ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Revision Notes

1. Magnetic flux. Magnetic flus if denoted by (ϕ)

$$\phi = \vec{A} . \vec{B}$$

$$\phi = AB \cos \theta$$

Unit. Wb. or Tesla m^2. Thus 1 T = 1 W b m^2.

1. Faraday's laws of electromagnetic induction. On the basis of his experiments, Faraday gave the following laws :

(*i*) *Whenever magnetic flux linked with a circuit changes, induced e.m.f. is produced.*

(*ii*) *The induced e.m.f. lasts as long as the change in the magnetic flux continues.*

(*iii*) *The magnitude of induced e.m.f. is directly proportional to the rate of change of magnetic flux.*

> **Key Point**
>
> The magnetic flux linked with a loop does not change with time when
>
> (*i*) When both loop and magnet move in same direction with same velocity.
>
> (*ii*) magnet is rotated about its axis keeping its position from loop unchanged.
>
> (*iii*) loop is rotated in a uniform magnet field keeping it fully within the field.

The magnitude of induced e.m.f. is given by

$$e = \frac{\phi_2 - \phi_1}{dt}.$$

where ϕ_1 and ϕ_2 are magnetic flux linked with the coil initially and after time t. The negative sign shows that induced e.m.f. opposes the change taking plane in magnetic flux/

In differential notation,

$$e = -\frac{d\phi}{dt}.$$

In CGS system, e is measured in *e.m.u.* and ϕ in maxwell while in SI system, e is measured in volt and ϕ in weber.

Note. It may be remembered that

$$1 \text{ volt} = 10^8 \text{ e.m.u. of potential,}$$

$$1 \text{ ampere} = \frac{1}{10} \text{ e.m.u. of current,}$$

$$1 \text{ coulomb} = \frac{1}{10} \text{ e.m.u. of charge,}$$

and $$1 \text{ ohm} = 10^9 \text{ e.m.u. of resistance.}$$

(*a*) **Induced current.** If the coil is a closed circuit and has a resistance R, then induced current

$$i = \frac{e}{R} = \frac{N}{R}\frac{d\phi}{dt} (N = \text{no. of turns in the coil})$$

(*b*) **Induced charge.** Incuded charge is given by :

$$q = i \times t = \frac{N\phi}{R} \text{ where } \phi = \text{change of flux.}$$

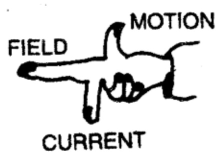
Also $$q = -\frac{N(\phi_2 - \phi_1)}{Rt} \times t$$

$$q = -\frac{N(\phi_2 - \phi_1)}{R}$$

$$q = \frac{e}{R} \times t \qquad q = \frac{N(\phi_2 - \phi_1)\,t}{R}$$

This shows that induced change is independent of **time interval.**

(*c*) **Len's law.** The direction of induced *e.m.f.* due to electromagnetic induction is such that its effect opposes the cause which has produced it.



(*d*) **Fleming's right hand rile.** *If thumb, fore–finger and the middle finger are spread perpendicular to one another (in two different⊥ planes) such that the fore–finger denotes the direction oif magnetic field and thumb, the direction of motion then the middle finger denotes the direction of induced e.m.f.*

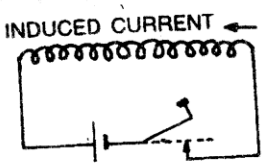
3. E.M.F. induced in a moving conductor. If a straight conductor of length l is moving perpendicular to a uniform magnetic field of flux density B with a velocity v, then

Infuced *e.m.f.* $e = Blv$

If R is the resistance of the conductor, then

Induced current $$i = \frac{e}{R} = \frac{Blv}{R}$$

4. Self inductance or coefficient of self induction. The magnetic flux linked with a coil through which current I is flowing is given by



$$\phi = LI$$

where L is called self inductance or coefficient of self induction of the coil.

The coefficient of self induction of a coil is numerically equal to the magnetic flux linked with it, when unit current flowes through it.

The instantaneous induced e.m.f. produced in the coil is given by

$$e = -L\frac{dI}{dt}$$

where $\frac{dI}{dt}$ is rate of change of current at that instant,

The coefficient of self induction of a coil is also numerically equal to the induced e.m.f set up in the coil, when the rate of change of current in the coil is unity.

