Square Root and Cube Root

SQUARE

A number multiplied by itself is known as the *square* of the given number. For example, square of 6 is $6 \times 6 = 36$.

Square Root

Square root of a given number is that number which when multiplied by itself is equal to the given number.

For example, square root of 81 is 9 because $9^2 = 9 \times 9 = 81$.

The square root of a number is denoted by the symbol $\sqrt{}$ or $\sqrt{\ }$, called the *radical sign*.

Thus, $\sqrt{81} = 9$, $\sqrt{64} = 8$ and so on.

Note that $\sqrt{1} = 1$.

Methods of Finding the Square Root

I. Prime Factorization Method

1. Find the prime factors of the given number.
2. Group the factors in pairs.
3. Take one number from each pair of factors and then multiply them together.

   This product is the square root of the given number.

Illustration 1 Find the square root of

(i) 4761 (ii) 207025

Solution: (i) $4761 = \underbrace{23 \times 23} \times \underbrace{3 \times 3}$

$\therefore \quad \sqrt{4761} = 23 \times 3 = 69$

(ii) $207025 = \underbrace{5 \times 5} \times \underbrace{7 \times 7} \times \underbrace{13 \times 13}$

$\therefore \quad \sqrt{207025} = 5 \times 7 \times 13 = 455$

Note:

The above method is used when the given number is a perfect square or when every prime factor of that number is repeated twice.

II. Method of Division

This method is used when the number is large and the factors cannot be easily determined.

The working rule is explained with the help of following example:

Step 1 The digits of a number, whose square root is required, are separated into periods of two beginning from the right. The last period may be either single digit or a pair.

```
          476
      ┌──────────
  8   │ 22  65  76
 87   │ 16
      │  665
      │  609
      │ ------
946   │  5676
      │  5676
      │ ------
      │    ×
```

Step 2 Find a number (here, 4) whose square may be equal to or less than the first period (here, 22).

Step 3 Find out the remainder (here, 6) and bring down the next period (here, 65).

Step 4 Double the quotient (here, 4) and write to the left (here, 8).

Step 5 The divisor of this stage will be equal to the above sum (here, 8) with the quotient of this stage (here, 7) suffixed to it (here, 87).

Step 6 Repeat this process (step 4 and step 5) till all the periods get exhausted.

The quotient (here, 476) is equal to the square root of the given number (here, 226576).

Illustration 2 Find the square root of

(i) 180625 (ii) 1498176

Solution: (i)

```
          425
      ┌──────────
    8 | 18  06  25
   82 | 16
      |  206
      |  164
  845 |   4225
      |   4225
      |   ×
```

Thus, $\sqrt{180625} = 425$

(ii)

```
          1224
      ┌──────────
    1 | 1  49  81  76
      |  1
   22 |    49
      |    44
  242 |     581
      |     484
 2444 |      9776
      |      9776
      |      ×
```

Thus, $\sqrt{1478176} = 1224$

Square Root of a Decimal

If the given number is having decimal, we separate the digits of that number into periods of two to the right and left starting from the decimal point and then proceed as in the following illustration:

Illustration 3 Find the square root of

(i) 12.1801 (ii) 127.0129

(iii) 0.1790136 (iv) 0.000625

Solution: (i)

```
          3.49
      ┌──────────
    3 | 12.  18  01
      |  16
   64 |  665
      |  609
  689 |   6201
      |   6201
      |   ×
```

$\therefore \sqrt{12.1801} = 3.49$

(ii)

```
          11.27
      ┌──────────
    1 | 1  27.  01  29
      | 1  .
   21 |    27
      |    21
  222 |     601
      |     444
 2447 |      15729
      |      15729
      |      ×
```

$\therefore \sqrt{127.0129} = 11.27$

(iii) Since the number of decimal places is odd, we make it even by affixing one 0 to the right.

```
          0.423
      ┌──────────
    4 | 0.  17  90  13  60
      |     16
   82 |     190
      |     164
  843 |      2613
      |      2529
  846 |       8460
```

In the above solution, after bringing down the last period, we note that the remainder is not zero. So, a pair of zeros can be annexed and process can be continued to find the square root up to 4 places of decimals. The above process can be continued still further and square root up to required number of decimal places can be obtained.

Note:

If a decimal has an odd number of decimal places, its square root cannot be found exactly.

(iv)

```
          .025
      ┌──────────
    0 | 0.  00  06  25
      |     00
   02 |     06
      |     04
   45 |      225
      |      225
      |      ×
```

$\therefore \sqrt{0.000625} = 0.025$

Square Root of a Fraction

(*a*) *If the denominator is a perfect square*: The square root is found by taking the square root of the numerator and denominator separately.

(*b*) *If the denominator is not a perfect square*: The fraction is converted into decimal and then square root is obtained or the denominator is made perfect square by multiplying and dividing by a suitable number and then its square root is obtained.

