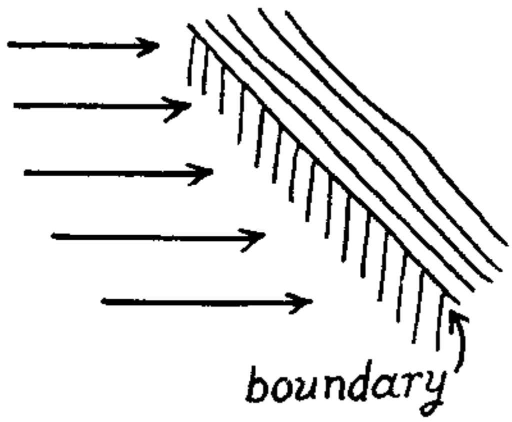
5.4 POLARIZATION OF LIGHT

5.157 Natural light can be considered to be an incoherent mixture of two plane polarized light of intensity $I_0/2$ with mutually perpendicular planes of vibration. The screen consisting of the two polaroid half-planes acts as an opaque half-screen for one or the other of these light waves. The resulting diffraction pattern has the alterations in intensity (in the illuminated region) characteristic of a straight edge on both sides of the boundary.

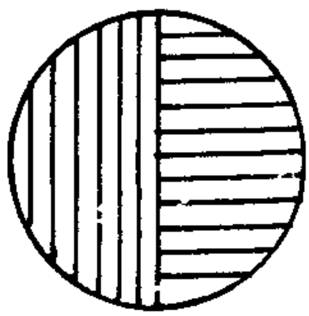


At the boundary the intensity due to either component is

$$\frac{(I_0/2)}{4}$$

and the total intensity is $\frac{I_0}{4}$. (Recall that when light of intensity I_0 is incident on a straight edge, the illuminance in front of the edge is $I_0/4$).

5.158 (a) Assume first that there is no polaroid and the amplitude due to the entire hole which extends over the first Fresnel zone is A_1



Then, we know, as usual, $I_0 = \frac{A_1^2}{4}$,

When the polaroid is introduced as shown above, each half transmits only the corresponding polarized light. If the full hole were covered by one polaroid the amplitude transmitted will be $(A_1/\sqrt{2})$.

Therefore the amplitude transmitted in the present case will be $\frac{A_1}{2\sqrt{2}}$ through either half.

Since these transmitted waves are polarized in mutually perpendicular planes, the total intensity will be

$$\left(\frac{A_1}{2\sqrt{2}}\right)^2 + \left(\frac{A_1}{2\sqrt{2}}\right)^2 = \frac{A_1^2}{4} = I_0.$$

(b) We interpret the problem to mean that the two polaroid pieces are separated along the circumference of the circle limiting the first half of the Fresnel zone. (This however is inconsistent with the polaroids being identical in shape; however no other interpretation makes sense.)

From (5.103) and the previous problems we see that the amplitudes of the waves transmitted through the two parts is

$$\frac{A_1}{2\sqrt{2}}(1+i) \text{ and } \frac{A_1}{2\sqrt{2}}(1-i)$$

and the intensity is

$$\left|\frac{A_1^2}{2\sqrt{2}}(1+i)\right|^2 + \left|\frac{A_1}{2\sqrt{2}}(1-i)\right|^2$$

$$= \frac{A_1^2}{2} = 2I_0$$

5.159 When the polarizer rotates with angular velocity ω its instantaneous principal direction makes angle ωt from a reference direction which we choose to be along the direction of vibration of the plane polarized incident light. The transmitted flux at this instant is

$$\Phi_0 \cos^2 \omega t$$

and the total energy passing through the polarizer per revolution is

$$\int_0^T \Phi_0 \cos^2 \omega t \, dt \, , \quad T = 2\pi/\omega$$

$$= \Phi_0 \frac{\pi}{\omega} = 0{\cdot}6 \text{ mJ} .$$

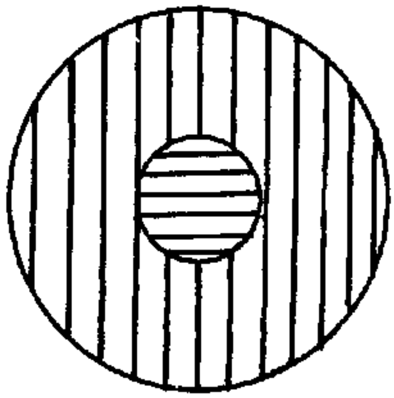
5.160 Let I_0 = intensity of the incident beam.

Then the intensity of the beam transmitted through the first Nicol prism is

$$I_1 = \frac{1}{2} I_0$$

and through the 2^{nd} prism is

$$I_2 = \left(\frac{1}{2} I_0\right) \cos^2 \varphi$$



Through the N^{th} prism it will be

$$I_N = I_{N-1} \cos^2 \varphi$$

$$= \frac{1}{2} I_0 \cos^{2(N-1)} \varphi$$

Hence fraction transmitted

$$= \frac{I_N}{I_0} = \eta = \frac{1}{2} \cos^{(2N-1)} \varphi = 0{\cdot}12 \text{ for } N = 6 .$$

and $\varphi = 30°$

5.161 When natural light is incident on the first polaroid, the fraction transmitted will be $\frac{1}{2}\tau$ (only the component polarized parallel to the principal direction of the polaroid will go).

The emergent light will be plane polarized and on passing through the second polaroid will be polarized in a different direction (corresponding to the principal direction of the 2nd polaroid) and the intensity will have decreased further by $\tau \cos^2 \varphi$.

In the third polaroid the direction of polarization will again have to change by φ thus only a fraction $\tau \cos^2 \varphi$ will go through.

Finally $$I = I_0 \times \frac{1}{2} \tau^3 \cos^4 \varphi$$

Thus the intensity will have decreased

$$\frac{I_o}{I} = \frac{2}{\tau^3 \cos^4 \varphi} = 60{\cdot}2 \text{ times}$$

for $$\tau = 0{\cdot}81, \ \varphi = 60°.$$

5.162 Suppose the partially polarized light consists of natural light of intensity I_1 and plane polarized light of intensity I_2 with direction of vibration parallel to, say, x - axis.

Then when a polaroid is used to transmit it, the light transmitted will have a maximum intensity

$$\frac{1}{2} I_1 + I_2,$$

when the principal direction of the polaroid is parallel to x - axis, and will have a minimum intensity $\frac{1}{2} I_1$ when the principal direction is $\perp^r$ to x - axis.

Thus $$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{I_2}{I_1 + I_2}$$

so $$\frac{I_2}{I_1} = \frac{P}{1-P} = \frac{0{\cdot}25}{0{\cdot}75} = \frac{1}{3}.$$

5.163 If, as above,

I_1 = intensity of natural component

I_2 = intensity of plane polarized component

then $$I_{max} = \frac{1}{2} I_1 + I_2$$

and $$I = \frac{I_{max}}{\eta} = \frac{1}{2} I_1 + I_2 \cos^2 \varphi$$

so $$I_2 = I_{max} \left(1 - \frac{1}{\eta} \right) \operatorname{cosec}^2 \varphi$$

$$I_1 = 2 I_{max} \left[1 - \left(1 - \frac{1}{\eta} \right) \operatorname{cosec}^2 \varphi \right] = \frac{2 I_{max}}{\sin^2 \varphi} \left[\frac{1}{\eta} - \cos^2 \varphi \right]$$

Then $$P = \frac{I_2}{I_1 + I_2} = \frac{1 - \frac{1}{\eta}}{2\left(\frac{1}{\eta} - \cos^2 \varphi \right) + 1 - \frac{1}{\eta}} = \frac{\eta - 1}{1 - \eta \cos 2\varphi}$$

On putting $\eta = 3{\cdot}0\,,\ \varphi = 60°$

we get $$P = \frac{2}{1 + 3 \times \frac{1}{2}} = \frac{4}{5} = 0{\cdot}8$$

5.164 Let us represent the natural light as a sum of two mutually perpendicular components, both with intensity I_0. Suppose that each polarizer transmits a fraction α_1 of the light with oscillation plane parallel to the prinicipal direction of the polarizer and a fraction α_2 with oscillation plane perpendicular to the principal direction of the polaizer. Then the intensity of light transmitted through the two polarizers is equal to

$$I_{\parallel} = \alpha_1^2 I_0 + \alpha_2^2 I_0$$

when their principal direction are parallel and

$$I_{\perp} = \alpha_1 \alpha_2 I_0 + \alpha_2 \alpha_1 I_0 = 2\,\alpha_1 \alpha_2 I_0$$

when they are crossed. But

$$\frac{I_{\perp}}{I_{\parallel}} = \frac{2\,\alpha_1 \alpha_2}{\alpha_1^2 + \alpha_2^2} = \frac{1}{\eta}$$

so $$\frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} = \sqrt{\frac{\eta - 1}{\eta + 1}}$$