The unit of self inductance is e.m.u. in CGS system and henry in SI.

Self inductance of a solenoid. Self inductance of a solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

$L = \mu_r\mu_0 n^2 lA$ where m_r is relative permeability of material of core used.

where μ_0 = permeability of free space.

N = Total no. of turns and n = no. of turns per unit length.

l = length of the coil.

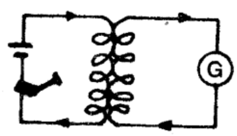
A = area of cross-section of coil.

Also $$L = \frac{\mu_0 n^2 l^2 A}{l} = \mu_0 n^2 lA.$$

5. Mutual inductance or coefficient of mutual induction. The magnetic flux linked with one coil when a current I flows through a neighbouring coil is given by

$$\phi = MI$$

where M is called *mutual inductance or coefficient of mutual induction between the two coils.*



The coefficient of mutual induction between two coils is numerically equal to the magnetic flux linked with one coil, when unit current flows through the neighbouring coil.

The instantaneous induced *e.m.f.* produced in one coil is given by

$$e = -M\frac{dI}{dt}$$

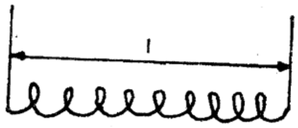
where $\frac{dI}{dt}$ is rate to change of current in the neighbouring coil at that instant.

> **Remember**
> Self induction is called inertia of electricity.

The coefficient of mutual induction between two coils is also numerically equal to the induced e.m.f. set up in one coil, when the rate of change of current in the neighbouring coil is unity.

In CGS system, the unit of mutual inductance is *e.m.u.* and in S.I., the unit is *henry*. 1 henry = 10^9 *e.m.u.* of mutual inductance.

Mutual inductance between two coils. The mutual inductance between two coils of area A, no. of turns N_1 and N_2 with length of secondary or primary as l is given by :



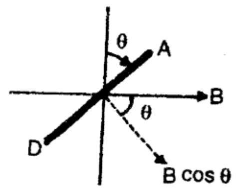
$$M = -\mu_0\frac{N_1N_2A}{l}$$

If n_1 and n_2 are the no. of turns per unit length in the two coils,

then $M = \mu_0 n_1 n_2 Al$. $(\because N_1 = n_1 l \text{ and } N_2 = n_2 l)$

Note. In CGS system I, e, ϕ and M are measured in *e.m.u.* of current, *e.m.u.* of *e.m.f.*, maxwell and e.m.u. of mutual induction respectively, whele in SI, they are respectively measured in ampere, volt, weber and henry.

6. Induced emf produced in a coil rotating inside a magnetic field (a.c generator). Consider coil of area A, number of induction turns N and rotating inside a magnetic field of induction B with angular velocity ω. At $t = 0$, the coil is vertical. At any time t, the plane of coil will make an angle θ equal to ωt with the vertical.



The induced *e.m.f.* produced in the coil at time t given by

$$e = NAB\omega \sin \omega t$$

where N is frequency and T is time period of rotation.

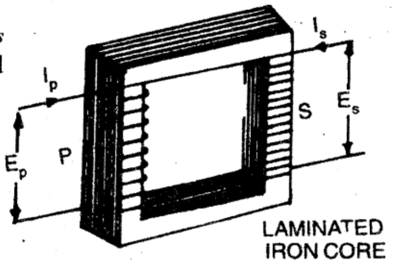
$$e_{max} = NAB\omega$$

Thus $$e = e_{max} \sin \omega t.$$

7. Transformer. *A transformer is a device of changing a low voltage alternating current into a high voltage alternating current or vice-versa.*

A transformer which increases the voltage (current will decrease), is called **step-up transformer** while another while which decreases the voltage (current will increases) is **step-down transformer.**

Suppose a transformer consists of a primary of n_p turns and the secondary coil of n_s turns [Fig.]. Let E_p and E_s be the values of *e.m.f.* across primary and secondary coil and I_p and I_s be the respective values of the current.



Then, $\frac{E_s}{E_p} = \frac{n_s}{n_p} = \frac{I_p}{I_s} = k$ where k is known as transformation ratio.

