# Class XI Session 2023-24
# Subject - Mathematics
# Sample Question Paper - 2

**Time Allowed: 3 hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ’s and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

**Section A**

1. tan 150° = ? **[1]**

a) $\frac{-1}{\sqrt{3}}$ b) $\frac{1}{\sqrt{3}}$

c) $-\sqrt{3}$ d) $\sqrt{3}$

2. Let f(x) = (x - 1) Then, **[1]**

a) f (|x|) = f(x) b) $f(x^2) = (f(x)]^2$

c) None of these d) f (x + y) = f(x) f(y)

3. Two dice are thrown simultaneously. The probability of obtaining total score of seven is **[1]**

a) $\frac{6}{36}$ b) $\frac{8}{36}$

c) $\frac{7}{36}$ d) $\frac{5}{36}$

4. $\lim\limits_{x \to 3} \frac{\sqrt{x^2+10}-\sqrt{19}}{x-3}$ is equal to **[1]**

a) 1 b) $\frac{6}{\sqrt{19}}$

c) $\frac{3}{\sqrt{19}}$ d) 0

5. The two lines ax + by = c and a′x + b′y = c′ are perpendicular if **[1]**

a) ab′ = ba′ b) aa′ + bb′ = 0

c) ab + a′b′ = 0 d) ab′ + ba′ = 0

6. The number of non-empty subsets of the set {1, 2, 3, 4} is: **[1]**

a) 14 b) 16

c) 17 | d) 15

7. Mark the correct answer for $\left(\frac{1-i}{1+i}\right)^2$ = ? **[1]**

a) $\frac{1}{\sqrt{2}}$ | b) -1

c) $\frac{-1}{2}$ | d) 1

8. The range of the function f(x) = |x - 1| is **[1]**

a) R | b) $(-\infty, 0)$

c) $(0, \infty)$ | d) $[0, \infty)$

9. If x belongs to set of integers, A is the solution set of 2(x - 1) < 3x - 1 and B is the solution set of 4x - 3 $\leq$ 8 + x, find A ∩ B **[1]**

a) {0, 2, 4} | b) {1, 2, 3}

c) {0, 1, 2} | d) {0, 1, 2, 3}

10. At 3 : 40, the hour and minute hands of a clock are inclined at **[1]**

a) $\frac{7\pi^c}{18}$ | b) $\frac{2\pi^c}{3}$

c) $\frac{3\pi^c}{18}$ | d) $\frac{13\pi^c}{18}$

11. Let A = {x : x ∈ R, x > 4} and B = {x ∈ R : x < 5}. Then, A ∩ B = **[1]**

a) [4, 5) | b) [4, 5]

c) (4, 5] | d) (4, 5)

12. If in an infinite G.P., first term is equal to 10 times the sum of all successive terms, then its common ratio is **[1]**

a) $\frac{1}{9}$ | b) $\frac{1}{11}$

c) $\frac{1}{10}$ | d) $\frac{1}{20}$

13. $(\sqrt{5}+1)^4 + (\sqrt{5}-1)^4$ is **[1]**

a) an irrational number | b) a negative real number

c) a rational number | d) a negative integer

14. Solve the system of inequalities -2 $\leq$ 6x - 1 < 2 **[1]**

a) $-\frac{1}{6} \leq x < \frac{1}{2}$ | b) $-\frac{1}{6} < x < \frac{3}{2}$

c) none of these | d) $-\frac{1}{7} \leq x > \frac{1}{2}$

15. If A = {1, 3, 5, B} and B = {2, 4}, then **[1]**

a) {4} ⊂ A | b) None of these

c) B ⊂ A | d) 4 ∈ A

16. The value of sec $\theta$ can **[1]**

a) can't lie between -1 and 1 | b) can't be less than 1

c) can't be greater than 1 | d) can't be equal to 1

17. Mark the correct answer for: $i^{326}$ = ? **[1]**

a) -i | b) i

c) -1

d) 1

18. If ${}^nC_{18} = {}^nC_{12}$, then ${}^{32}C_n$ = ? **[1]**

a) None of these

b) 248

c) 992

d) 496

19. **Assertion (A):** The expansion of $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$ . **[1]**

**Reason (R):** If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. **[1]**

**Reason (R):** The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

**Section B**

21. If A = (1, 2, 3), B = {4}, C = {5}, then verify that $A \times (B - C) = (A \times B) - (A \times C)$ . **[2]**

OR

Let A = {-2, -1, 0, 1, 2} and f: A $\rightarrow Z$ be given by f(x) = $x^2$ - 2x - 3 find pre image of 6. -3 and 5.

22. Evaluate: $\lim\limits_{x \rightarrow 0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$. **[2]**

23. Find the eccentricity of an ellipse whose latus rectum is one half of its major axis. **[2]**

OR

Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum: $x^2$ = -16y

24. Write the set in roster form: C = {x : x is a two-digit number such that the sum of its digits is 9}. **[2]**

25. Find the angles between the pairs of straight lines x - 4y = 3 and 6x - y = 11. **[2]**

**Section C**

26. Let A = {1, 2} and B = {2, 4, 6}. Let f = {(x, y) : x $\in$ A, y $\in$ B and y > 2x +1}. Write f as a set of ordered pairs. Show that f is a relation but not a function from A to B. **[3]**

27. Solve systems of linear inequation: $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, x > 0$ **[3]**

28. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(-4, 0, 0) is equal to 10. **[3]**

OR

Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled isosceles triangle.

29. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively. **[3]**

OR

Using g binomial theorem, expand $\left\{(x+y)^5 + (x-y)^5\right\}$ and hence find the value of $\{(\sqrt{2}+1)^5+(\sqrt{2}-1)^5\}$

30. If (a + ib) $= \frac{c+i}{c-i}$, where c is real, prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$.. **[3]**

OR

Evaluate: $\sqrt{5+12i}$.

31. Using the properties of sets and their complements prove that (A ∪ B) - C = (A - C) ∪ (B - C) **[3]**

**Section D**

32. A fair coin is tossed four times, and a person win Rs. 1 for each head and lose Rs. 1.50 for each tail that turns up. **[5]** Form the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

33. Differentiate $\frac{\sin x}{x}$ from first principle. **[5]**

OR

Differentiate log sin x from first principles.

34. In an increasing GP, the sum of the first and last terms is 66, the product of the second and the last but one is 128 **[5]** and the sum of the terms is 126. How many terms are there in this GP?