(a) Now the degree of polarization produced by either polarizer when used singly is

$$P_0 = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2}$$

(assuming, of course, $\alpha_1 > \alpha_2$)

Thus $$P_0 = \sqrt{\frac{\eta - 1}{\eta + 1}} = \sqrt{\frac{9}{11}} = 0{\cdot}905$$

(b) When both polarizer are used with their principal directions parallel, the transmitted light, when analysed, has

maximum intensity, $I_{max} = \alpha_1^2 I_0$ and minimum intensity, $I_{min} = \alpha_2^2 I_0$

so $$P = \frac{\alpha_1^2 - \alpha_2^2}{\alpha_1^2 + \alpha_2^2} = \frac{\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} \cdot \frac{(\alpha_1 + \alpha_2)^2}{\alpha_1^2 + \alpha_2^2}$$

$$= \sqrt{\frac{\eta - 1}{\eta + 1}} \cdot \left(1 + \frac{2\,\alpha_1 \alpha_2}{\alpha_1^2 + \alpha_2^2}\right)$$

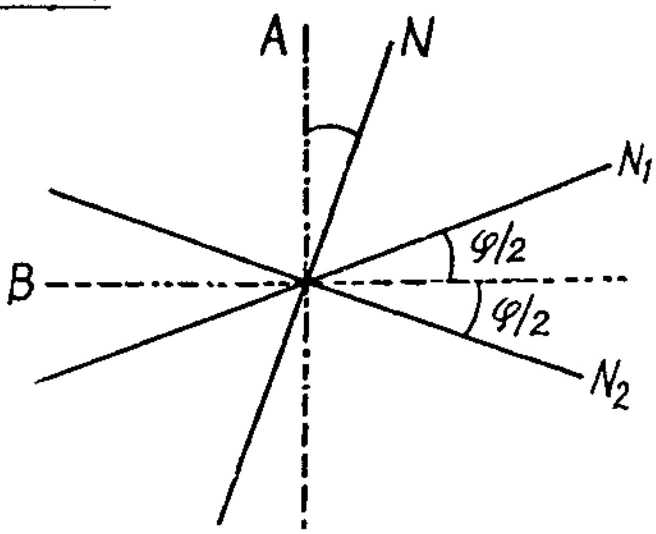
$$= \sqrt{\frac{\eta - 1}{\eta + 1}} \left(1 + \frac{1}{\eta}\right) = \frac{\sqrt{\eta^2 - 1}}{\eta} = \sqrt{1 - \frac{1}{\eta^2}} = 0.995.$$

5.165 If the principal direction N of the Nicol is along A or B, the intensity of light transmitted is the same whether the light incident is one with oscillation plane N_1 or one with N_2. If N makes an angle $\delta\varphi$ with A as shown then the fractional difference in intensity transmitted (when the light incident is N_1 or N_2) is

$$\left(\frac{\Delta I}{I}\right)_A = \frac{\cos^2\left(90° - \frac{\varphi}{2} - \delta\varphi\right) - \cos^2\left(90° + \frac{\varphi}{2} - \delta\varphi\right)}{\cos^2\left(90° - \frac{\varphi}{2}\right)}$$

$$= \frac{\sin^2\left(\frac{\varphi}{2} + \delta\varphi\right) - \sin^2\left(\frac{\varphi}{2} - \delta\varphi\right)}{\sin^2\frac{\varphi}{2}}$$

$$= \frac{2\sin\frac{\varphi}{2}\cdot 2\cos\frac{\varphi}{2}\,\delta\varphi}{\sin^2\frac{\varphi}{2}} = 4\cot\frac{\varphi}{2}\,\delta\varphi$$



If N makes an angle $\delta\varphi\ (<<\varphi)$ with B then

$$\left(\frac{\Delta I}{I}\right)_B = \frac{\cos^2(\varphi/2 - \delta\varphi) - \cos^2(\varphi/2 + \delta\varphi)}{\cos^2\varphi/2} = \frac{2\cos\frac{\varphi}{2}\cdot 2\sin\varphi/2\,\delta\varphi}{\cos^2\varphi/2} = 4\tan\varphi/2\,\delta\varphi$$

Thus
$$\eta = \left(\frac{\Delta I}{I}\right)_A \Big/ \left(\frac{\Delta I}{I}\right)_B = \cot^2\varphi/2$$

or
$$\varphi = 2\tan^{-1}\frac{1}{\sqrt{\eta}}$$

This gives $\varphi = 11{\cdot}4°$ for $\eta = 100$.

5.166 Fresnel equations read

$$I'_\perp = I_\perp\frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \text{ and } I'_{||} = I_{||}\frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}$$

At the boundary between vacuum and a dielectric $\theta_1 \neq \theta_2$ since by Snell's law

$$\sin\theta_1 = n\sin\theta_2$$

Thus $I'_\perp / I_\perp$ cannot be zero. However, if $\theta_1 + \theta_2 = 90°$, $I'_{||} = 0$ and the reflected light is polarized in this case. The condition for this is

$$\sin\theta_1 = n\sin\theta_2,\ = n\sin(90° - \theta_1)$$

or $\tan\theta_1 = n$ θ_1 is called Brewsta's angle.

The angle between reflected light and refracted light is 90° in this case.

5.167 (a) From Fresnel's equations

$$\left.\begin{aligned} I'_{\perp} &= I_{\perp}\frac{\sin^2(\theta_1-\theta_2)}{\sin^2(\theta_1+\theta_2)} \\ I'_{||} &= 0 \end{aligned}\right\} \text{ at Brewste's angle}$$

$$I'_{\perp} = I_{\perp}\sin^2(\theta_1-\theta_2)$$

$$= \frac{1}{2}I(\sin\theta_1\cos\theta_2-\cos\theta_1\sin\theta_2)^2$$

Now $$\tan\theta_1 = n,\ \sin\theta_1 = \frac{n}{\sqrt{n^2+1}}$$

$$\cos\theta_1 = \frac{1}{\sqrt{n^2+1}},\ \sin\theta_2 = \cos\theta_1$$

$$\cos\theta_2 = \sin\theta_1$$

$$I'_{\perp} = \frac{1}{2}I\left(\frac{n^2-1}{n^2+1}\right)^2$$

Thus reflection coefficient $= \rho = \dfrac{I'_{\perp}}{I}$

$$= \frac{1}{2}\left(\frac{n^2-1}{n^2+1}\right)^2 = 0{\cdot}074$$

on putting $n = 1{\cdot}5$

(b) For the refracted light

$$I''_{\perp} = I_{\perp}-I'_{\perp} = \frac{1}{2}I\left\{1-\left(\frac{n^2-1}{n^2+1}\right)^2\right\}$$

$$= \frac{1}{2}I\frac{4n^2}{(n^2+1)^2}$$

$$I'_{||} = \frac{1}{2}I$$

at the Brewster's angle.

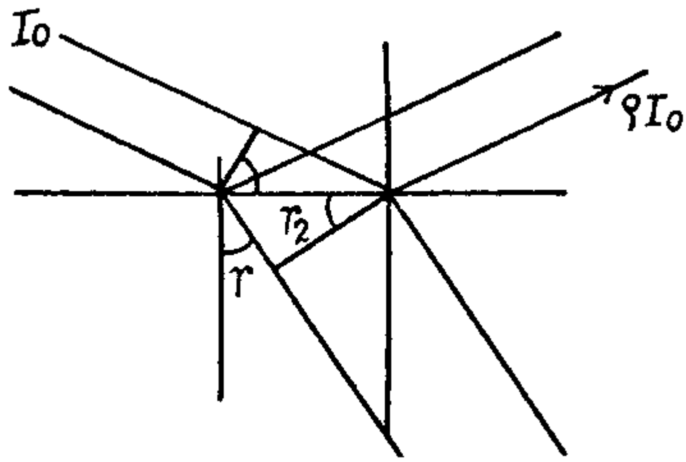
Thus the degree of polarization of the refracted light is

$$P = \frac{I''_{||}-I''_{\perp}}{I''_{||}+I''_{\perp}} = \frac{(n^2+1)^2-4n^2}{(n^2+1)^2+4n^2}$$

$$= \frac{(n^2-1)^2}{2(n^2+1)^2-(n^2-1)^2} = \frac{\rho}{1-\rho}$$

On putting $\rho = 0{\cdot}074$ we get $P = 0{\cdot}080$.

