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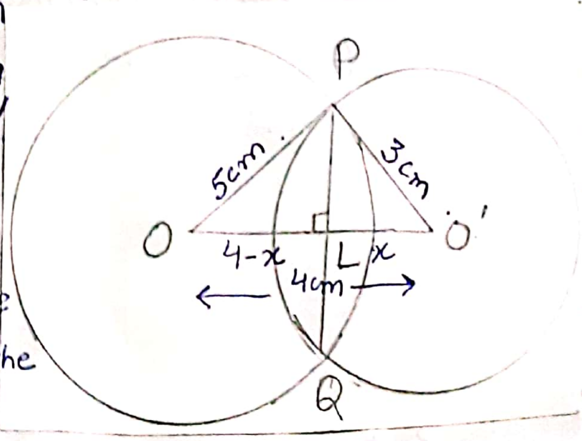
Ex-10.4.

1. Two circles of radii 5cm and 3cm intersect at two points and distance between their centre is 4cm. Find the length of common chord.

Sol:- Let PQ be the common chord of two intersecting circles with centre O and O'.



We know that
In two intersecting circles, the line joining their centre is perpendicular bisector of the common chord.

$\therefore \angle OLP = \angle OLQ = 90^\circ$ and $PL = LQ$.

Now in rt ΔOLP.

$PL^2 + OL^2 = OP^2$ —(1)

Let $O'L = x$ and $OL = 4-x$.

$PL^2 + (4-x)^2 = 5^2$

$PL^2 + (16 + x^2 - 8x) = 25$

$PL^2 = 25 - 16 - x^2 + 8x$

$= 9 - x^2 + 8x$

Now in rht $\Delta O'LP$.

$(PO')^2 = (PL)^2 + (LO')^2$

$\cancel{3^2} = \cancel{9} - \cancel{x^2} + 8x + \cancel{x^2}$.

$\therefore 8x = 0$

$\Rightarrow x = 0$.

$\therefore$ L and O' coincide

$\therefore$ PQ is a diameter of smaller circle.

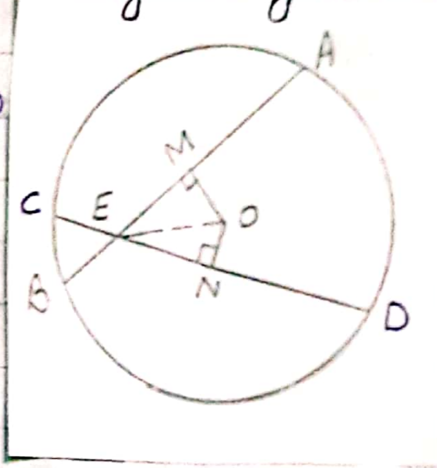
$\Rightarrow PL = 3cm$

$PL = LQ \Rightarrow LQ = 3cm$

Now $PQ = PL + LQ = 3cm + 3cm = 6cm$.

$\therefore$ Length of common chord $= 6cm$.

2. If two equal chords of a circle intersect within the circle, prove that the line segment of one chord are equal to corresponding segment of the other chord.



Sol:- Given:- A circle with centre O and equal chords AB and CD intersect at E.

To Prove:- $DE = AE$ and $CE = BE$.

Construction:- Draw $OM \perp AB$ and $ON \perp CD$. Join OE.

Proof:- $AB = CD$ (given)

$\therefore OM = ON$ (Equal chords are equidistant from centre). — (1)

In ΔOME and ΔONE

$\angle OME = \angle ONE$ (each $90°$)

$OM = ON$ (by (1))

$OE = OE$ (common)

$\therefore \Delta OME \cong \Delta ONE$ [By RHS congruence Rule]

$\Rightarrow ME = NE$ (c.p.c.t) — (2)

The perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore AM = \frac{1}{2} AB$ and $ND = \frac{1}{2} CD$.

But $AB = CD$

$\therefore AM = ND$. — (3).