35. $0 \leq x \leq \pi$ and x lies in the IInd quadrant such that sin x = $\frac{1}{4}$. Find the values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$. **[5]**

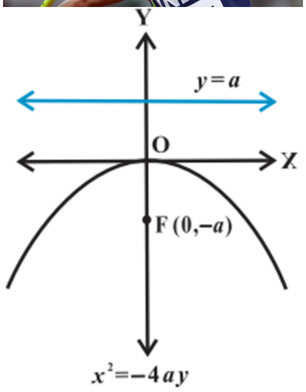
OR

Prove that: sin 20° sin 40° sin 80° = $\frac{\sqrt{3}}{8}$

**Section E**

36. **Read the text carefully and answer the questions:** **[4]**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.


(i) Name the shape of path followed by a javelin. If equation of such a curve is given by $x^2$ = -16y, then find the coordinates of foci.

(ii) Find the equation of directrix and length of latus rectum of parabola $x^2$ = -16y.

(iii) Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y-axis and also find equation of directrix.

**OR**

Find the equation of the parabola with focus (2, 0) and directrix x = -2 and also length of latus rectum.

37. **Read the text carefully and answer the questions:** **[4]**

Consider the data.

| Class | Frequency |
|---|---|
| 0-10 | 6 |
| 10-20 | 7 |

| | |
|---|---|
| 20-30 | 15 |
| 30-40 | 16 |
| 40-50 | 4 |
| 50-60 | 2 |

(i) Find the mean deviation about median.

(ii) Find the Median.

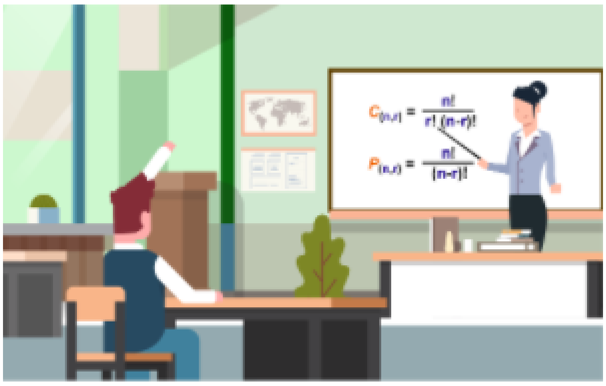
(iii) Write the formula to calculate the Mean deviation about median?

**OR**

Write the formula to calculate median?

38. **Read the text carefully and answer the questions:** **[4]**

During the math class, a teacher clears the concept of permutation and combination to the 11th standard students. After the class was over she asks the students some questions, one of the question was: how many numbers between 99 and 1000 (both excluding) can be formed such that:



(i) How many numbers between 99 and 1000 (both excluding) can be formed such that every digit is either 3 or 7.

(ii) How many numbers between 99 and 1000 (both excluding) can be formed such that without any restriction?

# Solution

**Section A**

1. **(a)** $\frac{-1}{\sqrt{3}}$

   **Explanation:** tan 150° = tan (180° - 30°) = - tan 30° $= \frac{-1}{\sqrt{3}}$

2. 

   **(c)** None of these

   **Explanation:** f(x) = x-1

   $f(x^2) = x^2 - 1$

   $[f(x)]^2 = (x-1)^2$

   $= x^2 + 1 - 2x$

   So, $f(x^2) \neq [f(x)]^2$

   f(x + y) = x + y - 1

   f(x)f(y) = (x - 1)(y - 1)

   So, f(x + y) ≠ f(x) f(y)

   f(|x|) = |x|-1 $\neq$ f(x)

3. **(a)** $\frac{6}{36}$

   **Explanation:** When two dices are thrown, there are (6 $\times$ 6) = 36 outcomes.

   The set of all these outcomes is the sample space given by

   S = (1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

   (2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

   (3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

   (4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

   (5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

   (6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

   $\therefore$ n(S) = 36

   Let E be the event of getting a total score of 7.

   Then E = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

   $\therefore$ n(E) = 6

   Hence, required probability = nEnS = $\frac{6}{36}$

4. 

   **(c)** $\frac{3}{\sqrt{19}}$

   **Explanation:** Using L'Hospital,

   $\lim\limits_{x \to 3} \frac{\frac{2x}{2\sqrt{x^2+10}}}{1}$

   Substituting x = 3 in $\frac{\frac{2x}{2\sqrt{x^2+10}}}{1}$

   We get $\frac{3}{\sqrt{19}}$

5. 

   **(b)** aa′ + bb′ = 0

   **Explanation:** We know that Slope of the line ax + by = c is $\frac{-a}{b}$, and the slope of the line a′x + b′y = c′ is $\frac{-a'}{b'}$ The lines are perpendicular if $\tan\theta = \frac{3}{5-x}$ (1)

   $\frac{-a}{b}\frac{-a'}{b'} = -1$ or $aa' + bb' = 0$

6. 

   **(d)** 15

**Explanation:** Total no. of subset including empty set = $2^n$

So total subset = $2^4$ = 16

The no. of non empty set = 16 - 1 = 15

7.

**(b)** -1

**Explanation:** $\frac{(1-i)}{(1+i)} = \frac{(1-i)}{(1+i)} \times \frac{(1-i)}{(1-i)} = \frac{(1-i)^2}{(1-i^2)} = \frac{1+i^2-2i}{(1+1)} = \frac{1-1-2i}{2} = \frac{-2i}{2}$ = -i

$\Rightarrow \left(\frac{1-i}{1+i}\right)^2 = (-i)^2 = i^2 = -1$

8.

**(d)** $[0, \infty)$

**Explanation:** A modulus function always gives a positive value

R(f) = $[0, \infty)$

9.