For a step-up transformer, $k > 1$ and for a step down transformer, $k > 1$

For a 100% efficient (ideal) transformer, Input power = Output power *i.e.*,

$$E_pI_p = E_sI_s.$$

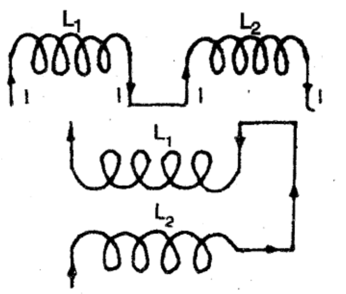
8. (*i*) An electric field produced by time varying magnetic field, which has non-vanishing closed line integral is called as non-conservative field.Here $\oint \vec{E}\,\vec{dl} \neq 0$,

(*ii*) In conservative fields $\oint E\,.\,dl = 0$.

(*iii*) The direction of induced current is given by Fleming's right hand rule.

(*iv*) The circulating currents induced in metal sheets, blocks when the magnetic flux linked with them changes are called eddy currents or Focault current

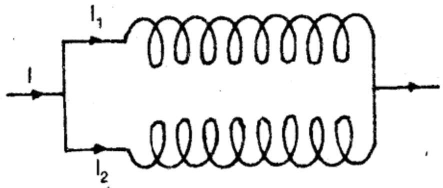
(*v*) If coils are in series as shown in (a) the



$$L = L_1 + L_2 + 2M$$

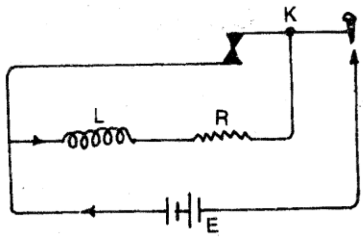
If the coils are as shown in (b), then

$$L = L_1 + L_2 - 2M.$$



$$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}.$$

9. (*i*) Growth and decay of current in L–R circuit. During growth of current in L–R circuit, let R be the inductanceless resistance and L the resistanceless inductance. If I is the instantaneous value of the current at any time *t*, then



The maximum value of current $I_m = \frac{E}{R}$

The current I at any instant is given by

$$I = I_m\left(1-e^{-\frac{R}{L}t}\right)$$

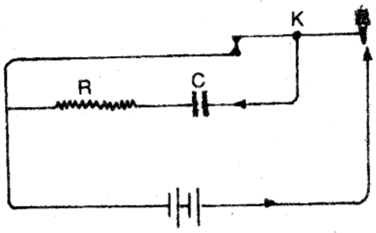
The expression L/R is called the time constant and is measured in seconds.

(*ii*) Decay of current. On switching off the circuit without introducing and additional resistance, the current takes some time to decay from maximum to zero value. The current at any instant is given by

$$I = I_m\, e^{-\frac{R}{L}t}$$

The value of current I after L/2, 2L/R, 3L/R ...... is given by 0.3679 I_m, 0.1357 I_m, 0.0498 I_m ...... etc.

10. Charging and discharging of a condenser. (*i*) When a circuit containing capacitance and resistance in series with a battery is switched on the charge grows from zero to maximum value through the capacitor in a certain time. If q is the instantaneous value of charge and q_m the maximum value of charge.



Maximum value of charge q_m = EC

The instantaneous value of charge q is given by

$$q = q_m\left(1-e^{\frac{-t}{RC}}\right)$$

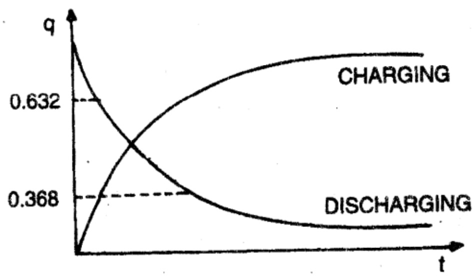
(*ii*) Similarly when the circuit is switched off without introducing any additional resistance, then the charge takes some time to decay from maximum to zero value. The value of charge at any instant is given by

$$q = q_m \cdot e^{\frac{-t}{RC}}$$

In both these cases RC is called the **time constant** as it has the dimensions of time.

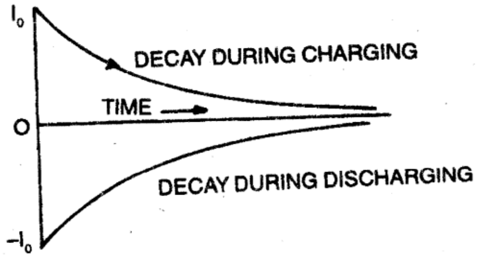
(*iii*) After t = 5RC, the capacitor gets almost fully charged.