Similarly $MB = NC$.

~~tto~~ Adding (2) & (3).

$ME + AM = NE + ND$

$\boxed{AE = DE}$ — (4).

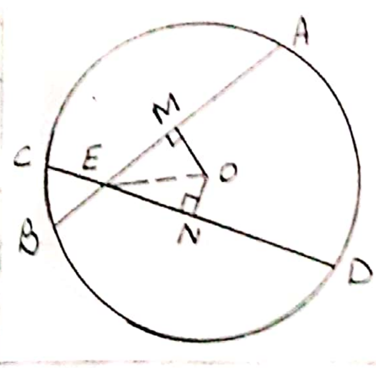
$AB = CD$

$AB - AE = CD - DE$. (subtracting (4)).

$\boxed{BE = CE}$

3. If two equal chords of circle intersect within the circle, Prove that the line joining the point of intersection ~~of~~ to the centre makes equal angles with the chords.

Sol:- Given A circle with centre O and equal chords AB and CD are intersecting at E.



To Prove:- $\angle OEA = \angle OED$.

Construction:- Draw $OM \perp AB$ and $ON \perp CD$. Join OE.

Proof:- In ΔOME and ΔONE,

$OM = ON$

[$\because$ Equal chords are equidistant from the centre]

$OE = OE$ (common)

$\angle OME = \angle ONE$ (each 90°).

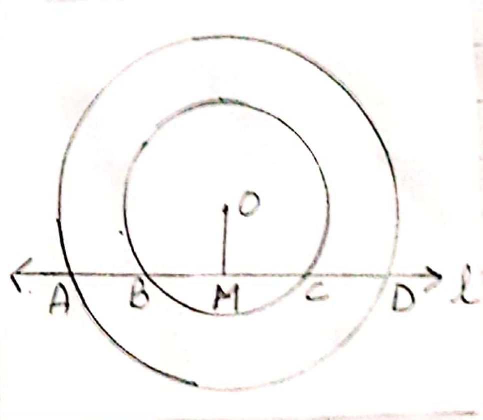
$\therefore \Delta OME \cong \Delta ONE$ (By RHS congruence Rule.)

$\therefore \angle OEM = \angle OEN$ (by c.p.c.t)

$\Rightarrow \angle OEA = \angle OED$.

4. If a line intersect two concentric circles (circles with same centre) with centre O at A, B, C and D. Prove that AB = CD. (see fig 10.25).

Sol:- Given:- Two circles with common centre O. A line l intersects the outer circle at A and D and the inner circle at B and C.



To Prove:- AB = CD.

Construction:- Draw $OM \perp l$.

Proof:- For outer circle, $OM \perp l$

$\therefore AM = MD$ — (1)

[Perpendicular from centre to the chord bisects the chord]

For inner circle $OM \perp l$

$\therefore BM = MC$ — (2)

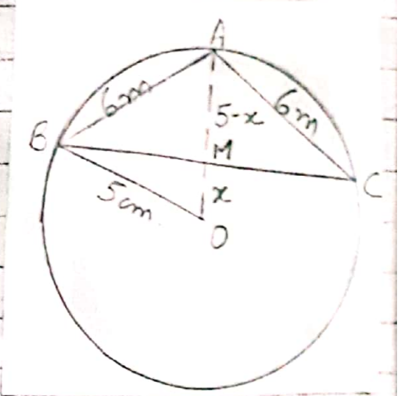
Subtracting (2) from (1).

$AM - BM = MD - MC$

$\Rightarrow AB = CD$.

5. Three girls Reshma, Salma & Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma & Salma and between Salma and Mandip is 6m each, what is distance between Reshma and Mandip?

Sol: Let positions of Reshma, Salma, Mandip be A, B and C.



Given:- $AB = AC = 6m$.

$\therefore$ Centre of the circle bisects $\angle BAC$.

Let M be point of intersection of BC and OA.