**(d)** {0, 1, 2, 3}

**Explanation:** Given 2(x - 1) < 3x - 1

$\Rightarrow$ 2x - 2 < 3x - 1

$\Rightarrow$ 2x - 2 + 2 < 3x - 1 + 2

$\Rightarrow$ 2x < 3x + 1

$\Rightarrow$ 2x - 3x < 3x + 1 - 3x

$\Rightarrow$ -x < + 1

$\Rightarrow$ x > -1 but x $\in$ Z

Hence A = {0, 1, 2, 3, 4, ....}

Now 4x - 3 $\leq$ 8 + x

$\Rightarrow$ 4x - 3 + 3 $\leq$ 8 + x + 3

$\Rightarrow$ 4x $\leq$ 11 + x

$\Rightarrow$ 4x - x $\leq$ 11 + x - x

$\Rightarrow$ 3x $\leq$ 11

$\Rightarrow \frac{3x}{3} \leq \frac{11}{3}$

$\Rightarrow$ x $\leq \frac{11}{3}$

$\Rightarrow$ x $\leq 3\frac{2}{3}$, but x $\in$ Z

Therefore B = {....., -2, -1, 0, 1, 2, 3}

Hence A $\cap$ B = {0, 1, 2, 3}

10.

**(d)** $\frac{13\pi^c}{18}$

**Explanation:** We know, in clock 1 rotation gives $360^o$

i.e. 60 minutes = $360^o$ and 12 hours = $360^o$

So,1 minute = $6^o$ and 1 hour = $30^o$

Now, For hour hand:

3 hours = 3 $\times$ $30^o$ = $90^o$ and for another 40 minute = $(\frac{30^\circ}{60}) \times 40 = 20^o$

i.e. angle traced by hour hand is $90^o + 20^o = 110^o$

Now, for minute hand:

40 minute = 40 $\times$ $6^o$ = $240^o$

i.e. angle traced by minute hand is $240^o$.

So, the angle between hour hand and minute hand = $240^o$ - $110^o$

= $130^o$

$= 130^\circ \times \frac{\pi^c}{180}$

$= \frac{13\pi^c}{18}$

11.

**(d)** (4, 5)

**Explanation:** We have,A ={x : x $\in$ R, x > 4} and B = {x $\in$ R : x < 5}

A $\cap$ B = (4, 5)

12.

**(b)** $\frac{1}{11}$

**Explanation:** Let the first term of the G.P. be a

Let its common ratio be r.

We are given that,

First term = 10 [Sum of all successive terms]

a = $10\left(\frac{ar}{1-r}\right)$

$\Rightarrow$ a - ar = 10ar

$\Rightarrow$ 11ar = a

$\Rightarrow r = \frac{a}{11a} = \frac{1}{11}$

13.

**(c)** a rational number

**Explanation:** We have $(a + b)^n + (a - b)^n$

= $\left[{}^nC_0a^n +^n C_1a^{n-1}b +^n C_2a^{n-2}b^2 +^n C_3 \quad a^{n-3}b^3\right.$ + ..... + ${}^nC_nb^n]$ +

$\left[{}^nC_0a^n -^n C_1a^{n-1}b +^n C_2a^{n-2}b^2 -^n C_3a^{n-3}b^3\right.$ + ..... + $(-1)^n \cdot^n C_n \quad b^n]$

= 2[${}^nC_0 \quad a^n +^n C_2 \quad a^{n-2}b^2 +$ ...]

Let a = $\sqrt{5}$ and b = 1 and n = 4

Now we get $(\sqrt{5}+1)^4 + (\sqrt{5}-1)^4 = 2\left[{}^4C_0(\sqrt{5})^4 +^4 C_2(\sqrt{5})^2 1^2 +^4 C_4(\sqrt{5})^0 1^4\right]$

= 2[25 + 30 + 1] = 112

14. **(a)** $-\frac{1}{6} \leq x < \frac{1}{2}$

**Explanation:** -2 $\leq$ 6x - 1 < 2

$\Rightarrow$ -2 + 1 $\leq$ 6x - 1 + 1 < 2 + 1

$\Rightarrow$ -1 $\leq$ 6x < 3

$\Rightarrow \frac{-1}{6} \leq \frac{6x}{6} < \frac{3}{6}$

$\Rightarrow \frac{-1}{6} \leq x < \frac{1}{2}$

15.

**(b)** None of these

**Explanation:** 4 $\notin$ A

{4} $\not\subset$ A

B $\not\subset$ A

Therefore, we can say that none of these options satisfy the given relation.

16. **(a)** can't lie between -1 and 1

**Explanation:** $|\sec\theta| \geq 1 \Rightarrow (\sec\theta \leq -1)$ or $(\sec\theta \geq 1)$

$\therefore$ value of sec $\theta$ can never lie between - 1 and 1

17.

**(c)** -1

**Explanation:** $i^{326} = (i^4)^{81} \times i^2 = 1^{81} \times (-1) = 1 \times (-1) = -1$

18.

**(d)** 496

**Explanation:** ${}^nC_{18} = {}^nC_{12}$

$\Rightarrow$ n = (18 + 12) = 30

$\therefore {}^{32}C_n = {}^{32}C_{30} = {}^{32}C_2 = \frac{32 \times 31}{2}$ = 496

19.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation: Assertion:**

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

**Reason:**

$(1 + (-1))^n = n_{c_0} 1^n + n_{c_1} (1)^{n-1} (-1)^1 + n_{c_2} (1)^{n-2} (-1)^2 + ... + {}^n c_n (1)^{n-n} (-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3} + ... (-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20.

**(c)** A is true but R is false.

**Explanation: Assertion** Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{4+7+8+9+10+12+13+17}{8} = 10$

| **xi** | **$\|xi - \bar{x}\|$** |
|---|---|
| 4 | \|4 - 10\| = 6 |
| 7 | \|7 - 10\| = 3 |
| 8 | \|8 - 10\| = 2 |
| 9 | \|9 - 10\| = 1 |
| 10 | \|10 - 10\| = 0 |
| 12 | \|12 - 10\| = 2 |
| 13 | \|13 - 10\| = 3 |
| 17 | \|17 - 10\| = 7 |
| $\sum x_i = 80$ | $\sum \|x_i - \bar{x}\| = 24$ |

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{24}{8} = 3$

**Reason** Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{38+70+48+40+42+55}{+63+46+54+44} = 50$

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i - \bar{x}|}{n}$

$= \frac{84}{10} = 8.4$

Hence, Assertion is true and Reason is false.

## Section B

21. As given in the question we have, A = {1, 2, 3}, B = {4} and C = {5}

From set theory, (B - C) = {4}

$\therefore \quad A \times (B - C) = \{1, 2, 3\} \times \{4\}$ = {(1, 4), (2, 4), (3, 4)}........(i)

Now,

$A \times B = \{1, 2, 3\} \times \{4\}$ = {(1, 4), (2, 4), (3, 4)}

and, $A \times C = \{1, 2, 3\} \times \{5\}$ = {(1,5), (2, 5), (3, 5)}

$\therefore \quad (A \times B) - (A \times C)$= {(1, 4), (2, 4), (3, 4)}.......(ii)

From equation (i) and equation (ii), we get

$A \times (B - C) = (A \times B) - (A \times C)$

We can see the equations (i) and (ii) have same ordered pairs.

