DAY EIGHTEEN

Electrostatics

Learning & Revision for the Day

- Electric Charge
- Coulomb's Law of Forces between Two Point Charges
- Superposition Principle
- Electric Field
- Motion of A Charged Particle in An Electric Field
- Electric Field due to a Point Charge
- Continuous Charge Distribution
- Electric Dipole
- Electric Flux (ϕ_E)
- Gauss Law
- Electric Potential
- Electric Potential Energy
- Equipotential Surface
- Conductors and Insulators
- Electrical Capacitance
- Capacitor

If the charge in a body does not move, then the fricitional electricity is known as static electricity. The branch of physics which deals with static electricity is called electrostatics.

Electric Charge

Electric charge is the property associated with matter due to which it produces and experiences electric and magnetic effects.

Conservation of Charge

We can neither create nor destroy electric charge. The charge can simply be transferred from one body to another. There are three modes of charge transfer:

(a) By friction (b) By conduction (c) By induction

Quantisation of Charge

Electric charge is quantised. The minimum amount of charge, which may reside independently is the electronic charge e having a value of 1.6×10^{-19} C, i.e. $Q = \pm\, ne$, where, n is any integer.

Important properties of charges are listed below

- Like charges repel while opposite charges attract each other.
- Charge is invariant i.e. charge does not change with change in velocity.
- According to theory of relativity, the mass, time and length change with a change in velocity but charge does not change.
- A charged body attracts a lighter neutral body.
- Electronic charge is additive, i.e. the total charge on a body is the algebraic sum of all the charges present in different parts of the body. For example, if a body has different charges as $+2q, +4q, -3q, -q$, then the total charge on the body is $+2q$.

Coulomb's Law of Forces between Two Point Charges

- If q_1 and q_2 be two stationary point charges in free space separated by a distance r, then the force of attraction / repulsion between them is

$$F = \frac{K|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_1||q_2|}{r^2} \qquad \left[K = \frac{1}{4\pi\varepsilon_0}\right]$$

$$= \frac{9 \times 10^9 \times |q_1||q_2|}{r^2} \qquad [K = 9 \times 10^9 \text{ N-m}^2/\text{c}^2]$$

- If some dielectric medium is completely filled between the given charges, then the Coulomb's force between them becomes

$$F_m = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2} \qquad \left[\because \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r \text{ or } k\right]$$

$$= \frac{1}{4\pi k\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Forces between Multiple Charges

When a number of point charges are present in a region then force acting between any two point charges remains unaffected by the presence of other charges and remains same as according to Coulomb's law. If four identical charges of magnitude q each are placed at the four corners of square of side a, then the force on any one charge due to the rest of the three charges is

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a^2} (2\sqrt{2} + 1)$$

Superposition Principle

It states that, the net force on any one charge is equal to the vector sum of the forces exerted on it by all other charges. If there are four charges q_1, q_2, q_3 and q_4, then the force on q_1 (say) due to q_2, q_3 and q_4 is given by $\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14}$, where $\mathbf{F}_{12}$ is the force on q_1 due to q_2, $\mathbf{F}_{13}$ that due to q_3 and $\mathbf{F}_{14}$ that due to q_4.

Electric Field

The space surrounding an electric charge q in which another charge q_0 experiences a force of attraction or repulsion, is called the electric field of charge q. The charge q is called the **source charge** and the charge q_0 is called the **test charge**. The test charge must be **negligibly small** so that it does not modify the electric field of the source charge.

Intensity (or Strength) of Electric Field (E)

The intensity of electric field at a point in an electric field is the ratio of the forces acting on the test charge placed at that point to the magnitude of the test charge.

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}, \qquad \text{where, } \mathbf{F} \text{ is the force acting on } q_0.$$

Electric field intensity ($\mathbf{E}$) is a vector quantity.

The direction of electric field is same as that of force acting on the positive test charge. Unit of E is NC^{-1} or Vm^{-1}.

Electric Field Lines

An electric field line in an electric field is a smooth curve, tangent to which, at any point, gives the direction of the electric field at that point.

Properties of electric field lines are given below

- Electric field lines come out of a positive charge and go into the negative charge.
- No two electric field lines intersect each other.
- Electric field lines are continuous but they never form a closed loop.
- Electric field lines cannot exist inside a conductor. **Electric shielding** is based on this property.

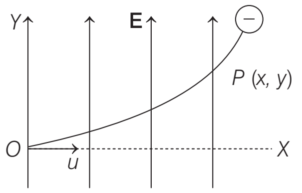
Motion of a Charged Particle in an Electric Field

Let a charged particle of mass m and charge q, enters the electric field along X-axis with speed u. The electric field E is along Y-axis is given by

$$F_y = qE$$

and force along X-axis remains zero, i.e.

$$F_x = 0$$



$\therefore$ Acceleration of the particle along Y-axis is given by

$$a_y = \frac{F_y}{m} = \frac{qE}{m}$$

The initial velocity is zero along Y-axis ($u_y = 0$).

$\therefore$ The deflection of charged particle along Y-axis after time t is given by $y = u_y t + \frac{1}{2} a_y t^2$

$$= \frac{qE}{2m} t^2$$

Along X-axis there is no acceleration, so the distance covered by particle in time t along X-axis is given by $x = ut$

Eliminating t, we have

$$y = \left(\frac{qE}{2mu^2} x^2\right)$$

$$y \propto x^2$$

This shows that the path of charged particle in perpendicular field is a **parabola**.

Electric Field due to a Point Charge

1. Electric Field due to a Point Charge at a Distance

Electric field at a distance r from a point charge q is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

- If q_1 and q_2 are two like point charges, separated by a distance r, a neutral point between them is obtained at a point distant r_1 from q_1, such that

$$r_1 = \frac{r}{\left[1 + \sqrt{\frac{q_2}{q_1}}\right]}$$

- If q_1 and q_2 are two charges of opposite nature separated by a distance r, a neutral point is obtained in the extended line joining them, at a distance r_1 from q_1, such that,

$$r_1 = \frac{r}{\left[\sqrt{\frac{q_2}{q_1}} - 1\right]}$$

2. Electric Field due to Infinitely Long Uniformly Charged Straight Wire

Electric field at a point situated at a normal distance r, from an infinitely long uniformly charged straight wire having a linear charge density λ, is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

3. Electric Field due to a Charged Cylinder

- For a conducting charged cylinder of linear charge density λ and radius R, the electric field is given by

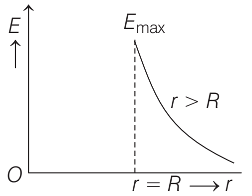
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}, \text{ for } r > R,$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}, \text{ for } r = R$$

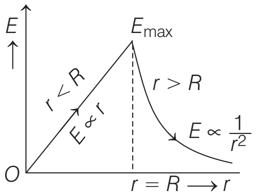
and $$E = 0, \text{ for } r < R$$

- For a non-conducting charged cylinder, for $r \le R$,

$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2} \text{ and } E = \frac{\lambda}{2\pi\varepsilon_0 r}, \text{ for } r > R$$



(a) Variation of electric field with distance for conducting cylinder



(b) Variation of electric field with distance for non-conducting cylinder

4. Electric Field due to a Uniformly Charged Infinite Plane Sheet

Electric field near a uniformly charged infinite plane sheet having surface charge density σ is given by

$$E = \frac{\sigma}{2\,\varepsilon_0}$$

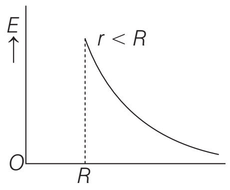
5. Electric Field due to a Uniformly Charged Thin Spherical Shell

For a charged conducting sphere/ shell of radius R and total charge Q, the electric field is given by

Case I $E = 0$, for $r < R$

Case II $E = \dfrac{Q}{4\pi\varepsilon_0 R^2}$, for $r = R$

Case III $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, for $r > R$



Variation of electric field with distance for uniformly charge spherical shell

Continuous Charge Distribution

The continuous charge distribution may be one dimensional, two dimensional and three-dimensional.

1. **Linear charge density** (λ) If charge is distributed along a line, *i.e.*, straight or curve is called linear charge distribution. The uniform charge distribution q over a length L of the straight rod.

   Then, the linear charge density, $\lambda = \dfrac{q}{L}$

   Its unit is coulomb metre^{-1} (Cm^{-1}).

2. **Surface charge density** (σ) If charge is distributed over a surface is called surface charge density, *i.e.*,

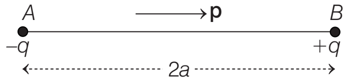
$$\sigma = q/A$$

   Its unit is coulomb m^{-2} (Cm^{-2})

3. **Volume charge density** (ρ) If charge is distributed over the volume of an object, is called volume charge density, *i.e.*, $p = \dfrac{q}{V}$. Its unit is coulomb metre^{-3} (Cm^{-3}).

Electric Dipole

An electric dipole consists of two equal and opposite charges separated by a small distance.