5.168 The energy transmitted is, by conservation of energy, the difference between incident energy and the reflected energy. However the intensity is affected by the change of the cross section of the beam by refraction. Let A_i, A_r, A_t be the cross sections of the incident, reflected and transmitted beams. Then



$$A_i = A_r$$

$$A_t = A_i \frac{\cos r}{\cos i}$$

But at Brewster's angle $r = 90 - i$

so $$A_t = A_i \tan i = n A_i$$

Thus $$I_t = \frac{(1-\rho) I_0}{n}$$

5.169 The amplitude of the incident component whose oscillation vector is perpendicular to the plane of incidence is

$$A_\perp = A_0 \sin \varphi$$

and similarly $$A_{||} = A_0 \cos \varphi$$

Then $$I'_\perp = I_0 \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} \sin^2 \varphi$$

$$= I_0 \left[\frac{\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2}{\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2} \right]^2 \sin^2 \varphi$$

$$= I_0 \left[\frac{n^2 - 1}{n^2 + 1} \right]^2 \sin^2 \varphi$$

Hence $$\rho = \frac{I'_\perp}{I_0} = \left[\frac{n^2 - 1}{n^2 + 1} \right]^2 \sin^2 \varphi$$

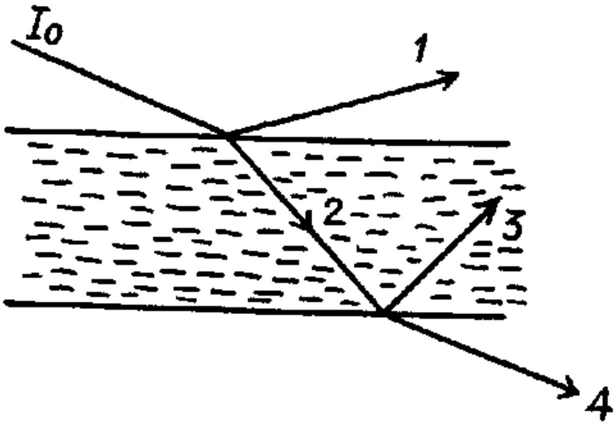
Putting $n = 1{\cdot}33$ for water we get $\rho = 0{\cdot}0386$

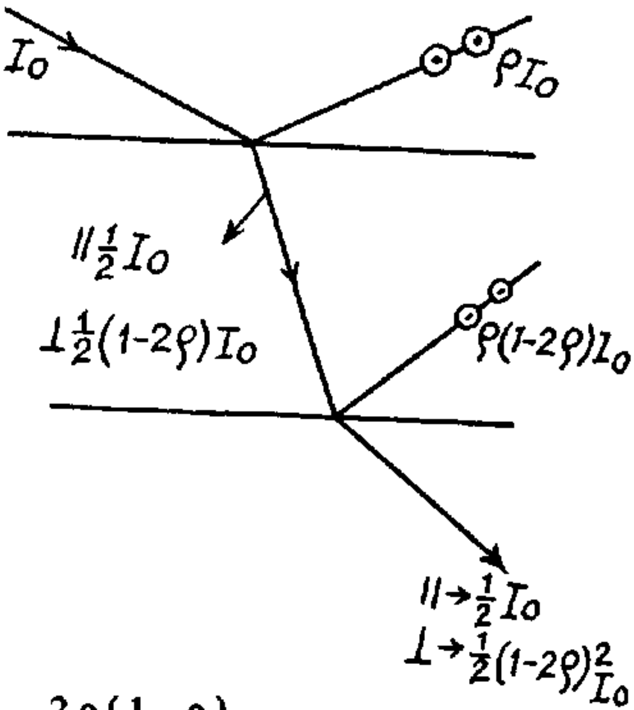
5.170 Since natural light is incident at the Brewster's angle, the reflected light 1 is completely polarized and $P_1 = 1$.

Similarly the ray 2 is incident on glass air surface at Brewster's angle $\left(\tan^{-1}\frac{1}{n}\right)$ so 3 is also completely polarized. Thus $P_3 = 1$

Now as in 5.167 (b)

$$P_2 = \frac{\rho}{1-\rho} = 0{\cdot}087 \text{ if } \rho = 0{\cdot}080$$





Finally as shown in the figure

$$P_4 = \frac{\frac{1}{2} - \frac{1}{2}(1 - 2\rho)^2}{\frac{1}{2} + \frac{1}{2}(1 - 2\rho)^2} = \frac{2\rho(1-\rho)}{1 - 2\rho(1-\rho)} = 0{\cdot}173$$

5.171 (a) In this case from Fresnel's equations

$$I'_\perp = I_\perp \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)}$$

we get $$I_1 = \left(\frac{n^2 - 1}{n^2 + 1}\right)^2 I_0 = \rho I_0 \text{ say}$$

then $$I_2 = (1 - \rho) I_0, \; I_3 = \rho(1 - \rho) I_0$$

(ρ is invariant under the substitution $n \to \frac{1}{n}$)

finally $$I_4 = (1 - \rho)^2 I_0 = \frac{16 n^4}{(n^2 + 1)^4} I_0 = 0{\cdot}726\, I_0 .$$

(b) Suppose ρ' = coefficient of reflection for the component of light whose electric vector oscillates at right angles to the incidence plane.

From Fresnel's equations $$\rho' = \left(\frac{n^2 - 1}{n^2 + 1}\right)^2$$

Then in the transmitted beam we have a partially polarized beam which is a superposition of two (|| & ⊥) components with intensities

$$\frac{1}{2} I_0 \;\&\; \frac{1}{2} I_0 (1 - \rho')^2$$

Thus $$P = \frac{1 - (1 - \rho')^2}{1 + (1 - \rho')^2} = \frac{(n^2 + 1)^4 - 16 n^4}{(n^2 + 1)^4 + 16 n^4} = \frac{1 - 0{\cdot}726}{1 + 0{\cdot}726} \approx 0{\cdot}158$$

5.172 (a) When natural light is incident on a glass plate at Brewster's angle, the transmitted light has

$$I''_{||} = I_0/2 \text{ and } I''_{\perp} = \frac{16\,n^4}{(n^2+1)^4} I_0/2 = \alpha^4 I_0/2$$

where I_0 is the incident intensity (see 5.171 a)

After passing through the 2^{nd} plate we find

$$I_{||}'''' = \frac{1}{2} I_0 \quad \text{and} \quad I_{|}'''' = (\alpha^4)^2 \frac{1}{2} I_0$$

Thus after N plates

$$I_{||}^{trans} = \frac{1}{2} I_0$$

$$I_{\perp}^{trans} = \alpha^{4N} \frac{1}{2} I_0$$

Hence

$$P = \frac{1-\alpha^{4N}}{1+\alpha^{4N}} \quad \text{where} \quad \alpha = \frac{2\,n}{1+n^2}$$

(b) $\alpha^4 = 0{\cdot}726$ for $n = \frac{3}{2}$.

Thus

$$P(N = 1) = 0{\cdot}158,\ P(N = 2) = 0{\cdot}310$$

$$P(N = 5) = 0{\cdot}663,\ \ P(N = 10) = 0{\cdot}922.$$

5.173 (a) We decompose the natural light into two components with intensity $I_{||} = \frac{1}{2} I_0 = I_{\perp}$ where $||$ has its electric vector oscillating parallel to the plane of incidence and $\perp$ has the same $\perp^r$ to it.

By Fresnel's equations for normal incidence

$$\frac{I'_{\perp}}{I_{\perp}} = \lim_{\theta_1 \to 0} \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)} = \lim_{\theta_1 \to 0} \left(\frac{\theta_1 - \theta_2}{\theta_1 + \theta_2}\right)^2 = \left(\frac{n-1}{n+1}\right)^2 = \rho$$

similarly

$$\frac{I'_{||}}{I_{||}} = \rho = \left(\frac{n-1}{n+1}\right)^2$$

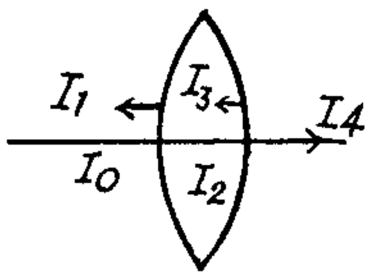
Thus

$$\frac{I'}{I} = \rho = \left(\frac{0{\cdot}5}{2{\cdot}5}\right)^2 = \frac{1}{25} = 0{\cdot}04$$

(b) The reflected light at the first surface has the intensity $I_1 = \rho I_0$

Then the transmitted light has the intensity $I_2 = (1-\rho) I_0$

At the second surface where light emerges from glass into air, the reflection coefficient is again ρ because ρ is invariant under the substitution $n \to \frac{1}{n}$.