Illustration 4 Find the square root of

(i) $\frac{2704}{49}$ (ii) $\frac{44}{25}$

(iii) $\frac{354}{43}$ (iv) $\frac{461}{32}$

Solution: (i) $\sqrt{\frac{2704}{49}} = \frac{\sqrt{2704}}{\sqrt{49}} = \frac{\sqrt{52\times52}}{\sqrt{7\times7}} = \frac{52}{7}$

$= 7\frac{3}{7}$

(ii) $\sqrt{\frac{44}{25}} = \frac{\sqrt{44}}{\sqrt{25}} = \frac{\sqrt{44}}{\sqrt{5\times5}} = \frac{\sqrt{44}}{5} = \frac{6.6332}{5}$

$= 1.3266$ (nearly)

(iii) $\sqrt{\frac{354}{43}} = \sqrt{8.2325} = 2.8692$ (nearly)

(iv) $\sqrt{\frac{461}{32}} = \sqrt{\frac{461\times2}{32\times2}} = \frac{\sqrt{922}}{\sqrt{64}} = \frac{30.3644}{8}$

$= 3.7955$ (nearly)

Cube

Cube of a number is obtained by multiplying the number itself thrice.

For example, 27 is the cube of 3 as $27 = 3\times3\times3$.

Cube Root

Cube root of a given number is that number which when raised to the third power produces the given number, that is the cube root of a number x is the number whose cube is x.

The cube root of x is written as $\sqrt[3]{x}$.

For example, cube root of 64 is 4 as $4\times4\times4 = 64$.

Methods to Find Cube Root

I. Method of Factorization

1. Write the given number as product of prime factors.
2. Take the product of prime numbers, choosing one out of three of each type.

This product gives the cube root of the given number.

Illustration 5 Find the cube root of 42875.

Solution: Resolving 42875 into prime factors, we get

$42875 = \underbrace{5\times5} \times \underbrace{5\times7} \times \underbrace{7\times7}$

$\therefore \quad \sqrt[3]{42875} = 5\times7 = 35$

II. Short-cut Method to Find Cube Roots of Exact Cubes Consisting of up to 6 Digits:

Before we discuss the method to find the cube roots of exact cubes, the following two remarks are very useful and must be remembered by heart.

1. $1^3 = 1$; $2^3 = 9$; $3^3 = 27$; $4^3 = 64$; $5^3 = 125$; $6^3 = 216$; $7^3 = 343$; $8^3 = 512$;
   $93 = 729$; $103 = 1000$.
2. If the cube ends in 1, then its cube root ends in 1
   If the cube ends in 2, then its cube root ends in 8
   If the cube ends in 3, then its cube root ends in 7
   If the cube ends in 4, then its cube root ends in 4
   If the cube ends in 5, then its cube root ends in 5
   If the cube ends in 6, then its cube root ends in 6
   If the cube ends in 7, then its cube root ends in 3
   If the cube ends in 8, then its cube root ends in 2
   If the cube ends in 9, then its cube root ends in 9
   If the cube ends in 0, then its cube root ends in 0

Clearly from above

$1 \leftrightarrow 1$, $4 \leftrightarrow 4$, $5 \leftrightarrow 5$, $6 \leftrightarrow 6$, $9 \leftrightarrow 9$, $0 \leftrightarrow 0$

$2 \leftrightarrow 8$, $3 \leftrightarrow 7$.

The method of finding the cube root of a number up to 6 digits which is actually a cube of some number consisting of 2 digits is best illustrated with the help of following examples:

Illustration 6 Find the cube roots of the following:

(i) 2744 (ii) 9261

(iii) 19684 (iv) 54872

(v) 614125

Solution: (i) Make groups of 3 digits from the right side.

2 744

2 lies between 1^3 and 2^3, so left digit is 1

744 ends in 4, so right digit is 4

Thus, cube root of 2744 is 14

(ii) 9 261

9 lies between 2^3 and 3^3, so left digit is 2

261 ends in 1, so right digit is 1

Thus, cube root of 9261 is 21

(iii) 19 683

19 lies between 2^3 and 3^3, so left digit is 2

683 ends in 3, so right digit is 7

Thus, cube root of 19683 is 27

(iv) 54 872

54 lies between 3^3 and 4^3, so left digit is 3

872 ends in 2, so right digit is 8

Thus, cube root of 19683 is 38

(iv) 614 125

614 lies between 8^3 and 9^3, so left digit is 8

125 ends in 5, so right digit is 5

Thus, cube root of 614125 is 85

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. Taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{6} = 2.449$, then the value of $\frac{9+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{6-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ to three places of decimals is:

(*a*) 9.231 (*b*) 13.716

(*c*) 11.723 (*d*) 15.892

[Based on MAT, 2002]

