# Chapter 4 Matrices and Determinants

## Ex 4.10

### Answer 1e.

We know that linear regression is performed on a data set to find a linear model for the data and the model given by quadratic regression is called the best-fitting quadratic model.

Thus, the given statement can be completed as "When you perform quadratic regression on a set of data, the quadratic model obtained is called the **best-fitting quadratic model**."

### Answer 1gp.

We know that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is $(h, k)$. It is given that the vertex is $(4, -5)$.

Thus, $y = a(x - 4)^2 - 5$.

Substitute 2 for $x$, and $-1$ for $y$ to find $a$ since the graph passes through $(2, -1)$.
$-1 = a(2 - 4)^2 - 5$

Solve for $a$.

$$-1 = a(-2)^2 - 5$$
$$-1 = 4a - 5$$
$$4 = 4a$$
$$1 = a$$

Replace $a$ with 1 in $y = a(x - 4)^2 - 5$.
$y = 1(x - 4)^2 - 5$.

The quadratic function is $y = (x - 4)^2 - 5$.

### Answer 1mr.

**a.** First, factor out $-0.12$ from the first two terms on the right side.

$$y = -0.12\left(x^2 - 10x\right) + 2$$

Prepare to complete the square.

$$y + ? = -0.12\left(x^2 - 10x + ?\right) + 2$$

Square half the coefficient of $x$.

$$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$$

Now, complete the square. For this, add $-0.12(25)$ to each side of the equation.

$$y + (-0.12)(25) = -0.12(x^2 - 10x) + 2 + (-0.12)(25)$$
$$y - 3 = -0.12(x^2 - 10x + 25) + 2$$

Write $x^2 - 10x + 25$ as a binomial squared.
$y - 3 = -0.12(x - 5)^2 + 2$

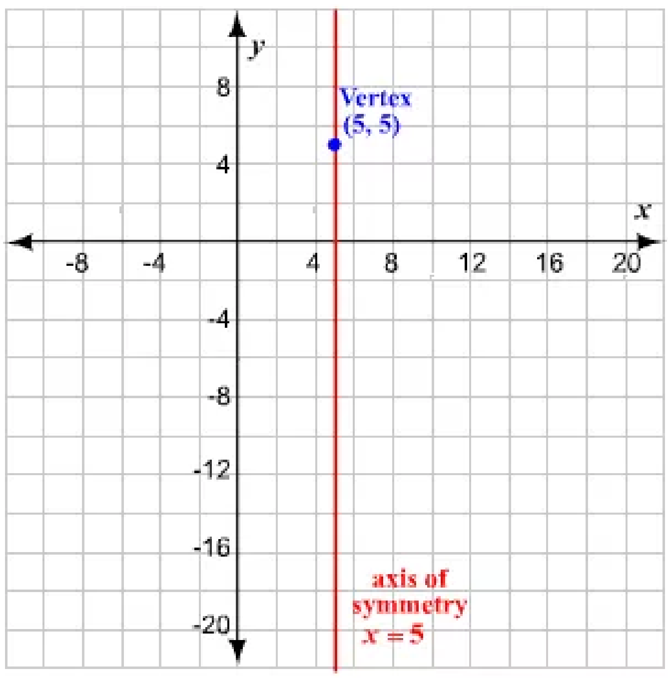
Solve for $y$. For this, add 3 to each side.

$$y - 3 + 3 = -0.12(x - 5)^2 + 2 + 3$$
$$y = -0.12(x - 5)^2 + 5$$

A quadratic function in the form $y = a(x - h)^2 + k$ is said to be in vertex form, where $(h, k)$ is the vertex.
Thus, the given function in vertex form is $y = -0.12(x - 5)^2 + 5$.

**b.** On comparing the equation with the vertex form, we find that $a = -0.12$, $h = 5$, and $k = 5$. Thus, the vertex is $(h, k) = (5, 5)$ and the axis of symmetry is $x = 5$.
Since $a < 0$, the parabola opens down.

Plot the vertex (5, 5) on a coordinate plane and draw the axis of symmetry $x = 5$.

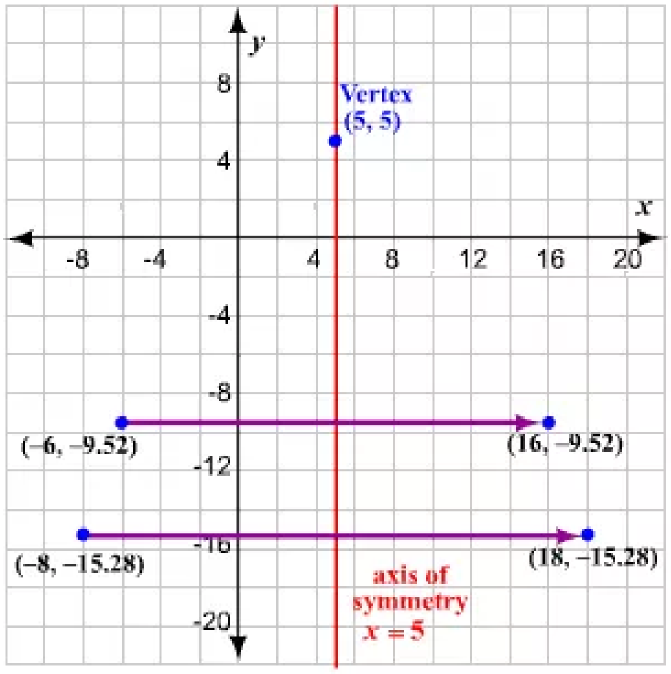



Evaluate the function for two values of $x$.

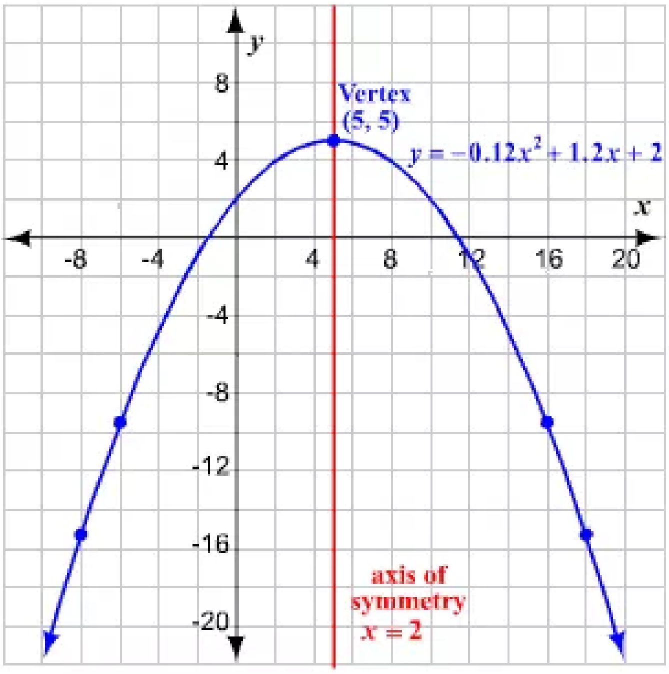
$$x = -8: \quad y = -0.12(-8 - 5)^2 + 5 = -15.28$$
$$x = -6: \quad y = -0.12(-6 - 5)^2 + 5 = -9.52$$

Thus, $(-8, -15.28)$ and $(-6, -9.52)$ are two points on the graph.

Now, plot the points (–8, –15.28) and (–6, –9.52) and their reflections in the axis of symmetry.




Connect the points with a smooth curve.




**c.** We know that the maximum height of the bean bag is the $y$-coordinate of the vertex of the graph of the function.
The vertex of the function's graph is (5, 5). Therefore, the maximum height of the bean bag is 5 ft.

## Answer 1q.

The given equation is in standard form.
The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$, where $a$, $b$, $c$ are real numbers and $a \neq 0$.

Substitute 1 for $a$, $-4$ for $b$, and 5 for $c$ in the formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

Evaluate.

$$\begin{aligned} x &= \frac{4 \pm \sqrt{16 - 20}}{2} \\ &= \frac{4 \pm \sqrt{-4}}{2} \\ &= \frac{4 \pm 2\sqrt{-1}}{2} \end{aligned}$$

We know that $\sqrt{-1} = i$. Thus,

$$\frac{4 \pm 2\sqrt{-1}}{2} = \frac{4 \pm 2i}{2}.$$

Simplify.

$$\frac{4 \pm 2i}{2} = 2 \pm i$$

Therefore, the solutions are $2 \pm i$.

The solutions can be checked using a graphing utility.

## Answer 2e.

The standard form of a quadratic function is $y = ax^2 + bx + c$.

Take the three points on the parabola that are not the vertex or $x-$ -intercepts.
Substitute the coordinates of these three points in the standard form of quadratic function and then will get system of three linear equations in three unknowns $a, b$ and $c$.

If we solve this system, then we can get a solution for $a, b$ and $c$.

Hence, we can obtain the required quadratic function in standard form.

## Answer 2gp.

Consider the vertex $(-3,1)$ and passes through $(0,-8)$.

Write a quadratic function.

The quadratic function in vertex form is $y=a(x-h)^2+k$ where the vertex is $(h,k)$.

Substitute the vertex $(h,k)=(-3,1)$,

$$y=a(x-h)^2+k$$
$$y=a(x+3)^2+1$$

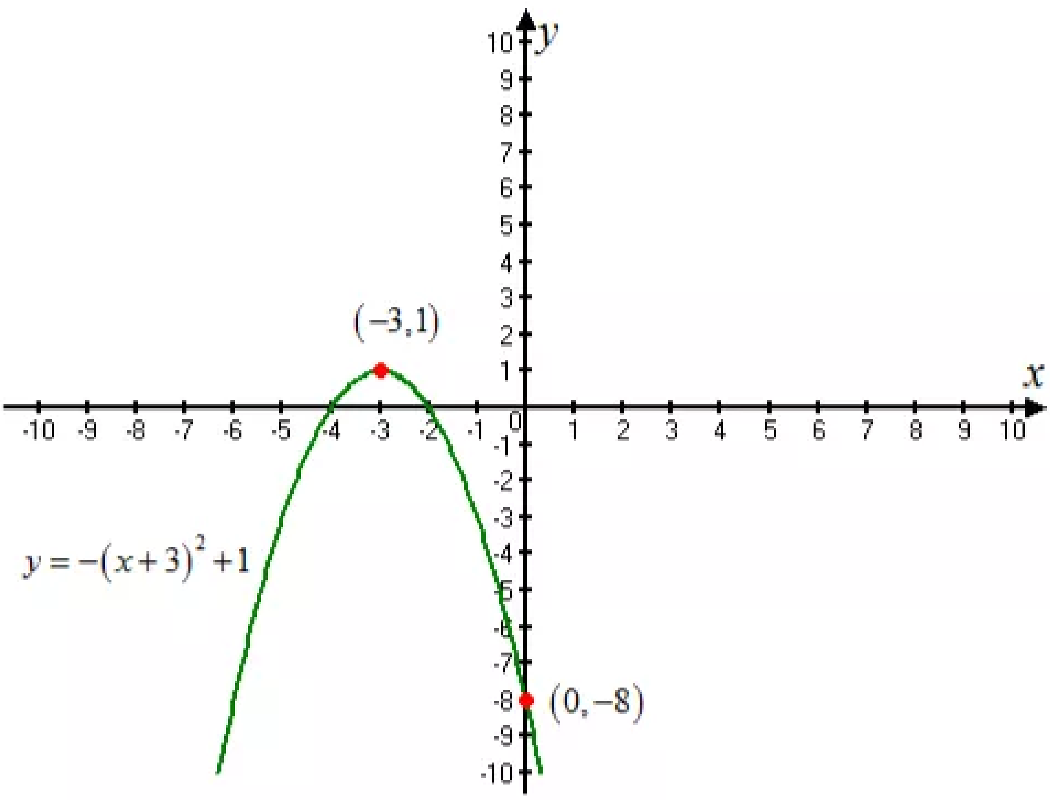
The quadratic function passes through the point $(0,-8)$.

Substitute the value $(x,y)=(0,-8)$,

$$y=a(x+3)^2+1$$
$$-8=a(0+3)^2+1$$
$$-8=9a+1$$
$$a=-1$$

Hence, the quadratic function is $y=-(x+3)^2+1$.

The graph of the function is as shown below.



## Answer 2mr.

Let $x$ represent the price decrease and $R(x)$ represent the monthly sale of drums.

We write a verbal model. Then write and simplify a quadratic function.

$$\begin{pmatrix}\text{Monthly revenue}\\ \text{dollars}\end{pmatrix} = (\text{Number of drum}) \cdot \begin{pmatrix}\text{Drum price}\\ \text{dollars/drum}\end{pmatrix}$$

a.

So, we have

$$R(x) = (40+2x)\cdot(120-x)$$

$$R(x) = (40+2x)(120-x)$$

$$R(x) = 4800 - 40x + 240x - 2x^2 \quad [\text{Multiply using FOIL method}]$$

$$R(x) = -2x^2 + 200x + 4800 \quad [\text{Combine like terms}]$$

Hence, the function that models the store's revenue from sales of the new drum model is

$$\boxed{R(x) = -2x^2 + 200x + 4800}$$

b.

We want to write the inequality we can use to find the prices that result in revenues over \$4830.

So, we have

$$R(x) > 4830$$

$$-2x^2 + 200x + 4800 > 4830 \quad [\text{Substitute } -2x^2 + 200x + 4800 \text{ for } R(x)]$$

$$-2x^2 + 200x + 4800 - 4830 > 4830 - 4830 \quad [\text{Subtract 4830 from each side}]$$

$$-2x^2 + 200x - 30 > 0$$

Hence, the inequality we can use to find the prices that result in revenues over \$4830 is

$$\boxed{-2x^2 + 200x - 30 > 0}.$$

c.

We first, write and solve the equation obtained by replacing $>$ with $=$ as given below.

$$-2x^2 + 200x - 30 = 0 \quad \begin{bmatrix}\text{Write equation that corresponds}\\ \text{to original inequality}\end{bmatrix}$$

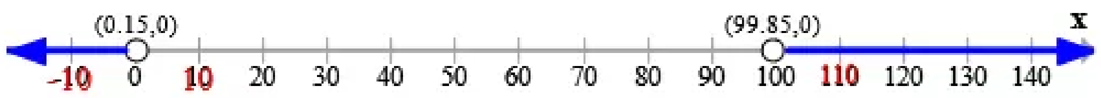
$$2x^2 - 200x + 30 = 0 \quad [\text{Multiply each side by } -1]$$

$$x^2 - 100x + 15 = 0 \quad [\text{Divide each side by 2}]$$

$$x = \frac{100 \pm \sqrt{(-100)^2 - 4(1)(15)}}{2(1)} \quad [\text{Use quadratic formula}]$$

$$x \approx 0.15, 99.85 \quad [\text{Simplify}]$$

The numbers 0.15 and 99.85 are the critical $x$-values of the inequality $-2x^2+200x-30>0$.
We plot 0.15 and 99.85 on a number line, using open dots because the values do not satisfy the inequality.
The critical $x$-values partition the number line into three intervals.
We test an $x$-value in each interval to see if it satisfies the inequality.