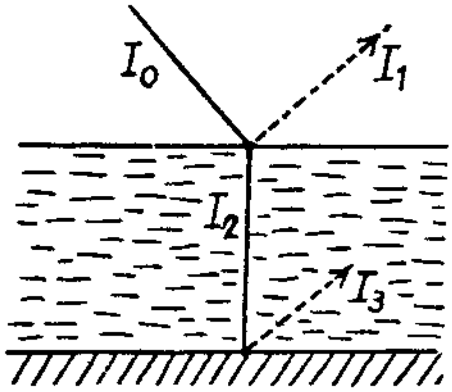


Thus $I_3 = \rho\,(1-\rho)\,I_0$ and $I_4 = (1-\rho)^2 I_0$.

For N lenses the loss in luminous flux is then

$$\frac{\Delta \Phi}{\Phi} = 1 - (1 - \rho)^{2N} = 0.335 \text{ for } N = 5$$

5.174 Suppose the incident light can be decomposed into waves with intensity $I_{||}$ & $I_{\perp}$ with oscillations of the electric vectors parallel and perpendicular to the plane of incidence.



For normal incidence we have from Fresnel equations

$$I_{\perp}' = I_{\perp}\left(\frac{\theta_1 - \theta_2}{\theta_1 + \theta_2}\right)^2 \longrightarrow I_{\perp}\left(\frac{n-1}{n+1}\right)^2$$

where we have used $\sin\theta \approx \theta$ for small θ.

Similarly $$I'_{||} = I_{||}\left(\frac{n'-1}{n'+1}\right)^2$$

Then the refracted wave will be

$$I''_{||} = I_{||}\frac{4n'}{(n'+1)^2} \quad \text{and} \quad I_{\perp}'' = I_{\perp}\frac{4n'}{(n'+1)^2}$$

At the interface with glass

$$I_{\perp}''' = I_{\perp}''\left(\frac{n'-n}{n'+n}\right)^2, \text{ similarly for } I_{||}'''$$

we see that

$$\frac{I_{\perp}'}{I_{\perp}} = \frac{I_{\perp}'''}{I_{\perp}''} \quad \text{if} \quad n' = \sqrt{n}, \text{ similarly for } || \text{ component.}$$

This shows that the light reflected as a fraction of the incident light is the same on the two surfaces if $n' = \sqrt{n}$.

Note:- The statement of the problem given in the book is incorrect. Actual amplitudes are not equal; only the reflectance is equal.

5.175 Here $\theta_1 = 45°$

$$\sin\theta_2 = \frac{1}{\sqrt{2}} \times \frac{1}{n} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3} = 0{\cdot}4714$$

$$\theta_2 = \sin^{-1} 0{\cdot}4714 = 28{\cdot}1°$$

Hence $$I'_{\perp} = I_{\perp}\frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)}$$

$$= \frac{1}{2}I_0\left(\frac{\sin 16{\cdot}9°}{\sin 73{\cdot}1°}\right)^2 = \frac{1}{2}I_0 \times 0{\cdot}0923$$

$$I'_{||} = \frac{1}{2} I_0 \left(\frac{\tan 16{\cdot}9}{\tan 73{\cdot}1} \right)^2 = \frac{1}{2} I_0 \times 0{\cdot}0085$$

Thus

(a) Degree of polarization P of the reflected light

$$= \frac{0{\cdot}0838}{0{\cdot}1008} = 0{\cdot}831$$

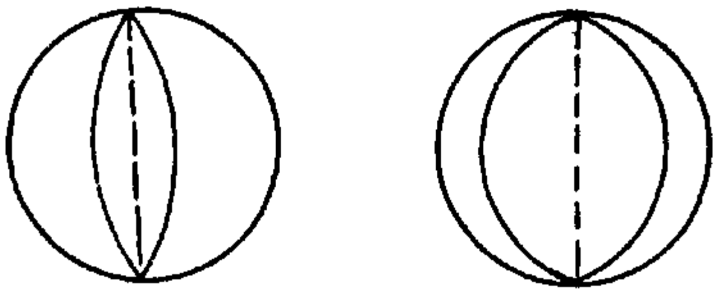
(b) By conservation of energy

$$I''_{\perp} = \frac{1}{2} I_0 \times 0{\cdot}9077$$

$$I''_{||} = \frac{1}{2} I_0 \times 0{\cdot}9915$$

Thus $$P = \frac{0{\cdot}0838}{1{\cdot}8982} = 0{\cdot}044$$

5.176 The wave surface of a uniaxial crystal consists of two sheets of which one is a sphere while the other is an ellipsoid of revolution.

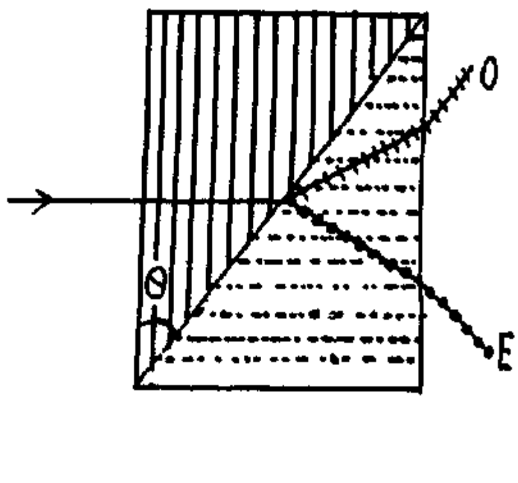


The optic axis is the line joining the points of contact.

To makes the appropriate Huyghen's construction we must draw the relevant section of the wave surface inside the crystal and determine the directions of the ordinary and extraordinary rays. The result is as shown in Fig. 42 (a, b & c) of the answers

5.177 In a uniaxial crystal, an unpolarized beam of light (or even a polarized one) splits up into O (for ordinary) and E (for extraordinary) light waves. The direction of vibration in the O and E waves are most easily specified in terms of the O and E principal planes. The principal plane of the ordinary wave is defined as the plane containing the O ray and the optic axis. Similarly the principal plane of the E wave is the plane containing the E ray and the optic axis. In terms of these planes the following is true : The O vibrations are perpendicular to the principal plane of the O ray while the E vibrations are in the principal plane of the E ray.

When we apply this definition to the wollaston prism we find the following :



(exaggerated.)

When unpolarized light enters from the left the O and E waves travel in the same direction but with different speeds. The O ray on the left has its vibrations normal to the plane of the paper and it becomes E ray on crossing the diagonal boundary of the two prism similarly the E ray on the left becomes O ray on the right. In this case Snell's law is applicable only approximately. The two rays are incident on the boundary at an angle θ and in the right prism the ray which we have called O ray on the right emerges at

$$\sin^{-1}\frac{n_e}{n_0}\sin\theta = \sin^{-1}\frac{1{\cdot}658}{1{\cdot}486}\times\frac{1}{2} = 33{\cdot}91°$$

where we have used

$$n_e = 1{\cdot}1658\,,\ n_0 = 1{\cdot}486 \text{ and } \theta = 30°\,.$$

Similarly the E ray on the right emerges within the prism at

$$\sin^{-1}\frac{n_0}{n_e}\sin\theta = 26{\cdot}62°$$

This means that the O ray is incident at the boundary between the prism and air at

$$33{\cdot}91 - 30° = 3{\cdot}91°$$

and will emerge into air with a deviation of

$$\sin^{-1} n_0 \sin 3{\cdot}91°$$

$$= \sin^{-1}(1{\cdot}658 \sin 3{\cdot}91°) = 6{\cdot}49°$$

The E ray will emerge with an opposite deviation of

$$\sin^{-1}(n_e \sin(30° - 26{\cdot}62°))$$

$$= \sin^{-1}(1{\cdot}486 \sin 3{\cdot}38°) = 5{\cdot}03°$$

Hence $\delta \approx 6{\cdot}49° + 5{\cdot}03° = 11{\cdot}52°$

This result is accurate to first order in $(n_e - n_0)$ because Snell's law holds when $n_e = n_0$.