The dipole moment of a dipole is defined as the product of the magnitude of either charges and the distance between them. Therefore, dipole moment

$$\mathbf{p} = q(2\mathbf{a})$$

Electric Field due to a Dipole

- At a point distant r from the centre of a dipole, along its axial line $\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}r}{(r^2 - a^2)^2}$

[direction of $\mathbf{E}$ is the same as that of $\mathbf{p}$]

For a short dipole,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}}{r^3} \qquad [r >> a]$$

- At a point distant r from the centre of a dipole, along its equatorial line

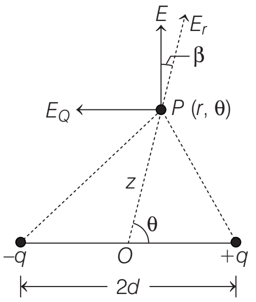
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p}}{(r^2 + a^2)^{3/2}}$$

[direction of $\mathbf{E}$ is opposite to that of $\mathbf{p}$]

For a short dipole $\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p}}{r^3}$ $\qquad [r >> a]$

- At a point distant r from the centre of a short dipole, along a line inclined at an angle θ with the dipole axis

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3} \sqrt{3\cos^2\theta + 1}$$



- $\mathbf{E}$ subtends an angle β from $\mathbf{r}$ such that

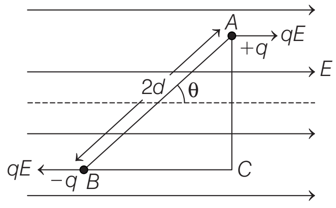
$$\tan\beta = \frac{1}{2}\tan\theta$$

Torque on a Dipole in a Uniform Electric Field

When a dipole is placed in an external electric field, making an angle θ with the direction of the uniform electric field E, it experiences a torque given by

$$\tau = qE \times AC$$
$$\tau = \mathbf{p} \times \mathbf{E}$$
$$\tau = pE \sin\theta$$

or $\quad qE \times 2\,d \sin\theta = (q \times 2\,d)\, E \sin\theta$



Work Done in Rotating a Dipole

If an electric dipole initially kept in an uniform electric field $\mathbf{E}$, making an angle θ_1, is rotated so as to finally subtend an angle θ_2, then the work done for rotating the dipole is,

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

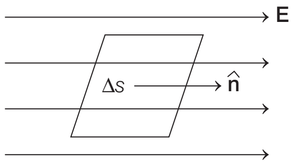
Potential Energy of a Dipole

It is the amount of work done in rotating an electric dipole from a direction perpendicular to electric field to a particular direction.

Hence, $\quad U = -pE\cos\theta \qquad$ or $U = -\mathbf{p}\cdot\mathbf{E}$

Electric Flux (ϕ_E)

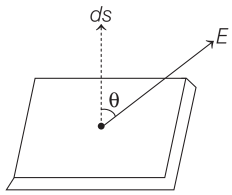
It is a measure of the flow of electric field through a surface. It can be defined as the total number of lines of electric field passes through a surface placed perpendicular to direction of field.



i.e. $\qquad \phi_E = \int EdS\cos\theta = \int \mathbf{E}\cdot d\mathbf{S} = \int \mathbf{E}\cdot\hat{\mathbf{n}}\, dS$

Gauss's Law

The total electric flux linked with a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the net charge enclosed by that surface. Thus,



$$\phi_E = \oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}[Q_{\text{enclosed}}]$$

where, $Q_{\text{enclosed}} \sum_{i=1}^{i=n} q_i$ is the algebraic sum of all the charges inside the closed surface.

Electric Potential

The amount of work done in bringing a unit positive charge, without any acceleration, from infinity to that point, along any arbitrary path.

$$V = \frac{W}{q_0}$$

Electric potential is a state function and does not depend on the path followed.

1. Electric Potential Due to a Point Charge

Potential due to a point charge Q, at a distance r is given by $\qquad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$

2. Electric Potential Due to a System of Charges

If a number of charges $q_1, q_2, q_3, \ldots$ are present in space, then the electric potential at any point will be

$$V = V_1 + V_2 + V_3 + \ldots$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right] = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\left(\frac{q_i}{r_i}\right)$$

3. Electric Potential Due to an Electric Dipole

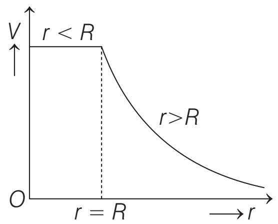
At any general point, $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$

On the dipole axis, $\theta = 0°$ and $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$

On the equitorial axis, $\theta = 90°$ and $V = 0$

4. Electric Potential due to Some Common Charge Distributions

Potential at a point distant r from an infinitely long wire having linear charge density λ, is



$$V = \frac{\lambda}{2\pi\varepsilon_0}\cdot\ln r$$

For a charged conducting sphere/shell having total charge Q and radius R, the potential at a point distant r from the centre of the sphere/shell is

(i) $V = \frac{Q}{4\pi\varepsilon_0 r}$, for $r > R$ (ii) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$, for $r = R$

(iii) $V = \frac{Q}{4\pi\varepsilon_0 R}$, for $r \le R$

For a charged non-conducting (dielectric) sphere of radius R, the charge Q is uniformly distributed over the entire volume.

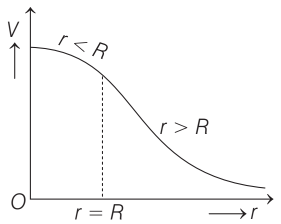
Hence, (i) $V = \frac{Q}{4\pi\varepsilon_0 r}$, for $r > R$

(ii) $V = \frac{Q}{4\pi\varepsilon_0 R}$, for $r = R$

and (iii) $V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3R^2 - r^2}{2R^3}\right]$, for $r < R$

At the centre of the sphere $(r = 0)$

$$V = \frac{3Q}{8\pi\varepsilon_0 R} = \frac{3}{2}V_s \qquad \left[V_s = \frac{Q}{4\pi\varepsilon_0 R}\right]$$



Electric Potential Energy

The electric energy of a system of charges is the work that has been done in bringing those charges from infinity to near each other to form the system. For two point charges q_1 and q_2 separated by distance r_{12}, the potential energy is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$.

In general, for a system of n charges, the electric potential energy is given by

$$U = \frac{1}{2}\Sigma\frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}},\ i \ne j$$

$\left[\frac{1}{2}\text{ is used as each term in summation will appear twice}\right]$

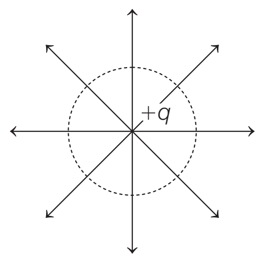
Relation between E and V

Because E is force per unit charge and V is work per unit charge. E and V are related in the same way as work and force.

Work done against the field to take a unit positive charge from infinity (reference point) to the given point $V_P = -\int_{\infty}^{P}\mathbf{E}\cdot d\mathbf{r}$ volt

where, the negative sign indicates that the work is done against the field.

Equipotential Surface



Equipotential surface is an imaginary surface joining the points of same potential in an electric field. So, we can say that the potential difference between any two points on an equipotential surface is **zero**.

The electric lines of force at each point of an equipotential surface are **normal** to the surface. Figure shows the electric lines of force due to point charge $+q$. The spherical surface will be the equipotential surface and the electrical lines of force emanating from the point charge will be radial and normal to the spherical surface.

Regarding equipotential surface, following points are worth noting

(i) Equipotential surface may be **planar**, solid etc. But equipotential surface can never be **point size**.

(ii) Equipotential surface is single valued. So, equipotential surfaces never **cross** each other.

(iii) Electric field is always **perpendicular** to equipotential surface.

(iv) Work done to move a point charge q between two points on equipotential surface is **zero**.

(v) The **surface** of a conductor in equilibrium is an equipotential.

Conductors and Insulators

Conductors are those materials through which electricity can pass through easily. e.g. metals like copper, silver, iron etc. **Insulators** are those materials through which electricity cannot pass through, e.g. rubber, ebonite, mica etc.

Dielectrics and Polarisation

Dielectrics are insulating materials which transmit electric effect without actually conducting electricity.

e.g. mica, glass, water etc.

When a dielectric is placed in an external electric field, so the molecules of dielectric gain a permanent electric dipole moment. This process is called polarisation.

Electrical Capacitance

Capacitance of a conductor is the amount of charge needed in order to raise the potential of the conductor by unity.

Mathematically, Capacitance $C = \dfrac{Q}{V}$

Sharing of Charges

- Let us have two charged conductors having charges Q_1 and Q_2 (or potentials V_1, V_2 and capacitances C_1, C_2 respectively). If these are joined together. In such a cases

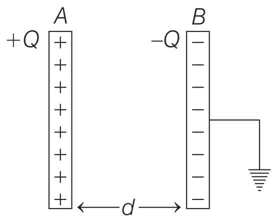
  Common potential, $V = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$
- During sharing of charges, there is some loss of electrostatic energy, which in turn reappears as heat or light. The loss of electrostatic energy

$$\Delta U = U_i - U_f = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

- When charges are shared between any two bodies, their potential become equal. The charges acquired are in the ratio of their capacitances.