Test for $x=10$.

$$-2(10)^2+200(10)-30>0$$
$$-200+2000-30>0$$
$$1770>0 \qquad [\text{True statement}]$$

Test for $x=-10$

$$-2(-10)^2+200(-10)-30>0$$
$$-200-2000-30>0$$
$$-2230>0 \qquad [\text{False statement}]$$

Test for $x=110$.

$$-2(110)^2+200(110)-30>0$$
$$-12100+22000-30>0$$
$$9870>0 \qquad [\text{True statement}]$$

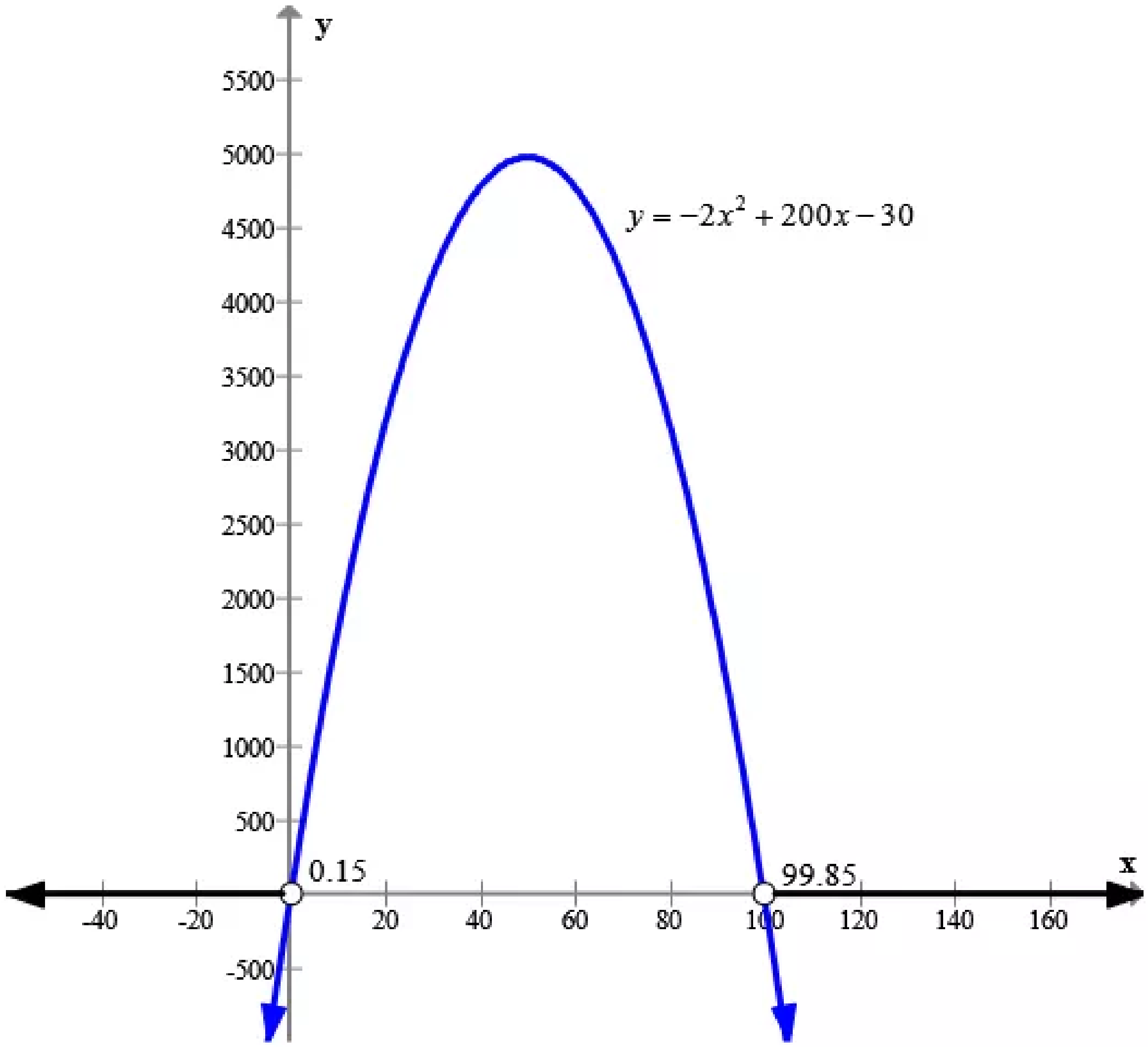
Hence, the solution of the inequality is approximately

$\boxed{x<0.15 \text{ or } x>99.85}$.

The solution consists of the $x$-values for which the graph of $y=-2x^2+200x-30$ lies on or above the $x$-axis.
We find the graph's $x$-intercepts by letting $y=0$ and using the quadratic formula to solve for $x$.
So, we get from the result

$x \approx 0.15, 99.85$

We sketch a parabola that opens down and has 0.15 and 99.85 as $x$-intercepts as shown below.




The graph lies on or above the $x$-axis to the left of (excluding) $x = 0.15$ and to the right of (excluding) $x = 99.85$.
Hence, the solution of the inequality is approximately

$$\boxed{x < 0.15 \text{ or } x > 99.85}$$

## Answer 2q.

Use the quadratic formula to solve the equation $2x^2 - 8x + 1 = 0$.

Compare the above equation with the standard form $ax^2 + bx + c = 0$.

$a = 2, b = -8, c = 1$

Therefore the roots are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{8 \pm \sqrt{64 - 4(2)(1)}}{4}$$

$$x = \frac{8 \pm \sqrt{56}}{4}$$

$$x = \frac{8 \pm 2\sqrt{14}}{4}$$

$$x = 2 \pm \frac{1}{2}\sqrt{14}$$

The solutions of $2x^2 - 8x + 1 = 0$ are $\boxed{x = 2 + \frac{1}{2}\sqrt{14} \text{ and } x = 2 - \frac{1}{2}\sqrt{14}}$.

## Answer 3e.

We know that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is $(h, k)$. It is given that the vertex is (3, 2).

Thus,

$y = a(x - 3)^2 + 2$.

Since the graph passes through (5, 6), substitute 5 for $x$, and 6 for $y$ in equation (1).

$6 = a(5 - 3)^2 + 2$

Solve for $a$.

$6 = 4a + 2$

$4 = 4a$

$1 = a$

Replace $a$ with 1 in $y = a(x - 3)^2 + 2$.

$y = (x - 3)^2 + 2$

The quadratic function is $y = (x - 3)^2 + 2$.

## Answer 3gp.

We know that the intercept form of a quadratic function is $y = a(x - p)(x - q)$, where $p$ and $q$ are the intercepts.

It is given that the $x$-intercepts are $-2$ and 5. Substitute $-2$ for $p$, and 5 for $q$.

$y = a[x - (-2)](x - 5)$

Simplify.

$y = a(x + 2)(x - 5)$

The parabola passes through the point (6, 2). In order to find the value of $a$, substitute 6 for $x$, and 2 for $y$ in the equation.

$2 = a(6 + 2)(6 - 5)$

Solve for $a$.

$$2 = a(8)(1)$$

$$2 = 8a$$

$$\frac{1}{4} = a$$

Substitute the value for $a$ in $y = a(x + 2)(x - 5)$.

$$y = \frac{1}{4}(x + 2)(x - 5)$$

Expand.

$$y = \frac{1}{4}(x^2 - 3x - 10)$$

The required quadratic equation is $y = \frac{1}{4}(x^2 - 3x - 10)$.

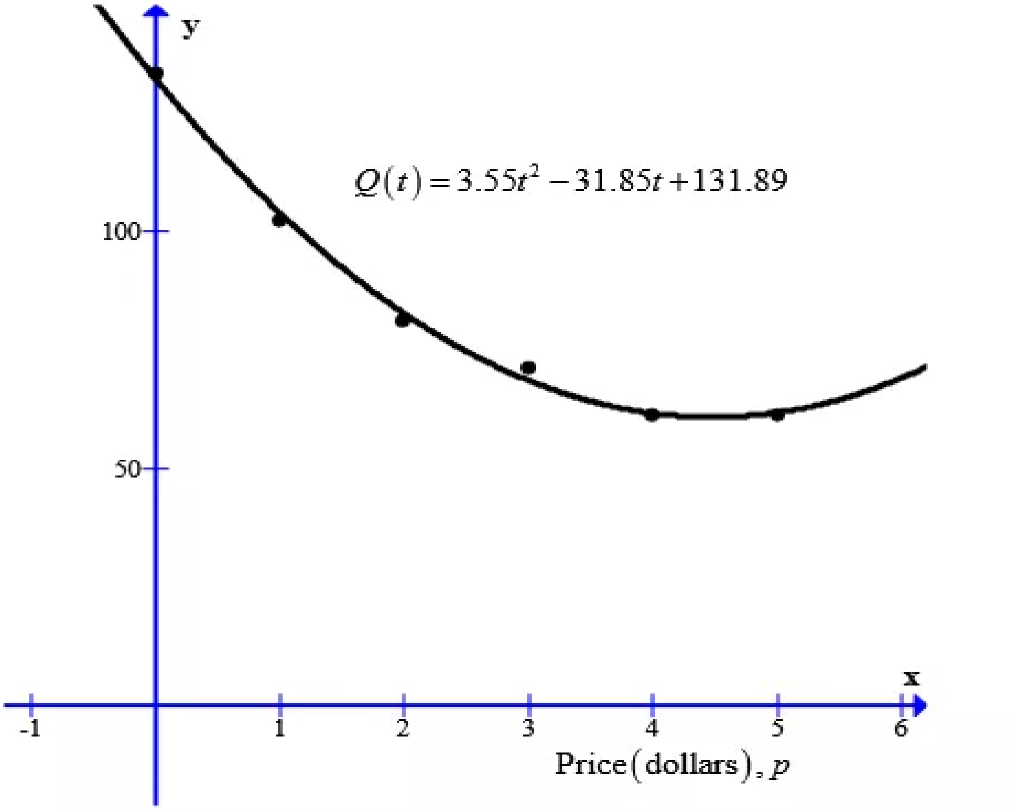
## Answer 3mr.

The table below shows the average price of a VCR from 1998 through 2003.

| Years since 1998, $t$ | Price(dollars), $p$ |
|---|---|
| 0 | 133 |
| 1 | 102 |
| 2 | 81 |
| 3 | 71 |
| 4 | 61 |
| 5 | 61 |

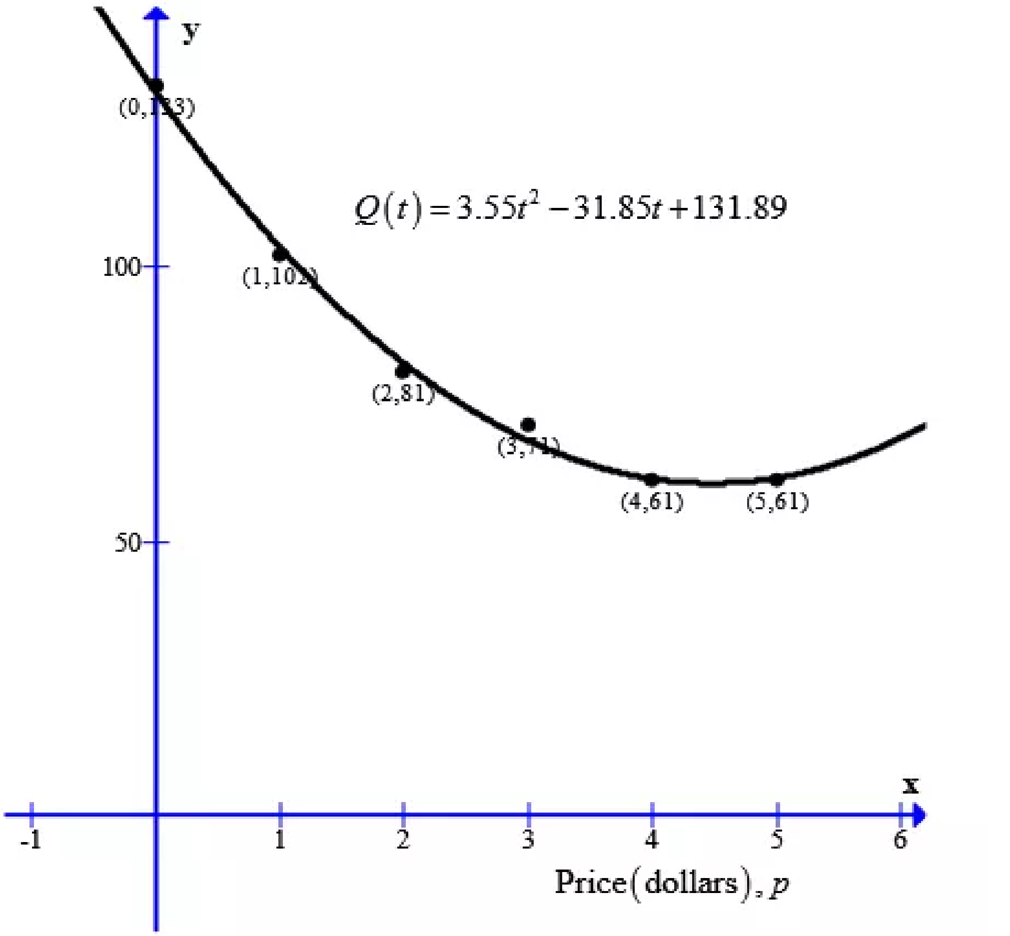
(a)

The best fitting quadratic model of the data is given by the quadratic equation is $Q(t)=3.55t^2-31.85t+131.89$ And the model is shown below:




(b)

The graph of the model with the data together is shown below:



(c)

Since, the quadratic model is $Q(t)=3.55t^2-31.85t+131.89$ ,where $t$ stands for years, like $t=0$ represents 1998.

Therefore, the price of a VCR in 2010 is given by:

When $x=12$ then

$$\begin{aligned} Q(t) &= 3.55t^2-31.85t+131.89 \\ &= 3.55(12)^2-31.85(12)+131.89 \\ &= 260.89 \end{aligned}$$

That is the price of a VCR in 2010 is $\boxed{\$260.89}$ which is a good estimation.

## Answer 3q.

The given equation is in standard form.

The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$, where $a$, $b$, $c$ are real numbers and $a \neq 0$.

Substitute 3 for $a$, 5 for $b$, and 4 for $c$ in the formula.

$$x = \frac{-5 \pm \sqrt{5^2-4(3)(4)}}{2(3)}$$

Evaluate.

$$x = \frac{-5 \pm \sqrt{25-48}}{6}$$

$$= \frac{-5 \pm \sqrt{-23}}{6}$$

We know that $\sqrt{-1} = i$. Thus,

$$\frac{-5 \pm \sqrt{-23}}{6} = \frac{-5 \pm i\sqrt{23}}{6}.$$

Therefore, the solutions are $\frac{-5 \pm i\sqrt{23}}{6}$.

The solutions can be checked using a graphing utility.

## Answer 4e.

Consider the vertex $(-2,1)$ and passes through $(-1,-1)$.

Write a quadratic function.

The quadratic function in vertex form is $y=a(x-h)^2+k$ where the vertex is $(h,k)$.

Substitute the vertex $(h,k)=(-2,1)$,

$$y=a(x-h)^2+k$$
$$y=a(x+2)^2+1$$

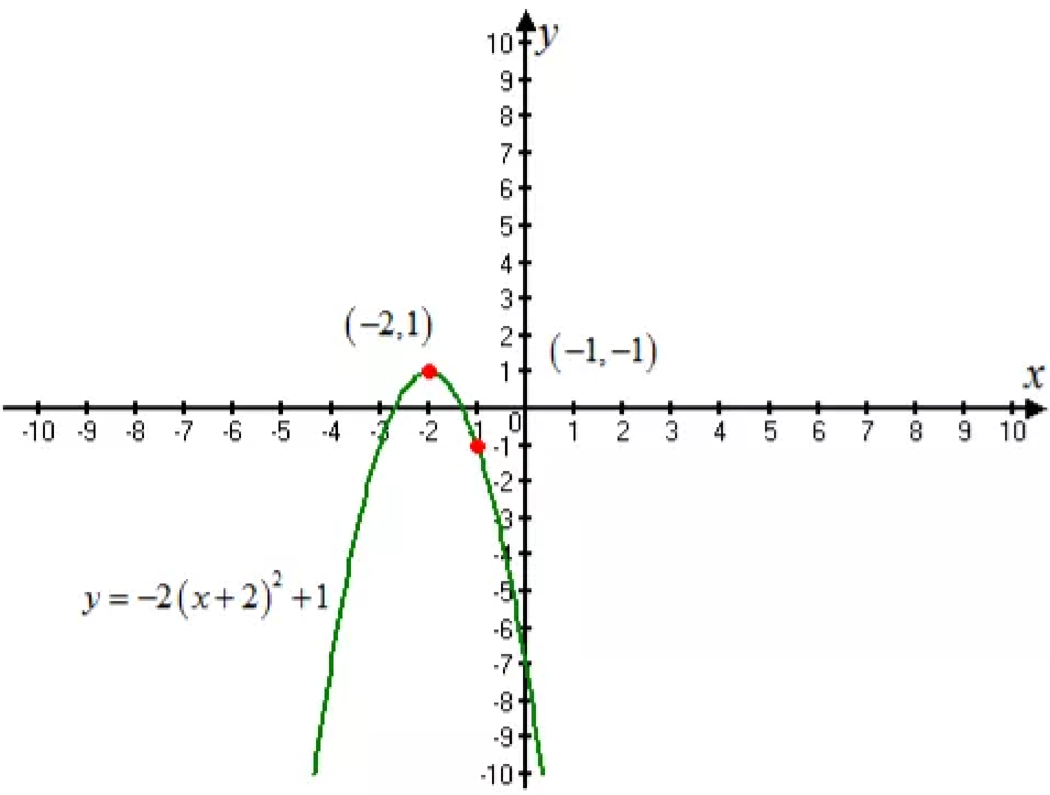
The quadratic function passes through the point $(-1,-1)$.

Substitute the value $(x,y)=(-1,-1)$,

$$y=a(x+2)^2+1$$
$$-1=a(-1+2)^2+1$$
$$-1=a+1$$
$$a=-2$$

Hence, the quadratic function is $y=-2(x+2)^2+1$.

The graph of the function is as shown below.



## Answer 4gp.

Consider the points $(-1,5),(0,-1),(2,11)$.

Write a quadratic function.

The standard form of a quadratic function is $y = ax^2 + bx + c$.