Hence verified.

OR

From the given we can assume,

Let x be a pre-image of 6 Then

f(x) = 6 = $x^2$ - 2x - 3 = 6 = $x^2$ - 2x - 9 = 0 = x = $1 \pm \sqrt{10}$

Since x = $1 \pm \sqrt{10} \notin A$ so there is nor pre image of 6 f(x) = -3 = $x^2$ - 2x - 3 = -3 = $x^2$ - 2x = 0 = x = 0.2

Clearly, 0.2 $\in A$ So 0 and 2 are pre image of -3

Let x be a pre image of 5 then

f(x) = 5 = $x^2$ - 2x - 3 = 5 = $x^2$ - 2x - 8 = 0 = (x - 4) (x + 2) = 0 = x = 4,

Since, -2A be 4A so, -2 is a pre image of 5

22. To evaluate: $\lim\limits_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$ or $\pm\infty$ then

$\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$

As x $\to$ 0, we have

$\lim\limits_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right) = \frac{0}{0}$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$\lim\limits_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right) = \lim\limits_{x\to 0}\frac{\frac{d}{dx}\left(e^{3x}-e^{2x}\right)}{\frac{d}{dx}(x)}$

$\lim\limits_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right) = \lim_{x\to 0}\frac{3e^{3x}-2e^{2x}}{1}$

$\lim\limits_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$ = 3 - 2

$\lim\limits_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$ = 1

Thus, the value of $\lim\limits_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$ is 1

23. Given that, Length of Latus Rectum $= \frac{1}{2}$ major Axis

Let the equation of the required ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ... (i)

As we know that,

Length of Latus Rectum $= \frac{2b^2}{a}$ and Length of Major Axis = 2a

So, according to the question,

$\frac{2b^2}{a} = \frac{1}{2}\times 2a \Rightarrow \frac{2b^2}{a} = a \Rightarrow$ $2b^2 = a^2$ ... (ii)

$\Rightarrow \mathrm{a} = \sqrt{2\mathrm{b}^2} \Rightarrow \mathrm{a} = \mathrm{b}\sqrt{2}$

Eccentricity $= \frac{c}{a}$ ... (iii)

where, $c^2 = a^2 - b^2$

So, $c^2 = 2b^2 - b^2$ [from (ii)]

$\Rightarrow c^2 = b^2$

Putting the value of c and a in eq. (iii), we get

Eccentricity $= \frac{c}{a} = \frac{b}{\sqrt{2}b} \Rightarrow \mathrm{e} = \frac{1}{\sqrt{2}}$

OR

The given equation of parabola is $x^2$ = 16y which is of the form $x^2$ = -4ay

$\therefore$ 4a = 16 $\Rightarrow$ a = 4

$\therefore$ Coordinates of focus are (0, -4)

Axis of parabola is x = 0

Equation of the directrix is y = 4 $\Rightarrow$y - 4 = 0

Length of latus rectum = 4 $\times$ 4 = 16

24. We have,

9 = 0 + 9, Numbers can be 09, 90

9 = 1 + 8, Numbers can be 18, 81

9 = 2 + 7, Numbers can be 27, 72

9 = 3 + 6, Numbers can be 36, 63

9 = 4 + 5, Numbers can be 45, 54

9 = 5 + 4, Numbers can be 54, 45

The elements of this set are 18, 27, 36, 45, 54, 63, 72, 81 and 90 and

Therefore, C = {18, 27, 36, 45, 54, 63, 72, 81, 90}

25. Given that equations of the lines are,

x - 4y = 3 .... (i)

6x - y = 11 .... (ii)

Let $m_1$ and $m_2$ be the slopes of these lines.

Here, $m_1 = \frac{1}{4}$, $m_2 = 6$

Let $\theta$ be the angle between the lines.

Then,

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{1}{4} - 6}{1 + \frac{3}{2}}\right|$

$= \frac{23}{10}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{23}{10}\right)$

Therefore, the acute angle between the lines is $\tan^{-1}\left(\frac{23}{10}\right)$

**Section C**

26. We have, A = f{1, 2} and B = {2, 4, 6}

Also it is given that, f = {(x, y) : x ∈ A, y ∈ B and y > 2x + 1}.

Put x = 1 in y > 2x + 1, we obtain

y > 2(1) + 1

⇒ y > 3

and y ∈ B

This means y = 4,6 if x = 1 because it satisfies the condition y > 3.

Put x = 2 in y > 2x + 1, we get

y > 2(2) + 1

⇒ y > 5

This means y = 6 if x = 2 because, it satisfies the condition y > 5.

∴ f = {(1, 4),(1, 6),(2, 6)}

(1,2),(2,2),(2,4) are not the members of 'f' because they do not satisfy the given condition y > 2x + 1

Firstly, we have to show that f is a relation from A to B.

First elements in F = 1, 2

All the first elements are in Set A. So, the first element is from set A

Second elements in F = 4, 6

All the second elements are in Set B

So, the second element is from set B

Since the first element is from set A and second element is from set B

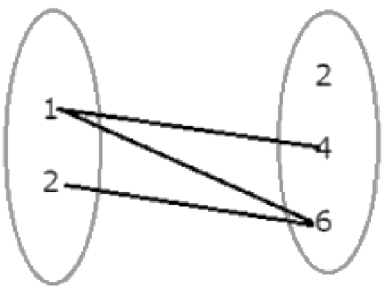
Hence, F is a relation from A to B.

All elements of the first set are associated with the elements of the second set.

i. An element of the first set has a unique image in the second set.

   Now, we have to show that $f$ is not a function from A to B

   f = {(1, 4),(1, 6),(2, 6)}

   f = {(1, 4),(1, 6),(2,6)}

   Here, 1 is coming twice.

Hence, it does not have a unique (one) image.

So, it is not a function.