2. If $n^2 = 12345678987654321$, what is n?

(*a*) 12344321 (*b*) 1235789

(*c*) 111111111 (*d*) 11111111

3. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

(*a*) 32 (*b*) 34

(*c*) 35 (*d*) 36

[Based on MAT, 2005]

4. The smallest number by which 3600 must be multiplied to make it a perfect cube is:

(*a*) 40 (*b*) 60

(*c*) 20 (*d*) 15

5. By what least number, 2450 be multiplied, so that the resulting number is perfect square?

(*a*) 8 (*b*) 10

(*c*) 5 (*d*) 2

6. The largest number of five digits which is a perfect square is:

(*a*) 97344 (*b*) 98596

(*c*) 99856 (*d*) None of these

7. The smallest number which when subtracted from the number 62512 makes it a perfect square is:

(*a*) 22 (*b*) 32

(*c*) 12 (*d*) 2

8. What least number should be subtracted from the square root of $21\frac{15}{289}$ so that the result is a whole number?

(*a*) 15/289 (*b*) 7/17

(*c*) 10/17 (*d*) 5/17

9. A general wishing to draw up his 16160 men in the form of a solid square found that he had 31 men over. The number of men in the front row is:

(*a*) 127 (*b*) 123

(*c*) 137 (*d*) 129

10. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ is:

(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$

(*c*) $\frac{3}{5}$ (*d*) $\frac{5}{3}$

11. $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{3}}}}}} = ?$

(*a*) 331/64 (*b*) 331/32

(*c*) 31/64 (*d*) None of these

12. A general wishing to draw up his 16160 men in the form of a solid square, found that he had 31 men left over. Find the number of men in the front.

(*a*) 126 (*b*) 125

(*c*) 128 (*d*) 127

13. A man plants his orchard with 5625 trees and arranges them so that there are as many rows as there are trees in a row. How many rows are there?

(*a*) 125 (*b*) 25

(*c*) 75 (*d*) None of these

14. Find the number whose square is equal to the difference between the squares of 75.15 and 60.12.

(*a*) 225.9 (*b*) 67.635

(*c*) 45.09 (*d*) 15.03

15. If $\sqrt{1+\sqrt{1-\frac{2176}{2401}}} = 1+\frac{x}{7}$, value of x is:

(*a*) 3 (*b*) 1

(*c*) 5 (*d*) 7

16. The areas of two square fields are 420.25 m^2 and 441 m^2 respectively. The ratio of their sides is:

(*a*) 20:21 (*b*) 40:41

(*c*) 41:42 (*d*) 40:42

17. If $\sqrt{12} = 3.464$, value of $\sqrt{\frac{3}{4}} + 2\sqrt{\frac{4}{3}}$ is:

(*a*) 3.17 (*b*) 3.464

(*c*) 3.1753 (*d*) None of these

18. What will come in place of the question mark (?) in the following equation:

$$25^{7.5} \times 5^{2.5} \div 125^{1.5} = 5\ ?$$

(*a*) 16 (*b*) 17.5

(*c*) 8.5 (*d*) 13

[Based on PNB Management Trainee Exam, 2003]

19. The least number by which 14175 be divided to make it a perfect square is:

(*a*) 3 (*b*) 5

(*c*) 7 (*d*) 15

20. Multiply the difference between the two lowest numbers with the difference between the two highest numbers in the following sequences:

89, 7, 91, 72, 31, 25, 18, 89, 16, 58, 38, 42, 86

(*a*) 18 (*b*) 77

(*c*) 81 (*d*) 16

[Based on NABARD, 1999]

21. One-fifth of a number is equal to five-eighths of the second number. If 35 is added to the first number it becomes four times of second number. What is the value of the second number?

(*a*) 125 (*b*) 70

(*c*) 40 (*d*) 25

[Based on NABARD, 1999]

22. If $\sqrt{x} + \sqrt{49} = 8.2$, then the value of x is equal to:

(*a*) 1.20 (*b*) 1.40

(*c*) 1.44 (*d*) 1.89

[Based on MAT, 1998]

23. The value of $\left(\frac{-1}{216}\right)^{-\frac{2}{3}}$ is:

(*a*) $\frac{1}{36}$ (*b*) $-\frac{1}{36}$

(*c*) -36 (*d*) 36

[Based on MAT, 1998]

24. If $\sqrt{5} = 2.236$, then the value of $\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125}$ is equal to:

(*a*) 7.826 (*b*) 8.944

(*c*) 5.59 (*d*) 10.062

[Based on MAT, 1998]

25. If $a = \left(\frac{1}{10}\right)^2$, $b = \frac{1}{5}$ and $c = \sqrt{\frac{1}{100}}$, then which of the following statements is correct?