The quadratic function passes through the points $(-1,5)$,

$$y = ax^2 + bx + c$$
$$5 = a(-1)^2 - b + c$$
$$5 = a - b + c \qquad ......(1)$$

The quadratic function passes through the points $(0,-1)$,

$$y = ax^2 + bx + c$$
$$-1 = a(0)^2 + b(0) + c$$
$$-1 = c$$

The quadratic function passes through the points $(2,11)$,

$$y = ax^2 + bx + c$$
$$11 = a(2)^2 + b(2) + c$$
$$11 = 4a + 2b + c \qquad ......(2)$$

Multiply by 2 to equation – (1) and the add to equation – (2),

$$2(5 = a - b + c) \quad \rightarrow \quad 2a - 2b + 2c = 10$$
$$11 = 4a + 2b + c \quad \rightarrow \quad \underline{4a + 2b + c \ = 11} \quad (\text{Add})$$
$$6a + 3c = 21 \qquad ......(3)$$

Substitute the value $c = -1$ in equation – (3),

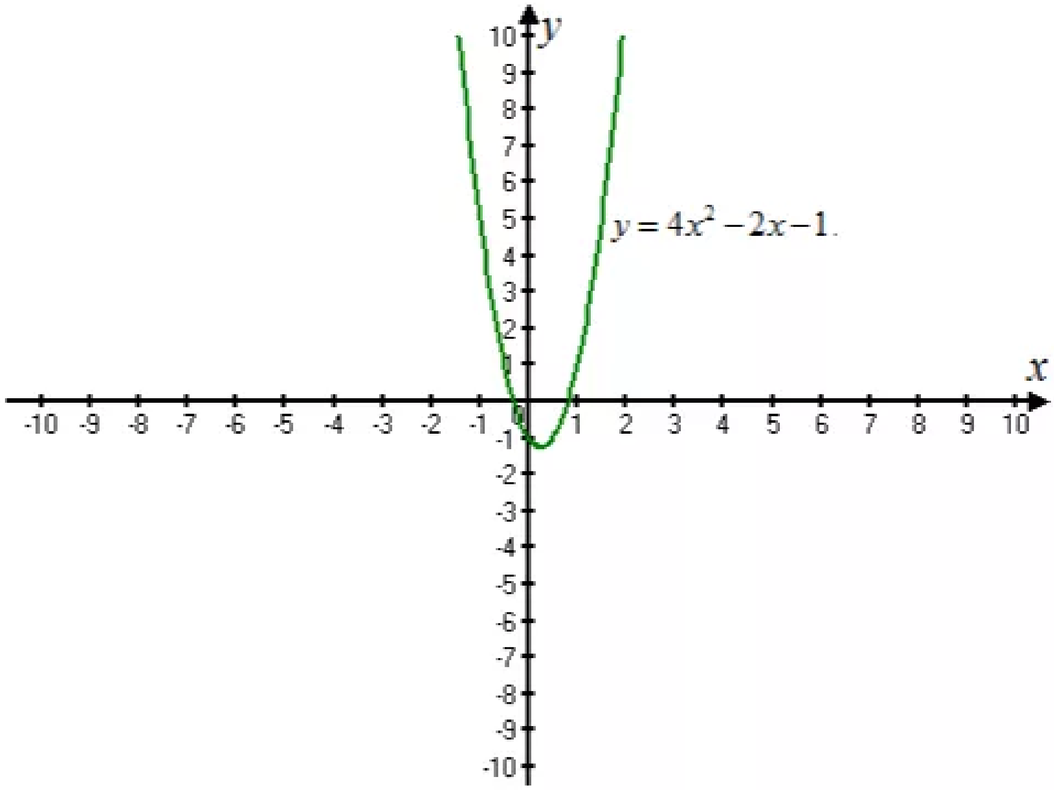
$$6a + 3c = 21$$
$$6a + 3(-1) = 21$$
$$6a = 24$$
$$a = 4$$

Substitute the values $c = -1, a = 4$ in equation – (1),

$$a - b + c = 5$$
$$4 - b - 1 = 5$$
$$-b + 3 = 5$$
$$b = -2$$

Hence, the required quadratic equation is $y = 4x^2 - 2x - 1$.

The graph of the function is as shown below.

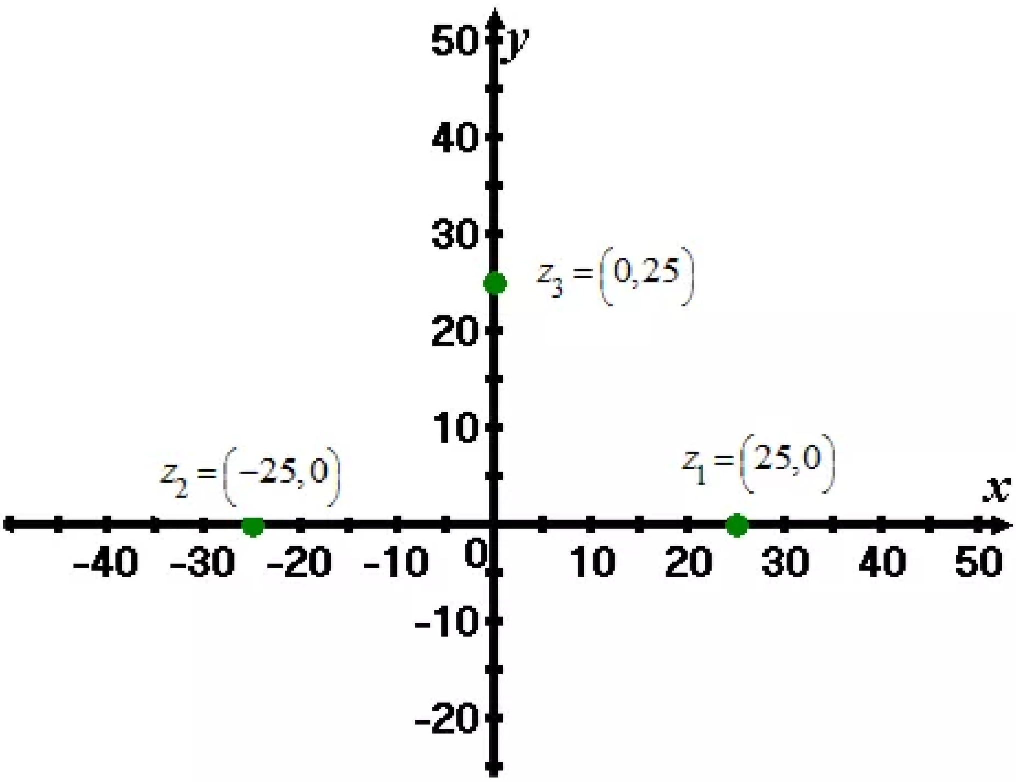



**Answer 4mr.**

$z_1 = 25, z_2 = -25, z_3 = 25i$ are three distinct complex numbers with absolute value $25$.

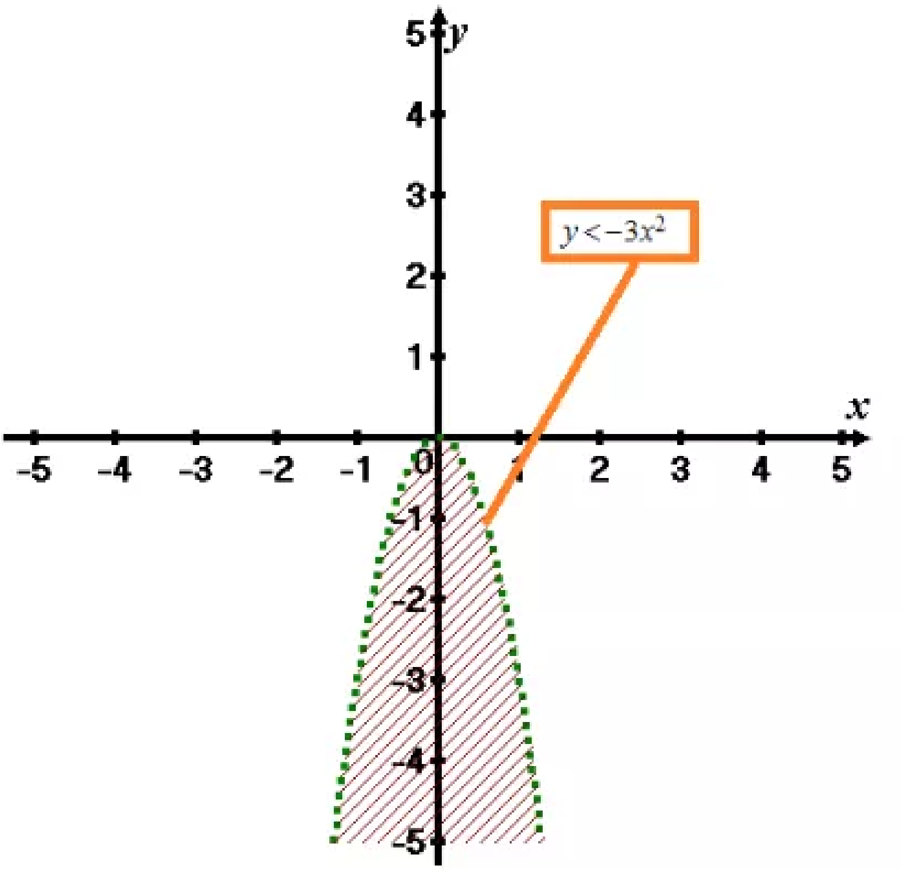
These numbers can be written in the form $(25,0),(-25,0)$ and $(0,25)$.

The following diagram contains the above three complex numbers $z_1, z_2$ and $z_3$.



## Answer 4q.

The graph of the inequality $y < -3x^2$ is shown below:




## Answer 5e.

We know that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is $(h, k)$. It is given that the vertex is $(-1, -3)$.

Thus,
$y = a(x + 1)^2 - 3$.

Substitute 1 for $x$, and $-1$ for $y$ since the graph passes through $(-1, -3)$.
$-1 = a(1 + 1)^2 - 3$

Solve for $a$.

$$-1 = 4a - 3$$
$$2 = 4a$$
$$\frac{1}{2} = a$$

Replace $a$ with $\frac{1}{2}$ in $y = a(1+1)^2 - 3$.

$$y = \frac{1}{2}(x+1)^2 - 3$$

The quadratic function is $y = \frac{1}{2}(x+1)^2 - 3$.

**Answer 5gp.**

***Step 1*** **Substitute** the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute $-2$ for $x$, and $-1$ for $y$ in $y = ax^2 + bx + c$.

$$-1 = a(-2)^2 + b(-2) + c$$

Simplify.

$$-1 = 4a - 2b + c \quad \text{Equation 1}$$

Replace $x$ with 0, and $y$ with 3 in $y = ax^2 + bx + c$ and simplify.

$$3 = a(0)^2 + b(0) + c$$

$$3 = c \quad \text{Equation 2}$$

Substitute 4 for $x$, and 1 for $y$ in $y = ax^2 + bx + c$ and simplify.

$$1 = a(4)^2 + b(4) + c$$

$$1 = 16a + 4b + c \quad \text{Equation 3}$$

***Step 2*** **Rewrite** the system of three equations as a system of two equations.

Substitute for $c$ in Equation 1 and simplify.

$$-1 = 4a - 2b + 3$$

$$-4 = 4a - 2b \quad \text{Revised Equation 1}$$

Now, substitute for $c$ in Equation 3 and simplify.

$$1 = 16a + 4b + 3$$

$$-2 = 16a + 4b \quad \text{Revised Equation 3}$$

***Step 3*** **Solve** the system consisting Revised Equation 1 and Revised Equation 3.

Add 2 times Revised Equation 1 to Revised Equation 3.

$$-4 = 4a - 2b \quad \times \quad 2 \qquad -8 = 8a - 4b$$

$$-2 = 16a + 4b \qquad \underline{-2 = 16a + 4b}$$

$$-10 = 24a \quad \text{Equation 4}$$

Divide each side of the equation by 24.

$$\frac{-10}{24} = \frac{24a}{24}$$

$$-\frac{5}{12} = a$$

Now, substitute the values for $a$ and $c$ in Equation 1 to find the value of $b$.

$$-1 = 4\left(-\frac{5}{12}\right) - 2b + 3$$

Solve for $b$.

$$-1 = -\frac{5}{3} - 2b + 3$$
$$-\frac{7}{3} = -2b$$
$$\frac{7}{6} = b$$

Substitute the values for $a$, $b$, and $c$ in $y = ax^2 + bx + c$.

$$y = -\frac{5}{12}x^2 + \frac{7}{6}x - 3$$

The required quadratic equation is $y = -\frac{5}{12}x^2 + \frac{7}{6}x - 3$.

## Answer 5mr.

We know that the conjugate of $5 - 9i$ is $5 + 9i$. The product of $5 - 9i$ and its conjugate can be written as $(5 - 9i)(5 + 9i)$.

Apply the FOIL method and multiply.

$$(5 - 9i)(5 + 9i) = (5)(5) + (5)(9i) + (-9i)(5) + (-9i)(9i)$$
$$= 25 + 40i - 40i + 81i^2$$

We know that $i^2 = -1$. Thus,
$25 + 40i - 40i + 81i^2 = 25 + 40i - 40i + 81(-1)$.

Simplify.
$25 + 40i - 40i + 81(-1) = -56$

Thus, the product of $5 - 9i$ and its conjugate is $-56$.

## Answer 5q.

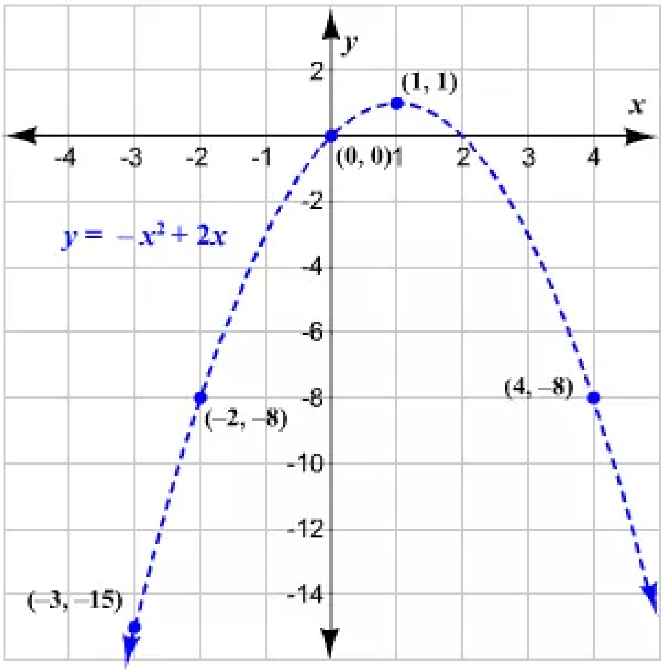
***Step1*** **Graph** $y = -x^2 + 2x$.
For this, substitute some values for $x$, say, 1 and find the corresponding values for $y$.

$$y = (1)^2 + 5(1)$$
$$= 1 + 5$$
$$= 6$$

Organize the results in a table.

| $x$ | 4 | 1 | 0 | −2 | −3 |
|---|---|---|---|---|---|
| $y = -x^2 + 2x$ | −8 | 1 | 0 | −8 | −15 |

Plot these points and join them using a smooth curve. Since > is the inequality symbol, use a dashed line to draw the curve.

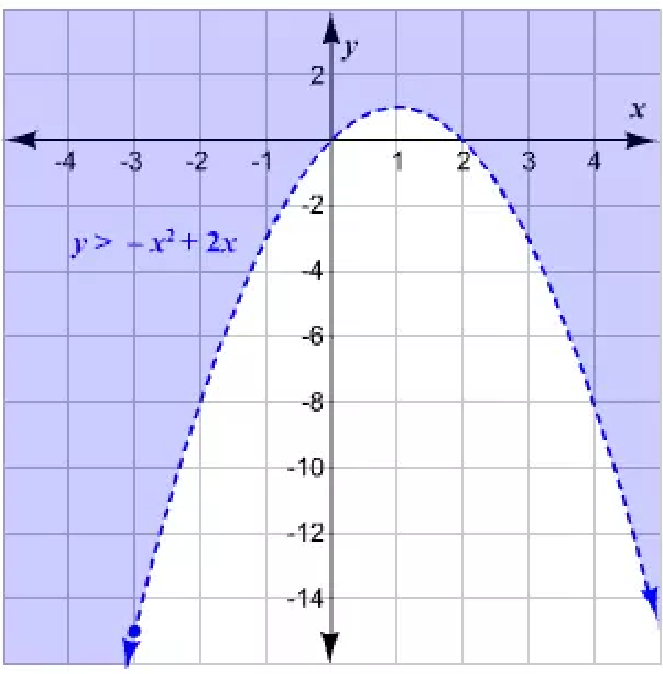



***Step 2*** **Test** a point inside the parabola, say, $(1, -2)$.

$$y > -x^2 + 2x$$
$$-2 \overset{?}{>} -(1)^2 + 2(1)$$
$$-2 \overset{?}{>} -1 + 2$$
$$-2 > -1 \qquad \times$$

Thus, $(1, -2)$ is not a solution of the inequality.

***Step 3*** **Shade** the region outside the parabola since $(1, -2)$.is not a solution of the inequality.



## Answer 6e.

Consider the vertex $(-4,1)$ and passes through $(-2,5)$.

Write a quadratic function.

The quadratic function in vertex form is $y=a(x-h)^2+k$ where the vertex is $(h,k)$.