27. Given that,

$\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, x > 0$

==> 4 $\leq$ 3(x+1) < 6 [multiply by (x+1)]

==> 4 $\leq$ 3x + 3 < 6

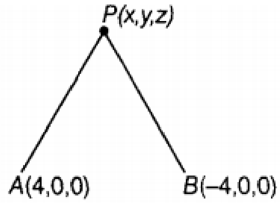
now, 3x + 3 $\geq$ 4 and 3x + 3 < 6

==> 3x $\geq$ 1 and 3x < 3

==> x $\geq \frac{1}{3}$ and x < 1

==> $\frac{1}{3}$ $\leq$ x < 1

28. Let a point P(x, y, z) such that PA + PB = 10




$\Rightarrow \sqrt{(x-4)^2+(y-0)^2+(z-0)^2}+\sqrt{(x+4)^2+(y-0)^2+(z-0)^2}=10$

[$\because$ distance $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$]

$\Rightarrow \sqrt{x^2-8x+16+y^2+z^2}+\sqrt{x^2+8x+16+y^2+z^2}=10$

$\Rightarrow \sqrt{x^2+y^2+z^2-8x+16}=10-\sqrt{x^2+y^2+z^2+8x+16}$

On squaring sides, we get

$x^2 + y^2 + z^2 - 8x + 16 = 100 + x^2 + y^2 + z^2 + 8x + 16$

$-20\sqrt{x^2+y^2+z^2+8x+16}$

$\Rightarrow -16x-100=-20\sqrt{x^2+y^2+z^2+8x+16}$

$\Rightarrow \quad 4x+25=5\sqrt{x^2+y^2+z^2+8x+16}$ [dividing both sides by -4]

Again squaring on both sides, we get

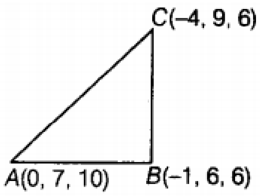
$16x^2 + 200x + 625 = 25(x^2 + y^2 + z^2 + 8x + 16)$

$\Rightarrow 16x^2 + 200x + 625 = 25x^2 + 25y^2 + 25z^2 + 200x + 400$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

OR

Let A (0, 7, 10), B (-1, 6,6) and C (-4, 9, 6) be the given points. We have,




Now, $AB=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$ [$\because$ distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$]

$=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

$BC=\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}$

$=\sqrt{9+9+0}=\sqrt{18}=3\sqrt{2}$

and $AC=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}$

$=\sqrt{16+4+16}$

$\therefore \quad AC=\sqrt{36}=6$ ...... (i)

Now, $AB^2 + BC^2 = (3\sqrt{2})^2+(3\sqrt{2})^2 = 18 + 18 = 36$

$\therefore AB^2 + BC^2 = AC^2$ [from Eq. (i)]

Also, AB = BC $= 3\sqrt{2}$

Hence, ABC is a right isosceles triangle.

29. We have $T_1={}^nC_0a^nb^0=729\ \ldots$ (i)

$T_2={}^nC_1a^{n-1}b=7290\ \ldots$ (ii)

$T_3={}^nC_2a^{n-2}b^2=30375\ \ldots$ (iii)

From (i) $a^n = 729$ . . . (iv)

From (ii) $na^{n-1} b = 7290$ . . . (v)

From (iii) $\frac{n(n-1)}{2}a^{n-2}b^2 = 30375$ . . . (vi)

Multiplying (iv) and (vi), we get

$\frac{n(n-1)}{2}a^{2n-2}b^2 = 729 \times 30375$ . . . (vii)

Squaring both sides of (v) we get

$n^2a^{2n-2}b^2 = (7290)(7290)$(viii)

Dividing (vii) by (viii), we get

$\frac{n(n-1)a^{2n-2}b^2}{2n^2a^{2n-2}b^2} = \frac{729\times 30375}{7290\times 7290}$

$\Rightarrow \frac{(n-1)}{2n} = \frac{30375}{72900} \Rightarrow \frac{n-1}{2n} = \frac{5}{12} \Rightarrow 12n - 12 = 10n$

$\Rightarrow 2n = 12 \Rightarrow n = 6$

From (iv) $a^6 = 729 \Rightarrow a^6 = (3)^6 \Rightarrow a = 3$

From (v) $6 \times 3^5 \times b = 7290 \Rightarrow b = 5$

Thus a = 3, b = 5 and n = 6.

OR

$We\ have$

$(x+y)^5 + (x-y)^5 = 2\left[{}^5C_0\ x^5 + {}^5C_2\ x^3y^2 + {}^5C_4\ x^1y^4\right]$

$= 2\ (x^5 + 10x^3y^2 + 5xy^4$

$Putting\ x = \sqrt{2}\ and\ y = 1,\ we\ get$

$(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5 = 2\left[(\sqrt{2})^5 + 10(\sqrt{2})^3 + 5\sqrt{2}\right]$

$= 2\left[4\sqrt{2} + 20\sqrt{2} + 5\sqrt{2}\right]$

$= 58\sqrt{2}$

30. Here $a + ib = \frac{c+i}{c-i}$

$= \frac{c+i}{c-i} \times \frac{c+i}{c+i} = \frac{(c+i)^2}{c^2-i^2}$

$= \frac{c^2+2ci+i^2}{c^2+1}$

$= \frac{c^2-1}{c^2+1} + \frac{2c}{c^2+1}i$

Comparing real and imaginary parts on both sides, we have

$a = \frac{c^2-1}{c^2+1}$ and $b = \frac{2c}{c^2+1}$

Now $a^2 + b^2 = \left(\frac{c^2-1}{c^2+1}\right)^2 + \left(\frac{2c}{c^2+1}\right)^2$

$= \frac{(c^2-1)^2+4c^2}{(c^2+1)^2} = \frac{(c^2+1)^2}{(c^2+1)^2} = 1$

Also $\frac{b}{a} = \frac{\frac{2c}{c^2+1}}{\frac{c^2-1}{c^2+1}} = \frac{2c}{c^2-1}$

OR

Let, $(a + ib)^2 = 5 + 12i$

$\Rightarrow a^2 + (bi)^2 + 2abi = 5 + 12i$ $[(a + b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow a^2 - b^2 + 2abi = 5 + 12i$ $[i^2 = -1]$

now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 5$…………..eq.1

$\Rightarrow 2ab = 12$

$\Rightarrow a = \frac{6}{b}$……..eq.2

now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{6}{b}\right)^2 - b^2 = 5$

$\Rightarrow 36 - b^4 = 5b^2$

$\Rightarrow b^4 + 5b^2 - 36 = 0$

=> $(b^2 + 9)(b^2 - 4) = 0$

$\Rightarrow b^2 = -9$ or $b^2 = 4$

As b is real no. so, $b^2 = 4$

b = 2 or b= -2

put value of b in equation (2) ===> a = 3 or a= -3

Hence the square root of the complex no. is 3 + 2i and -3 - 2i.

