linear equation is

$$kx+y-k^2=0$$

put $x=1$ and $y=2$, we get

$$k(1)+(2)-k^2=0$$
$$k^2-k-2=0$$
$$k^2-2k+k-2=0$$
$$k(k-2)+(k-2)=0$$
$$(k-2)(k+1)=0$$
$$k=2,-1.$$

OR

Given: $C=\dfrac{5F-160}{9}$

Rearranging the given equation

$$C=\frac{5F-160}{9}$$
$$\Rightarrow \quad 9C=5F-160$$
$$\Rightarrow \quad 5F=9C+160$$
$$\Rightarrow \quad F=\frac{9C+160}{5}$$

(A) For 86° F

$$C=\frac{5\times 86-160}{9}$$
$$\Rightarrow \quad C=\frac{430-160}{9}$$
$$\Rightarrow \quad C=\frac{270}{9}=30°C$$

(B) For 35°C

$$F=\frac{9\times 35+160}{5}$$
$$\Rightarrow \quad F=\frac{315+160}{5}=\frac{475}{5}$$
$$=95° F$$

(C) For 0°C

$$F=\frac{9\times 0+160}{5}$$
$$\Rightarrow \quad F=\frac{160}{5}=32° F$$

For 0° F

$$C=\frac{5\times 0-160}{9}$$
$$\Rightarrow \quad C=\frac{-160}{9}=-\left(\frac{160}{9}\right)^{o} C$$

(D) $C=F$

From the given condition above the relation satisfies,

So,

$$\Rightarrow \quad C=\frac{5C+160}{9}$$
$$\Rightarrow \quad 9C=5C-160$$
$$\Rightarrow \quad 9C-5C=-160$$
$$\Rightarrow \quad 4C=-160$$
$$\Rightarrow \quad C=\frac{-160}{4}$$
$$\Rightarrow \quad C=F=-40.$$

SECTION - E

36. (A)
$$p(y) = 2y^2 - 18y + 40$$
$$= 2y^2 - 10y - 8y + 40$$
$$= 2y(y-5) - 8(y-5)$$
$$= (2y-8)(y-5)$$
$$2y - 8 = 0 \quad y - 5 = 0$$
$$2y = 8 \quad y = 5$$
$$y = 4$$

As $\alpha > \beta$, so $\beta = 4$

$\therefore$ Number of students who take healthy food

$= 4$

(B)
$$p(y) = 2y^2 - 18y + 40$$
$$= 2y^2 - 10y - 8y + 40$$
$$= 2y(y-5) - 8(y-5)$$
$$= (2y-8)(y-5)$$
$$2y - 8 = 0 \quad y - 5 = 0$$
$$2y = 8 \quad y = 5$$
$$y = 5$$

As $\alpha > \beta$, So $\alpha = 5$

$\therefore$ Number of students who take junk food = 5

OR

$$p(y) = 2y^2 - 18y + 40$$
$$p(0) = 2(0) - 18 \times 0 + 40$$
$$p(0) = 40$$

$$p(1) = 2(1)^2 - 18(1) + 40$$
$$= 2 - 18 + 40$$
$$= 42 - 18$$
$$= 24$$
$$p(2) = 2(2)^2 - 18(2) + 40$$
$$= 2 \times 4 - 36 + 40$$
$$= 48 - 36$$
$$= 12$$
$$p(0) + p(1) = k.\, p(2)$$
$$40 + 24 = k.(12)$$
$$64 = k.(12)$$
$$k = \frac{64}{12}$$
$$k = \frac{16}{3}$$

(C)
$$p(y) = 2y^2 - 18y + 40$$
$$p(-1) = 2(-1)^2 - 18(-1) + 40$$
$$= 2 + 18 + 40$$
$$p(-1) = 60$$

37. (A) From the given figure, Hudson Lane forms 57° with M.G road and Akbar Road and M.G Road form a 90° at their intersection point.

Therefore, the required angle between Akbar Road and Hudson lane = 90° – 57° = 33°.

(B) Since, Hudson lane makes 57° with M.G. road and H.M. Road makes 37° with M.G. Road.

So, the angle between Hudson lane and H.M. road = 57° + 37° = 94°

Which is greater than 90°.

Hence, the angle formed by Hudson lane and H.M road is an obtuse angle.

OR

Priya travels from H.M road to M.G road [East] to Hudson to Akbar road and then to M.G road west.

So, the measure of angle she cover

=37° + 90° + 90° = 217°.

But if she goes from H.M road to south of BT road and then to M.G. road [west],

Then, the measure of angle, she cover

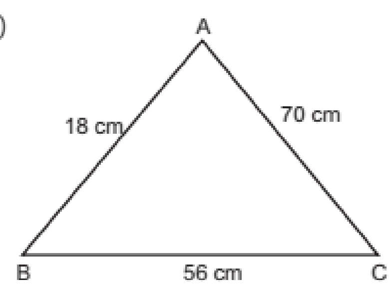
= 53° + 90°= 143°

Hence, the shortest angle she has to cover will be 143°.

(C) The required measure of reflex angle formed between M.G Road [in east direction] with Hudson lane

= 360° – 57° = 303°.

38. (A)



Perimeter of one triangular piece

= AB + BC + CA

= 18 + 56 + 70

= 144 cm

(B) Semi-perimeter of the triangular piece

$= \frac{\text{Perimeter}}{2}$

= 144 cm

= 72 cm

(C) Semi-perimeter of a triangular piece, $s = 72$ cm [From (B)]

Now, area of one triangular piece

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{72(72-18)(72-56)(72-70)}$$

$$= \sqrt{36 \times 2 \times 27 \times 2 \times 16 \times 2}$$

$$= 144\sqrt{6}\ \text{cm}^2$$

OR

Area of 32 triangular pieces

$$= 32 \times \text{Area of 1 piece}$$

$$= 32 \times 144\sqrt{6}\ \text{cm}^2$$

$$= 4608\sqrt{6}\ \text{cm}^2$$