Capacitor

A capacitor is a device which stores electrostatic energy. It consists of conductors of any shape and size carrying charges of equal magnitudes and opposite signs and separated by an insulating medium.

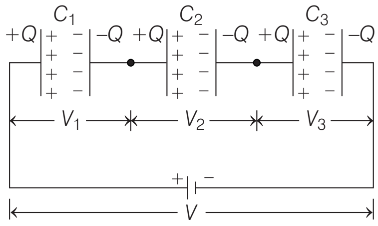


There are two types of combination of capacitors:

1. Series Grouping

In a series arrangement, $V = V_1 + V_2 + V_3 + \ldots$

and $\quad V_1 : V_2 : V_3 \ldots = \dfrac{1}{C_1} : \dfrac{1}{C_2} : \dfrac{1}{C_3} : \ldots$



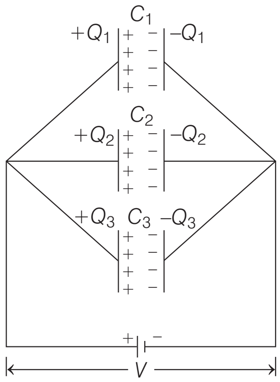
The equivalent capacitance C_s is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots = \sum_{i=1}^{i=n} \frac{1}{C_i}$$

2. Parallel Grouping

In a parallel arrangement, $Q = Q_1 + Q_2 + Q_3 + \ldots$

and $\quad Q_1 : Q_2 : Q_3 \ldots = C_1 : C_2 : C_3 \ldots$



The equivalent capacitance is given by

$$C_p = C_1 + C_2 + C_3 + \ldots = \sum_{i=1}^{i=n} C_i$$

Capacitance of a Parallel Plate Capacitor

1. Capacitor without Dielectric Medium between the Plates

If the magnitude of charge on each plate of a parallel plate capacitor be Q and the overlapping area of plates be A, then

- Electric field between the plates,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

- Potential difference between the plates

  $V = E \cdot d = \dfrac{\sigma d}{\varepsilon_0} = \dfrac{Qd}{\varepsilon_0 A}$, where d = separation between the two plates.
- Capacitance, $C = \dfrac{Q}{V} = \dfrac{\varepsilon_0 A}{d}$

2. Capacitor with Dielectric Medium between the Plates

- If a dielectric medium of dielectric constant K is completely filled between the plates of a capacitor, then its capacitance becomes,

$$C' = \frac{K\varepsilon_0 A'}{d} = KC_0 \qquad \left[\text{where, } C_0 = \frac{\varepsilon_0 A'}{d}\right]$$

- If a dielectric slab/sheet of thickness t (where, $t < d$) is introduced between the plates of the capacitor, then

$$C' = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{K}\right)}$$

- Magnitude of the attractive force between the plates of a parallel plate capacitor is given by

$$F = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2A\varepsilon_0} = \frac{CV^2}{2d}$$

- The energy density between the plates of a capacitor

$$u = \frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

Energy Stored in a Capacitor

If a capacitor of capacity C is charged to a potential V, the electrostatic energy stored in it is,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Energy Loss During Parallel Combination

When two capacitor of C_1 capacitance charge to potential V_1, whereas another of C_2 charge to potential of V_2, then after parallel combination.

$$\text{Loss in energy} = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

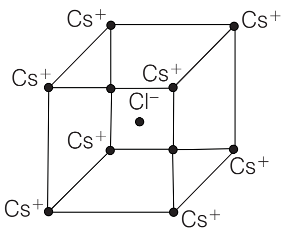
DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 Two balls of same mass and carrying equal charge are hung from a fixed support of length l. At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, X between the balls is proportional **→ JEE Main (Online) 2013**

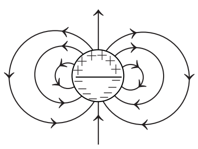
(a) l (b) l^2 (c) $l^{2/3}$ (d) $l^{1/3}$

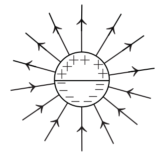
2 In the basic CsCl crystal structure, $^+$Cs and $^-$Cl ions are arranged in a bcc configuration as shown below. The net electrostatic force exerted by the 8 Cs^+ on the the Cl^- ion is

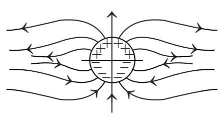


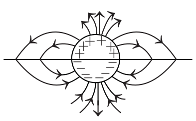
(a) $\frac{1}{4\pi\varepsilon_0}\frac{4e^2}{3a^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{16e^2}{3a^2}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{32a^2}{3a^2}$ (d) zero

3 A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) **→ JEE Main 2015**

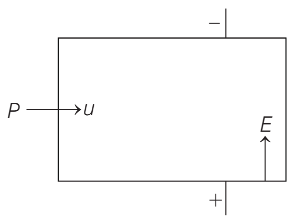
(a) 

(b) 

(c) 

(d) 

4 A positively charged particle P enters the region between two parallel plates with a velocity u, in a direction parallel to the plates. There is a uniform electric field in this region. P emerges from this region with a velocity v. Taking C as a constant, v will depend on u as



(a) $v = Cu$ (b) $v = \sqrt{u^2 + Cu}$

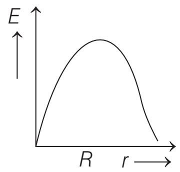
(c) $v = \sqrt{u^2 + \frac{C}{u}}$ (d) $v = \sqrt{u^2 + \frac{C}{u^2}}$

5 An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

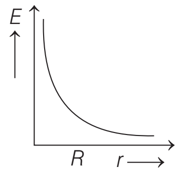
(a) 10^{-3} C/m (b) 10^{-4} C/m (c) 10^{-5} C/m (d) 10^{-7} C/m

6 In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be **→ AIEEE 2012**

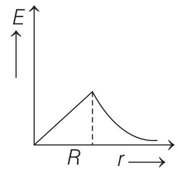
(a)



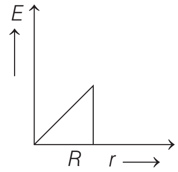
(b)



(c)



(d)



7 Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is **→ AIEEE 2009**

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$ (c) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$

8 Two points dipoles of dipole moment $\mathbf{P}_1$ and $\mathbf{P}_2$ are at a distance x from each other an $\mathbf{P}_1 \| \mathbf{P}_2$. The force between the dipoles is **→ JEE Main (Online) 2013**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{4P_3P_2}{x^3}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{3P_3P_2}{x^3}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{6P_1P_2}{x^3}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{3P_1P_2}{x^3}$

9 If the electric flux entering and leaving an enclosed surface respectively are ϕ_1 and ϕ_2, the electric charge inside the surface will be

(a) $\frac{\phi_2 - \phi_1}{\varepsilon_0}$ (b) $\frac{\phi_1 + \phi_2}{\varepsilon_0}$ (c) $\frac{\phi_1 - \phi_2}{\varepsilon_0}$ (d) $\varepsilon_0(\phi_1 + \phi_2)$

10 A cylinder of radius R and length L is placed in a uniform electric field E parallel to the axis of the cylinder, the total electric flux for the surface of the cylinder is

(a) $2\pi R^2 E$ (b) $\frac{\pi R^2}{E}$ (c) $\frac{\pi R^2 + \pi R^2}{E}$ (d) zero

11 A large insulated sphere of radius r, charged with Q units of electricity, is placed in contact with a small insulated uncharged sphere of radius r' and is then separated. The charge on the smaller sphere will now be

(a) $Q(r + r')$ (b) $Q(r - r')$

(c) $\frac{Q}{r' + r}$ (d) $\frac{Qr'}{r' + r}$

12 Consider a finite insulated, uncharged conductor placed near a finite positively charged conductor. The uncharged body must have a potential **→ JEE Main (Online) 2013**

(a) less than the charged conductor and more than at infinity
(b) more than the charged conductor and less than at infinity
(c) more than the charged conductor and more than at infinity
(d) less than the charged conductor and less than at infinity

13 The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where r is the distance from the centre where a, b are constants. Then the charge density inside the ball is **→ AIEEE 2011**

(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0$

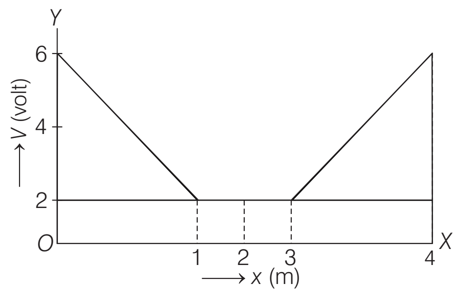
(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_0 r$

14 Two points P and Q are maintained at the potentials of 10 V and −4 V, respectively. The work done in moving 100 electrons from P to Q is **→ AIEEE 2009**

(a) -19×10^{-17} J (b) 9.60×10^{-17} J

(c) -2.24×10^{-16} J (d) 2.24×10^{-16} J

15 The variation of electric potential with distance from a fixed point is shown in the figure. What is the value of electric field at $x = 2$ m?