31. $(A \cup B) - C = (A - C) \cup (B - C)$

Let $x \in [(A \cup B) - C]$

$x \in (A \cup B)$ and $x \notin C$

$(x \in A$ or $x \in B)$ and $x \notin C)$

$(x \in A$ and $x \notin C)$ or $(x \in B$ and $x \notin C)$

$x \in \{(A - C)$ or $x \in (B - C)\}$

$x \in \{(A - C) \cup (B - C)\}$

$(A \cup B) - C \subseteq (A - C) \cup (B - C)$ ...(i)

Again, let $y \in [(A - C) \cup (B - C)]$

$y \in (A - C)$ or $y \in (B - C)$

$(y \in A$ and $y \notin C)$ or $(y \in B$ and $y \notin C)$

$(y \in A$ or $y \in B)$ and $y \notin C$

$y \in \{(A \cup B)$ and $y \notin C$

$y \in \{(A \cup B) - C\}$

$(A - C) \cup (B - C) \subseteq (A \cup B) - C$ ...(ii)

From eqs. (i) and (ii),

$(A \cup B) - C = (A - C) \cup (B - C)$ Hence proved

**Section D**

32. Here a coin is tossed four times. So number of elements in the sample space (S) will be $2^4 = 16$. n(S) = 16.

The sample space,

S = {HHHH, HHHT, HHTH, HTHH, HTTH, HTHT, HHTT, HTTT, THHH, THHT, THTH, TTHH, TTTH, TTHT, THTT, TTTT}

Amounts:

i. When 4 heads turns up = Rs( 1 + 1+ 1 + 1)= Rs. 4. i.e., Person wins Rs. 4
ii. When 3 heads and 1 tail turns up = Rs(1+ 1+1 –1.50 =Rs. 1.50. i.e.,Person wins Rs. 1.50
iii. When2 heads and 2 tails turns up = Rs(1 + 1– 1.50– 1.50) =– Rs. 1. i.e., Person losesRs. 1
iv. When1 head and 3 tails turns up = Rs(1– 1.50– 1.50– 1.50 )=– Rs 3.50. i.e., Person losesRs. 3.50
v. When4 tails turns up= Rs(– 1.50– 1.50– 1.50– 1.50) =– Rs 6. i.e., Person loses Rs. 6

Let the events for which the person wins Rs 4, wins Rs 1.50, loses Re1, loses Rs 3.50 and loses Rs 6 be denoted by $E_1$, $E_2$, $E_3$, $E_4$ and $E_5$.

i.e., $E_1$ = {HHHH}, $E_2$ = {HHHT, HHTH, HTHH, THHH} $E_3$ = {HHTT, HTHT, HTTH, THTH, THHT, TTHH}

$E_4$ = {HTTT, TTTH, THTT, TTHT}, $E_5$ = {TTTT}

Here, $n(E_1) = 1$, $n(E_2) = 4$, $n(E_3) = 6$, $n(E_4) = 4$ and $n(E_5) = 1$.

Hence, $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{16}$,

$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{16} = \frac{3}{8}$

$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

and $P(E_5) = \frac{n(E_5)}{n(S)} = \frac{1}{16} = \frac{1}{16}$

33. Let $f(x) = \frac{\sin x}{x}$

By using first principle of derivative,

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\therefore f'(x) = \lim_{h \to 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h}$

$= \lim_{h \to 0} \frac{x \sin(x+h) - (x+h) \sin x}{x(x+h) \times h}$

$= \lim_{h \to 0} \frac{x[\sin(x+h) - \sin x] - h \sin x}{h \cdot x(x+h)}$

$= \lim_{h \to 0} \frac{x\left[2 \cdot \cos\left(\frac{x+h+x}{2}\right) \cdot \sin\left(\frac{x+h-x}{2}\right)\right] - h \sin x}{h \cdot x(x+h)}$

$\left[\because \sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)\right]$

$= \lim_{h \to 0} \frac{x\left[2 \cdot \sin \frac{h}{2} \cdot \cos\left(x + \frac{h}{2}\right)\right] - h \sin x}{h \cdot x(x+h)}$

$= \lim_{\frac{h}{2} \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \cdot \lim_{h \to 0} \frac{\cos\left(x + \frac{h}{2}\right)}{(x+h)} - \lim_{h \to 0} \frac{\sin x}{x(x+h)}$

$= (1) \cdot \frac{\cos x}{x} - \frac{\sin x}{x^2} \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$

$= \frac{\cos x}{x} - \frac{\sin x}{x^2}$

OR

Let f (x) = log sin x. Then, f (x + h) = log sin (x + h)

$\therefore \frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log \sin(x+h) - \log \sin x}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{\frac{\sin(x+h)}{\sin x}\right\}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1 + \frac{\sin(x+h)}{\sin x} - 1\right\}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1 + \frac{\sin(x+h) - \sin x}{\sin x}\right\}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1 + \frac{\sin(x+h) - \sin x}{\sin x}\right\}}{h\left\{\frac{\sin(x+h) - \sin x}{\sin x}\right\}} \times \left\{\frac{\sin(x+h) - \sin x}{\sin x}\right\}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1 + \frac{\sin(x+h) - \sin x}{h}\right\}}{\left\{\frac{\sin(x+h) - \sin x}{h}\right\}} \times \frac{\sin(x+h) - \sin x}{h} \times \frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1 + \frac{\sin(x+h) - \sin x}{h}\right\}}{\left\{\frac{\sin(x+h) - \sin x}{h}\right\}} \times \lim_{h \to 0} \frac{2 \sin \frac{h}{2} \cos\left(x + \frac{h}{2}\right)}{h} \times \frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1 + \frac{\sin(x+h) - \sin x}{h}\right\}}{\left\{\frac{\sin(x+h) - \sin x}{h}\right\}} \times \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right) \cos\left(x + \frac{h}{2}\right)}{\frac{h}{2}} \times \frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = 1 $\times$ cos x $\times$ $\frac{1}{\sin x}$ = cot x.