Substitute the vertex $(h,k)=(-4,1)$,

$$y=a(x-h)^2+k$$
$$y=a(x+4)^2+1$$

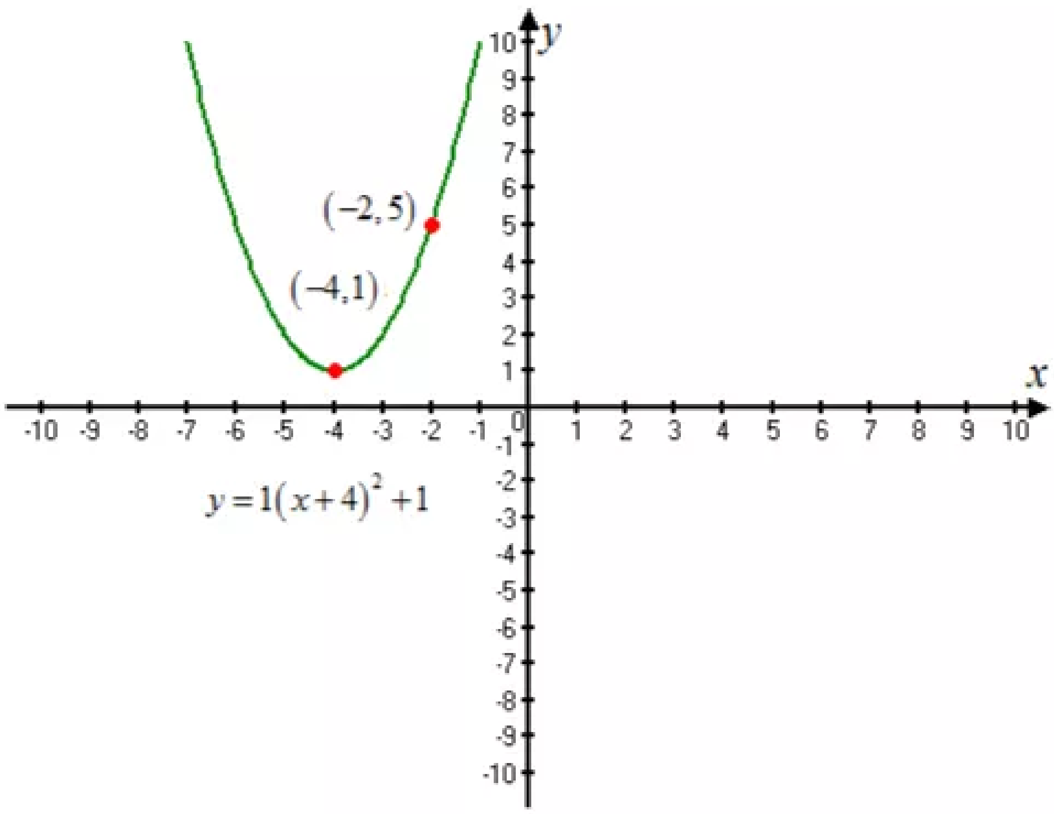
The quadratic function passes through the point $(-2,5)$.

Substitute the value $(x,y)=(-2,5)$,

$$y=a(x+4)^2+1$$
$$5=a(-2+4)^2+1$$
$$5=4a+1$$
$$a=1$$

Hence, the quadratic function is $y=1(x+4)^2+1$.

The graph of the function is as shown below.



## Answer 6gp.

Consider the points $(-1,0),(1,-2),(2,-15)$.

Write a quadratic function.

The standard form of a quadratic function is $y=ax^2+bx+c$.

The quadratic function passes through the points $(-1,0)$,

$$y=ax^2+bx+c$$
$$0=a(-1)^2-b+c$$
$$0=a-b+c \qquad ......(1)$$

The quadratic function passes through the points $(1,-2)$,

$$y=ax^2+bx+c$$
$$-2=a(1)^2+b(1)+c$$
$$-2=a+b+c \qquad ......(2)$$

The quadratic function passes through the points $(2,-15)$,

$$y=ax^2+bx+c$$
$$-15=a(2)^2+b(2)+c$$
$$-15=4a+2b+c \qquad ......(3)$$

Multiply by 2 to equation – (1) and the add to equation – (3),

$$\begin{aligned} 2(0=a-b+c) &\rightarrow & 2a-2b+2c&=0 \\ -15=4a+2b+c &\rightarrow & \underline{4a+2b+c\ }&\underline{=-15} \quad (\text{Add}) \\ && 6a+3c&=-15 \\ && 2a+c&=-5 \quad ......(3) \end{aligned}$$

Add the equations – (1) and – (2),

$$\begin{aligned} 0=a-b+c &\rightarrow & a-b+c&=0 \\ -2=a+b+c &\rightarrow & \underline{a+b+c\ }&\underline{=-2} \quad (\text{Add}) \\ && 2a+2c&=-2 \\ && a+c&=-1 \quad ......(4) \end{aligned}$$

Subtract equation – (3) and (4),

$$\begin{aligned} 2a+c&=-5 \\ \underline{a+c}&\underline{=-1} \quad (\text{Subtract}) \\ a&=-4 \end{aligned}$$

Substitute the values $a=-4$ in equation – (4),

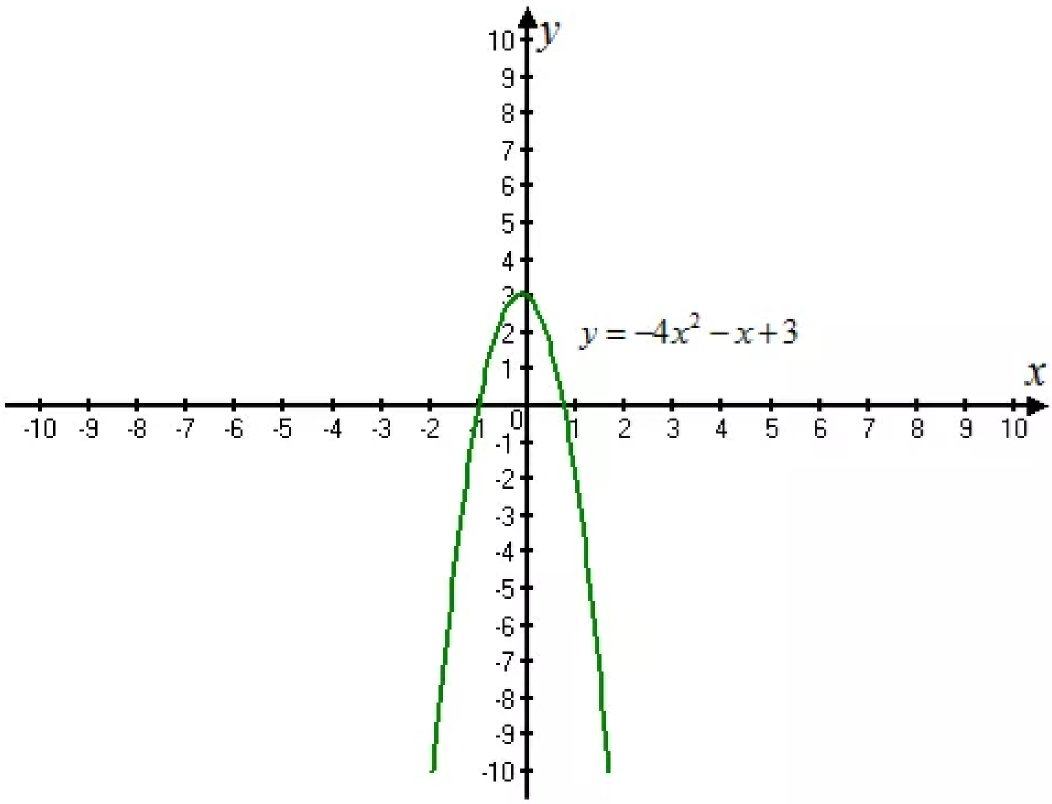
$$a+c=-1$$
$$-4+c=-1$$
$$c=3$$

Substitute the values $a=-4, c=3$ in equation – (1),

$$a-b+c=0$$
$$-4-b+3=0$$
$$-b-1=0$$
$$b=-1$$

Hence, the required quadratic equation is $y=-4x^2-x+3$.

The graph of the function is as shown below.




**Answer 6mr.**

The absolute value of a complex number $z=a+ib$, denoted $|z|$ is a nonnegative real number defined as $|z|=\sqrt{a^2+b^2}$.

Hence, the absolute value of $-15+20i$ is as given below.

$$\begin{aligned} |-15+20i| &= \sqrt{(-15)^2+(20)^2} && \left[\begin{array}{l}\text{Substitute } -15 \text{ and } 20 \text{ for } a \text{ and } b \\ \text{respectively in } \sqrt{a^2+b^2}\end{array}\right] \\ &= \sqrt{625} && [\text{Simplify}] \\ &= 25 \end{aligned}$$

Hence, the absolute value of $15-20i$ is as given below.

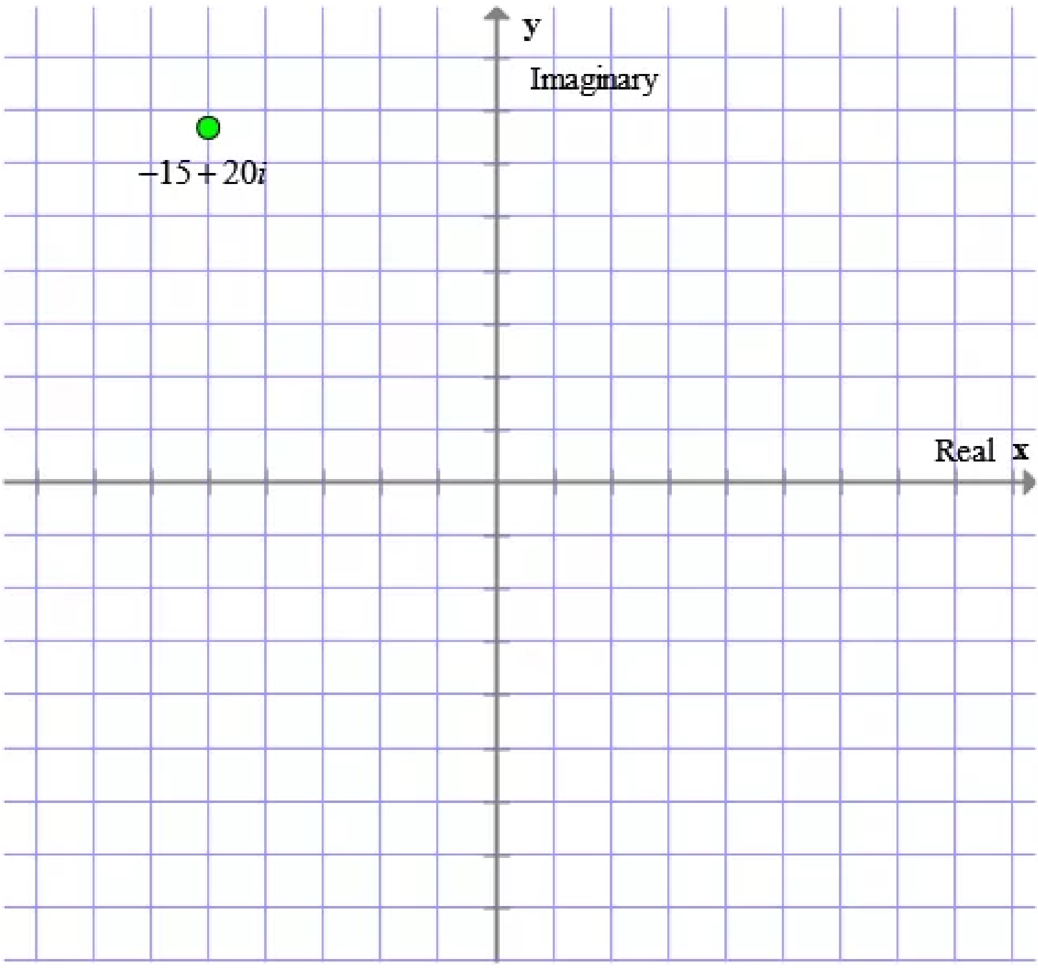
$$\begin{aligned} |15-20i| &= \sqrt{(15)^2+(-20)^2} && \left[\begin{array}{l}\text{Substitute } 15 \text{ and } -20 \text{ for } a \text{ and } b \\ \text{respectively in } \sqrt{a^2+b^2}\end{array}\right] \\ &= \sqrt{625} && [\text{Simplify}] \\ &= 25 \end{aligned}$$

Hence, the absolute value of $-20+15i$ is as given below.

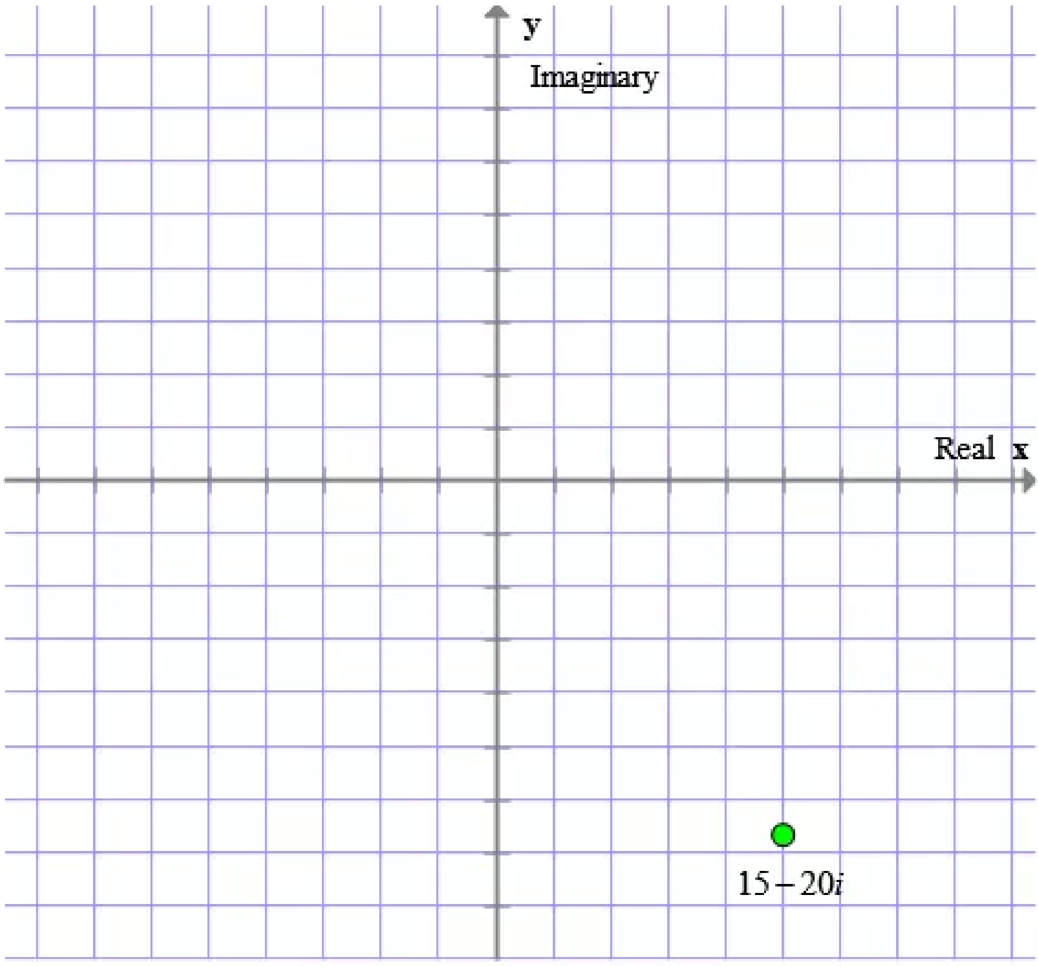
$$\begin{aligned} |-20+15i| &= \sqrt{(-20)^2+(15)^2} && \left[\begin{array}{l}\text{Substitute } -20 \text{ and } 15 \text{ for } a \text{ and } b \\ \text{respectively in } \sqrt{a^2+b^2}\end{array}\right] \\ &= \sqrt{625} && [\text{Simplify}] \\ &= 25 \end{aligned}$$

Hence, the three different complex numbers with an absolute value of 25 are $\boxed{-15+20i, 15-20i, -20+15i}$.