(a) Zero (b) 6/2

(c) 6/1 (d) 6/3

16 The electric potential V at any point (x, y, z) in space is given by $V = 4x^2$ volt. The electric field at (1, 0, 2) m in Vm^{-1} is

(a) 8, along the positive x-axis
(b) 8, along the negative x-axis
(c) 16, along the x-axis
(d) 16, along the z-axis

17 Assume that an electric field $\mathbf{E} = 30x^2\,\mathbf{i}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m/s **→ JEE Main 2014**

(a) 120 J/C (b) −120 J/C

(c) −80 J/C (d) 80 J/C

18 Figure shows some equipotential lines distributed in space. A charged object is move from point A to point B.

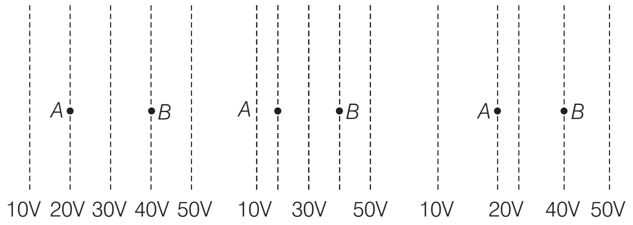
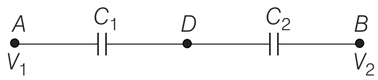


Fig. (i) Fig. (ii) Fig. (iii)

(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig.(i), Fig. (ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater that Fig. (ii) but equal to that in

19 Two condensers C_1 and C_2 in a circuit are joined as shown in the figure. The potential of point A is V_1 and that of B is V_2. The potential of point D will be

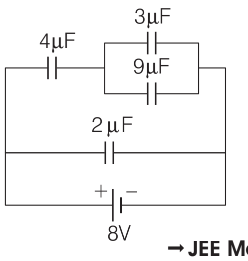


(a) $\frac{1}{2}(V_1 + V_2)$
(b) $\frac{C_1V_2 + C_2V_1}{C_1 + C_2}$
(c) $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$
(d) $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

20 Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then **➞ JEE Main 2013**

(a) $5C_1 = 3C_2$
(b) $3C_1 = 5C_2$
(c) $3C_1 + 5C_2 = 0$
(d) $9C_1 = 4C_2$

21 A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4\mu F$ and $9\mu F$ capacitors), at a point distance 30 m from it, would equal to



➞ JEE Main 2016 (Offline)

(a) 240 N/C (b) 360 N/C (c) 420 N/C (d) 480 N/C

22 An uncharged parallel plate capacitor having a dielectric of constant K is connected to a similar air cored parallel capacitor charged to a potential V. The two share the charge and the common potential is V'. The dielectric constant K is

(a) $\frac{V' - V}{V' + V}$ (b) $\frac{V' - V}{V'}$ (c) $\frac{V' - V}{V}$ (d) $\frac{V - V'}{V'}$

23 A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to **➞ JEE Main 2014**

(a) 6×10^{-7} C/m^2
(b) 3×10^{-7} C/m^2
(c) 3×10^4 C/m^2
(d) 6×10^4 C/m^2

24 A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be **➞ JEE Main 2018**

(a) 1.2 nC (b) 0.3 nC (c) 2.4 nC (d) 0.9 nC

25 A parallel plate capacitor having a separation between the plates d, plate area A and material with dielectric constant K has capacitance C_0. Now one-third of the material is replaced by another material with dielectric constant $2K$, so that effectively there are two capacitors one with area $\frac{1}{3}A$, dielectric constant $2K$ and another with area $\frac{2}{3}A$ and dielectric constant K. If the capacitance of this new capacitor is C then C/C_0 is **➞ JEE Main (Online) 2013**

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

26 A parallel plate capacitor of area 60 cm^2 and separation 3 mm is charged initially to $90\,\mu C$. If the medium between the plate gets slightly conducting and the plate loses the charge initially at the rate of 2.5×10^{-8} C/s, then what is the magnetic field between the plates? **➞ JEE Main (Online) 2013**

(a) 2.5×10^{-8} T
(b) 2.0×10^{-7} T
(c) 1.63×10^{-11} T
(d) Zero

27 **Case I** Identical point charges of magnitude Q are kept at the corners of a regular pentagon of side a.
Case II One charge is now removed. Match the following for above two cases.

	Column I		Column II
A.	Electric field as the centre of pentagon in case I	1.	$\frac{1}{4\pi\varepsilon_0}\frac{Q \times 5}{a}$
B.	Electric potential at the centre of pentagon in case I	2.	Zero
C.	Electric field at the centre of pentagon in case II	3.	$\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2}$
D.	Electric potential at the centre of pentagon in case II	4.	$\frac{1}{4\pi\varepsilon_0}\frac{Q}{a} \times 4$

Codes

	A	B	C	D		A	B	C	D
(a)	2	1	3	4	(b)	1	2	3	4
(c)	2	3	4	2	(d)	4	1	2	4

Direction (Q. Nos. 28-29) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below:*

(a) Statement I is true, Statement II is false
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is false, Statement II is true

28 **Statement I** For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement II The net work done by a conservative force on an object moving along a closed loop is zero.

29 **Statement I** No work is required to be done to move a test charge between any two points on an equipotential surface.

Statement II Electric lines of force at the equipotential surfaces are mutually perpendicular to each other.

→ JEE Main (Online) 2013

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A capacitance of 2μ**F** is required in an electrical circuit across a potential difference of 1kV. A large number of 1μ**F** capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

→ JEE Main 2017 (Offline)

(a) 16 (b) 24 (c) 32 (d) 2

2 Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$, respectively. The potential of shell B is

→ JEE Main 2018

(a) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (b) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

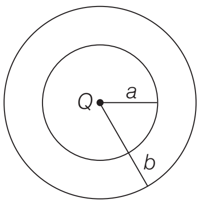
(c) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (d) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

3 An electric dipole has a fixed dipole moment $\mathbf{p}$, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E}_1 = E\,\hat{\mathbf{i}}$, it experiences a torque $\mathbf{E}_1 = \tau\,\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\,\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is

→ JEE Main 2017 (Offline)

(a) 45° (b) 60° (c) 90° (d) 30°

4 The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is



→ JEE Main 2016 (Offline)

(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2-a^2)}$

(c) $\frac{2Q}{\pi(a^2-b^2)}$ (d) $\frac{2Q}{\pi a^2}$

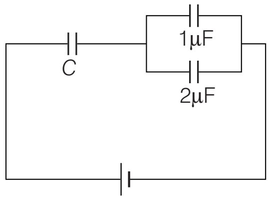
5 A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then $\frac{Q}{q}$ equals

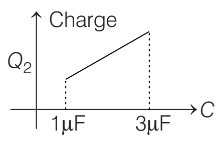
→ AIEEE 2009

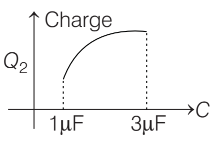
(a) $-2\sqrt{2}$ (b) -1 (c) 1 (d) $-\frac{1}{\sqrt{2}}$

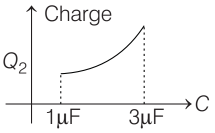
6 In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of C is given properly by (figures are drawn schematically and are not to scale)

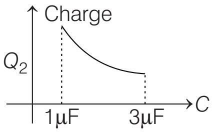
→ JEE Main 2015



(a)


(b)


(c) 

(d) 

7 A uniform electric field **E** exists between the plates of a charged condenser. A charged particle enters the space between the plates and perpendicular to **E**. The path of the particle between the plates is a

→ JEE Main (Online) 2013

(a) straight line (b) hyperbola
(c) parabola (d) circle

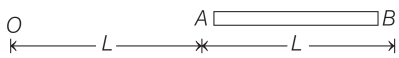
8 The surface charge density of a thin charged disc of radius R is σ. The value of the electric field at the centre of the disc is $\frac{\sigma}{2\varepsilon_0}$. With respect to the field at the centre, the electric field along the axis at a distance R from the centre of the disc **→ JEE Main (Online) 2013**

(a) reduces by 71% (b) reduces by 29.3%
(c) reduces by 9.7% (d) reduces by 14.6%

9 A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3, and R_4 respectively. Then, **→ JEE Main 2015**

(a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(d) $2R < R_4$

10 A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance **→ JEE Main 2013**



(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$
(c) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$

11 Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement $(y < a)$ along the y-axis, the net force acting on the particle is proportional to **→ JEE Main 2013**

(a) y (b) $-y$ (c) $1/y$ (d) $-1/y$

12 Two small equal point charges of magnitude q are suspended from a common point on the ceiling by insulating massless strings of equal lengths. They come to equilibrium with each string making angle θ from the vertical. If the mass of each charge is m, then the electrostatic potential at the centre of line joining them will be $\left(\frac{1}{4\pi\varepsilon_0} = k\right)$.