5.178 The wave is moving in the direction of z– axis

(a) Here $E_x = E\cos(\omega t - kz)$, $E_y = E\sin(\omega t - kz)$

$$\frac{E_x^2}{E^2} + \frac{E_y^2}{E^2} = 1$$

so the tip of the electric vector moves along a circle. For the right handed coordinate system this represents circular anticlockwise polarization when observed towards the incoming wave.

(b) $E_x = E\cos(\omega t - kz)$, $E_y = E\cos\left(\omega t - kz + \frac{\pi}{4}\right)$

so
$$\frac{E_y}{E} = \frac{1}{\sqrt{2}}\cos(\omega t - kz) - \frac{1}{\sqrt{2}}\sin(\omega t - kz)$$

or
$$\left(\frac{E_y}{E} - \frac{1}{\sqrt{2}}\frac{E_x}{E}\right)^2 = \frac{1}{2}\left(1 - \frac{E_x^2}{E^2}\right)$$

or
$$\frac{E_y^2}{E^2} + \frac{E_x^2}{E^2} - \sqrt{2}\,\frac{E_y E_x}{E^2} = \frac{1}{2}$$

This is clearly an ellipse. By comparing with the previous case (compare the phase of E_y in the two cases) we see this represents elliptical clockwise polarization when viewed towards the incoming wave.

We write the equations as

$$E_x + E_y = 2E\cos\left(\omega t - kz + \frac{\pi}{8}\right)\cos\frac{\pi}{8}$$

$$E_x - E_y = +2E\sin\left(\omega t - kz + \frac{\pi}{8}\right)\sin\frac{\pi}{8}$$

Thus

$$\left(\frac{E_x + E_y}{2E\cos\frac{\pi}{8}}\right)^2 + \left(\frac{E_x - E_y}{2E\sin\frac{\pi}{8}}\right)^2 = 1$$

Since $\cos\frac{\pi}{8} > \sin\frac{\pi}{8}$, the major axis is in the direction of the straight line $y = x$.

(c) $E_x = E\cos(\omega t - kz)$

$E_y = E\cos(\omega t - kz + \pi) = -E\cos(\omega t - kz)$

Thus the top of the electric vector traces the curve

$$E_y = -E_x$$

which is a straight line ($y = -x$). It corresponds to plane polarization.

5.179 For quartz

$$\left.\begin{aligned} n_e &= 1{\cdot}553 \\ n_0 &= 1{\cdot}544 \end{aligned}\right\} \text{ for } \lambda = 589\text{ nm}.$$

In a quartz plate cut parallel to its optic axis, plane polarized light incident normally from the left divides itself into O and E waves which move in the same direction with different speeds and as a result acquire a phase difference. This phase difference is

$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)d$$

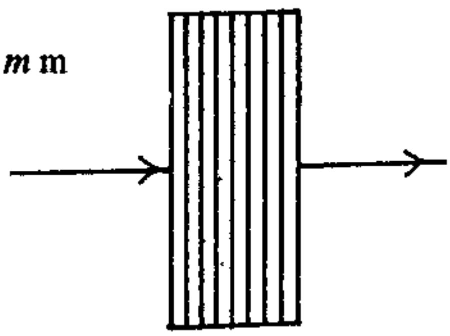
where d = thickness of the plate. In general this makes the emergent light elliptically polarized.

(a) For emergent light to experience only rotation of polarization plane

$\delta = (2k+1)\pi,\ k = 0, 1, 2, 3 \ldots$

For this $d = (2k+1)\dfrac{\lambda}{2(n_e - n_0)}$.

$$= (2k+1)\frac{{\cdot}589}{2\times{\cdot}009}\,\mu\text{m} = (2k+1)\frac{{\cdot}589}{18}\,\text{mm}$$



The maximum value of $(2k+1)$ for which this is less than 0·50 is obtained from

$$\frac{0{\cdot}50\times 18}{0{\cdot}589} = 15{\cdot}28$$

Then we must take $k = 7$ and $d = 15\times\dfrac{{\cdot}589}{18} = 0{\cdot}4908$ mm

(b) For circular polarization $\delta = \frac{\pi}{2}$

modulo 2π i.e. $\delta = (4k+1)\frac{\pi}{2}$

so $$d = (4k+1)\frac{\lambda}{4(n_e - n_0)} = (4k+1)\frac{0{\cdot}589}{36}$$

Now $$\frac{0{\cdot}50 \times 36}{0{\cdot}589} = 30{\cdot}56$$

The nearest integer less than this which is of the form $4k+1$ is 29 for $k = 7$. For this

$$d = 0{\cdot}4749 \text{ mm}$$

5.180 As in the previous problem the quartz plate introduces a phase difference δ between the O & E components. When $\delta = \pi/2$ (modulo π) the resultant wave is circularly polarized. In this case intensity is independent of the rotation of the rear prism. Now

$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\,d$$

$$= \frac{2\pi}{\lambda}0{\cdot}009 \times 0{\cdot}5 \times 10^{-3}\text{ m}$$

$$= \frac{9\pi}{\lambda},\ \lambda \text{ in } \mu\text{m}$$

For $\lambda = 0{\cdot}50\,\mu m$. $\delta = 18\pi$. The relevant values of δ have to be chosen in the form $\left(k + \frac{1}{2}\right)\pi$. For $k = 17, 16, 15$ we get

$$\lambda = 0{\cdot}5143\,\mu m,\ 0{\cdot}5435\,\mu\text{m} \text{ and } 0{\cdot}5806\,\mu\text{m}$$

These are the values of λ which lie between $0{\cdot}50\,\mu$m and $0{\cdot}60\,\mu$m.

5.181 As in the previous two problems the quartz plate will introduce a phase difference δ. The light on passing through the plate will remain plane polarized only for $\delta = 2k\pi$ or $(2k+1)\pi$. In the latter case the plane of polarization of the light incident on the plate will be rotated by 90° by it so light passing through the analyser (which was originally crossed) will be a maximum. Thus dark bands will be observed only for those λ for which

$$\delta = 2k\pi$$

Now $$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\,d \doteq \frac{2\pi}{\lambda} \times {\cdot}009 \times 1{\cdot}5 \times 10^{-3}\text{ m}$$

$$= \frac{27\pi}{\lambda}(\lambda \text{ in } \mu\text{m})$$

For $$\lambda = 0{\cdot}55 \text{ we get } \delta = 49{\cdot}09\,\pi$$

Choosing $\delta = 48\pi, 46\pi, 44\pi, 42\pi$ we get $\lambda = 0{\cdot}5625\,\mu$m, $\lambda = 0{\cdot}5870\,\mu$m, $\lambda = 0{\cdot}6136\,\mu$m and $\lambda = 0{\cdot}6429\,\mu$m. These are the only values between $0{\cdot}55\,\mu$m and $0{\cdot}66\,\mu$m. Thus there are four bands.

5.182 Here

$$\delta = \frac{2\pi}{\lambda} \times 0{\cdot}009 \times 0{\cdot}25 \text{ m}$$

$$= \frac{4{\cdot}5\,\pi}{\lambda}, \ \lambda \text{ in } \mu\text{m}.$$

We check that for

$$\lambda = 428{\cdot}6 \text{ nm} \quad \delta = 10{\cdot}5\,\pi$$

$$\lambda = 529{\cdot}4 \text{ nm} \quad \delta = 8{\cdot}5\,\pi$$

$$\lambda = 692{\cdot}3 \text{ nm} \quad \delta = 6{\cdot}5\,\pi$$

These are the only values of λ for which the plate acts as a quarter wave plate.

5.183 Between crossed Nicols, a quartz plate, whose optic axis makes 45° with the principal directions of the Nicols, must introduce a phase difference of $(2k+1)\pi$ so as to transmit the incident light (of that wavelength) with maximum intensity. For in this case the plane of polarization of the light emerging from the polarizer will be rotated by 90° and will go through the analyser undiminished. Thus we write for light of wavelengths 643 nm

$$\delta = \frac{2\pi \times 0{\cdot}009}{0{\cdot}643 \times 10^{-6}} \times d\,(\text{mm}) \times 10^{-3}$$

$$= \frac{18\,\pi\, d}{0{\cdot}643} = (2k+1)\,\pi \qquad (1)$$

To nearly block light of wavelength 564 nm we require

$$\frac{18\,\pi\, d}{0{\cdot}564} = (2k')\,\pi \qquad (2)$$

We must have $2k' > 2k+1$. For the smallest value of d we take $2k' = 2k+2$.