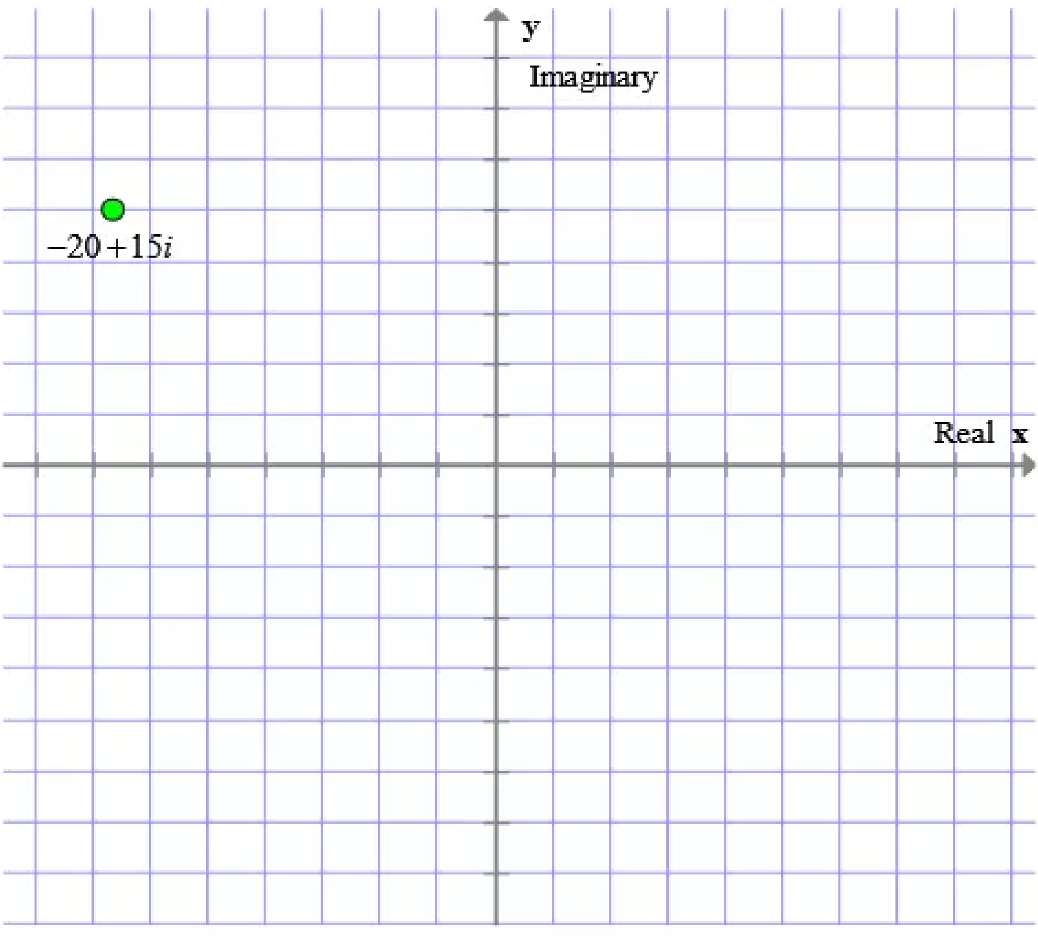
To plot $-15+20i$, we start at the origin, move to 15 units to the left, an then move 20 units up as shown below.



To plot $15-20i$ , we start at the origin, move to 15 units to the right, and then move 20 units down as shown below.

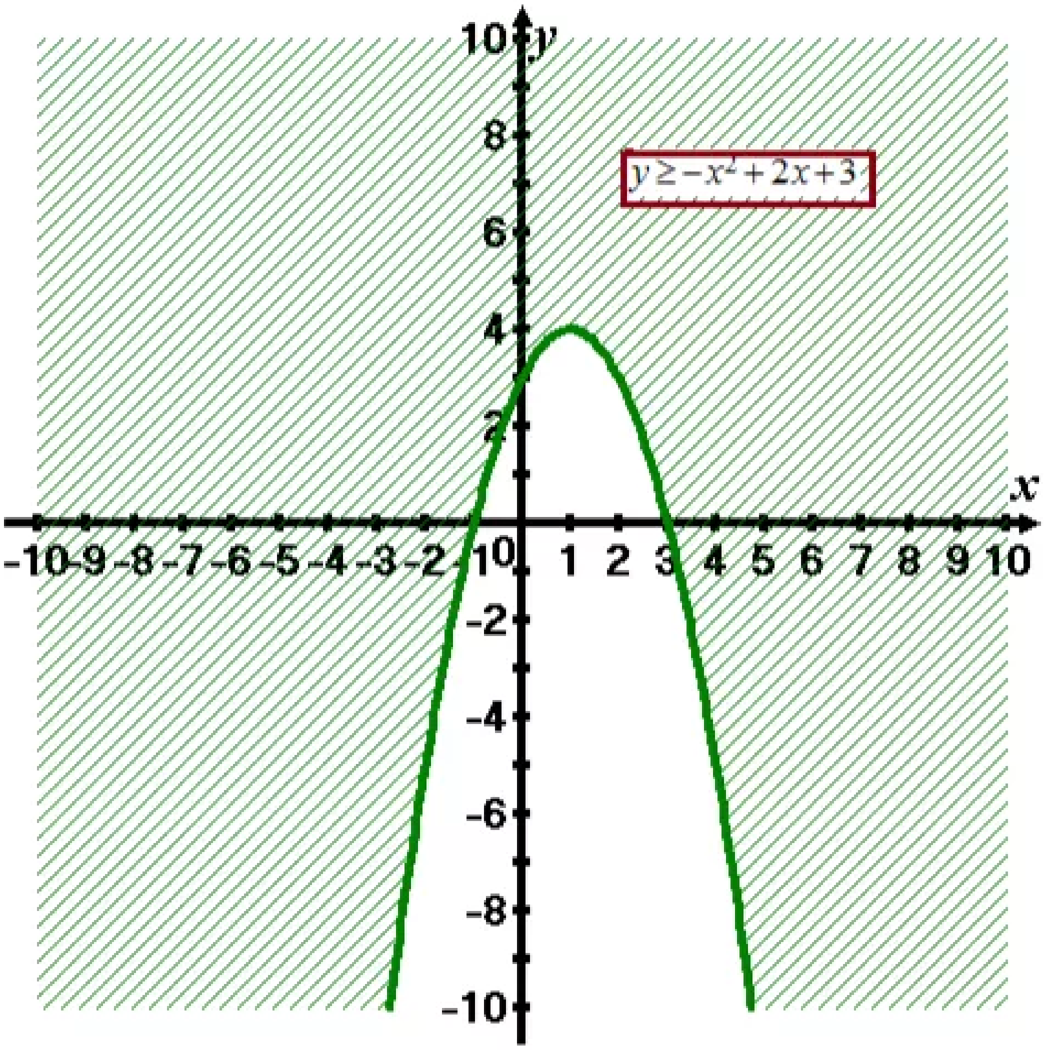



To plot $-20+15i$ , we start at the origin, move to 20 units to the left, and then move 15 units up as shown below.



## Answer 6q.

The graph of the inequality $y \geq -x^2 + 2x + 3$ is shown below:




## Answer 7e.

We know that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is $(h, k)$. It is given that the vertex is (1, 6).

Thus, $y = a(x - 1)^2 + 6$.

Substitute −1 for $x$, and 2 for $y$ to find $a$ since the graph passes through (−2, 5).
$2 = a(-1 - 1)^2 + 6$

Solve for $a$.

$$2 = 4a + 6$$
$$-4 = 4a$$
$$-1 = a$$

Replace $a$ with −1 in $y = a(x - 1)^2 + 6$.
$y = -1(x - 1)^2 + 6$

The quadratic function is $y = -1(x - 1)^2 + 6$.

**Answer 7mr.**

**a.** We know that the equation for an object that is launched or thrown is $h = -16t^2 + v_0 t + h_0$, where $h$ is the height, $t$ is the time in motion, $v_0$ is the object's initial vertical velocity, and $h_0$ is the initial height.

Substitute 50 for $v_0$, and 5 for $h_0$ in the equation.

$h = -16t^2 + 50t + 5$

A function that gives the ball's height after being thrown is $h = -16t^2 + 50t + 5$.

**b.** Replace $h$ with 3 to find the time the ball is in the air.

$3 = -16t^2 + 50t + 5$

Subtract 3 from each side.

$$3 - 3 = -16t^2 + 50t + 5 - 3$$
$$0 = -16t^2 + 50t + 2$$
$$-16t^2 + 50t + 2 = 0$$

Solve the equation using the quadratic formula.

$$t = \frac{-50 \pm \sqrt{(50)^2 - 4(-16)(2)}}{2(-16)}$$

Evaluate.

$$\begin{aligned} t &= \frac{-50 \pm \sqrt{2628}}{-32} \\ &\approx \frac{-50 \pm 51.3}{-32} \\ &\approx -0.041 \quad \text{or} \quad 3.2 \end{aligned}$$

Since time cannot be negative, discard $-0.041$. Therefore, about 3.2 seconds the ball is in the air.

**c.** We know that for a function $y = ax^2 + bx + c$, the vertex's $y$-coordinate is the maximum value if $a < 0$. In this case, the maximum height of the ball is the $y$-coordinate of the vertex.

The first method to find the vertex is as follows. The vertex has $x$-coordinate $-\frac{b}{2a}$. Thus, we can find the vertex by finding the $x$-coordinate first, and then evaluate the $y$-coordinate by substituting the value for $x$ in the function.

Another method is to convert the function in vertex form. Then, we can identify the vertex and its $y$-coordinate.

Other method is to find the intercepts of the function and then use the equation $x = \frac{p+q}{2}$ to find the $x$-coordinate. Now, substituting the value for $x$ in the function, we can find the vertex's $y$-coordinate.

A function that gives the ball's height after you throw it is $h = -16t^2 + 50t + 5$. Use the first method to find the coordinates of the vertex.

The vertex of the graph of $y = ax^2 + bx + c$ has $x$-coordinate $-\frac{b}{2a}$. Find the $x$-coordinate by substituting $-16$ for $a$, and 50 for $b$ and evaluate.

$$\begin{aligned} -\frac{b}{2a} &= -\frac{50}{2(-16)} \\ &\approx 1.6 \end{aligned}$$

The $x$-coordinate of the vertex is about 1.6. Substitute 1.6 for $x$ in the function to find the $y$-coordinate.

$$\begin{aligned} h &= -16(1.6)^2 + 50(1.6) + 5 \\ &= -40.96 + 80 + 5 \\ &\approx 44 \end{aligned}$$

The maximum height of the ball is about 44 ft.

Now, use the second method to find the coordinates of the vertex. First, take out $-1$ from the first two terms.

$$h = -16\left(t^2 - \frac{50}{16}t\right) + 5$$

Prepare to complete the square.

$$h + ? = -16\left(t^2 - \frac{50}{16}t + ?\right) + 5$$

Square half the coefficient of $t$.

$$\left(\frac{\frac{-50}{16}}{2}\right)^2 = \left(-\frac{25}{16}\right)^2 = \frac{625}{256}$$

Now, complete the square. For this, add $-16\left(\frac{625}{256}\right)$ to each side of the equation.

$$\begin{aligned} h + \left[-16\left(\frac{625}{256}\right)\right] &= -16\left(t^2 - \frac{50}{16}t\right) + 5 - 16\left(\frac{625}{256}\right) \\ h - \frac{625}{16} &= -16\left(t^2 - \frac{50}{16}t + \frac{625}{256}\right) + 5 \end{aligned}$$

Write $t^2 - \frac{50}{16}t + \frac{625}{256}$ as a binomial squared.

$$h - \frac{625}{16} = -16\left(t - \frac{25}{16}\right)^2 + 5$$

Solve for $h$. For this, add $\frac{625}{16}$ to each side.

$$h - \frac{625}{16} + \frac{625}{16} = -16\left(t - \frac{25}{16}\right)^2 + 5 + \frac{625}{16}$$

$$h = -16\left(t - \frac{25}{16}\right)^2 + \frac{705}{16}$$

A quadratic function in the form $y = a(x-h)^2 + k$ is said to be in vertex form, where $(h, k)$ is the vertex.

On comparing the equation obtained with the vertex form, we get $h = \frac{25}{16}$ or about 1.6, and $k = \frac{705}{16}$ or about 44. Thus, the vertex of the function's graph is (1.6, 44).

The maximum height of the ball is about 44 ft.

Use the third method to find the coordinates of the vertex. First, factor the right side of the function.

$h = (t + 0.10)(t - 3.2)$

A quadratic function of the form $y = a(x-p)(x-q)$ is said to be in intercept form. The $x$-coordinate of the vertex of the graph of a function in this form is $x = \frac{p+q}{2}$.

On comparing the given function with the intercept form, we get $a = 1$, $p = -0.1$, and $q = 3.2$.

Substitute for $p$ and $q$ in $x = \frac{p+q}{2}$ and simplify.

$$x = \frac{-0.10 + 3.2}{2}$$
$$\approx 1.6$$

The $x$-coordinate of the vertex is about 1.6.

Now, substitute 1.6 for $t$ in $h = -16t^2 + 50t + 5$ to find the $y$-coordinate and evaluate.

$$h = -16(1.6)^2 + 50(1.6) + 5$$
$$= -40.96 + 80 + 5$$
$$\approx 44$$

Therefore, the maximum height of the ball is about 44 ft.

## Answer 7q.

Rewrite the given inequality.

$x^2 + 5 \leq 0$

***Step1*** **Graph** $y = x^2 + 5$.

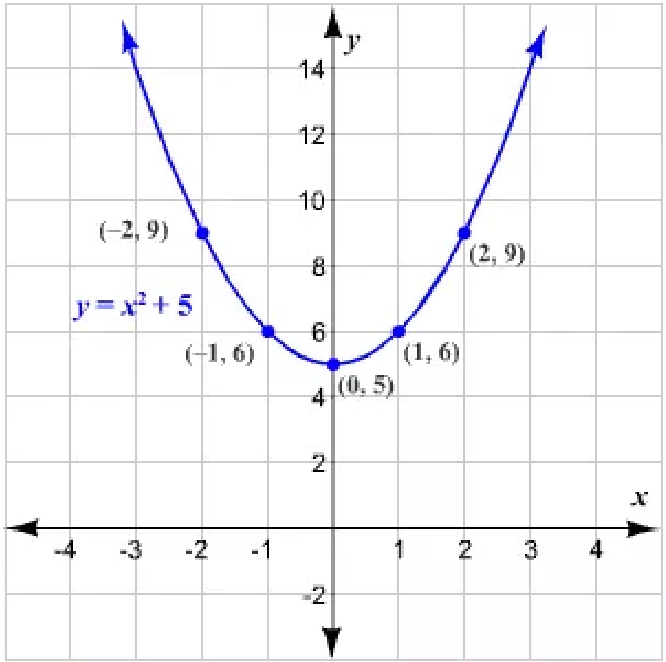
For this, substitute some values for $x$, say, 1 and find the corresponding values for $y$.

$$\begin{aligned} y &= (1)^2 + 5 \\ &= 1 + 5 \\ &= 6 \end{aligned}$$

Organize the results in a table.

| $x$ | 1 | 0 | −1 | −2 | 2 |
|---|---|---|---|---|---|
| $y = x^2 + 5$ | 6 | 5 | 6 | 9 | 9 |

Plot these points and join them using a smooth curve. Since the inequality is $\leq$ use a sold line to draw the curve.




From the graph, we can see that for no $x$-values $x^2 + 5 \leq 0$. Therefore, the given inequality has no solution.

## Answer 8e.

Consider the vertex $(5,-4)$ and passes through $(1,20)$.

Write a quadratic function.

The quadratic function in vertex form is $y=a(x-h)^2+k$ where the vertex is $(h,k)$.

Substitute the vertex $(h,k)=(5,-4)$,

$$y=a(x-h)^2+k$$
$$y=a(x-5)^2-4$$

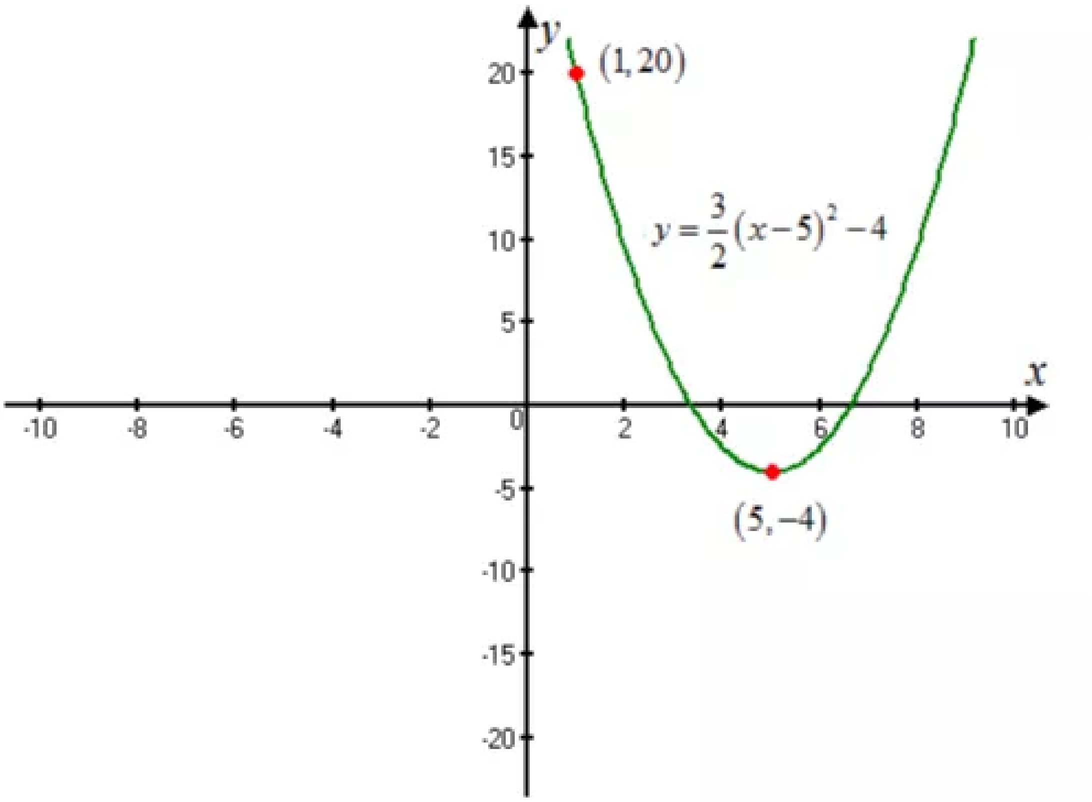
The quadratic function passes through the point $(1,20)$.