(*iv*) Discharging of a capacitor $q = q_0 e^{-t/RC}$ is graphically shows as



RC is time constant = 0.368 q_0

(*v*) Decay of current during charging and discharging is shown in Fig. below



11. Energy stored in an inductance coil. When the current grows in an inductance, work has to be done in establishing the current in it. This work is stored into the inductance as magnetic energy. When the circuit is broken, this energy is liberated.The energy stored at the make and liberated at the break is given by

$$\text{Energy} = \frac{1}{2} LI_m^2$$

$$\text{Energy at any instant} = \frac{1}{2} LI^2$$

If L is in henry and I in amperes, then energy is in Joules.

12. Circuit containing inductance and capacitance. When a condenser is allowed to discharge through an inductance, the discharge is oscillatory and is according to S.H.M. equation.

$$q = q_m \cos \omega t$$

where $$\omega = \frac{1}{\sqrt{LC}}$$

L being the inductance and C the capacitance of the circuit. The period of oscillation is given by

$$T = 2\pi \sqrt{LC}$$

and the frequency $n = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}$ and is called the natural frequency of L-C circuit.

13. Maximum or peak value of alternating voltage and alternating current. *The maximum value of e.m.f. in either direction is called the* **peak value** *of alternating e.m.f. It is given by*

$$E = E_0 \sin \omega t$$

where E is the instantaneous value and $\omega = 2\pi n$ in which n is the frequency of A.C.

Maximum value of the current I_0 *in either direction is called the* **peak value** *of the alternating current.*

$$I = I_0 \sin \omega t$$

where I is the instantaneous value of current.

14. Mean or average value of A.C. voltage and current. *The average or mean value of current is that steady current which sends the same charge through a circuit in the same time as the alternating current does in half the time period.* If I_m denotes the mean value, then

$$I_m = \frac{2I_0}{\pi} = 0.637\, I_0$$

Similarly the mean or average voltage E_m is given by

$$E_m = \frac{2E_0}{\pi} = 0.637\, E_m$$

15. Root mean square value or virtual value. *It is that steady current or voltage which produces the same heating effect in a resistance in a given time as the A.C. does in the same resistance in the same time.*

Key Point

D.C. ammeter and volt-meter are not capable of measuring A.C. currents of voltages. They will give zero reading when used in a.c. circuit. It is due to reason that mean value of A.C. current and voltages is zero over complete cycle.

If I_p denotes the virtual value of current, then

$$I_p = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Similarly $$E_p = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$

16. Relation between virtual and mean values. $I_v = I_m \frac{\pi}{2\sqrt{2}}$ and $E_v = E_m \frac{\pi}{2\sqrt{2}}$

17. Impedance and reactance. The ratio of the applied voltage to the current is called the impedance (Z) of the A.C. circuit if all the three elements (R, L, C) are present in general.

When only inductance or only capacitance is present in the circuit, then the ratio of E_{rms} and i_{rms} is called reactance of inductance or of capacitance respectively (represented by X_L and X_C).

Alternating currents/voltage a always measured by a.c. ammete and voltmeters. They alwa record their virtual values only.

18. Different types of alternating circuits : (*i*) Circuits containing only resistance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin \omega t$$

Instantaneous current and voltage are in phase always.

(*ii*) Circuit containing only inductance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \pi/2)$$

The current lags behind the voltage by phase $\pi/2$. The inductive reactance $X_L = \omega L = 2\pi n L$

(*iii*) Circuit containing only capacitance :

$$E = E_0 \sin \omega t \text{ then } i = i_0 \sin (\omega t + \pi/2)$$

The current leads the voltage by a phase angle $\pi/2$ radian.