→ JEE Main (Online) 2013

(a) $2\sqrt{k\, mg \tan\theta}$ (b) $\sqrt{k\, mg \tan\theta}$
(c) $4\sqrt{k\, mg / \tan\theta}$ (d) $4\sqrt{k\, mg \tan\theta}$

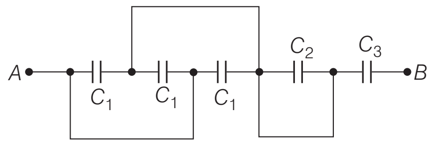
13 A point charge of magnitude $+1\,\mu C$ is fixed at $(0, 0, 0)$. An isolated uncharged spherical conductor, is fixed with its centre at $(4, 0, 0)$. The potential and the induced electric field at the centre of the sphere is **→ JEE Main (Online) 2013**

(a) 1.8×10^5 V and -5.625×10^6 V/m
(b) 0 V and 0 V / m
(c) 2.25×10^3 V and 5.625×10^2 V/m
(d) 2.25×10^5 V and 0 V/m

14 Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 16 gcm^{-3}, the dielectric constant of the liquid is **→ AIEEE 2010**

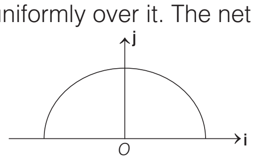
(a) 4 (b) 3 (c) 2 (d) 1

15 A circuit is shown in figure for which $C_1 = (3 \pm 0.011)\,\mu F$, $C_2 = (5 \pm 0.01)\,\mu F$ and $C_3 = (1 \pm 0.01)\,\mu F$. If C is the equivalent capacitance across AB, then C is given by



(a) $(0.9 \pm 0.114)\,\mu F$ (b) $(0.9 \pm 0.01)\,\mu F$
(c) $(0.9 \pm 0.023)\,\mu F$ (d) $(0.9 \pm 0.09)\,\mu F$

16 A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field **E** at the centre O is **→ AIEEE 2010**



(a) $\frac{q}{4\pi^2\varepsilon_0 r^2}\,\mathbf{j}$ (b) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\,\mathbf{j}$
(c) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\,\mathbf{j}$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}\,\mathbf{j}$

17 Two positive charges of magnitude q are placed at the ends of a side 1 of a square of side $2a$. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **→ AIEEE 2011**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$ (b) zero
(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 + \frac{1}{\sqrt{5}}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 - \frac{2}{\sqrt{5}}\right)$

18 Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d $(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then as a function of distance x between them, **→ AIEEE 2011**

(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$ (c) $v \propto x$ (d) $v \propto x^{-1/2}$

19 Combination of two identical capacitors, a resistor R and a DC voltage source of voltage 6 V is used in an experiment on C-R circuit. It is found that for a parallel combination of the capacitor, the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 s. For series combination, the time needed for reducing the voltage of the fully charged series combination by half is **→ AIEEE 2011**

(a) 20 s (b) 10 s (c) 5 s (d) 2.5 s

20 A resistor R and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed (take, $\log_{10} 2.5 = 0.4$) **→ AIEEE 2011**

(a) $1.7 \times 10^5 \, \Omega$ (b) $2.7 \times 10^6 \, \Omega$
(c) $3.3 \times 10^7 \, \Omega$ (d) $1.3 \times 10^4 \, \Omega$

21 Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0 \left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where, r is the distance from the origin. The electric field at a distance $r (r < R)$ from the origin is given by **→ AIEEE 2010**

(a) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (b) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(c) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

22 Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then, the ratio t_1 / t_2 will be **→ AIEEE 2010**

(a) 1 (b) 1/2
(c) 1/4 (d) 2

23 The question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniform positive charge density ρ. As a result of this uniform charge distribution, there is a finite value of electric potential, at the surface of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement I When a charge q is taken from the centre of the surface of the sphere its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement II The electric field at a distance $r (r < R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$. **→ AIEEE 2012**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement II
(d) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I

ANSWERS

SESSION 1

1 (d)	**2** (d)	**3** (a)	**4** (d)	**5** (d)	**6** (c)	**7** (c)	**8** (c)	**9** (d)	**10** (d)
11 (d)	**12** (a)	**13** (c)	**14** (d)	**15** (a)	**16** (b)	**17** (c)	**18** (c)	**19** (b)	**20** (b)
21 (c)	**22** (d)	**23** (a)	**24** (a)	**25** (b)	**26** (d)	**27** (a)	**28** (b)	**29** (b)	

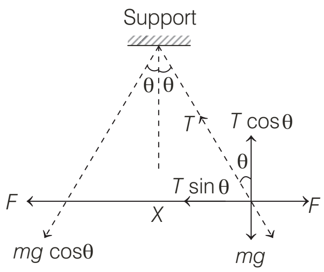
SESSION 2

1 (c)	**2** (b)	**3** (b)	**4** (a)	**5** (a)	**6** (d)	**7** (c)	**8** (a)	**9** (c,d)	**10** (d)
11 (a)	**12** (c)	**13** (c)	**14** (c)	**15** (c)	**16** (c)	**17** (a)	**18** (d)	**19** (d)	**20** (b)
21 (b)	**22** (c)	**23** (a)							

Hints and Explanations

SESSION 1

1 $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{x^2} = T\sin\theta$



The resultant of components $mg\cos\theta$ and force of repulsion balances, the tension in the string for the equilibrium massive change.

$$T\cos\theta = mg$$

$$\tan\theta = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{x^2 mg}$$

$$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg}$$

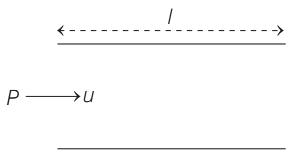
$\Rightarrow$ $\dfrac{x}{2l} \propto \dfrac{ql}{x^2}$ or $x^3 \propto l$ or $x \propto l^{1/3}$

Thus, we find the separation between the balls is proportional to $l^{1/3}$, where, l is length of string.

2. By symmetry resultant force applied by eight charges on corners is zero.

3 Field lines should originate from positive charge and terminate to negative charge. Thus, (b) and (c) are not possible. Electric field lines cannot form corners as shown in (d). Thus, correct option is (a).

4 $v_x = u, t = \dfrac{l}{u},\ a_y = \dfrac{Eq}{m} = a$



$$v_y = at = \frac{al}{u}$$

As, $v^2 = v_x^2 + v_y^2$

$$v^2 = u^2 + \frac{a^2l^2}{u^2}$$

$$= u^2 + \frac{C}{u^2},$$

$$v = \sqrt{u^2 + \frac{C}{u^2}}$$

5 Let λ be the linear charge density.

Given, distance $r = 2\text{ cm} = 2\times10^{-2}$ m

Electric field $E = 9\times10^4$ N/C

Using the formula of electric field due to an infinite line charge.

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Dividing and multiplying by 2 to get $\dfrac{1}{4\pi\varepsilon_0}$ because, we have the value of $\dfrac{1}{4\pi\varepsilon_0}$

$$E = \frac{2}{2}\times\frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\lambda}{4\pi\varepsilon_0 r}$$

Putting the values, we get

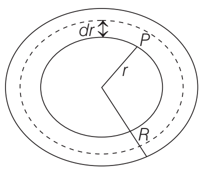
$$9\times10^4 = \frac{2\times9\times10^9\times\lambda}{2\times10^{-2}}$$

$$\lambda = \frac{9\times10^4\times2\times10^{-2}}{2\times9\times10^9} = 10^{-7}\text{ C/m}$$

Thus, the linear charge density is 10^{-7} C/m.

6 Electric field inside the uniformly charged sphere varies linearly, as $E = \dfrac{kQ}{R^3}\cdot r, (r \le R)$, while outside the sphere, it varies as inverse square of distance, $E = \dfrac{kQ}{r^2}; (r \ge R)$ which is correctly represented in option (c).

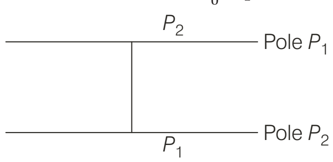
7 According to Gauss' theorem,



$$E\,4\pi r_1^2 = \frac{\int_0^{r_1}\frac{Q}{\pi R^4}\,r\,4\pi r^2\,dr}{\varepsilon_0}$$

$$\Rightarrow\quad E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$

8 We know that, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2P_1P_2}{r^3}$



With the help of this relation, we find the force between dipole is $\dfrac{1}{4\pi\varepsilon_0}\dfrac{6P_1P_2}{x^3}$.

9 Let $-q_1$ be the charge, due to which flux ϕ_1 is entering the surface.

$$\phi_1 = \frac{-q_1}{\varepsilon_0}\quad\text{or}\quad q_1 = -\phi_1\varepsilon_0$$

Let $+q_2$ be the charge, due to which flux ϕ_2, is leaving the surface.

$$\phi_2 = \frac{q_2}{\varepsilon_0}\quad\text{or}\quad q_2 = \varepsilon_0\phi_2$$

Electric charge inside the surface

$$= q_2 - q_1 = \varepsilon_0\phi_2 + \varepsilon_0\phi_1 = \varepsilon_0(\phi_2 + \phi_1)$$

10 As uniform electric field is parallel to the cylindrical axis

$$\int \mathbf{E}\cdot d\mathbf{S} = \int E\,dS\cos 90^\circ = 0$$

Further flux entering the cylinder at one end = flux leaving the cylinder at other end. Therefore, total electric flux is zero.