Substitute the value $(x,y)=(1,20)$,

$$y=a(x-5)^2-4$$
$$20=a(1-5)^2-4$$
$$24=16a$$
$$a=\frac{3}{2}$$

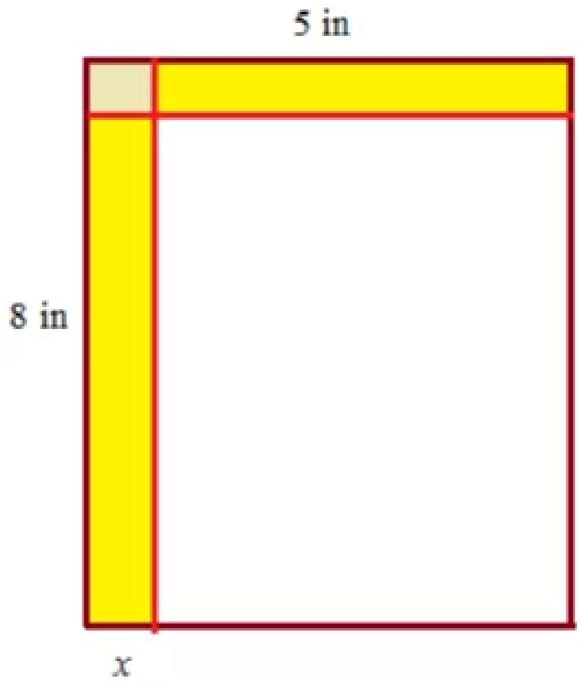
Hence, the quadratic function is $y=\frac{3}{2}(x-5)^2-4$.

The graph of the function is as shown below.



**Answer 8mr.**

A note paper measuring 5 inches by 8 inches contains a solid stripe along the paper's top and left sides with width $x$ inches.
The diagram is shown below:




If the stripes will take up one-third of the area of the paper .
Then we need to find $x$ .

$$8x+(5-x)x=\frac{1}{3}(8)(5)$$

$$8x+5x-x^2=\frac{40}{3}$$

$$x^2-13x+\frac{40}{3}=0$$

$$3x^2-39x+40=0$$

$$x=\frac{-(-39)\pm\sqrt{(-39)^2-4(3)(40)}}{2(3)} \qquad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{39\pm\sqrt{1041}}{6}$$

$$x=\frac{39-\sqrt{1041}}{6}\approx\boxed{1.12}$$

Discard, $x=\frac{39+\sqrt{1041}}{6}>11$ .

Since no side of the note paper has length greater than 11.
This note paper cannot have a stripe of width greater than 11 inches.
Therefore, $\boxed{x=1.12}$ .

## Answer 8q.

Solve the inequality $12 \le x^2 - 7x$

Consider,

$12 \le x^2 - 7x$

$x^2 - 7x - 12 \ge 0$

$x^2 - 7x + \left(-\frac{7}{2}\right)^2 \ge 12 + \left(-\frac{7}{2}\right)^2$ Add $12 + \left(-\frac{7}{2}\right)^2$ on both sides

$\left(x - \frac{7}{2}\right)^2 \ge \frac{97}{4}$ Write left side as a perfect square

The inequality becomes

$x - \frac{7}{2} \le -\sqrt{\frac{97}{4}}$ or $x - \frac{7}{2} \ge \sqrt{\frac{97}{4}}$

$x - \frac{7}{2} \le -\frac{\sqrt{97}}{2}$ or $x - \frac{7}{2} \ge \frac{\sqrt{97}}{2}$

$x \le \frac{7 - \sqrt{97}}{2}$ or $x \ge \frac{7 + \sqrt{97}}{2}$

$x \in \left(-\infty, \frac{7 - \sqrt{97}}{2}\right)$ or $x \in \left(\frac{7 + \sqrt{97}}{2}, \infty\right)$

$x \in \left(-\infty, \frac{7 - \sqrt{97}}{2}\right) \cup \left(\frac{7 + \sqrt{97}}{2}, \infty\right)$

Therefore, $\boxed{\left(-\infty, \frac{7 - \sqrt{97}}{2}\right) \cup \left(\frac{7 + \sqrt{97}}{2}, \infty\right)}$ is the solution set of the given inequality.

## Answer 9e.

We know that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is $(h, k)$. It is given that the vertex is $(-3, 3)$.

Thus,

$y = a(x + 3)^2 + 3.$

Substitute 1 for $x$, and $-1$ for $y$ to find $a$ since the graph passes through $(1, -1)$.

$-1 = a(1 + 3)^2 + 3$

Solve for $a$.

$-1 = 16a + 3$

$-4 = 16a$

$-\frac{1}{4} = a$

Replace $a$ with $-\frac{1}{4}$ in $y = a(1+3)^2 + 3$.

$$y = -\frac{1}{4}(x+3)^2 + 3$$

The quadratic function is $y = -\frac{1}{4}(x+3)^2 + 3$.

## Answer 9mr.

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the discriminant.

Substitute 3 for $a$, 5 for $b$, and $-2$ for $c$ in $b^2 - 4ac$ and evaluate.

$$\begin{aligned} b^2 - 4ac &= 5^2 - 4(3)(-2) \\ &= 25 + 24 \\ &= 49 \end{aligned}$$

The discriminant of the given quadratic equation is 49.

## Answer 9q.

We can solve the given inequality using a table. First, rewrite the given inequality.

$2x^2 + 5x + 2 > 0$

Now, find the $x$-values for which the value of quadratic expression is 0. Solve using the quadratic formula.

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4(2)(2)}}{2(2)} \\ &= \frac{-5 \pm \sqrt{9}}{4} \\ &= \frac{-5 \pm 3}{4} \end{aligned}$$

Evaluate.

$x = -2$ or $x \approx -0.5$

Choose some values for $x$ including $-2$ and $-0.5$, and evaluate the expression $2x^2 + 5x + 2$. Then, organize the results in a table.

| $x$ | $-6$ | $-4$ | $-2$ | $-1$ | $-0.5$ | 0 | 1 | 2 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $2x^2 + 5x + 2$ | 44 | 14 | 0 | $-1$ | 0 | 2 | 9 | 20 | 54 |

From the table, we note that the $2x^2 + 5x + 2 > 0$ when $x < -2$ or $x > -0.5$. Therefore, the solution of the given inequality is $x < -2$ or $x > -0.5$.

## Answer 10e.

Consider the vertex $(5,0)$ and passes through $(2,-27)$.

Write a quadratic function.

The quadratic function in vertex form is $y=a(x-h)^2+k$ where the vertex is $(h,k)$.

Substitute the vertex $(h,k)=(5,0)$,

$$y=a(x-h)^2+k$$
$$y=a(x-5)^2+0$$

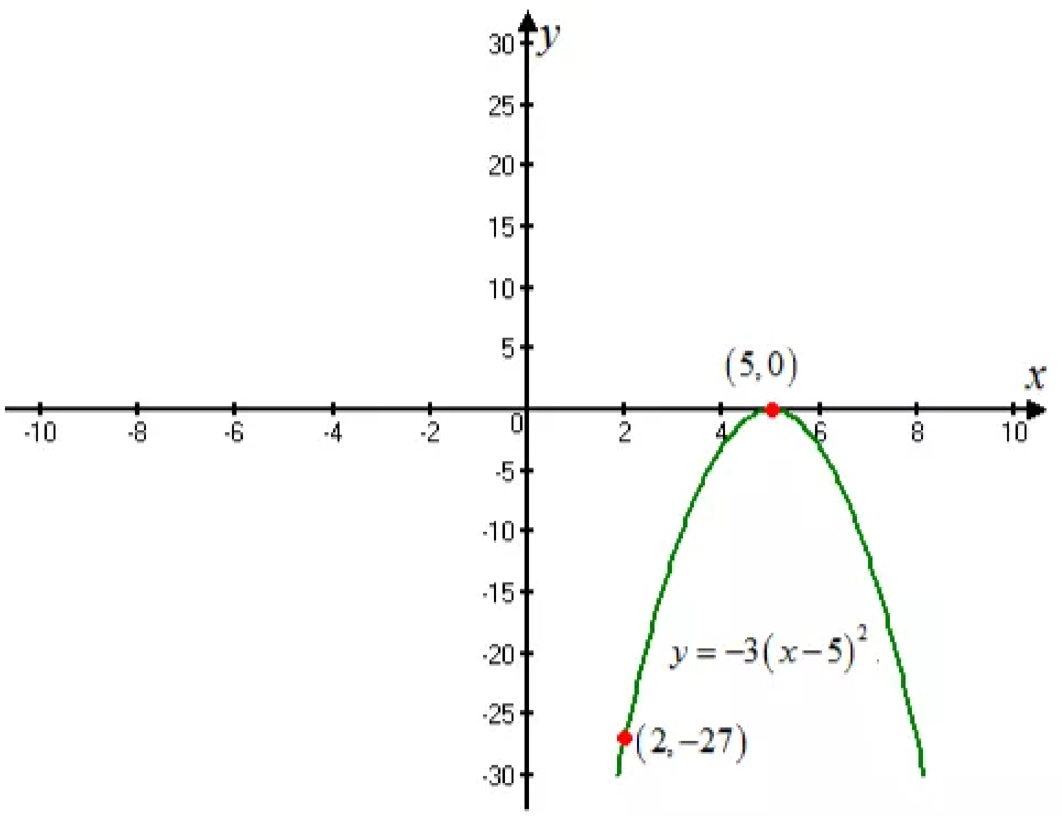
The quadratic function passes through the point $(2,-27)$.

Substitute the value $(x,y)=(2,-27)$,

$$y=a(x-5)^2+0$$
$$-27=a(2-5)^2$$
$$-27=9a$$
$$a=-3$$

Hence, the quadratic function is $y=-3(x-5)^2$.

The graph of the function is as shown below.



## Answer 10q.

Write a quadratic function with vertex $(5,7)$ and passing through the point $(3,11)$.

The required quadratic function will be of the form.

$y=a(x-5)^2+7$ ...... (1)

(Since, the quadratic function in vertex form is $y=a(x-h)^2+k$ where $(h,k)$ is the vertex)

The above function $y=a(x-5)^2+7$ passes through the points $(3,11)$.

$11=a(3-5)^2+7$ Substitute $(3,11)$

$11=4a+7$

$4=4a$

$a=1$

Substitute $a=1$ in equation (1)

Therefore $\boxed{y=(x-5)^2+7}$ is the required quadratic function.

## Answer 11e.

We know that the vertex form of a quadratic function is $y=a(x-h)^2+k$, where the vertex is $(h, k)$. It is given that the vertex is $(-4, -2)$.

Thus,

$y=a(0+4)^2-2$.

Substitute 1 for $x$, and $-1$ for $y$ to find $a$ since the graph passes through (0, 30).

$30=a(0+4)^2-2$

Solve for $a$.

$30 = 16a - 2$

$32 = 16a$

$2 = a$

Replace $a$ with 2 in $y=a(0+4)^2-2$.

$y=2(x+4)^2-2$

The quadratic function is $y=2(x+4)^2-2$.

## Answer 11q.

We know that the intercept form of a quadratic function is $y=a(x-p)(x-q)$, where $p$ and $q$ are the intercepts.

It is given that the $x$-intercepts are $-3$ and 5. Substitute $-3$ for $p$, and 5 for $q$.

$y=a[x-(-3)](x-5)$

Simplify.

$y=a(x+3)(x-5)$

The parabola passes through the point (7, –40). In order to find the value of $a$, substitute 7 for $x$, and –40 for $y$ in the equation.
$-40 = a(7+3)(7-5)$

Solve for $a$.
$$-40 = 20a$$
$$-2 = a$$

Substitute the value for $a$ in $y = a(x+3)(x-5)$.
$y = -2(x+3)(x-5)$

The quadratic equation is $y = -2(x+3)(x-5)$.

## Answer 12e.

Consider the vertex $(2,1)$ and passes through $(4,-2)$.

Write a quadratic function.

The quadratic function in vertex form is $y = a(x-h)^2 + k$ where the vertex is $(h,k)$.

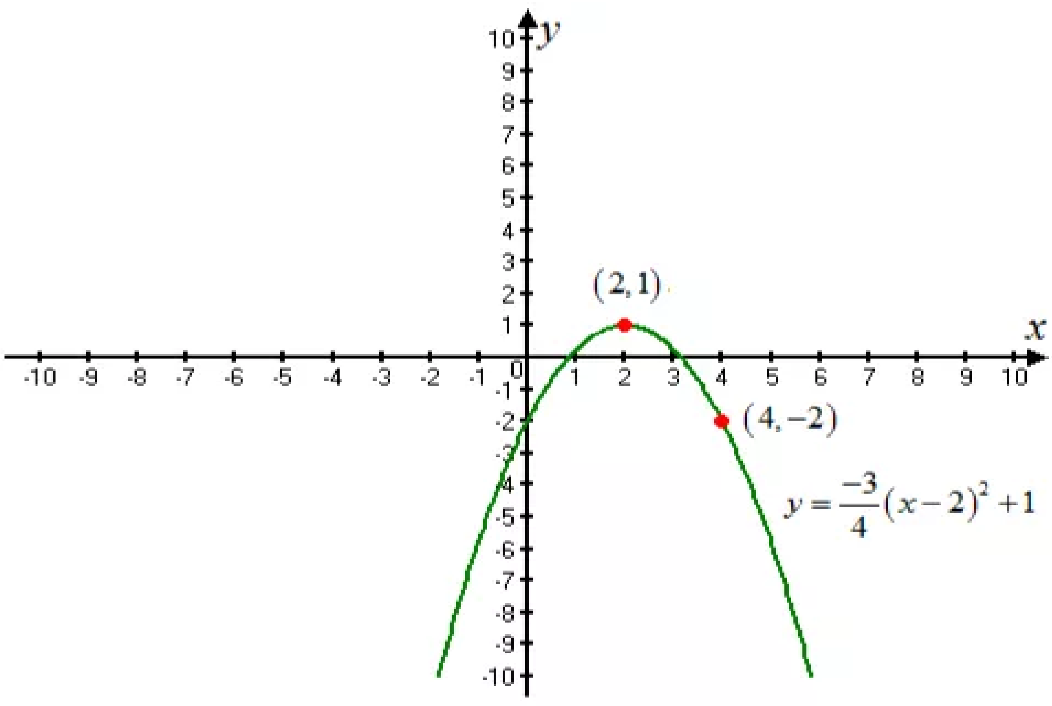
Substitute the vertex $(h,k) = (2,1)$,
$$y = a(x-h)^2 + k$$
$$y = a(x-2)^2 + 1$$

The quadratic function passes through the point $(4,-2)$.

Substitute the value $(x,y) = (4,-2)$,
$$y = a(x-2)^2 + 1$$
$$-2 = a(4-2)^2 + 1$$
$$-3 = 4a$$
$$a = \frac{-3}{4}$$

Hence, the quadratic function is $y = \frac{-3}{4}(x-2)^2 + 1$.

The graph of the function is as shown below.



## Answer 12q.

618-4.10-12Q

AID: 484 | 24/11/2012
RID: 1372 | 05/12/2012

Write a quadratic function that passes through the points $(-1,2),(4,-23),(2,-7)$

The standard form of a quadratic function is $y = ax^2 + bx + c$ ...... (1).

Since, the above function $y = ax^2 + bx + c$ passes through the points $(-1,2),(4,-23),(2,-7)$.

$$\begin{cases} a-b+c=2 \\ 16a+4b+c=-23 \\ 4a+2b+c=-7 \end{cases}$$ Substitute the points $(-1,2),(4,-23),(2,-7)$

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Solve the above system.

Consider,

$a-b+c=2$

$16a+4b+c=-23$ subtract

$-15a-5b=25$

$3a+b=-5$ ...... (2)

Also

$a-b+c=2$

$4a+2b+c=-7$ Subtract

$-3a-3b=9$

$a+b=-3$ ...... (3)

Subtract (3) from (2)

$2a=-2$

$a=-1$

Equation (3) becomes

$-1+b=-3$

$b=-2$

Substitute

$a=-1, b=-2$ in equation $a-b+c=2$

$c=1$

Equation (1) becomes

Therefore $\boxed{y=-x^2-2x+1}$ is the required quadratic function.

## Answer 13e.

We know that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is $(h, k)$. It is given that the vertex is $(-1, -4)$.

Thus,

$y = a(x + 1)^2 - 4$.

Substitute 2 for $x$, and $-1$ for $y$ to find $a$ since the graph passes through (2, −1).
$-1 = a(2+1)^2 - 4$

Solve for $a$.

$$-1 = 9a - 4$$
$$3 = 9a$$
$$\frac{1}{3} = a$$

Replace $a$ with $\frac{1}{3}$ in equation $y = a(2+1)^2 - 4$.

$$y = \frac{1}{3}(x+1)^2 - 4$$

The quadratic function is $y = \frac{1}{3}(x+1)^2 - 4$.

## Answer 13q.

We know that the equation for an object that is launched or thrown is $h = -16t^2 + v_0 t + h_0$, where $h$ is the height, $t$ is the time in motion, $v_0$ is the object's initial vertical velocity, and $h_0$ is the initial height.