(*a*) $a < b < c$ (*b*) $a < c < b$

(*c*) $b < c < a$ (*d*) $c < a < b$

[Based on MAT, 1998]

26. Which is the smallest of the following numbers?

(*a*) $\sqrt{7}$ (*b*) $1/\sqrt{7}$

(*c*) $\sqrt{7}/7$ (*d*) 1/7

[Based on MAT, 1999]

27. The value of $3\sqrt{\sqrt{0.000064}}$ is:

(*a*) 0.02 (*b*) 0.2

(*c*) 2.0 (*d*) None

[Based on MAT, 1999]

28. Square root of 117649 is:

(*a*) 347 (*b*) 343

(*c*) 353 (*d*) 357

[Based on MAT, 2000]

29. Cube root of 658503 is:

(*a*) 83 (*b*) 77

(*c*) 87 (*d*) 97

[Based on MAT, 2000]

30. $\sqrt{110.25} \times \sqrt{0.01} \div \sqrt{0.0025} - \sqrt{420.25}$ equals to:

(*a*) 0.75 (*b*) 0.50

(*c*) 0.64 (*d*) 0.73

[Based on SNAP, 2010]

31. In a class of 40 students and 5 teachers, each student got sweets that are 25 per cent of the total number of students and each teacher got sweets that are 15 per cent of the total number of students. How many sweets were there?

(*a*) 480 (*b*) 440

(*c*) 430 (*d*) 450

[Based on IRMA, 2008]

32. $\sqrt{915849} + \sqrt{795664} = (?)^2$:

(*a*) 1849 (*b*) 79

(*c*) 33 (*d*) None of these

[Based on IRMA, 2009]

33. $\sqrt{956240} = ?$

(*a*) 979 (*b*) 864

(*c*) 1009 (*d*) 647

[Based on IRMA, 2009]

34. Simplify $\frac{\sqrt{5}\ \ \sqrt{3}}{\sqrt{5}\ \ \sqrt{3}}$:

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) None of these

[Based on NMAT, 2006]

35. If $x = 0.888$, $y = \sqrt{0.888}$ and $z = (0.888)^2$, then which of the following is true?

(*a*) $y < x < z$

(*b*) $y < z < x$

(*c*) $x < z < y$

(*d*) $z < x < y$

[Based on ATMA, 2005]

36. The positive integer n is divisible by 25. If $\sqrt{n}$ is greater than 25, which of the following could be the value of $\frac{n}{25}$?

(*a*) 26 (*b*) 23

(*c*) 25 (*d*) 22

[Based on ATMA, 2008]

37. If $x = 3 + 2\sqrt{2}$, then the value of $(x^{1/2} - x^{-1/2})$ is:

(*a*) 2 (*b*) $\sqrt{2}$

(*c*) $\sqrt{2} - 1$ (*d*) $\sqrt{2} + 1$

[Based on ATMA, 2008]

38. $(32)^2 + \sqrt{?} - (23)^2 = 536$:

(*a*) 1764 (*b*) 1849

(*c*) 1521 (*d*) 1681

[Based on NMAT, 2008]

39. $\sqrt{1562000} = ?$

(*a*) 1175 (*b*) 1200

(*c*) 1250 (*d*) 1325

[Based on NMAT, 2008]

40. The value of $\sqrt[3]{1.001001001} - \sqrt[3]{1.001001}$ is closest to:

(*a*) 0 (*b*) 10^{-6}

(*c*) 3.10^{-9} (*d*) $(3.3).10^{-10}$

[Based on GBO Delhi University, 2011]

41. If the square root of a number is between 6 and 7, then the cube root of the number will be between:

(*a*) 1 and 2 (*b*) 2 and 3

(*c*) 3 and 4 (*d*) 4 and 5

[Based on GBO Delhi University, 2011]

42. V is inversely proportional to the square root of m and m is inversely proportional to the square of t. The relationship between V and t is:

(*a*) $V \propto t^2$ (*b*) $V \propto t$

(*c*) $V \propto \frac{1}{t}$ (*d*) $V \propto \sqrt{t}$

[Based on GBO Delhi University, 2011]

DIFFICULTY LEVEL-2 (BASED ON MEMORY)

1. Let n (> 1) be a composite integer such that $\sqrt{n}$ is not an integer. Consider the following statements:

   A: n has a perfect integer–valued divisor which is greater than 1 and less than $\sqrt{n}$

   B: n has a perfect integer–valued divisor which is greater than $\sqrt{n}$ but less than n

   (*a*) Both *A* and *B* are false

   (*b*) *A* is true but *B* is false

   (*c*) *A* is false but *B* is true

   (*d*) Both *A* and *B* are true

2. An operation '$' is defined as follows:
   For any two positive integers x and y,
   $x\$y = \sqrt{\left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}\right)}$ then which of the following is an integer?