The capacitance reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C}$

(*iv*) Circuit containing resistance and inductance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \phi)$$

where $\tan \phi = \frac{\omega L}{R}$ and $i_0 = \frac{E_0}{Z}$ and $I_v = \frac{E_v}{Z}$

The current lags behind the voltage by phase angle ϕ radian. The impedance

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X_L^2}$$

(*v*) Circuit containing resistance and capacitance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \phi)$$

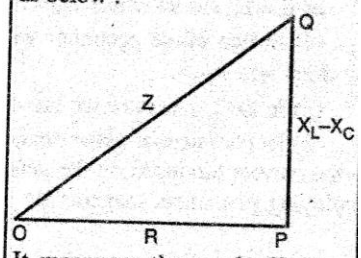
where $\tan \phi = \frac{I/\omega C}{R}$

$$i_0 = \frac{E_0}{Z} \text{ and } I_v = \frac{E_v}{Z}$$

The current leads the voltage by phase angle ϕ. The impedance

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2} = \sqrt{R^2 + X_C{}^2}$$

The impedance triangle is given as below :



It measures the total effective resistance offered by LCR-circuit.

(*vi*) Circuit containing resistance, inductance and capacitance in series (LCR circuit) :

The impedance $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

or $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

When $E = E_0 \sin \omega t$, then $i = i_0 \sin(\omega t - \phi)$

where $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{\omega L - 1/\omega C}{R}$

and $i_0 = \dfrac{E_0}{Z}$ and $I_v = \dfrac{E_v}{Z}$

At resonance : $n = \dfrac{1}{2\pi\sqrt{LC}}$ and $X_L = X_C$.

$Z = R$, $\phi = 0$ and I_0 is maximum.

(*vii*) Coefficient of coupling of two coils :

$$k = \sqrt{\frac{M}{L_1 L_2}}$$, k is always less than one

(*viii*) In case of L-C circuit :

$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

here $\omega^2 = \dfrac{1}{LC}$.

Power of an A.C. circuit. If $E = E_0 \sin \omega t$ is the applied *e.m.f.* and $I = I_0 \sin(\omega t + \phi)$ is the corresponding value of the current, then

Power of circuit $P = E_v \cdot I_v \cos\phi$

$\cos\phi$ is called the **power factor** of the circuit and is given by the ratio $\dfrac{R}{Z}$ where Z is the impedance of the circuit.

(*i*) For pure resistance $\phi = 0$ and $\cos\phi = 1$

$\therefore$ Power $P = E_v I_v$

(*ii*) For pure inductance and capacitance circuit $\phi = \pi/2$ and $\cos\phi = 0$. Such circuits are called **wattless circuit for which P = 0.**

(*iii*) For L–R circuit $\cos\phi = \dfrac{R}{\sqrt{R^2 + L^2\omega^2}}$.

Key Point

(*i*) The reciprocal of reactance X_L of a coil is called susceptance.

(*ii*) The reciprocal of impedance Z of a.c. circuit is called admittance.

19. (1) A genegator or a dynamo is a machine used for generating electric current by mechanical means. Here mechanical energy is converted into electrical energy.

(2) The frequency of A.C. in India is 50 Hz. In certain other countries it is 30 Hz, 50 Hz or 60 Hz.

(3) EMF is A.C. dynamo, $E = NBA\,\omega \sin\theta$ where N is the number of turns in the armature and θ is the angle between B and A (A is the area)

or $E = E_0 \sin \omega t$ where $E_0 = NBA\omega$

(4) In two phase generator we use two armatures and in three phase generator we use three armatures.

(5) In D.C. generator we use commutator. Here $E = E_0 \sin \omega t$.

(6) In commercial generators, we make use of electromagnets which are energised by the current produced by the generator itself. These are called dynamos whereas those employing permanent magnets are called magnetos.

(7) In two phase generator $E_1 = E_0 \sin \omega t$ and $E_2 = E_0 \sin\left(\omega t \pm \dfrac{\pi}{2}\right)$. In three phase generator $E_1 = E_0 \sin \omega t$, $E_2 = E_0 \sin\left(\omega t \pm \dfrac{2\pi}{3}\right)$, $E_3 = E_0 \sin\left(\omega t \pm \dfrac{4\pi}{3}\right)$.

(8) The flux at time t in the case of a generator is given by

$$\phi = NBA \cos \omega t$$

Key Point

Q factor of a resonant LCR circuit is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is voltage multiplication factor of a a.c. circuit.

(9) The choke coil does a wonderful job in A.C. It finds extensive use in battery eliminators, ratio and T.V. sets, mercury, fluorescent lamps etc.

(10) Capacitor can also do the job done by an inductor but it is inferior to choke.