34. Let the given GP contain n terms. Let abe the first term and r be the common ratio of this GP.

Since the given GP is increasing, we have r > 1

Now, $T_1 + T_n = 66 \Rightarrow a + ar^{(n-1)} = 66$ ...(i)

And, $T_2 \times T_{n-1} = 128 \Rightarrow ar \times ar^{(n-2)} = 128$

$\Rightarrow a^2r^{(n-1)} = 128 \Rightarrow ar^{(n-1)} = \frac{128}{a}$ ...(ii)

Using (ii) and (i), we get

$a + \frac{128}{a} = 66 \Rightarrow a^2 - 66a + 128 = 0$

$\Rightarrow a^2 - 2a - 64a + 128 = 0$

$\Rightarrow a(a - 2) - 64(a - 2) = 0$

$\Rightarrow (a - 2)(a - 64) = 0$

$\Rightarrow a = 2$ or $a = 64$

Putting a = 2 in (ii), we get

$r^{(n-1)} = \frac{128}{a^2} = \frac{128}{4}$ = 32 ...(iii)

Putting a = 64 in (ii), we get

$r^{(n-1)} = \frac{128}{a^2} = \frac{128}{64\times64} = \frac{1}{32}$ , which is rejected, since r > 1.

Thus, a = 2 and $r^{(n-1)}$ = 32

Now, $S_n$ = 126 $\Rightarrow \frac{a(r^n-1)}{(r-1)}$ = 126

$\Rightarrow 2\left(\frac{r^n-1}{r-1}\right)$ = 126 $\Rightarrow \frac{r^n-1}{r-1}$ = 63

$\Rightarrow \frac{r^{(n-1)}\times r-1}{r-1}$ = 63 $\Rightarrow \frac{32r-1}{r-1}$ = 63

$\Rightarrow$ 32r - 1 = 63r - 63 $\Rightarrow$ 31r = 62 $\Rightarrow$ r = 2

$\therefore r^{(n-1)}$ = 32 = 25 $\Rightarrow$ n - 1 = 5 $\Rightarrow$ n = 6

Hence, there are 6 terms in the given GP

35. We know,

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos^2 x = 1 - \left(\frac{1}{4}\right)^2$ ... [$\because \sin x = \frac{1}{4}$]

$\cos^2 x = 1 - \frac{1}{16} = \frac{16-1}{16} = \frac{15}{16}$

$\cos x = \pm\frac{\sqrt{15}}{4}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow$ cos x will be negative in second quadrant

So, $\cos x = -\frac{\sqrt{15}}{4}$

We know,

$\cos 2x = 2\cos^2 x - 1$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$-\frac{\sqrt{15}}{4} = 2\cos^2\frac{x}{2} - 1$ ... [$\because \cos x = -\frac{\sqrt{15}}{4}$]

$2\cos^2\frac{x}{2} = -\frac{\sqrt{15}}{4} + 1 = \frac{-\sqrt{15}+4}{4}$

$\cos^2\frac{x}{2} = \frac{-\sqrt{15}+4}{8}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{-\sqrt{15}+4}{8}}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\cos\frac{x}{2}$ will be positive in first quadrant

So, $\cos\frac{x}{2} = \sqrt{\frac{-\sqrt{15}+4}{8}}$

We know,

$\cos 2x = 1 - 2\sin^2 x$

$\cos x = 1 - 2\sin^2\frac{x}{2}$ ... [$\because \cos x = -\frac{\sqrt{15}}{4}$]

$-\frac{\sqrt{15}}{4} = 1 - 2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} = \frac{\sqrt{15}}{4} + 1 = \frac{\sqrt{15}+4}{4}$

$\sin^2\frac{x}{2} = \frac{\sqrt{15}+4}{8} = \pm\sqrt{\frac{\sqrt{15}+4}{8}}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\sin\frac{x}{2}$ will be positive in first quadrant

So, $\sin\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{8}}$

We know,

$\tan\frac{x}{2} = \frac{\sqrt{\frac{\sqrt{15}+4}{8}}}{\sqrt{\frac{-\sqrt{15}+4}{8}}}$

$\tan\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{8} \times \frac{8}{-\sqrt{15}+4}}$

$\tan\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{-\sqrt{15}+4}}$

On rationalising:

$\tan\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}}$

$\tan\frac{x}{2} = \sqrt{\frac{(4+\sqrt{15})^2}{4^2-(\sqrt{15})^2}}$ ... {$\because$ (a + b)(a - b) = $a^2$ - $b^2$}

$\tan\frac{x}{2} = \sqrt{\frac{(4+\sqrt{15})^2}{16-15}} = \sqrt{\frac{(4+\sqrt{15})^2}{1}} = 4 + \sqrt{15}$

Hence, values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\sqrt{\frac{-\sqrt{15}+4}{8}}$, $\sqrt{\frac{\sqrt{15}+4}{8}}$ and $4 + \sqrt{15}$ respectively

OR

Given, LHS $= sin 20^o sin 40^o sin 80^o$

= $\frac{1}{2}$ [2 sin 20° · sin 40° ] sin $80^o$ [multiplying and dividing by 2]

= $\frac{1}{2}[cos(20^o - 40^o) - cos(20^o + 40^o)] \cdot \sin 80^\circ$ [$\because 2\sin x \cdot \sin y$ = cos (x - y) - cos (x + y)]

= $\frac{1}{2}$ $[cos(-20^o) - cos 60^o] sin 80^o$

= $\frac{1}{2}$ [cos $20^o$ - $\frac{1}{2}$]· sin 80° [$\because$ cos (- $\theta$) = cos $\theta$ and cos $60^o$ = $\frac{1}{2}$]

= $\frac{1}{2} \times \frac{1}{2}\left[2\left(\cos 20^\circ - \frac{1}{2}\right) \cdot \sin 80^\circ\right]$ [again multiplying and dividing by 2]

= $\frac{1}{4}[2\cos 20^\circ \cdot \sin 80^\circ - \sin 80^\circ]$

= $\frac{1}{4}[sin(20^o + 80^o) - sin(20^o - 80^o) - sin 80^o]$ [$\because 2\cos x \cdot \sin y = sin(x + y) - sin(x - y)$]