Thus $\quad 0{\cdot}643\,(2k+1) = 0{\cdot}564 \times (2k+2)$

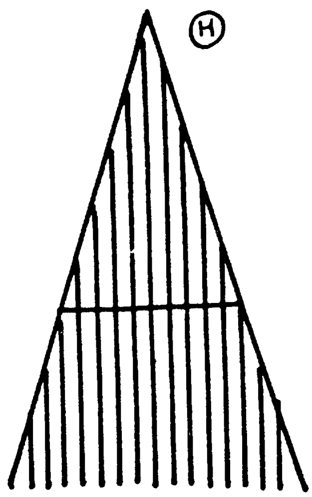
so $\quad 0{\cdot}079 \times 2k = 0{\cdot}564 \times 2 - 0{\cdot}643$

or $\quad 2k = 6{\cdot}139$

This is not quite an integer but is close to one. This means that if we take $2k = 6$ equations (1) can be satisfied exactly while equation (2) will hold approximately. Thus

$$d = \frac{7 \times 0{\cdot}643}{18} = 0{\cdot}250 \text{ mm}$$

5.184 If a ray traverses the wedge at a distance x below the joint, then the distance that the ray moves in the wedge is $2x \tan\frac{\Theta}{2}$ and this cause a phase difference



$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\, 2x \tan\frac{\Theta}{2}$$

between the E and O wave components of the ray. For a general x the resulting light is elliptically polarized and is not completely quenched by the analyser polaroid. The condition for complete quenching is

$$\delta = 2k\pi \text{ — dark fringe}$$

That for maximum brightness is

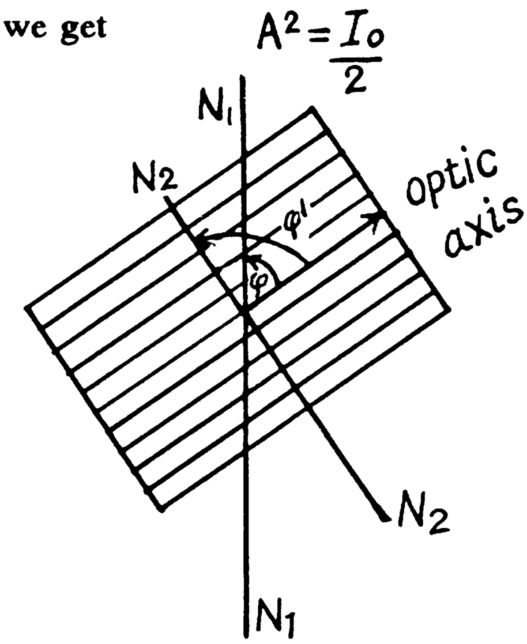
$$\delta = (2k+1)\pi - \text{bright fringe.}$$

The fringe width is given by

$$\Delta x = \frac{\lambda}{2(n_e - n_0)\tan\frac{\Theta}{2}}$$

Hence
$$(n_e - n_0) = \frac{\lambda}{2\,\Delta x \tan\Theta/2}$$

using
$$\tan(\Theta/2) = \tan 175° = 0{\cdot}03055\,,$$
$$\lambda = 0{\cdot}55\,\mu m \text{ and } \Delta x = 1\,mm\text{, we get}$$
$$n_e - n_0 = 9{\cdot}001 \times 10^{-3}$$



5.185 Light emerging from the first polaroid is plane polarized with amplitude A where N_1 is the principal direction of the polaroid and a vibration of amplitude can be resolved into two vibration : E wave with vibration along the optic axis of amplitude $A\cos\varphi$ and the O wave with vibration perpendicular to the optic axis and having an amplitude $A\sin\varphi$. These acquire a phase difference δ on passing through the plate. The second polaroid transmits the components :

$$A\cos\varphi\cos\varphi'$$

and
$$A\sin\varphi\sin\varphi'$$

What emerges from the second polaroid is a set of two plane polarized waves in the same direction and same plane of polarization but phase difference δ. They interfere and produce a wave of amplitude squared

$$R^2 = A^2\left[\cos^2\varphi\cos^2\varphi' + \sin^2\varphi\sin^2\varphi' + 2\cos\varphi\cos\varphi'\sin\varphi\sin\varphi'\cos\delta\right],$$

using
$$\cos^2(\varphi - \varphi') = (\cos\varphi\cos\varphi' + \sin\varphi\sin\varphi')^2$$
$$= \cos^2\varphi\cos^2\varphi' + \sin^2\varphi\sin^2\varphi' + 2\cos\varphi\cos\varphi'\sin\varphi\sin\varphi'$$

we easily find

$$R^2 = A^2\left[\cos^2(\varphi - \varphi') - \sin 2\varphi\sin 2\varphi'\sin^2\frac{\delta}{2}\right]$$

Now
$$A^2 = I_0/2 \text{ and } R^2 = I \text{ so the result is}$$

$$I = \frac{1}{2}I_0\left[\cos^2(\varphi - \varphi') - \sin 2\varphi\sin 2\varphi'\sin^2\frac{\delta}{2}\right]$$

Special cases :- Crossed polaroids : Here $\varphi - \varphi' = 90°$ or $\varphi' = \varphi - 90°$ and $2\varphi' = 2\varphi - 180°$

Thus in this case

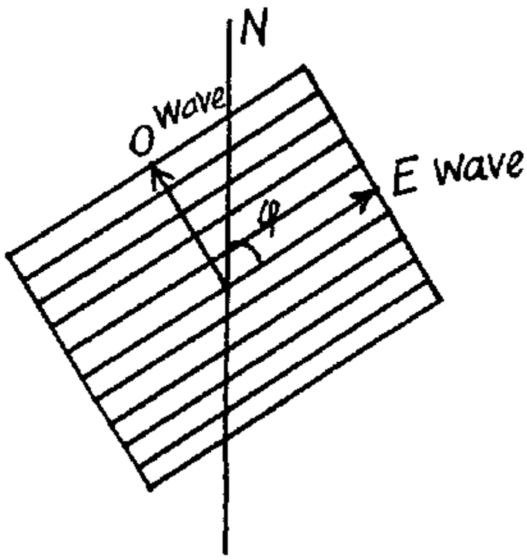
$$I = I_{\perp} = \frac{1}{2}I_0\sin^2 2\varphi\sin^2\frac{\delta}{2}$$

Parallel polaroids : Here $\varphi = \varphi'$ and

$$I = I_{||} = \frac{1}{2} I_0 \left(1 - \sin^2 2\varphi \sin^2 \frac{\delta}{2} \right)$$

With $\delta = \frac{2\pi}{\lambda}\Delta$, the conditions for the maximum and minimum are easily found to be that shown in the answer.

5.186 Let the circularly polarized light be resolved into plane polarized components of amplitude A_0 with a phase difference $\frac{\pi}{2}$ between then.



On passing through the crystal the phase difference becomes $\delta + \frac{\pi}{2}$ and the components of the E and O wave in the direction N are respectively
$A_0 \cos\varphi$ and $A_0 \sin\varphi$

They interfere to produce the amplitude squared

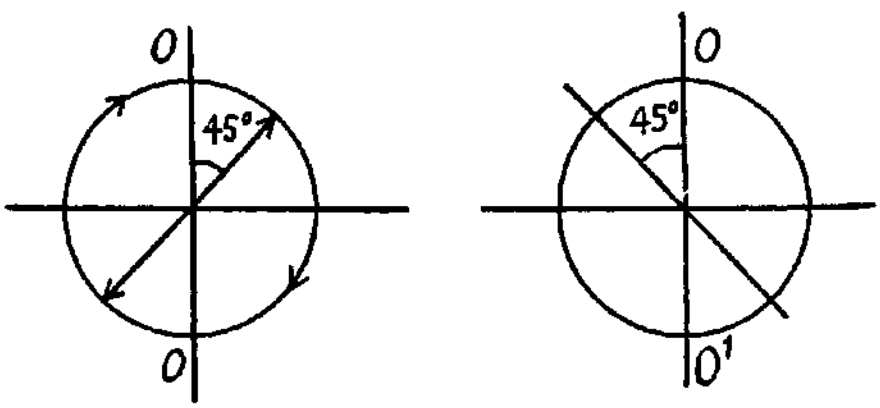
$$R^2 = A_0^2 \cos^2\varphi + A_0^2 \sin^2\varphi + 2A_0^2 \cos\varphi \sin\varphi \cos\left(\delta + \frac{\pi}{2}\right)$$

$$= A_0^2 (1 + \sin 2\varphi \sin\delta)$$

Hence $\qquad I = I_0 (1 + \sin 2\varphi \sin\delta)$

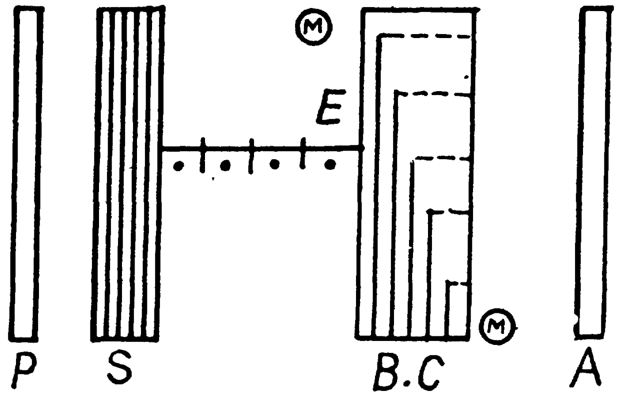
Here I_0 is the intensity of the light transmitted by the polaroid when there is no crystal plate.