   (*a*) 4$9 (*b*) 4$16

   (*c*) 4$1 (*d*) None of the above

3. A certain number of people agree to subscribe as many rupees each as there are subscribers. The whole subscription is ₹2582449. Find the number of subscribers.

   (*a*) 1607 (*b*) 1291225

   (*c*) 1503 (*d*) 1603

4. Which of the following numbers, wherein some of the digits have been suppressed by symbols, can possibly be the perfect square of a three-digit odd number?

(*a*) 65 ××× 1

(*b*) 9 ×× 1

(*c*) 10 ××× 4

(*d*) 9 ×××××× 5

5. Three cubes of iron whose edges are 6 cm, 8 cm and 10 cm, respectively, are melted and formed into a single cube. The edge of the new cube formed is:

(*a*) 12 cm (*b*) 14 cm

(*c*) 16 cm (*d*) 18 cm

[Based on FMS (MS), 2006]

6. If $\sqrt{841} = 29$, then the value of:

$$\left(\sqrt{841} + \sqrt{8.41 + \sqrt{0.0841 + \sqrt{0.000841}}}\right) \text{ is}$$

(*a*) 322.19 (*b*) 32.219

(*c*) 34.179 (*d*) 31.129

[Based on FMS (MS), 2006]

7. If $\sqrt{(0.05 \times 0.5 \times a)} = 0.5 \times 0.05 \times \sqrt{b}$, then $\frac{a}{b}$ is equal to:

(*a*) 0.0025 (*b*) 0.025

(*c*) 0.25 (*d*) None of these

[Based on FMS (MS), 2006]

8. If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$, than $\frac{a}{b}$ is :

(*a*) 0.016 (*b*) 0.16

(*c*) 1 (*d*) 16

[Based on FMS, 2005]

9. A gardener plants 17956 trees in such a way that there are as many rows as there are trees in a row. The number of trees in a row are:

(*a*) 136 (*b*) 134

(*c*) 144 (*d*) 154

[Based on FMS, 2006]

10. $[1/(\sqrt{9} - \sqrt{8}) - 1/(\sqrt{8} - \sqrt{7}) + 1/(\sqrt{7} - \sqrt{6})$

$-1/(\sqrt{6} - \sqrt{5}) + 1/(\sqrt{5} - \sqrt{4})] = ?$

(*a*) 0 (*b*) 1

(*c*) 5 (*d*) 1/3

[Based on FMS, 2006]

11. The number $\sqrt{8} + 3\sqrt{7}$ is equal to:

(*a*) $\left(\frac{8+\sqrt{7}}{\sqrt{2}}\right)$ (*b*) $8 - 3\sqrt{7}$

(*c*) $2\sqrt{2} + 3\sqrt{7}$ (*d*) $\left(\frac{8-\sqrt{7}}{\sqrt{2}}\right)$

[Based on FMS, 2009]

12. The expression $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}}$ is equal to:

(*a*) $\frac{\sqrt{3}}{6}$ (*b*) $-\frac{\sqrt{3}}{6}$

(*c*) $\frac{\sqrt{-3}}{6}$ (*d*) $\frac{5\sqrt{3}}{6}$

[Based on FMS, 2010]

13. $2 - \frac{\sqrt{6407522209}}{\sqrt{3600840049}}$ = is equal to:

(*a*) 0.666039 (*b*) 0.666029

(*c*) 0.666009 (*d*) None of these

[Based on IIFT, 2008]

14. If $n^2 = 12345678987654321$, what is n?

(a) 12344321

(b) 1235789

(c) 111111111

(d) 11111111

[Based on CAT, 1999]

15. Find the sum:

$$\sqrt{1 + \frac{1}{1^2} + \frac{1}{1^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \cdots + \sqrt{1 + \frac{1}{2007^2} + \frac{1}{2008^2}}$$

(a) $2008 - \frac{1}{2008}$ (b) $2007 - \frac{1}{2007}$

(c) $2007 - \frac{1}{2008}$ (d) $2008 - \frac{1}{2007}$

(e) $2008 - \frac{1}{2009}$

[Based on CAT, 2008]

Answer Keys

Difficulty Level-1

1. (*c*)	**2.** (*d*)	**3.** (*d*)	**4.** (*b*)	**5.** (*d*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*b*)	**11.** (*b*)	**12.** (*d*)	**13.** (*c*)
14. (*c*)	**15.** (*b*)	**16.** (*c*)	**17.** (*c*)	**18.** (*d*)	**19.** (*c*)	**20.** (*a*)	**21.** (*c*)	**22.** (*c*)	**23.** (*d*)	**24.** (*a*)	**25.** (*b*)	**26.** (*d*)
27. (*b*)	**28.** (*b*)	**29.** (*c*)	**30.** (*b*)	**31.** (*c*)	**32.** (*d*)	**33.** (*a*)	**34.** (*d*)	**35.** (*d*)	**36.** (*a*)	**37.** (*a*)	**38.** (*d*)	**39.** (*c*)
40. (*a*)	**41.** (*c*)	**42.** (*b*)										