11 Common potential,

$$V = \frac{Q + 0}{4\pi\varepsilon_0(r + r')}$$

Charge on smaller sphere,

$$4\pi\varepsilon_0 r'\times V = \frac{Qr'}{r + r'}$$

12 The uncharged body must have a potential less than the charged conductor and more than at infinity.

i.e. $V_\infty < V$ or $V > V_\infty$

13 Electric field, $E = \dfrac{d\phi}{dr} = -2ar$

By Gauss' theorem,

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$\Rightarrow$ $q = -8\pi\varepsilon_0 ar^3$

$$\rho = \frac{dq}{dV} = \frac{dq}{dr}\times\frac{dr}{dV}$$

$$= (-24\pi\varepsilon_0 ar^2)\left(\frac{1}{4\pi r^2}\right)$$

$$= -6\varepsilon_0 a$$

14 $W = QdV = Q(V_q - V_p)$

$$= -100\times(1.6\times10^{-19})\times(-4-10)$$

$$= +100\times1.6\times10^{-19}\times14$$

$$= +2.24\times10^{-16}\text{ J}$$

15 As, $E = \dfrac{dV}{dr}$ and around $x = 2$ m,

V = constant

$\therefore$ $dV = 0$ and $E = 0$

16 As E and V are related as,

$$E = \frac{-dv}{dx} = \frac{-d}{dx}(4x^2) = -8x$$

At (1, 0, 2), $E = -8(1) = -8$

17 As we know that, potential difference $V_A - V_O$ is

$$dV = -Edx, \int_{V_o}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_O = -30 \times \left[\frac{x^3}{3}\right]_0^2$$
$$= -10 \times [2^3 - (0)^3]$$
$$= -10 \times 8 = -80 \text{ J/C}$$

18 The work done by a electrostatics force is given by $W_{12} = q(V_2 - V_1)$. Here, initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

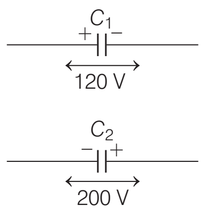
19 Let the potential of point D be V.
If q is charge on each condenser, then

$$V_1 - V = qC_1$$
$$\Rightarrow \quad V - V_2 = qC_2$$

Divide $\quad \dfrac{V_1 - V}{V - V_2} = \dfrac{C_1}{C_2}$

$$VC_1 - V_2C_1 = V_1C_2 - VC_2$$
$$V(C_1 + C_2) = C_1V_2 + C_2V_1;$$
$$V = \frac{C_1V_2 + C_2V_1}{C_1 + C_2}$$

20



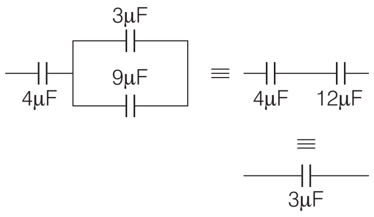
For potential to be made zero after connection, the charge on both the capacitors are equal.

i.e. $\quad q_1 = q_2$

$\therefore \quad C_1V_1 = C_2V_2, 120C_1 = 200C_2$

$\Rightarrow \quad 3C_1 = 5C_2$

21 Resultant circuit,



As, charge on $3\mu F = 3\mu F \times 8V = 24\mu C$

$\therefore$ Charge on $4\mu F$ = Charge on $12\mu F$
$= 24\mu C$

Charge on $3\mu F = 3\mu F \times 2V = 6\mu C$

Charge on $9\mu F = 9\mu F \times 2V = 18\mu C$

Charge on $4\mu F$ + Charge on $9\mu F$
$= (24 + 18)\mu C = 42\mu C$

$\therefore$ Electric field at a point distant 30 m

$$= \frac{9 \times 10^3 \times 42 \times 10^{-6}}{30 \times 30} = 420 \text{N/C}$$

22 Initial charge $= CV$ and

Final charge $= CV' + KCV'$

Since, initial charge = final charge

$$K = \frac{V - V'}{V'} K = \frac{V - V'}{V'}$$

23 When free space between parallel plates of capacitor, the

eletric field, $E = \dfrac{\sigma}{\varepsilon_0}$

When dielectric is introduced between parallel plates of capacitor,

$$E' = \frac{\sigma}{K\varepsilon_0}$$

Electric field inside dielectric

$$\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$$

where, K = dielectric constant of medium = 2.2

ε_0 = permittivity of free space
$= 8.85 \times 10^{-12}$

$\Rightarrow \quad \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$
$= 6.6 \times 8.85 \times 10^{-8}$
$= 5.841 \times 10^{-7} = 6 \times 10^{-7} \text{ C/m}^2$

24 Magnitude of induced charge is given by

$$Q' = (K - 1)CV_0$$
$$= \left(\frac{5}{3} - 1\right) 90 \times 10^{-12} \times 20$$
$$= 1.2 \times 10^{-9} \text{C}$$

$\Rightarrow \quad Q' = 1.2 \text{ nC}$

25 As capacitance,

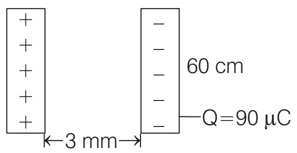
$$C_0 = k\frac{\varepsilon_0 A}{d}$$

Now equivalent capacitance,

$$C = \frac{2K}{3}\frac{\varepsilon_0 A}{d} + \frac{2k}{3}\frac{\varepsilon_0 A}{d} = \frac{4k}{3}\frac{\varepsilon_0 A}{d}$$

$$\therefore \quad \text{Ratio} = \frac{C}{\varepsilon_0} = \frac{\frac{4k}{3}\frac{\varepsilon_0 A}{d}}{k\frac{\varepsilon_0 A}{d}} = \frac{4}{3}$$

26



Given, $\quad \dfrac{dQ}{dt} = 2.5 \times 10^{-8}$ C/s

But in case of capacitor, there is no magnetic field inside the capacitor i.e. zero.

27 Point charges are uniformly distributed around the centre O. Hence, electric field E is zero.

$\therefore \quad A \rightarrow 2$

Electric potential $V = 5 \times \left(\dfrac{Q}{4\pi\varepsilon_0 a}\right)$

$\therefore \quad B \rightarrow 1$

In case II, the electric field of three identically placed charges is zero. Net field at the centre O is due to a single point charge and is given as

$$E = \frac{Q}{4\pi\varepsilon_0 a^2}$$

$\therefore \quad C \rightarrow 3$

In case II, the electric potential, being scalar, becomes

$$V = 4 \times \frac{Q}{4\pi\varepsilon_0 a}$$

$\therefore \quad D \rightarrow 4$

28 Work done by conservative force does not depend on the path. Electrostatic force is a conservative force.

29 As, $W = q(V_A - V_B)$. At equipotential surface $V_A = V_B$ so, $W = 0$

Now, we know that field lines makes an angle of 90° with the equipotential surfaces but these are parallel to one-another.

SESSION 2

1 As each capacitors cannot withstand more than 300 V, so there should be four capacitors in each row became in this condition 1 kV i.e. 1000 V will be divided by 4 (i.e. 250 not more than 300 V).

Now, equivalent capacitance of one row
$= \dfrac{1}{4} \times 1\,\mu F = 0.25\mu F$

[$\because$ in series combination, $C_{eq} = C/n$]

Now, we need equivalent of $2\mu F$, so let we need n such rows

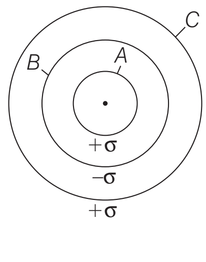
$\therefore \quad n \times 0.25 = 2\mu F$

[$\because$ in parallel combination $C_{eq} = nC$]

$$n = \frac{2}{0.25} = 8$$

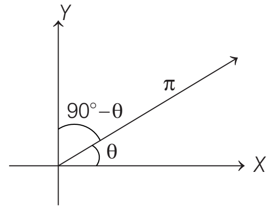
$\therefore$ Total number of capacitors
= number of rows × number of capacitors in each row
$= 8 \times 4 = 32$

2 Potential of B = Potential due to charge on A + Potential due to charge on B + Potential due to charge on C.



$$\therefore\ V_B = \frac{k(Q_A + Q_B)}{b} + \frac{kQ_C}{c}$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$
$$= \frac{\sigma\varepsilon}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + \frac{c^2}{c}\right)$$
$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$
$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

3



Torque applied on a dipole $\tau = pE \sin\theta$ where, θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\hat{\mathbf{i}}$

it means field is directed along positive X direction, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1 = pE_1 \sin\theta$$

In electric field $E_2 = \sqrt{3}\ E\hat{\mathbf{j}}$,

it means field is directed along positive Y-axis, so angle between dipole and field will be $90° - \theta$.