5.187 (a) The light with right circular polarization (viewed against the oncoming light, this means that the light vector is moving clock wise.) becomes plane polarized on passing through a quarter-wave plate. In this case the direction of oscillations of the electric vector of the electromagnetic wave forms an angle of $+45°$ with the axis of the crystal OO′ (see Fig (a) below). In the case of left hand circular polarizations, this angle will be $-45°$ (Fig (b)).



(b) If for any position of the plate the rotation of the polaroid (located behind the plate) does not bring about any variation in the intensity of the transmitted light, the incident light is unpolarized (i.e. natural). If the intensity of the transmitted light can drop to zero on rotating the analyzer polaroid for some position of the quarter wave plate, the incident light is circularly polarized. If it varies but does not drop to zero, it must be a mixture of natural and circularly polarized light.

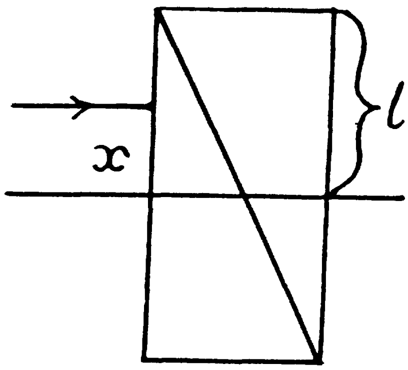
5.188 The light from P is plane polarized with its electric vector vibrating at 45° with the plane of the paper. At first the sample S is absent. Light from P can be resolved into components vibrating in and perpendicular to the plane of the paper. The former is the E ray in the left half of the Babinet compensator and the latter is the O ray. In the right half the nomenclature is the opposite. In the compensator the two components acquire a pahse difference which depends on the relative position of the ray. If the ray is incident at a distance x above the central line through the compensator then the E ray acquires a phase



$$\frac{2\pi}{\lambda}\Big(n_E(l-x)+n_0(l+x)\Big)\tan\Theta$$

while the O ray acquires

$$\frac{2\pi}{\lambda}\Big(n_o(l-x)+n_E(l+x)\Big)\tan\Theta$$



so the phase difference between the two reays is

$$\frac{2\pi}{\lambda}(n_o-n_E)\,2x\tan\Theta = \delta$$

we get dark fringes when ever $\delta = 2k\pi$

because then the emergent light is the same as that coming from the polarizer and is quenched by the analyser. {If $\delta = (2k+1)\pi$, we get bright fringes because in this case, the plane of polarizaton of the emergent hight has rotated by 90° and is therefore fully transmitted by the analyser.}

If follows that the fringe width Δx is given by

$$\Delta x = \frac{\lambda}{2\,|n_0-n_E|\tan\Theta}$$

(b) If the fringes are displaced upwards by δx, then the path difference introduced by the sample between the O and the E rays must be such as to be exactly cancelled by the compensator. Thus

$$\frac{2\pi}{\lambda}\Big[d(n'_O-n'_E)+(n_E-n_O)\,2\,\delta x\tan\Theta\Big] = 0$$

or

$$d(n'_O-n'_E) = -2(n_E-n_O)\,\delta x\,\Theta$$

using $\tan\Theta \approx \Theta$.

5.189 Light polarized along the x-direction (i.e. one whose electric vector has only an x component) and propagating along the z-direction can be decomposed into left and right circularly polarized light in accordance with the formula

$$E_x = \frac{1}{2}(E_x + iE_y) + \frac{1}{2}(E_x - iE_y)$$

On passing through a distance l of an active medium these acquire the phases $\delta_R = \frac{2\pi}{\lambda} n_R l$ and $\delta_l = \frac{2\pi}{\lambda} n_l l$ so we get for the complex amplitude

$$E' = \frac{1}{2}(E_x + iE_y)e^{i\delta_R} + \frac{1}{2}(E_x - iE_y)e^{i\delta_L}$$

$$= e^{i\frac{\delta_R+\delta_l}{2}}\left[\frac{1}{2}(E_x + iE_y)e^{i\delta/2} + \frac{1}{2}(E_x - iE_y)e^{-i\delta/2}\right]$$

$$= e^{i\frac{\delta_R+\delta_l}{2}}\left[E_x \cos\frac{\delta}{2} - E_y \sin\frac{\delta}{2}\right], \ \delta = \delta_R - \delta_L .$$

Apart from an over all phase $(\delta_R + \delta_l)/2$ (which is irrelevant) this represents a wave whose plane of polarization has rotated by

$$\frac{\delta}{2} = \frac{\pi}{\lambda}(\Delta n)l, \quad \Delta n = |n_R - n_l|$$

By definition this equals αl so

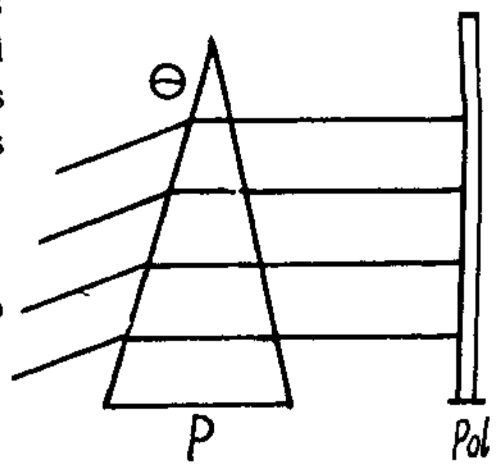
$$\Delta n = \frac{\alpha\lambda}{\pi}.$$

$$= \frac{589{\cdot}5 \times 10^{-6}\,\text{m m} \times 21{\cdot}72\,\text{deg/m m}}{\pi} \times \frac{\pi}{180}(\text{rad})$$

$$= \frac{{\cdot}5895 \times 21{\cdot}72}{180} \times 10^{-3}$$

$$= 0{\cdot}71 \times 10^{-4}$$

5.190 Plane polarized light on entering the wedge decomposes into right and left circularly polarized light which travel with different speeds in P and the emergent light gets its plane of polarization rotated by an angle which depends on the distance travelled.



Given that Δx = fringe width

$\Delta x \tan\theta$ = difference in the path length traversed by two rays which form successive bright or dark fringes.

Thus $\quad \frac{2\pi}{\lambda}|n_R - n_l|\Delta x \tan\theta = 2\pi$

Thus $\quad \alpha = \frac{\pi\Delta n}{\lambda} = \pi/\Delta x \tan\theta$

$= 20{\cdot}8$ ang deg/m m

Let x = distance on the polaroid Pol as measured from a maximum. Then a ray that falls at this distance traverses an extra distance equal to

$$\pm\, x \tan\theta$$

and hence a rotation of $$\pm\, \alpha x \tan\theta = \pm\, \frac{\pi x}{\Delta x}$$

By Malus' law the intensity at this point will be $\sim \cos^2\left(\frac{\pi x}{\Delta x}\right)$.

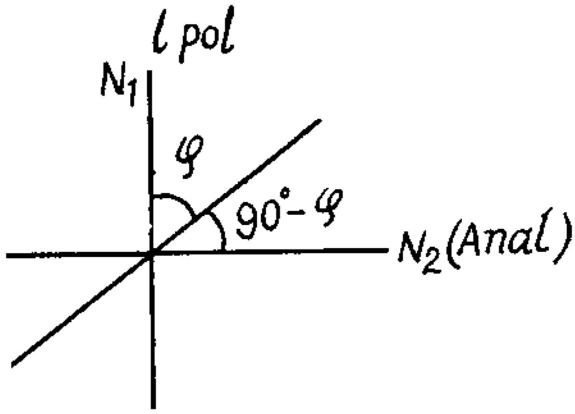
5.191 If I_0 = intensity of natural light then

$\frac{1}{2} I_0$ = intensity of light emerging from the polarizer nicol.