= $\frac{1}{4}$ $[sin 100^o - sin(-60^o) - sin 80^o]$

= $\frac{1}{4}$ [sin $100^o$ + sin $60^o$ - sin $80^o$] [$\because$ sin (- $\theta$) = - sin $\theta$]

= $\frac{1}{4}$ [sin ($180^o$ - $80^o$) + sin $60^o$ - sin $80^o$] [ $\because$ sin $100^o$ = sin ($180^o$ - $80^o$)]

= $\frac{1}{4}$ [sin $80^o$ + sin $60^o$ - sin $80^o$] [$\because$ sin ($\pi - \theta$) = sin $\theta$]

= $\frac{1}{4}$ × sin $60^o$ = $\frac{1}{4} \times \frac{\sqrt{3}}{2}$ [$\because$ sin $60^o$ = $\frac{\sqrt{3}}{2}$]

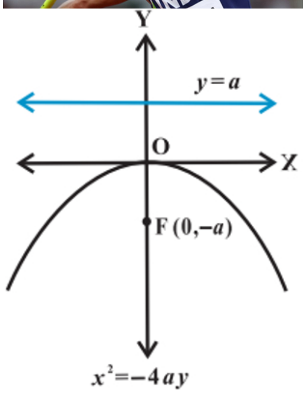
= $\frac{\sqrt{3}}{8}$ = RHS

Hence proved.

**Section E**

36. **Read the text carefully and answer the questions:**

Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.


(i) The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare $x^2$ = -16y with $x^2$ = -4ay

$\Rightarrow$ - 4a = -16

$\Rightarrow$ a = 4

coordinates of focus for parabola $x^2 = -4ay$ is (0, -a)

⇒ coordinates of focus for given parabola is (0, -4)

(ii) compare $x^2 = -16y$ with $x^2 = -4ay$

⇒ $-4a = -16$

⇒ $a = 4$

Equation of directrix for parabola $x^2 = -4ay$ is $y = a$

⇒ Equation of directrix for parabola $x^2 = -16y$ is $y = 4$

Length of latus rectum is $4a = 4 \times 4 = 16$

(iii) Equation of parabola with axis along y - axis

$x^2 = 4ay$

which passes through (5, 2)

⇒ $25 = 4a \times 2$

⇒ $4a = \frac{25}{2}$

hence required equation of parabola is

$x^2 = \frac{25}{2}y$

⇒ $2x^2 = 25y$

Equation of directrix is $y = -a$

Hence required equation of directrix is $8y + 25 = 0$.

OR

Since the focus (2,0) lies on the x-axis, the x-axis itself is the axis of the parabola.

Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$.

Since the directrix is $x = -2$ and the focus is (2,0), the parabola is to be of the form $y^2 = 4ax$ with $a = 2$.

Hence the required equation is $y^2 = 4(2)x = 8x$

length of latus rectum = 4a = 8

37. **Read the text carefully and answer the questions:**

Consider the data.

| **Class** | **Frequency** |
|---|---|
| 0-10 | 6 |
| 10-20 | 7 |
| 20-30 | 15 |
| 30-40 | 16 |
| 40-50 | 4 |
| 50-60 | 2 |

(i) We make the table from the given data.

| Class | $f_i$ | cf | Mid-point($x_i$) | $\|x_i - M\|$ | $f_i\|x_i - M\|$ |
|---|---|---|---|---|---|
| 0-10 | 6 | 6 | 5 | 23 | 138 |
| 10-20 | 7 | 13 | 15 | 13 | 91 |
| 20-30 | 15 | 28 | 25 | 3 | 45 |
| 30-40 | 16 | 44 | 35 | 7 | 112 |
| 40-50 | 4 | 48 | 45 | 17 | 68 |
| 50-60 | 2 | 50 | 55 | 27 | 54 |
| | 50 | | | | 508 |

Here, $\frac{N}{2} = \frac{50}{2} = 25$

Here, 25th item lies in the class 20-30. Therefore, 20-30 is the median class.

Here, l = 20, cf = 13, f = 15, b = 10 and N = 50

$\because$ Median, M = $l+\frac{\frac{N}{2}-cf}{f}\times b$

$\Rightarrow$ M = 20 + $\frac{25-13}{15}\times$ 10 = 20 + 8 = 28

Thus, mean deviation about median is given by

MD (M) = $\frac{1}{N}\sum_{i=1}^{6} f_i\left|x_i-M\right|$ = $\frac{1}{50}\times$ 508 = 10.16

Hence, mean deviation about median is 10.16.

(ii) Here, l = 20, cf = 13, f = 15, b = 10 and N = 50

$\because$ Median, M = $l+\frac{\frac{N}{2}-cf}{f}\times b$

$\Rightarrow$ M = 20 + $\frac{25-13}{15}\times$ 10 = 20 + 8 = 28

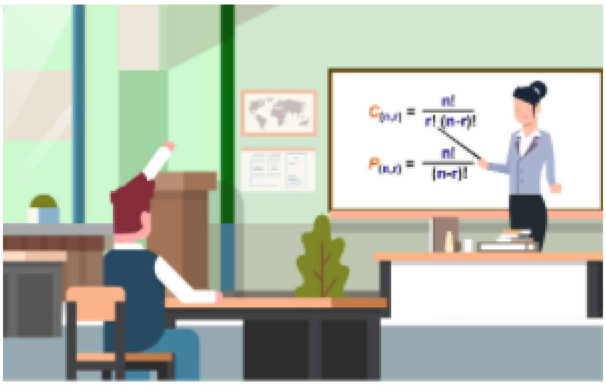
(iii) MD = $\frac{\Sigma f_i|x_i-M|}{N}$

OR

M = $1+\frac{\frac{N}{2}-cf}{f}\times h$

38. **Read the text carefully and answer the questions:**

During the math class, a teacher clears the concept of permutation and combination to the 11th standard students. After the class was over she asks the students some questions, one of the question was: how many numbers between 99 and 1000 (both excluding) can be formed such that:




(i) Here we need to get a 3-digit number

Three vacant paces are fixed with 3 or 7. Therefore, by the multiplication principle, the required number of three-digit numbers with every digit 3 or 7 3 or 7 is 2 $\times$ 2 $\times$ 2 =8

(ii) Three vacant paces are fixed with all 10 digits, but first place is fixed with 9 digits excluding 0.

Therefore, by the multiplication principle, the required number of three digits numbers without any restriction = 9 $\times$ 10 $\times$ 10 = 900