Substitute 30 for $v_0$, 0 for $h$, and 5 for $h_0$ in the equation.
$0 = -16t^2 + 50t + 5$

Solve the equation using the quadratic formula.

$$t = \frac{-50 \pm \sqrt{(50)^2 - 4(-16)(5)}}{2(-16)}$$

Evaluate.

$$t = \frac{-50 \pm \sqrt{2820}}{-32}$$
$$\approx \frac{-50 \pm 53.1}{-32}$$
$$\approx -0.10 \quad \text{or} \quad 3.2$$

Since time cannot be negative, discard −0.10. Therefore, for about 3.2 seconds the ball is in the air.

**Answer 14e.**

Consider the vertex $(3,5)$ and passes through $(7,-3)$.

Write a quadratic function.

The quadratic function in vertex form is $y=a(x-h)^2+k$ where the vertex is $(h,k)$.

Substitute the vertex $(h,k)=(3,5)$,

$$y=a(x-h)^2+k$$

$$y=a(x-3)^2+5$$

The quadratic function passes through the point $(7,-3)$.

Substitute the value $(x,y)=(7,-3)$,

$$y=a(x-3)^2+5$$

$$-3=a(7-3)^2+5$$

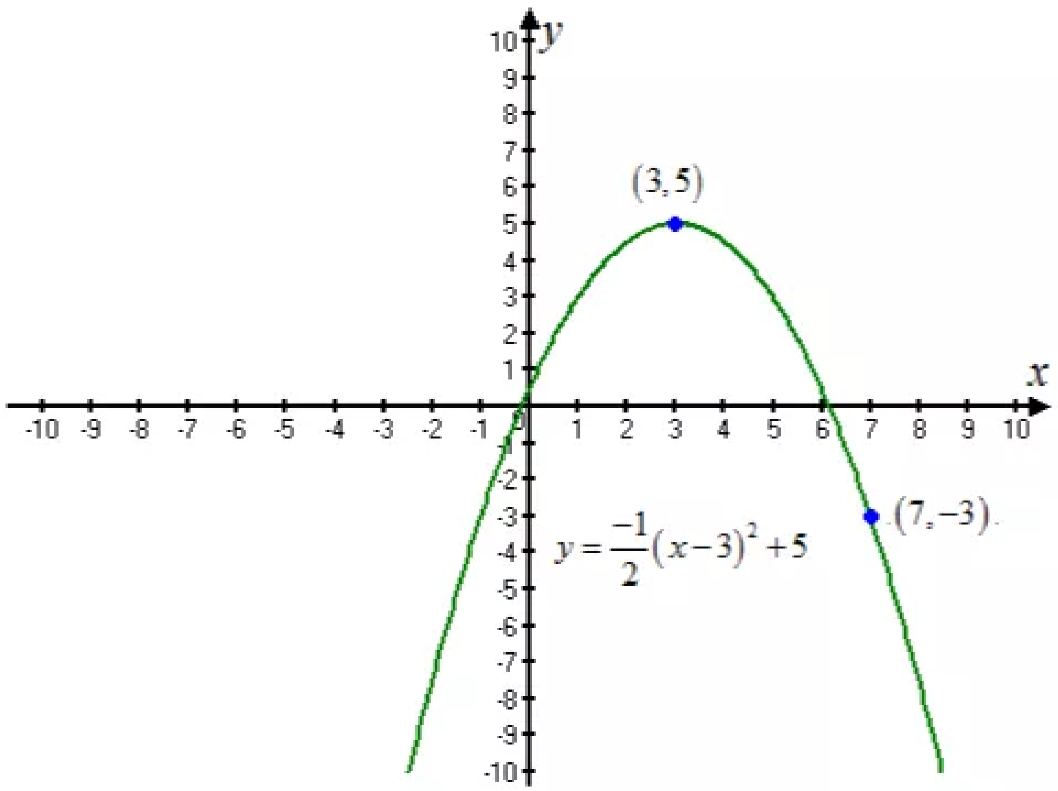
$$-8=16a$$

$$a=\frac{-8}{16}$$

$$a=\frac{-1}{2}$$

Hence, the quadratic function is $y=\frac{-1}{2}(x-3)^2+5$.

The graph of the function is as shown below.



## Answer 15e.

We know that the vertex form of a quadratic function is $y = a(x-h)^2 + k$, where the vertex is $(h, k)$. It is given that the vertex is $(5, -3)$.

Thus,
$y = a(x-5)^2 - 3$.

Substitute 1 for $x$, and 5 for $y$ to find $a$.
$5 = a(1-5)^2 - 3$

Solve for $a$.

$$5 = 16a - 3$$
$$8 = 16a$$
$$\frac{1}{2} = a$$

Replace $a$ with $\frac{1}{2}$ in equation (1).

$$y = \frac{1}{2}(x-5)^2 - 3$$

The quadratic function is $y = \frac{1}{2}(x-5)^2 - 3$.

In order to check whether the point (0, 3) lies on the given parabola, substitute 0 for $x$, and 3 for $y$ and evaluate.

$$3 \stackrel{?}{=} \frac{1}{2}(0-5)^2 - 3$$
$$3 \neq \frac{19}{2}$$

The point (0, 3) is not on the given parabola.

Similarly, check for the other points given in the choices. We note that the point (−1, 15) satisfies the quadratic function. Thus, the point (−1, 15) lie on the given parabola.

The correct answer is choice **C**.

## Answer 16e.

Consider a parabola that has the x-intercepts 4 and 7 passing through the point $(2,-20)$

The required quadratic function will be of the form.

$$y = a(x-4)(x-7)$$

(Since, the quadratic function in intercept form is $y = a(x-p)(x-q)$ where $p$ and $q$ are x-intercepts)

Since, the above function also passes through the point $(2,-20)$.

$$\Rightarrow -20 = a(-2)(-5)$$
$$a = -2$$

Therefore $y = -2(x-4)(x-7)$ is the required quadratic function.

The other point on parabola is $(5,4)$

## Answer 17e.

We know that the intercept form of a quadratic function is $y = a(x - p)(x - q)$, where $p$ and $q$ are the intercepts.

From the figure it is noted that the $x$-intercepts are $-2$ and 3. Substitute $-2$ for $p$, and 3 for $q$.
$y = a[x - (-2)](x - 3)$

Simplify.
$y = a(x + 2)(x - 3)$

The parabola passes through the point (0, 6). In order to find the value of $a$, substitute 0 for $x$, and 6 for $y$ in the equation.
$6 = a(0 + 2)(0 - 3)$

Solve for $a$.

$$6 = -6a$$
$$-1 = a$$

Substitute the value for $a$ in $y = a(x + 2)(x - 3)$.
$y = -1(x + 2)(x - 3)$

The quadratic equation is $y = -(x + 2)(x - 3)$.

## Answer 18e.

Consider a quadratic function in intercept form has the x-intercepts $-6$ and $-4$
Passing through the point $(-3,3)$
The required quadratic function will be of the form.

$$y = a(x+6)(x+4)$$

(Since, the quadratic function in intercept form is $y = a(x-p)(x-q)$ where $p$ and $q$ are x-intercepts)
Since, the above function also passes through the point $(-3,3)$. We get

$$\Rightarrow 3 = a(3)(1)$$
$$a = 1$$

Therefore $y = (x+6)(x+4)$ is the required quadratic function.

## Answer 19e.

We know that the intercept form of a quadratic function is $y = a(x - p)(x - q)$, where $p$ and $q$ are the intercepts.

From the figure it is noted that the $x$-intercepts are $-2$ and 3. Substitute $-3$ for $p$, and 3 for $q$.
$y = a[x - (-3)](x - 3)$

Simplify.
$y = a(x + 3)(x - 3)$

The parabola passes through the point (1, –4). In order to find the value of $a$, substitute 1 for $x$, and –4 for $y$ in the equation.
$-4 = a(1+3)(1-3)$

Solve for $a$.
$-4 = -8a$
$\frac{1}{2} = a$

Substitute the value for $a$ in $y = a(x+3)(x-3)$.
$y = \frac{1}{2}(x+3)(x-3)$

The quadratic equation is $y = \frac{1}{2}(x+3)(x-3)$.

## Answer 20e.

Consider a quadratic function in intercept form has the x-intercepts 2 and 5 , passing through the point $(4,-2)$
The required quadratic function will be of the form.
$$y = a(x-2)(x-5)$$
(Since, the quadratic function in intercept form is $y = a(x-p)(x-q)$ where $p$ and $q$ are x-intercepts)
Since, the above function also passes through the point $(4,-2)$.
$$-2 = a(2)(-1)$$
$$a = 1$$
Therefore $y = (x-2)(x-5)$ is the required quadratic function.

## Answer 21e.

We know that the intercept form of a quadratic function is $y = a(x-p)(x-q)$, where $p$ and $q$ are the intercepts.

It is given that the $x$-intercepts are –3 and 0. Substitute –3 for $p$, and 0 for $q$.
$y = a[x-(-3)](x-0)$

Simplify.
$y = a(x+3)x$

The parabola passes through the point (2, 10). In order to find the value of $a$, substitute 2 for $x$, and 10 for $y$ in the equation.
$10 = a(2+3)(2)$

Solve for $a$.
$10 = 10a$
$1 = a$

Substitute the value for $a$ in $y = a(x+3)x$.
$y = (x+3)x$

The quadratic equation is $y = (x+3)x$.

## Answer 22e.

Consider a quadratic function in intercept form that has the x-intercepts $-1$ and $4$ passing through the point $(2,4)$

The required quadratic function will be of the form.

$$y = a(x+1)(x-4)$$

(Since, the quadratic function in intercept form is $y = a(x-p)(x-q)$ where $p$ and $q$ are x-intercepts)

Since, the above function also passes through the point $(2,4)$

$$\Rightarrow 4 = a(3)(-2)$$

$$a = -\frac{2}{3}$$

Therefore $y = -\frac{2}{3}(x+1)(x-4)$ is the required quadratic function.

## Answer 23e.

We know that the intercept form of a quadratic function is $y = a(x-p)(x-q)$, where $p$ and $q$ are the intercepts.

It is given that the $x$-intercepts are 3 and 7. Substitute 3 for $p$, and 7 for $q$.
$y = a(x-3)(x-7)$

The parabola passes through the point $(6, -9)$. In order to find the value of $a$, substitute 6 for $x$, and $-9$ for $y$ in the equation.
$-9 = a(6-3)(6-7)$

Solve for $a$.
$-9 = -3a$
$3 = a$

Substitute 3 for $a$ in $y = a(x-3)(x-7)$.
$y = 3(x-3)(x-7)$

The quadratic equation is $y = 3(x-3)(x-7)$.

## Answer 24e.

Consider a quadratic function in intercept form that has the x-intercepts $-5$ and $-1$ passing through the point $(-7,-24)$

The required quadratic function will be of the form.

$$y=a(x+5)(x+1)$$

(Since, the quadratic function in intercept form is $y=a(x-p)(x-q)$ where $p$ and $q$ are x-intercepts)

Since, the above function also passes through the point $(-7,-24)$

$$\Rightarrow -24=a(-2)(-6)$$

$$a=-2$$

Therefore $y=-2(x+5)(x+1)$ is the required quadratic function.

## Answer 25e.

We know that the intercept form of a quadratic function is $y=a(x-p)(x-q)$, where $p$ and $q$ are the intercepts.

It is given that the $x$-intercepts are $-6$ and 3. Substitute $-6$ for $p$, and 3 for $q$.

$$y = a[x-(-6)](x-3)$$

$$y = a(x+6)(x-3)$$

The parabola passes through the point $(0,-9)$. In order to find the value of $a$, substitute 0 for $x$, and $-9$ for $y$ in the equation.

$$-9=a(0+6)(0-3)$$

Solve for $a$.

$$-9 = -18a$$

$$\frac{1}{2} = a$$

Substitute $\frac{1}{2}$ for $a$ in $y=a(x+6)(x-3)$.

$$y=\frac{1}{2}(x+6)(x-3)$$

The quadratic equation is $y=\frac{1}{2}(x+6)(x-3)$.

## Answer 26e.

The error in deriving the required quadratic function is, 5 and $-5$ are taken as x-intercepts and $(4,-3)$ is taken as a point through which the graph is passing, both of them are wrong.

We need to write a quadratic function in intercept form that has the x-intercepts 4 And $-3$ passing through the point $(5,-5)$

The required quadratic function will be of the form.

$$y = a(x-4)(x+3)$$

(Since, the quadratic function in intercept form is $y = a(x-p)(x-q)$ where $p$ and $q$ are x-intercepts)

Since, the above function also passes through the point $(5,-5)$

$$\Rightarrow -5 = a(1)(8)$$

$$a = -\frac{5}{8}$$

Therefore $y = -\frac{5}{8}(x-4)(x+3)$ is the required quadratic function.

## Answer 27e.

The vertex and a point on the graph are given. We can use the vertex form of a quadratic function. In the given solution, the error is that the intercept form of a quadratic function is used instead of vertex form.

Substitute 2 for $h$, and 3 for $k$ in the vertex form of a quadratic function.
The vertex form of a quadratic function is $y = a(x-h)^2 + k$, where $(h, k)$ is the vertex.

Substitute 2 for $h$, and 3 for $k$.

$$y = a(x-2)^2 + 3$$

In order to find $a$, use the other given point (1, 5).
Substitute 1 for $x$, and 5 for $y$.

$$5 = a(1-2)^2 + 3$$

Solve for $a$.

$$5 = a + 3$$

$$2 = a$$

## Answer 28e.

The standard form of a quadratic function is

$$y = ax^2 + bx + c.$$

The above quadratic function that passes through the points $(1,-6),(2,-1),(4,-3)$

$$\begin{cases} a+b+c=-6 \\ 4a+2b+c=-1 \\ 16a+4b+c=-3 \end{cases}$$

By solving the above system

$$a=-2, b=11, c=-15$$

Therefore $y = -2x^2 + 11x - 15$ is the requited quadratic function.

**Answer 29e.**

***Step 1*** Substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute $-3$ for $x$, and 4 for $y$ in $y = ax^2 + bx + c$.
$4 = a(-3)^2 + b(-3) + c$

Simplify.
$4 = 9a - 3b + c$ Equation 1

Now, substitute $-6$ for $x$, and $-2$ for $y$.
$-2 = a(-6)^2 + b(-6) + c$

Simplify.
$-2 = 36a - 6b + c$ Equation 2

Replace $x$ with $-4$, and $y$ with $-2$.
$-2 = a(-4)^2 + b(-4) + c$

Simplify.
$-2 = 16a - 4b + c$ Equation 3

***Step 2*** Rewrite the system of three equations in step 1 as a system of two equations.

Multiply each side of Equation 2 by $-1$.
$2 = -36a + 6b - c$ Revised Equation 2

Add Equation 1 and Revised equation 2 to eliminate $c$.

$$\begin{array}{rcl} 4 &=& 9a - 3b + c \\ 2 &=& -36a + 6b - c \\ \hline 6 &=& -27a + 3b \end{array}$$

Equation 4

Multiply each side of Equation 3 by $-1$.
$2 = -16a + 4b - c$ Revised Equation 3

Add Equation 2 and Revised equation 3 to eliminate $c$ again.

$$\begin{array}{rcl} -2 &=& 36a - 6b + c \\ 2 &=& -16a + 4b - c \\ \hline 0 &=& 20a - 2b \end{array}$$

Equation 5

***Step 3*** Now, solve the system consisting of Equations 4 and 5. For this, multiply each side of Equation 4 by 2 and Equation 5 by 3.

$6 = -27a + 3b \times 2 \rightarrow 12 = -54a + 6b$ Revised Equation 4

$0 = 20a - 2b \times 3 \rightarrow 0 = 60a - 6b$ Revised Equation 5

Add Revised Equations 4 and 5.

$$\begin{aligned} 12 &= -54a + 6b \\ 0 &= 60a - 6b \\ \hline 12 &= 6a \end{aligned}$$

Solve for $a$.

$a = 2$

First substitute 2 for $a$ in Equation 5 to find the value of $b$.

$0 = 20(2) - 2b$

Solve for $b$.

$$\begin{aligned} 0 &= 40 - 2b \\ 2b &= 40 \\ b &= 20 \end{aligned}$$

In order to find the value of $c$, substitute 2 for $a$, and 20 for $b$ in Equation (1).

$4 = 9(2) - 3(20) + c$

Solve for $c$.

$$\begin{aligned} 4 &= 18 - 60 + c \\ 4 &= -42 + c \\ 46 &= c \end{aligned}$$

Substitute 2 for $a$, 20 for $b$, and 46 for $c$ in $y = ax^2 + bx + c$.

$y = 2x^2 + 20x + 46$

Therefore, the quadratic equation is $y = 2x^2 + 20x + 46$.

## Answer 30e.

The standard form of a quadratic function is

$$y = ax^2 + bx + c.$$

The above quadratic function that passes through the points $(-4,-6),(0,-2),(2,6)$

$$\begin{cases} 16a - 4b + c = -6 \\ c = -2 \\ 4a + 2b + c = 6 \end{cases}$$

By solving the above system

$$a = \frac{1}{2}, b = 3, c = -2$$

The quadratic function is

$$y = \frac{1}{2}x^2 + 3x - 2$$

## Answer 31e.

***Step 1*** Substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute –4 for $x$, and –3 for $y$ in $y = ax^2 + bx + c$ and simplify.