Suppose the quartz plate rotates this light by φ, then the analyser will transmit

$$\frac{1}{2} I_0 \cos^2(90 - \varphi)$$

$$= \frac{1}{2} I_0 \sin^2\varphi$$



of this intensity. Hence $$\eta I_0 = \frac{1}{2} I_0 \sin^2\varphi$$

or $$\varphi = \sin^{-1}\sqrt{2\eta}$$

But $$\varphi = \alpha d \text{ so}$$

$$d_{min} = \frac{1}{\alpha}\sin^{-1}\sqrt{2\eta}$$

For minimum d we must take the principal value of inverse sine. Thus using $\alpha = 17$ ang deg/m m.

$$d_{min} = 2{\cdot}99 \text{ m m}.$$

5.192 For light of wavelength 436 nm

$$41{\cdot}5° \times d = k \times 180° = 2k \times 90°$$

(Light will be completely cut off when the quartz plate rotates the plane of polarization by a multiple of 180°.) Here d = thickness of quartz plate in mm.

For natural incident light, half the light will be transmitted when the quartz rotates light by an odd multiple of 90°. Thus

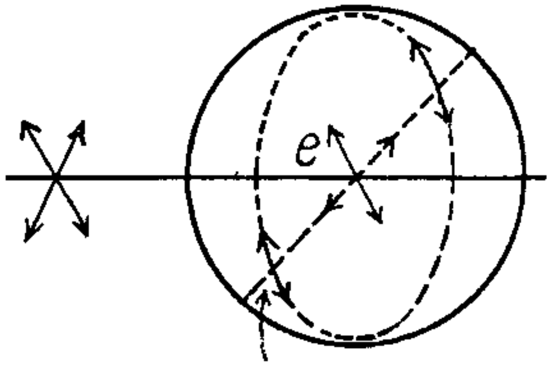
$$31{\cdot}1° \times d = (2k' + 1) \times 90°$$

Now $$\frac{41{\cdot}5}{31{\cdot}1} = 1{\cdot}3344 \approx \frac{4}{3}$$

Thus $$k = 2 \text{ and } k' = 1 \text{ and}$$

$$d = \frac{4 \times 90}{41{\cdot}5} = 8{\cdot}67 \text{ m m}.$$

5.193 Two effects are involved here : rotation of plane of polarizatin by sugar solution and the effect of that rotation on the scattering of light in the transverse direction. The latter is shown in the figure given below. It is easy to see from the figure that there will be no scattering of light in this transverse direction if the incident light has its electric vector parallel to the line of sight. In such a situation, we expect fringes to occur in the given experiment.



From the given data we see that in a distance of 50 cm, the rotation of plane of polarization must be 180°. Thus the specific rotation constant of sugar

$$= \frac{\text{rotation constant}}{\text{concentration}}$$

$$= \frac{180/50}{500\,\frac{g}{l}}\ \text{ang/deg/cm} = \frac{180}{5{\cdot}0\ \text{d m} \times (\cdot 500\ \text{gm/cc})}$$

$$= 72^\circ\ \text{ang deg/(dm}\cdot\text{gm/cc)}\ \ (1\ \text{d m} = 10\ \text{cm})$$

5.194 (a) In passing through the Kerr cell the two perpendicular components of the electric field will acquire a phase difference. When this phase difference equals 90° the emergent light will be circularly polarized because the two perpendicular components O & E have the same magnitude since it is given that the direction of electric field E in the capacitor forms an angle of 45° with the principal directions of then icols. In this case the intensity of light that emerges from this system will be independent of the rotation of the analyser prism.

Now the phase difference introduced is given by

$$\delta = \frac{2\pi}{\lambda}(n_e - n_0)\, l$$

In the present case $\delta = \frac{\pi}{2}$ (for minimum electric field)

$$n_e - n_0 = \frac{\lambda}{4\,l}$$

Now $$n_e - n_0 = B\lambda E^2$$

so $$E_{min} = \sqrt{\frac{1}{4Bl}}\,. = 10^5/\sqrt{88} = 10{\cdot}66\ \text{kV/cm}\,.$$

(b) If the applied electric field is

$$E = E_m \sin\omega t,\ \omega = 2\pi\nu$$

than the Kerr cell introduces a time varying phase difference

$$\delta = 2\pi B\,|\,E_m^2 \sin^2\omega t$$

$$= 2\pi \times 2{\cdot}2 \times 10^{-10} \times 10 \times (50 \times 10^3)^2 \sin^2\omega t$$

$$= 11\,\pi \sin^2\omega t$$

In one half-cycle $\left(\text{i.e. in time } \frac{\pi}{\omega} = T/2 = \frac{1}{2\nu}\right)$

this reaches the value $2k\pi$ when

$$\sin^2 \omega t = 0, \frac{2}{11}, \frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{10}{11}$$

$$\frac{2}{11}, \frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \frac{10}{11}$$

i.e. 11 times. On each of these occasions light will be interrupted. Thus light will be interrupted

$$2\nu \times 11 = 2{\cdot}2 \times 10^8 \text{ times per second}$$

(Light will be interrupted when the Kerr cell (placed between crossed Nicols) introduces a phase difference of $2k\pi$ and in no other case.)

5.195 From problem 189, we know that

$$\Delta n = \frac{\alpha \lambda}{\pi}$$

where α is the rotation constant. Thus

$$\Delta n = \frac{2\alpha}{2\pi/\lambda} = \frac{2\alpha c}{\omega}$$

On the other hand $\alpha_{mag} = VH$

Thus for the magnetic rotations $\Delta n = \frac{2cVH}{\omega}$.

5.196 Part of the rotations is due to Faraday effect and part of it is ordinary optical rotation. The latter does not change sign when magnetic field is reversed. Thus

$$\varphi_1 = \alpha l + VlH$$

$$\varphi_2 = \alpha l - VlH$$

Hence $2VlH = (\varphi_1 - \varphi_2)$

or $V = \left(\frac{\varphi_1 - \varphi_2}{2}\right) \Big/ lH$

Putting the values

$$V = \frac{510 \text{ ang min}}{2 \times {\cdot}3 \times 56{\cdot}5} \times 10^{-3} \text{ per } A = 0{\cdot}015 \text{ ang min/A}$$

5.197 We write

$$\varphi = \varphi_{\text{chemical}} + \varphi_{\text{magnetic}}$$

We look against the transmitted beam and count the positive direction clockwise. The chemical part of the rotation is annulled by reversal of wave vector upon reflection.

Thus $\varphi_{\text{chemical}} = \alpha l$

Since in effect there is a single transmission.

On the other hand

$$\varphi_{mag} = -NHVl$$

To get the signs right recall that dextro rotatory compounds rotate the plane of vibration in a clockwise direction on looking against the oncoming beam. The sense of rotation of light vibration in Faraday effect is defined in terms of the direction of the field, positive rotation being that of a right handed screw advancing in the direction of the field. This is the opposite of the definition of $\varphi_{chemical}$ for the present case. Finally

$$\varphi = (\alpha - VNH)\,l$$

(Note : If plane polarized light is reflected back & forth through the same active medium in a magnetic field, the Faraday rotation increases with each traveresal.)

5.198 There must be a Faraday rotation by 45° in the opposite direction so that light could pass through the second polaroid. Thus

$$V l H_{min} = \pi/4$$

or

$$H_{min} = \frac{\pi/4}{Vl} = \frac{45 \times 60}{2{\cdot}59 \times 0{\cdot}26}\,\frac{A}{m}$$

$$= 4{\cdot}01\,\frac{\text{k A}}{\text{m}}$$

If the direction of magnetic field is changed then the sense of rotation will also change. Light will be completely quenched in the above case.

5.199 Let r = radius of the disc

then its moment of inertia about its axis $= \frac{1}{2} m r^2$

In time t the disc will acquire an angular momentum

$$t \cdot \pi r^2 \cdot \frac{I}{\omega}$$

when circularly polarized light of intensity I falls on it. By conservation of angular momentum this must equal

$$\frac{1}{2} m r^2 \cdot \omega_0$$

where ω_0 = final angular velocity.

Equating $$t = \frac{m\,\omega\,\omega_0}{2\pi I}.$$

But $$\frac{\omega}{2\pi} = \nu = \frac{c}{\lambda} \quad \text{so } t = \frac{m\,c\,\omega_0}{I\lambda}.$$

Substituting the values of the various quantities we get

$$t = 11{\cdot}9 \text{ hours}$$