Difficulty Level-2

1. (*d*)	**2.** (*d*)	**3.** (*a*)	**4.** (*a*)	**5.** (*a*)	**6.** (*b*)	**7.** (*b*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)	**11.** (*c*)	**12.** (*a*)	**13.** (*a*)
14. (*d*)	**15.** (*a*)											

Explanatory Answers

Difficulty Level-1

1. (*c*) $\dfrac{9+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{6-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$=\frac{9\sqrt{5}-9\sqrt{3}+\sqrt{10}-\sqrt{6}+6\sqrt{5}+6\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$=\frac{15\sqrt{5}-3\sqrt{3}-2\sqrt{6}}{2}$$

$$=\frac{15\times 2.236-3\times 1.732-2\times 2.449}{2}$$

$$=\frac{33.54-5.196-4.898}{2}=11.723.$$

2. (*d*) Square root of 12345678987654321 is 1111111.

3. (*d*) No. of camels seen in the forest $=\dfrac{36}{4}=9$

No. of camels gone to the mountains $=2\sqrt{36}=12$

No. of camels seen on the bank of river = 15

$\therefore$ Total number of camels = 9 + 12 + 15 = 36.

4. (*b*) $3600=2\times2\times2\times2\times3\times3\times5\times5$

$=22\times22\times32\times52$

$\therefore$ 3600 should be multiplied by $2\times2\times3\times5$. That is 60 to make it a perfect cube.

5. (*d*) $2450=5\times5\times7\times7\times2$

$\therefore$ 2450 must be multiplied by 2.

6. (*c*) Largest number of 5 digits = 99999

$$\begin{array}{c|l} & 316 \\ \hline 3 & 9\,\overline{99}\;\;\overline{99} \\ & 9 \\ \hline 61 & 99 \\ & 61 \\ \hline 625 & 3899 \\ & 3756 \\ \hline & 143 \end{array}$$

Required number = $(316)^2 = 99856$.

7. (*c*)

$$\begin{array}{c|l} & 250 \\ \hline 2 & \overline{62}\,5\,\overline{12} \\ & 4 \\ \hline 45 & 225 \\ & 225 \\ \hline 50 & 12 \end{array}$$

So, 12 is the smallest number which when subtracted from 62512 makes it a perfect square

$\sqrt{62500}=250.$

8. (*c*) $21\dfrac{15}{289}=\dfrac{6084}{289}=\left(\dfrac{78}{17}\right)^2$

$\therefore$ Square root $=\dfrac{78}{17}=4\dfrac{10}{17}$

$\therefore$ Least fraction to be subtracted $=\dfrac{10}{17}$.

9. (*a*) The number of men in the front row is the square root of 16160 – 31, that is 16129 which is 127.

10. (*b*) $a = \dfrac{\sqrt{5}+1}{\sqrt{5}-1} \times \dfrac{\sqrt{5}+1}{\sqrt{5}+1} = \dfrac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-(1)^2}$

$= \dfrac{5+1+2\sqrt{5}}{5-1}$

$= \dfrac{6+2\sqrt{5}}{4} = \dfrac{3+\sqrt{5}}{2}$

$b = \dfrac{\sqrt{5}-1}{\sqrt{5}+1} \times \dfrac{\sqrt{5}-1}{\sqrt{5}-1} = \dfrac{(\sqrt{5}-1)^2}{(\sqrt{5})^2-(1)^2}$

$= \dfrac{5+1-2\sqrt{5}}{5-1} = \dfrac{6+2\sqrt{5}}{4} = \dfrac{3-\sqrt{5}}{2}$

$a^2+b^2 = \dfrac{(3+\sqrt{5})^2+(3-\sqrt{5})^2}{4}$

$= \dfrac{9+5+6\sqrt{5}+9+5-6\sqrt{5}}{5} = \dfrac{28}{4} = 7$

$ab = 1$

$\therefore \dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{7+1}{7-1} = \dfrac{8}{6} = \dfrac{4}{3}.$

11. (*b*) $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{3.3^{1/2}}}}} = \sqrt{\sqrt[3]{\sqrt[3]{3.3^{3/4}}}} = \sqrt{\sqrt[3]{3.3^{7/8}}}$