AM bisects $\angle CAB$ and $AB = AC$.

$\therefore$ $AM \perp CB$ and M is the mid-point of CB.

Let $OM = x$.

$MA = 5 - x$ ($\because$ Radius $= 5m$)

In $\triangle OMB$ (rht $\angle$ed $\triangle$).

$(OB)^2 = OM^2 + MB^2$.

$5^2 = x^2 + MB^2$.

$\boxed{MB^2 = 5^2 - x^2}$

Now In $\triangle AMB$.

$AB^2 = AM^2 + MB^2$.

$6^2 = (5-x)^2 + 5^2 - x^2$

$36 = 25 + \cancel{x^2} - 10x + 25 - \cancel{x^2}$

$36 - 50 = -10x$

$\cancel{-}14 = \cancel{-}10x$

$x = \frac{14}{10}$

$MB^2 = 5^2 - x^2 = 5^2 - \left(\frac{14}{10}\right)^2 = \left(5 + \frac{14}{10}\right)\left(5 - \frac{14}{10}\right)$

$= \left(\frac{50+14}{10}\right)\left(\frac{50-14}{10}\right) = \frac{64}{10} \times \frac{36}{10}$.

$MB^2 = \frac{64}{10} \times \frac{36}{10}$

$MB = \sqrt{\frac{64 \times 36}{100}} = \frac{8 \times 6}{10} = \frac{48}{10} = 4.8m$.

$BC = 2 \times MB = 2 \times 4.8 = 9.6m$.

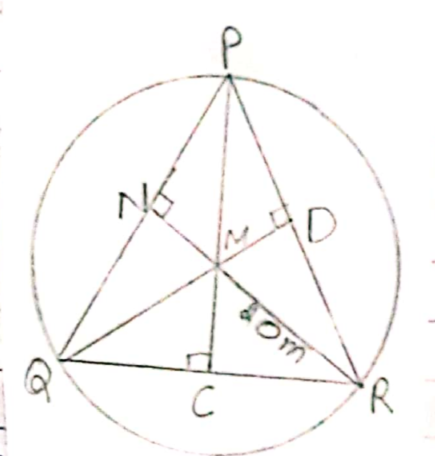
6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol: Let Ankur, Syed and David standing on point P, Q and R

Let $PQ = QR = PR = x$.

$\therefore$ ΔPQR is an equilateral Δ

Drawn altitudes PC, QD and RN from vertices to the sides of a Δ and intersect these altitudes at centre of circle M



ΔPQR is an equilateral Δ,

$\therefore$ these altitudes bisects their sides

In ΔPQC (rht angled)

$$PQ^2 = PC^2 + QC^2$$

$$x^2 = PC^2 + \left(\frac{x}{2}\right)^2 \qquad \left(\because QC = \frac{1}{2}QR = \frac{x}{2}\right).$$

$$PC^2 = x^2 - \frac{x^2}{4} = \frac{4x^2 - x^2}{4} = \frac{3x^2}{4}.$$

$$PC = \frac{\sqrt{3}}{2}x.$$

Now $MC = PC - PM = \frac{\sqrt{3}}{2}x - 20$. $\qquad (\because PM = \text{radius} = 20\,m)$

In rht $\angle$ed ΔQCM.

$$QM^2 = QC^2 + MC^2$$

$$(20)^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}x - 20\right)^2$$

$$\cancel{400} = \frac{x^2}{4} + \frac{3x^2}{4} + \cancel{400} - \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} \times 20x$$

$$0 = \frac{x^2 + 3x^2}{4} - 20\sqrt{3}x$$

$$\cancel{+}\frac{\cancel{4}x^2}{\cancel{4}} = \cancel{+}20\sqrt{3}x$$

$$x^2 = 20\sqrt{3}x \quad \Rightarrow x = 20\sqrt{3}$$

$PQ = QR = PR = x = 20\sqrt{3}\,m$.