(11) Induction coil is an apparatus for obtaining high potential difference from a low potential difference supply (D.C.). It is based on the principal of mutual induction.

(12) An induction coil is used in laboratory as high voltage supply for studying discharge through gases and as a high tension supply for the spark plugs in car engines.

(13) An electric motor is a machine for converting electrical energy into mechanical energy.

(14) Efficiency of a D.C. motor

$$\eta = \frac{\text{output mechanical power}}{\text{input electrical power}} = \frac{\text{back e.m.f.}}{\text{applied e.m.f.}}$$

For efficiency to be maximum, the back e.m.f. should be half of the applied e.m.f.

(15) Since $I_0 = \frac{E_0}{\omega L}$ so for low frequency AC, choke coild with laminated soft iron cores are used and for reducing high frequency A.C., air core chokes are used.

(16) Impedance triangle is a right angled triangle whose base is ohmic resistance R, normal $(X_L - X_C)$ and hypotenuse is impedance Z.

(17) When LCR are connected in parallel

$$\frac{1}{|Z|} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

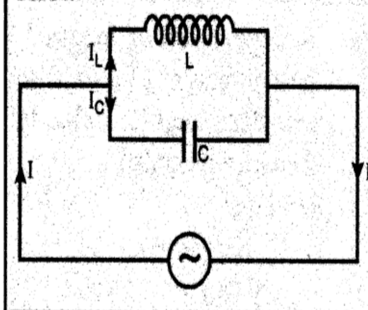
At $\omega = \omega_r = \frac{1}{\sqrt{LC}}, \frac{1}{|Z|}$ is minimum or $|Z|$ is maximum.

(18) For C-R circuit $\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$.

(19) For L-C-R circuit $\cos\phi = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$

Remember

Parallel resonant circuit is shown below



Formulas

2. $E = -\frac{N(\phi_2 - \phi_1)}{t} = -\frac{N\, d\phi}{dt}$

3. $E = Blv$

4. $E = E_0 \sin \omega t, E_0 = BAN\omega$

5. $E = -L\frac{dI}{dt}, \phi = LI$

6. $E = -M\frac{dI}{dt}, \phi = MI$

7. $\frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$

8. $\eta = \frac{\text{Output power}}{\text{Input power}}$

9. $L = \frac{\mu_0 N^2 A}{l}$

10. $M = \frac{\mu_0 N_1 N_2 A}{l}$

11. $f = \frac{1}{2\pi\sqrt{LC}}$

12. $I = I_0(1 - e^{-R/L\, t})$

    $I = I_0 e^{-R/L\, t}, \tau = \frac{L}{R}$

13. $Q = Q_0(1 - e^{-t/C}R), Q = Q_0 e^{-t/C}R$

    $\tau = CR$

14. $I = I_0 \sin \omega t$

15. $I_{rms} = \frac{I_0}{\sqrt{2}}$

16. $X_L = \omega L$

17. $X_C = \frac{1}{\omega C}$

18. LR circuit, $I = I_0 \sin(\omega t - \theta)$

$$= \frac{E_0}{\sqrt{R^2 + X_L^2}} \sin(\omega t - \theta)$$

$$\theta = \frac{\tan^{-1} \omega L}{R}$$

$$Z = \sqrt{R^2 + X_L^2}$$

19. CR-circuit, $I = I_0 \sin(\omega t + \theta)$

$$= \frac{E_0}{\sqrt{R^2 + X_C^2}} \sin(\omega t + \theta)$$

$$\theta = \tan^{-1} \frac{1}{\omega CR}$$

$$Z = \sqrt{R^2 + X_C^2}$$

20. **L-C-R circuit**

    $I = I_0 \sin(\omega t - \theta)$

$$\Rightarrow \quad I = \frac{E_0}{\sqrt{R^2 + (X_L - X_C^2)}} \sin(\omega t - \theta)$$

$$\theta = \tan^{-1} \frac{X_L - X_C}{R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

21. $P = 1/2\, E_0 I_0$

    $= E_{rms}.I_{rms}$ (Non-inductive circuit)

22. $P = 1/2\, E_0 I_0 \cos\theta$

    $= E_{rms}.I_{rms} \cos\theta$ (inductive circuit)

23. $\cos\theta = \frac{\text{True power}}{\text{Apparent power}} = \frac{R}{Z}$

24. $Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$

    = Quality factor