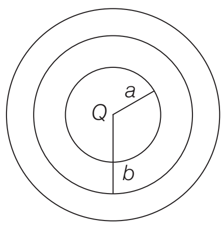
Torque in this direction

$$\tau_2 = pE \sin(90° - \theta)$$
$$= p\sqrt{3}\ E_1 \cos\theta$$

According to question,

$$\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$$
$$\therefore\ pE_1 \sin\theta = p\sqrt{3}\ E_1 \cos\theta$$
$$\tan\theta = \sqrt{3}$$
$$\Rightarrow \tan\theta = \tan 60°$$
$$\therefore \theta = 60°$$

4 As, Gaussian surface at distance r from centre,



$$\frac{Q + \int_a^r \frac{A}{r} 4\pi r^2 dr}{\varepsilon_0} = E 4\pi r^2$$
$$E\ 4\pi\varepsilon_0 r^2 = Q + A\frac{4\pi}{r^2}\left(\frac{r^2 - a^2}{2}\right)$$
$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r^2} + A\ 2\pi\left(\frac{r^2 - a^2}{r^2}\right)\right]$$
$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2} + A 2\pi - \frac{A 2\pi a^2}{r^2}\right)$$
$$E = \frac{1}{4\pi\varepsilon_0} \times A \times 2\pi$$

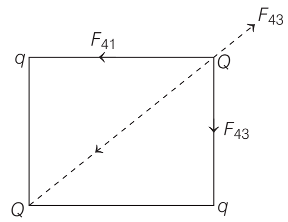
At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is

As, $Q = 2\pi A a^2$ i.e. $A = \dfrac{Q}{2\pi a^2}$

5 Three forces F_{41}, F_{42} and F_{43} acting on Q as shown.

Resultant of $F_{41} + F_{43} = \sqrt{2} F_{\text{each}}$

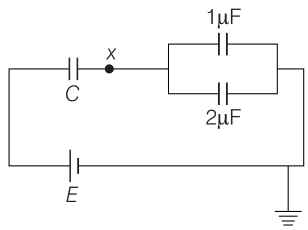
$$= \sqrt{2}\ \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}$$



Resultant on Q becomes zero only when q charges are of negative nature.

$$\therefore\ F = \frac{1}{4\pi\varepsilon_0}\frac{Q \times Q}{(\sqrt{2}d)^2}$$
$$\Rightarrow \sqrt{2}\frac{dQ}{d^2} = \frac{Q \times Q}{2d^2}$$
$$\Rightarrow \sqrt{2} \times q = \frac{Q \times Q}{2}$$
$$\therefore\ q = \frac{Q}{2\sqrt{2}} \quad \text{or} \quad \frac{Q}{q} = -2\sqrt{2}$$

6 Assume negative terminal of the battery as grounded (0 V). Suppose, potential of point x is V.



From the circuit diagram, we can write

$$Q_C = Q_1 + Q_2$$

or $C(E - V) = 1 \times V + 2 \times V$

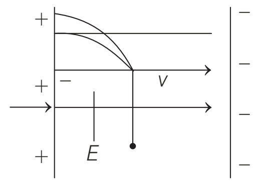
or $V[C + 3] = CE$ or $V = \dfrac{CE}{3 + C}$

$$\therefore\ Q_2 = C_2 (V) = \frac{2\,CE}{3 + C} = \frac{2E}{1 + 3/C}$$

As, C varied from 1μF to 3 μF, charge increases with decreasing slope.

Note As $C \to \infty, Q_2 \to 2E$ = constant

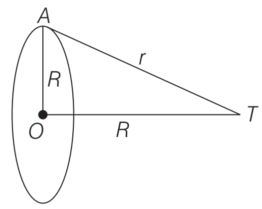
7



As the speed of particle is far to the intensity vector, and there is no acceleration in the direction for to **E**, but there is a electric force exerting on the particle (charge) whenever, it motion is in electric field.

Hence, continuously a force exerting on the particle for to its velocity or speed. Hence, path of particle must be parabola. As in the projectile motion.

8 Given, $E_0 = \dfrac{\sigma}{2\varepsilon_0}$



Also, we know that

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{qR}{(R^2 + R^2)^{3/2}}$$

but $q = \sigma A = \sigma\pi R^2$

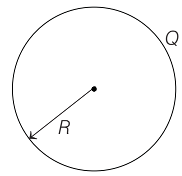
$$\Rightarrow E_p = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi R^3}{2\sqrt{2}R^3} = \frac{\sigma}{8\sqrt{2}\,\varepsilon_0}$$

$\therefore$ Reduction in electric field

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{4\sqrt{2}}\right]$$

Clearly, reduction is approx 71%.

9 Potential at the surface of the charged sphere



Charged sphere

$$V_0 = \frac{KQ}{R},\ V = \frac{KQ}{r}, r \geq R$$
$$= \frac{KQ}{2R^3}(3R^2 - r^2); r \leq R$$
$$V_{\text{centre}} = V_c = \frac{KQ}{2R^3} \times 3R^2$$
$$= \frac{3KQ}{2R} = \frac{3V_0}{2} \Rightarrow R_1 = 0$$

As potential decreases for outside points. Thus, according to the question, we can write

$$V_{R_2} = \frac{5V_0}{4} = \frac{KQ}{2R^3}(3R^2 - R_2^2)$$

$$\Rightarrow \quad \frac{5V_0}{4} = \frac{V_0}{2R^2}(3R^2 - R_2^2)$$

or $\quad \frac{5}{2} = 3 - \left(\frac{R_2}{R}\right)^2$

or $\left(\frac{R_2}{R}\right)^2 = 3 - \frac{5}{2} = \frac{1}{2}$ or $R_2 = \frac{R}{\sqrt{2}}$

Similarly,

$$V_{R_3} = \frac{3V_0}{4} \Rightarrow \frac{KQ}{R_3} = \frac{3}{4} \times \frac{KQ}{R}$$

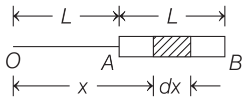
or $\quad R_3 = \frac{4}{3}R$

$$V_{R4} = \frac{KQ}{R_4} = \frac{V_0}{4}$$

$$\Rightarrow \quad \frac{KQ}{R_4} = \frac{1}{4} \times \frac{KQ}{R} \text{ or } R_4 = 4R$$

Thus, both options (d) and (c) are correct.

10



$$V = \int_L^{2L} \frac{kd\,Q}{x} = \int_L^{2L} \frac{\left(\frac{Q}{L}\right)dx}{x}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\left(\frac{1}{x}\right)dx$$

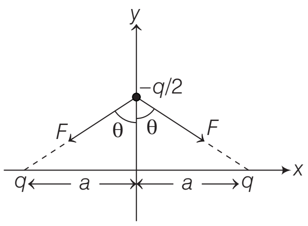
$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e x]_L^{2L}$$

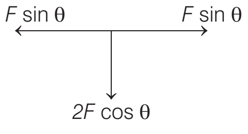
$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e 2L - \log_e L]$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\left[\log_e \frac{2L}{L}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\text{In}(2)$$

11 $F_{\text{net}} = 2F\cos\theta$

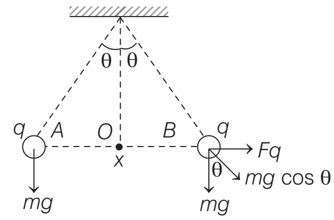




$$F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2 + a^2})^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

$$F_{\text{net}} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2 + a^2)^{3/2}} \Rightarrow \frac{kq^2 y}{a^3} \propto y$$

12



Force of attraction $= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}$

Potential at the mid-point O

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\frac{q}{x}$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{q}{x} \qquad \ldots\text{(i)}$$

From the figure,

$$\tan\theta = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}}{mg}$$

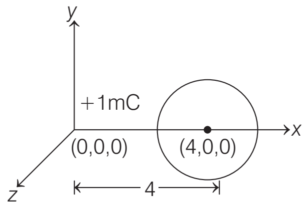
$$\therefore mg\tan\theta = k\frac{q^2}{x^2}, \quad x = \sqrt{\frac{kq^2}{mg\tan\theta}}$$

$$\Rightarrow \qquad x = q\sqrt{\frac{k}{mg\tan\theta}} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\text{Potential} = 4\sqrt{K\,mg/\tan\theta}$$