$= \sqrt{3.3^{15/16}} = 331/32.$

12. (*d*) $16160 - 31 = 16129$; $\sqrt{16129} = 127.$

13. (*c*) Let the number of trees be n

$\therefore \quad n^2 = 5625$

$\Rightarrow \quad n = \sqrt{5625} = 75.$

14. (*c*) $x^2 = (75.15)^2 - (60.12)^2$

$= 5647.52 - 3614.41$

$= 2033.11$

$\therefore \quad x = 45.09.$

15. (*b*) $\sqrt{1+\sqrt{1-\dfrac{2176}{2401}}} = 1 + \dfrac{x}{7}$

$\Rightarrow \quad 1 + \dfrac{x}{7} = \sqrt{1+\sqrt{\dfrac{225}{2401}}} = \sqrt{1+\dfrac{15}{49}}$

$= \sqrt{\dfrac{64}{49}} = \dfrac{8}{7} = 1 + \dfrac{1}{7}$

$\therefore \quad x = 1.$

16. (*c*) Ratio of their sides is the ratio of their square roots

$= \sqrt{420.25} : \sqrt{441}$

$= 20.5:21$

$= 41:42.$

17. (*c*) $\sqrt{\dfrac{3}{4}} + 2\sqrt{\dfrac{4}{3}} = \sqrt{12}\left(\dfrac{1}{4}+\dfrac{2}{3}\right)$

$= \dfrac{3.464 \times 11}{12} = 3.1753$

18. (*d*) $25^{7.5} \times 5^{2.5} \div 125^{1.5} = 5?$

or, $5^{2\times7.5} \times 5^{2.5} \div 5^{3\times1.5} = 5?$

or, $5^{15} \times 5^{2.5} \div 5^{4.5} = 5?$

or, $5^{17.5} \times \dfrac{1}{5^{4.5}} = 5?$ or, $5^{13} = 5?$

or, $? = 13.$

19. (*c*) $14175 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 7$

$= 52 \times 34 \times 7$

It must be multiplied by 7.

20. (*a*) Difference between two lowest numbers

$= 16 - 7 = 9$

Difference between two highest numbers

$= 91 - 89 = 2$

$\therefore$ Product of these two numbers $= 9 \times 2 = 18.$

21. (*c*) $\dfrac{1}{5}\ I = \dfrac{5}{8}\ II$

$\therefore \quad \dfrac{I}{II} = \dfrac{25}{8}$

$I + 35 = 4\ II$

or, $\dfrac{25}{8}\ II + 35 = 4\ II$

$\therefore \quad II = 40.$

22. (*c*) $\sqrt{x} = 8.2 - 7 = 1.2 \Rightarrow x = 1.44.$

23. (*d*) $\left(\dfrac{-1}{216}\right)^{-\frac{2}{3}} = \left(\dfrac{-1}{6^3}\right)^{-\frac{2}{3}} = \left(-\dfrac{1}{6}\right)^{-2}$

$= \dfrac{1}{\left(-\dfrac{1}{6}\right)^2} = \dfrac{1}{\dfrac{1}{36}} = 36.$

24. (*a*) $\dfrac{\sqrt{5}}{2} - \dfrac{10}{\sqrt{5}} + \sqrt{125}$

$= \dfrac{5 - 20 + 2\sqrt{5}\sqrt{125}}{2\sqrt{5}} = \dfrac{5 - 20 + 60}{2\sqrt{5}}$

$= \dfrac{35 \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}}$

$= \dfrac{7}{2} \times \sqrt{5} = \dfrac{7}{2} \times 2.236 = 7 \times 1.118 = 7.826.$

25. (*b*) $a = \left(\frac{1}{10}\right)^2 = \frac{1}{100} = 0.1$

$b = \frac{1}{5} = .2$

$c = \sqrt{\frac{1}{100}} = \sqrt{.01} = .1$

$\therefore \quad a < c < b.$

26. (*d*)

27. (*b*) Given expression $= 3\sqrt{0.008} = 0.2.$

28. (*b*) $117649 = 7 \times 7 \times 7 \times 7 \times 7 \times 7$

$\therefore \quad \sqrt{117649} = 7 \times 7 \times 7 = 343.$

29. (*c*) $658503 = 3 \times 3 \times 3 \times 24389$

$= 3 \times 3 \times 3 \times 29 \times 29 \times 29$

$\therefore \quad \sqrt[3]{658503} = 3 \times 29 = 87.$

30. (*b*) $\sqrt{110.25} = 10.5$

$\sqrt{0.01} = 0.1$

$\sqrt{0.0025} = 0.05$

$\sqrt{420.25} = 20.5$

$\therefore \quad \sqrt{110.25} \times \sqrt{0.01} \div \sqrt{0.0025} \times \sqrt{420.25}$

$= 10.5 \times \frac{0.1}{0.05} - 20.5$

$= \frac{1.05}{0.05} - 20.5$

$= 21 - 20.5 = 0.5$

Hence, option (*b*).

31. (c) $40 \times \frac{40 \times 25}{100} + 5 \times \frac{40 \times 15}{100} = 400 + 30 = 430.$

32. (*d*) It will be square of 43.