13



Potential at the centre of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{1\times 10^{-6}}{4}$$

$$= \frac{9\times 10^9 \times 10^{-6}}{4} = 2.25\times 10^3 \text{ V}$$

Now, electric field at the centre of sphere

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = 9\times 10^9 \frac{1\times 10^{-6}}{4^2}$$

$$= 0.5625\times 10^3$$

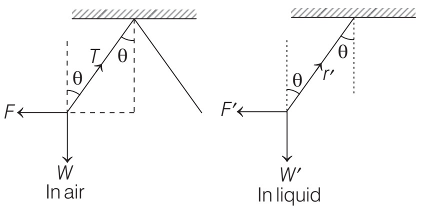
$$= 5.625\times 10^2 \text{ V/m}$$

14 Each ball in equilibrium have following three forms

(i) tension (ii) repulsive force

(iii) weight

By applying Lami's theorem,



In liquid $F' = \frac{F}{K}$

where, K = dielectric constant of liquid

$$W' = W - \text{upthrust} = V\rho g - V\sigma g$$

where, ρ = density of material,

σ = density of liquid

In air, using Lami's theorem,

$$\frac{W}{\sin(90°-\theta)} = \frac{F}{\sin(180°-\theta)}$$

or $\quad \frac{W}{\cos\theta} = \frac{F}{\sin\theta} \qquad \ldots\text{(i)}$

In liquid,

$$\frac{W'}{\cos\theta} = \frac{F'}{\sin\theta}$$

(As angles are same)…(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{W}{W'} = \frac{F}{F'}$$

or $\quad K = \frac{W}{W - \text{upthrust}}$

$$\Rightarrow K = \frac{V\rho g}{V\rho g - V\sigma g} = \frac{\rho}{\rho - \sigma} = \frac{1.6}{1.6 - 0.8} = 2$$

15 The capacitor C_2 is shorted, so it is not playing any role in circuit and can be removed. The 3 capacitors each of C_1 are connected in parallel and this is connected to C_3 in series.

$$C_{\text{eq}} = \frac{3C_1C_3}{3C_1 + C_3} = C = \frac{3\times 3\times 1}{3\times 3 + 1}$$

$$= 0.9\ \mu\text{F}$$

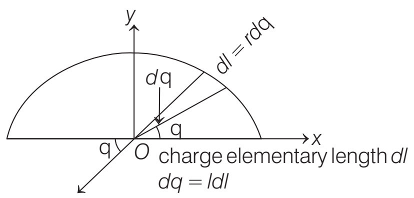
So, $\frac{\Delta C}{C} = \frac{3\Delta C_1}{C_1} + \frac{\Delta C_3}{C_3} + \frac{3\Delta C_1 + \Delta C_3}{3C_1 + C_3}$

[For computation of errors worst has to be taken]

$$\Rightarrow \quad \frac{\Delta C}{0.9} = \frac{3\times 0.011}{3} + \frac{0.01}{1} + \frac{(0.033 + 0.01)}{10}$$

$$\Rightarrow \quad \Delta C = \pm\ 0.023\ \mu\text{F}$$

16 Linear charge density $\lambda = \left(\frac{q}{\pi r}\right)$



Net field at O,

$$E = \int dE\sin\theta\,(-\,\mathbf{j})$$

$$= \int \frac{K\cdot dq}{r^2}\sin\theta\,(-\,\mathbf{j})$$

$$E = \frac{K}{r^2}\int \frac{qr}{\pi r}d\theta\sin\theta\,(-\,\mathbf{j}) \quad (\because\ dl = rd\theta)$$

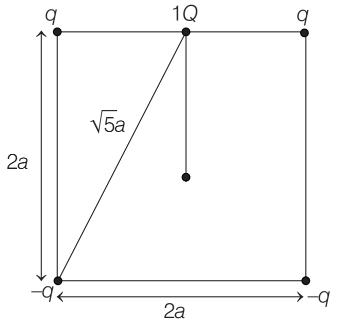
$$= \frac{K}{r^2}\frac{q}{\pi}\int_0^{\pi}\sin\theta\,(-\,\mathbf{j}) = \frac{q}{2\pi^2\varepsilon_0 r^2}(-\,\mathbf{j})$$

17 $U_i = \dfrac{2kqQ}{a} + \dfrac{2k(-q)Q}{\sqrt{5}a}$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

$U_f = 0$

By conservation of energy,



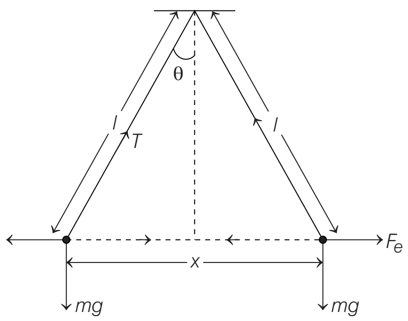
Gain in KE = loss in PE,

$$K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

18 At any instant,

$$T\cos\theta = mg \qquad \ldots\text{(i)}$$

$$T\sin\theta = F_e = \frac{kq^2}{x^2} \qquad \ldots\text{(ii)}$$



From Eqs. (i) and (ii), we get

$$\frac{kq^2}{x^2} = mg\tan\theta$$

(q = magnitude of charge)

$$\Rightarrow \quad q^2 = \frac{mg}{k} \cdot \frac{x}{2l} \cdot x^2 \left(\because \tan\theta = \frac{x}{2l}\right)$$

$$\Rightarrow \quad q^2 = \frac{mg}{2kl}x^3 \qquad \ldots\text{(iii)}$$

$$\Rightarrow \quad 2q\frac{dq}{dt} = \frac{3mg}{2kl}x^2\frac{dx}{dt}$$

$$\Rightarrow \quad 2\left(\frac{mg}{2kl}x^3\right)^{1/2}\frac{dq}{dt} = \frac{3mg}{2kl}x^2 v$$

$$\left[\because q = \left(\frac{mg}{2kl}x^3\right)^{1/2}\right]$$

$$\Rightarrow \quad vx^{1/2} = \text{constant}$$

$$\Rightarrow \quad v \propto x^{-1/2}$$

19 If C_e is the effective capacitance, then

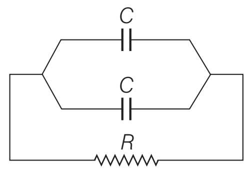
$$V_C = \frac{1}{2}V_0, \ \frac{q}{C_e} = \frac{q_0}{2C_e}$$

As $\quad q = q_0\left(1 - e^{\frac{-t}{RC_e}}\right)$

$$\Rightarrow \quad q_0\left(1 - e^{-t/RC_e}\right) = \frac{q_0}{2}$$

$$\Rightarrow \quad t = RC_e \ln 2$$

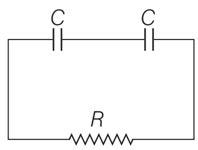
For parallel grouping,



$$C_e = 2C$$

$$\therefore \quad t_2 = 2\,RC\ln 2$$

For series grouping,



$$C_e = \frac{C}{2}$$

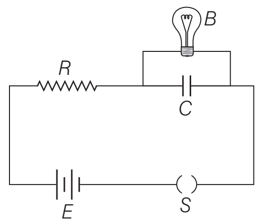
$$\therefore \quad t_1 = \frac{RC}{2}\ln 2$$

$$\therefore \quad \frac{t_2}{t_1} = \frac{1}{4}$$

$$\Rightarrow \quad t_2 = \frac{t_1}{4} = \frac{10}{4}$$

$$\Rightarrow \quad t_2 = 2.5\,\text{s}$$

20 Neon bulb is filled with gas, so its resistance is infinite, hence no current flows through it.



Now, $\quad V_c = E\left(1 - e^{-t/RC}\right)$

$$\Rightarrow \quad 120 = 200\left(1 - e^{-t/RC}\right)$$

$$\Rightarrow \quad e^{-t/RC} = \frac{2}{5} \Rightarrow \quad t = RC\ \ln 2.5$$

$$\Rightarrow \quad R = \frac{t}{C\ln 2.5}$$

$$= \frac{t}{2.303C\ \log 2.5}$$

$$= 2.7 \times 10^6\,\Omega$$

21 Apply shell theorem, the total charge upto distance r can be calculated as followed

$$dq = 4\pi r^2 \cdot dr \cdot \rho$$

$$= 4\pi r^2 \cdot dr \cdot \rho_0\left[\frac{5}{4} - \frac{r}{R}\right]$$

$$= 4\pi\rho_0\left[\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right]$$

$$\int dq = q = 4\pi\rho_0\int_0^r\left(\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right)$$

$$= 4\pi\rho_0\left[\frac{5}{4}\frac{r^3}{3} - \frac{1}{R}\frac{r^4}{4}\right]$$

$$E = \frac{kq}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2} \cdot 4\pi\rho_0\left[\frac{5}{4}\left(\frac{r^3}{3}\right) - \frac{r^4}{4R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

22 Energy stored in capacitor,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2C}(q_0 e^{-t/\tau})^2$$

$$= \frac{q_0^2}{2C}e^{-2t/\tau} \qquad \text{[where, } \tau = CR\text{]}$$

$$U = U_i e^{-2t/\tau}$$

$$\frac{1}{2}U_i = U_i e^{-2t_1/\tau}$$

$$\frac{1}{2} = e^{-2t_1/\tau}$$

$$\Rightarrow \quad t_1 = \frac{\tau}{2}\ln 2$$

Now, $\quad q = q_0 e^{-t/\tau}$

$$\frac{1}{4}q_0 = q_0 e^{-t_2/\tau}$$

$$t_2 = \tau\ln 4 = 2\tau\ln 2$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{1}{4}$$

23 Statement I is dimensionally wrong while from Gauss's law,

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0}$$

Given Statement II is correct.