33. (*a*)

34. (*d*) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{8-2\sqrt{15}}{2}$

$= 4 - \sqrt{15}.$

35. (*d*) $x = 0.888,$

$y = \sqrt{0.888} = 0.942$

$z = (0.888)^2 = 0.78$

$\therefore \quad z < x < y.$

36. (*a*) Since, $\sqrt{n} > 25$ and also n is divisible by 25 then the number could be 650

$\therefore \quad \frac{n}{25} = \frac{650}{25} = 26.$

37. (*a*) $x = 3 + 2\sqrt{2} = (1 + \sqrt{2})^2$

Now, taking $(x^{1/2} - x^{-1/2})$

$= (1+\sqrt{2}) - \left(\frac{1}{1+\sqrt{2}}\right)$

$= (1+\sqrt{2}) - \left(\frac{\sqrt{2}-1}{2-1}\right)$

$= 1 + \sqrt{2} - \sqrt{2} + 1$

$= 2.$

38. (*d*)

39. (*c*) $1250 \times 1250 = 1562500$ (1562000 approx.)

40. (*a*)

41. (*c*)

42. (*b*) $V = \frac{a}{\sqrt{m}}, m = \frac{b}{t^2},$

where a and b are constants

$V = \frac{a}{\sqrt{m}} = \frac{a}{\sqrt{\frac{b}{c^2}}} = \frac{a}{\sqrt{b}} \times t$

$\Rightarrow \quad V \propto t.$

Difficulty Level-2

1. (*d*) Consider a number $n = 6$

$\sqrt{n} = 245$

A: We have a divisor 2 which is greater than 1 and less than $\sqrt{6}$.

B: We have a divisor 3 which is greater than but less than 6.

Thus, both statements are true.

2. (*d*) By direct substitution.

3. (*a*) Let the number of subscribers be n

$\Rightarrow \quad n^2 = 258249$

$\therefore \quad n = \sqrt{2582449} = 1607.$

4. (*a*) The square of an odd number cannot have 4 as the unit digit. The square of a 3-digit number will have at least 5 digits and at the most 6 digits.

5. (*a*) $6^3 + 8^3 + 10^3 = \text{New cube}$

$216 + 512 + 1000 = \text{New cube}$

$\Rightarrow \quad \sqrt[3]{1728} = 12 \text{ cm}.$

6. (*b*) $29 + \frac{29}{10} + \frac{29}{100} + \frac{29}{1000}$

$= 29 + 2.9 + 0.29 + 0.029$

$= 32.219.$

7. (*b*) On squaring both the sides, we get

$0.05 \times 0.5 \times a = (0.5)^2 \times (0.05)^2 \times b$

$\Rightarrow \quad \frac{a}{b} = 0.025.$

8. (*a*) $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$

On squaring both the sides, we get

$0.04 \times 0.4 \times a = (0.4)^2 \times (0.04)^2 \times b$

$\therefore \quad \frac{a}{b} = 0.016.$

9. (*b*) $\sqrt{17956} = 134.$

10. (*c*) $\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}}$

$= (\sqrt{9}-\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+\sqrt{4})$

$= (\sqrt{9}+\sqrt{4}) = 3 + 2 = 5.$

11. (*c*) $\sqrt{8} + 3\sqrt{7} = 2\sqrt{2} + 3\sqrt{7}.$

12. (*a*) $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = \frac{\sqrt{4} \times \sqrt{4} - \sqrt{3} \times \sqrt{3}}{\sqrt{3} \times \sqrt{4}}$

$= \frac{4-3}{\sqrt{12}} = \frac{1}{\sqrt{12}}$

$= \frac{\sqrt{12}}{12} = \frac{2\sqrt{3}}{12}$

$= \frac{\sqrt{3}}{6}.$

13. (*a*) $2 - \frac{\sqrt{6407522209}}{\sqrt{3600840049}} = 2 - \frac{80047}{60007}$

$= 2 - 1.3339610$

$= 0.666039.$

14. (*d*) Square root of 12345678987654321 is 11111111.

15. (*a*) First term is

$\sqrt{1+1+\frac{1}{4}} = \frac{3}{2} = 2 - \frac{1}{2}$

Sum of first two terms is

$\frac{3}{2} + \sqrt{1+\frac{1}{4}+\frac{1}{9}}$

$= \frac{3}{2} + \sqrt{\frac{36+9+4}{36}} = \frac{3}{2} + \frac{7}{6} = \frac{16}{6} = \frac{8}{3} = 3 - \frac{1}{3}$

Sum of first three terms is

$= \frac{8}{3}\sqrt{\frac{169}{144}} = \frac{8}{3} + \frac{13}{12} = \frac{45}{12} = \frac{15}{4} = 4 - \frac{1}{4}$

Similarly, sum of the given terms is $2008 - \frac{1}{2008}.$