$-3 = a(-4)^2 + b(-4) + c$

Simplify.

$-3 = 16a - 4b + c$ Equation 1

Now, substitute 0 for $x$, and –2 for $y$ in $y = ax^2 + bx + c$.

$-2 = a(0)^2 + b(0) + c$

Simplify.

$-2 = c$ Equation 2

Replace $x$ with 1, and $y$ with 7 in $y = ax^2 + bx + c$.

$7 = a(1)^2 + b(1) + c$

Simplify.

$7 = a + b + c$ Equation 3

***Step 2*** Rewrite the system of three equations in Step 1 as a system of two equations.

Substitute –2 for $c$ in Equation 1.

$-3 = 16a - 4b - 2$

Simplify.

$-1 = 16a - 4b$ Revised Equation 1

Substitute –2 for $c$ in equation (3).

$7 = a + b - 2$

Simplify.

$9 = a + b$ Revised Equation 3

***Step 3*** Now, solve the system consisting of Revised Equations 1 and 3.
In order to eliminate $b$, multiply each side of Revised Equation 3 by 4.

$9 = a + b \quad \times \quad 4 \qquad 36 = 4a + 4b$ Equation 4

Add Revised Equation 1 and Equation 4.

$$\begin{aligned} -1 &= 16a - 4b \\ 36 &= 4a + 4b \\ \hline 35 &= 20a \end{aligned}$$

Solve for $a$.

$$\begin{aligned} a &= \frac{35}{20} \\ &= \frac{7}{4} \end{aligned}$$

Substitute $\frac{7}{4}$ for $a$ in Revised Equation 3.

$$9 = \frac{7}{4} + b$$

Solve for $b$.

$$\frac{29}{4} = b$$

Substitute $\frac{7}{4}$ for $a$, $\frac{29}{4}$ for $b$, and $-2$ for $c$ in $y = ax^2 + bx + c$.

$$y = \frac{7}{4}x^2 + \frac{29}{4}x + (-2)$$

Therefore, the quadratic equation is $y = \frac{7}{4}x^2 + \frac{29}{4}x - 2$.

## Answer 32e.

The standard form of a quadratic function is

$$y = ax^2 + bx + c.$$

Above quadratic function that passes through the points $(-2,-4),(0,10),(3,-7)$

$$\begin{cases} 4a - 2b + c = -4 \\ c = 10 \\ 9a + 3b + c = -7 \end{cases}$$

By solving the above system

$$a = -\frac{38}{15}, b = \frac{29}{15}, c = 10$$

The quadratic function is

$$y = -\frac{38}{15}x^2 + \frac{29}{15}x + 10$$

## Answer 33e.

***Step 1*** **Substitute** the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute $-2$ for $x$, and 4 for $y$ in $y = ax^2 + bx + c$.

$$4 = a(-2)^2 + b(-2) + c$$

Simplify.

$$4 = 4a - 2b + c \quad \text{Equation 1}$$

Replace $x$ with 0, and $y$ with 5 in $y = ax^2 + bx + c$ and simplify.

$$5 = a(0)^2 + b(0) + c$$

$$5 = c \qquad \text{Equation 2}$$

Substitute 1 for $x$, and $-11$ for $y$ in $y = ax^2 + bx + c$ and simplify.

$$-11 = a(1)^2 + b(1) + c$$

$$-11 = a + b + c \qquad \text{Equation 3}$$

***Step 2*** **Rewrite** the system of three equations as a system of two equations.
For this, substitute 5 for $c$ in Equation 1.

$$4 = 4a - 2b + 5$$

Simplify

$$-1 = 4a - 2b \qquad \text{Revised Equation 1}$$

Now, substitute 5 for $c$ in Equation 3 and simplify.

$$-16 = a + b \qquad \text{Revised Equation 3}$$

***Step 3*** **Solve** the system consisting revised Equations 1 and 3.

$$\begin{aligned} -1 &= 4a - 2b & \qquad & & -1 &= 4a - 2b \\ -16 &= a + b & \times\ 2 & & -32 &= 2a + 2b \\ & & & & -33 &= 6a \end{aligned}$$

Solve for $a$.

$$a = -\frac{33}{6}$$

$$= -\frac{11}{2}$$

In order to find the value of $b$, substitute $-\frac{11}{2}$ for $a$ in Revised Equation 3.

$$-16 = -\frac{11}{2} + b$$

Solve for $b$.

$$-\frac{21}{2} = b$$

Substitute the values for $a$, $b$, and $c$ in $y = ax^2 + bx + c$.

$$y = -\frac{11}{2}x^2 - \frac{21}{2}x + 5$$

The quadratic equation is $y = -\frac{11}{2}x^2 - \frac{21}{2}x + 5$.

## Answer 34e.

The standard form of a quadratic function is

$$y = ax^2 + bx + c.$$

The above quadratic function that passes through the points $(-1,-1),(1,11),(3,7)$

$$\begin{cases} a - b + c = -1 \\ a + b + c = 11 \\ 9a + 3b + c = 7 \end{cases}$$

By solving the above system

$$a = -2, b = 6, c = 7$$

The quadratic function is

$$y = -2x^2 + 6x + 7$$

## Answer 35e.

***Step 1*** **Substitute** the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute −1 for $x$, and 9 for $y$ in $y = ax^2 + bx + c$.

$$9 = a(-1)^2 + b(-1) + c$$

Simplify.

$$9 = a - b + c \quad \text{Equation 1}$$

Replace $x$ with 1, and $y$ with 1 in $y = ax^2 + bx + c$ and simplify.

$$1 = a(1)^2 + b(1) + c$$

$$5 = a + b + c \qquad \text{Equation 2}$$

Substitute 3 for $x$, and 17 for $y$ in $y = ax^2 + bx + c$ and simplify.

$$17 = a(3)^2 + b(3) + c$$

$$17 = 9a + 3b + c \qquad \text{Equation 3}$$

***Step 2*** **Rewrite** the system of three equations as a system of two equations.

Add Equations 1 and 2.

$$\begin{array}{rcl} 9 &=& a - b + c \\ 1 &=& a + b + c \\ \hline 10 &=& 2a \quad + 2c \end{array} \quad \text{Equation 4}$$

Add 3 times Equation 1 to Equation 3.

$$\begin{array}{lcl} 9 = a - b + c & \times \; 3 & 27 = 3a - 3b + 3c \\ 17 = 9a + 3b + c & & 17 = 9a + 3b + c \\ \hline & & 44 = 12a \quad + 4c \end{array} \quad \text{Equation 5}$$

***Step 3*** **Solve** the system consisting revised Equations 1 and 3.

Add −2 times Equation 4 to Equation 5.

$$\begin{aligned} 44 &= 12a + 4c \\ 10 &= 2a + 2c \quad \times -2 \end{aligned} \qquad \begin{aligned} 44 &= 12a + 4c \\ -20 &= -4a - 4c \\ \hline 24 &= 8a \end{aligned}$$

Divide both the sides by 8.

$$\frac{24}{8} = \frac{8a}{8}$$

$$3 = a$$

Substitute 3 for $a$ in Equation 4.

$10 = 2(3) + 2c$

Simplify.

$$\begin{aligned} 10 &= 6 + 2c \\ 4 &= 2c \\ 2 &= c \end{aligned}$$

Now, substitute the values for $a$ and $c$ in Equation 1 to find the value of $b$.

$9 = 3 - b + 2$

Solve for $b$.

$b = -4$

Substitute the values for $a$, $b$, and $c$ in $y = ax^2 + bx + c$.

$y = 3x^2 - 4x + 2$

The quadratic equation is $y = 3x^2 - 4x + 2$.

## Answer 36e.

Consider the points

$(-6,-1)$, $(-3,-4)$ and $(3,8)$

It is need to write quadratic function in standard form for the parabola that passes through the given points.

**Step-1**: substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations shown below.

$-1 = a(-6)^2 + b(-6) + c$ Substitute -6 for $x$ and -1 for $y$.

$-1 = 36a - 6b + c$ Equation 1

$-4 = a(-3)^2 + b(-3) + c$ Substitute -3 for $x$ and -4 for $y$.

$-4 = 9a - 3b + c$ Equation 2

$8 = a(3)^2 + b(3) + c$ Substitute 3 for $x$ and 8 for $y$.

$8 = 9a + 3b + c$ Equation 3

**Step-2:** solve equations 1 and 2 to eliminate $b$

$$\begin{array}{r} 36a-6b+c=-1 \times 1 \\ 9a-3b+c=-4 \times -2 \\ \hline 18a-c=7 \end{array}$$

Equation 4

Solve the equations 2 and 3 to eliminate $b$

$$\begin{array}{r} 9a-3b+c=-4 \\ 9a+3b+c=8 \\ \hline 9a+c=2 \end{array}$$

Equation 5

**Step-3:** solve equations 4 and 5 to get values of $a$ and $c$

$$\begin{array}{r} 18a-c=7 \\ 9a+c=2 \\ \hline 27a=9 \\ a=\frac{1}{3} \end{array}$$

Substitute $a=1/3$ in equation 5 to get value of $c$

$$9\left(\frac{1}{3}\right)+c=2$$
$$3+c=2$$
$$c=-1$$

Thus, $a=1/3$ and $c=-1$

Substitute values of $a$ and $c$ in equation 3 to get value of $b$

$$9\left(\frac{1}{3}\right)+3b-1=8$$
$$3+3b=9$$
$$3b=6$$
$$b=2$$

Therefore, $a=1/3, b=2$ and $c=-1$

Hence, a quadratic function for the parabola passing through the given points is

$$\boxed{y=\frac{1}{3}x^2+2x-1}$$

## Answer 37e.

***Step 1*** **Substitute** the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute $-2$ for $x$, and $-13$ for $y$ in $y = ax^2 + bx + c$.
$-13 = a(-2)^2 + b(-2) + c$

Simplify.
$-13 = 4a - 2b + c$ Equation 1

Replace $x$ with 2, and $y$ with 3 in $y = ax^2 + bx + c$ and simplify.
$3 = a(2)^2 + b(2) + c$
$3 = 4a + 2b + c$ Equation 2

Substitute 4 for $x$, and 5 for $y$ in $y = ax^2 + bx + c$ and simplify.
$5 = a(4)^2 + b(4) + c$
$5 = 16a + 4b + c$ Equation 3

***Step 2*** **Rewrite** the system of three equations as a system of two equations.

Add Equations 1 and 2 to eliminate $b$.

$$\begin{array}{rl} -13 = & 4a - 2b + c \\ 3 = & 4a + 2b + c \\ \hline -10 = & 8a \quad\quad + 2c \end{array}$$

Equation 4

Add $-2$ times Equation 1 to Equation 3 to eliminate $b$ again.

$$\begin{array}{rl} 5 = & 16a + 4b + c \\ 3 = & 4a + 2b + c \quad \times \quad -2 \end{array} \qquad \begin{array}{rl} 5 = & 16a + 4b + c \\ -6 = & -8a - 4b - 2c \\ \hline -1 = & 8a \quad\quad - c \end{array}$$

Equation 5

Multiply each side of the Equation by $-1$.

$$\frac{-1}{-1} = \frac{8a - c}{-1}$$

$1 = -8a + c$ Equation 6

***Step 3*** **Solve** the system consisting of Equations 4 and 6.

$$\begin{array}{rl} -10 = & 8a + 2c \\ 1 = & -8a + c \\ \hline -9 = & 3c \end{array}$$

Solve for $c$. For this, divide both the sides by 3.

$$\frac{-9}{3} = \frac{3c}{3}$$

$-3 = c$

Substitute $-3$ for $c$ in Equation 5.
$-1 = 8a - (-3)$

Solve for $a$.

$$-1 = 8a + 3$$
$$-4 = 8a$$
$$-\frac{1}{2} = a$$

Now, substitute the values for $a$ and $c$ in Equation 1 to find the value of $b$.

$$-13 = 4\left(-\frac{1}{2}\right) - 2b - 3$$

Solve for $b$.

$$-13 = -2 - 2b - 3$$
$$-8 = -2b$$
$$4 = b$$

Substitute the values for $a$, $b$, and $c$ in $y = ax^2 + bx + c$.

$$y = -\frac{1}{2}x^2 + 4x - 3$$

The required quadratic equation is $y = -\frac{1}{2}x^2 + 4x - 3$.

## Answer 38e.

Consider the points

$(-6,29)$, $(-4,12)$ and $(2,-3)$

It is need to write quadratic function in standard form for the parabola that passes through the given points.

**Step-1:** substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations shown below.

$29 = a(-6)^2 + b(-6) + c$ Substitute -6 for $x$ and 29 for $y$.

$29 = 36a - 6b + c$ Equation 1

$12 = a(-4)^2 + b(-4) + c$ Substitute -4 for $x$ and 12 for $y$.

$12 = 16a - 4b + c$ Equation 2

$-3 = a(2)^2 + b(2) + c$ Substitute 2 for $x$ and -3 for $y$.

$-3 = 4a + 2b + c$ Equation 3

**Step-2:** solve equations 1 and 2 to eliminate $b$

$$36a-6b+c=-29 \quad \times 2$$
$$16a-4b+c=12 \quad \times 3$$

$$24a-c=22 \text{ Equation 4}$$

Solve the equation 2 and 3 to eliminate $b$

$$16a-4b+c=12 \quad \times 1$$
$$4a+2b+c=-3 \quad \times 2$$

$$8a+c=2 \text{ Equation 5}$$

**Step-3:** solve equations 4 and 5 to get values of $a$ and $c$

$$24a-c=22$$
$$8a+c=2$$

$$32a=24$$
$$a=\frac{3}{4}$$

Substitute $a=3/4$ in equation 5 to get value of $c$

$$8\left(\frac{3}{4}\right)+c=2$$
$$6+c=2$$
$$c=-4$$

Thus, $a=3/4$ and $c=-4$

Substitute values of $a$ and $c$ in equation 3 to get value of $b$

$$4\left(\frac{3}{4}\right)+2b-4=-3$$
$$3+2b=1$$
$$2b=-2$$
$$b=-1$$

Therefore, $a=3/4, b=-1$ and $c=-4$

Hence, a quadratic function for the parabola passing through the given points is

$$\boxed{y=\frac{3}{4}x^2-x-4}$$

## Answer 39e.

***Step 1*** **Substitute** the coordinates of each point in $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute $-3$ for $x$, and $-2$ for $y$ in $y = ax^2 + bx + c$.
$-2 = a(-3)^2 + b(-3) + c$

Simplify.
$-2 = 9a - 3b + c$ Equation 1

Replace $x$ with 3, and $y$ with 10 in $y = ax^2 + bx + c$ and simplify.
$10 = a(3)^2 + b(3) + c$
$10 = 9a + 3b + c$ Equation 2

Substitute 6 for $x$, and $-2$ for $y$ in $y = ax^2 + bx + c$ and simplify.
$-2 = a(6)^2 + b(6) + c$
$-2 = 36a + 6b + c$ Equation 3

***Step 2*** **Rewrite** the system of three equations as a system of two equations.

Add $-1$ times Equation 2 to Equation 1 to eliminate $c$.

$-2 = 9a - 3b + c$ → $-2 = 9a - 3b + c$
$10 = 9a + 3b + c$ $\times\ -1$ → $-10 = -9a - 3b - c$
$-12 = -6b$ Equation 4

Divide each side by $-6$.
$$\frac{-12}{-6} = \frac{-6b}{-6}$$
$$2 = b$$

***Step 3*** **Solve** the system consisting of Equations 1 and 3.
Add $-1$ times Equation 3 to Equation 1.

$-2 = 9a - 3b + c$ → $-2 = 9a - 3b + c$
$-2 = 36a + 6b + c$ $\times\ -1$ → $2 = -36a - 6b - c$
$0 = -27a - 9b$ Equation 5

Solve for $a$. For this, substitute the value for $b$ in Equation 5.
$0 = -27a - 9(2)$

Simplify.
$$0 = -27a - 18$$
$$18 = -27a$$
$$-\frac{2}{3} = a$$

Now, substitute the values for $a$ and $b$ in Equation 1 to find the value of $c$.

$$-2 = 9\left(-\frac{2}{3}\right) - 3(2) + c$$

Solve for $b$.

$$-2 = -6 - 6 + c$$

$$10 = c$$

Substitute the values for $a$, $b$, and $c$ in $y = ax^2 + bx + c$.

$$y = -\frac{2}{3}x^2 + 2x + 10$$

The required quadratic equation is $y = -\frac{2}{3}x^2 + 2x + 10$.

## Answer 40e.

Consider the points

$(-0.5,-1)$, $(2,8)$ and $(11,25)$

It is need to write quadratic function in standard form for the parabola that passes through the given points.

**Step-1:** substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations shown below.

$-1 = a(-0.5)^2 + b(-0.5) + c$ Substitute -0.5 for $x$ and -1 for $y$.

$-1 = 0.25a - 0.5b + c$ Equation 1

$8 = a(2)^2 + b(2) + c$ Substitute 2 for $x$ and 8 for $y$.

$8 = 4a + 2b + c$ Equation 2

$25 = a(11)^2 + b(11) + c$ Substitute 11 for $x$ and 25 for $y$.

$25 = 121a + 11b + c$ Equation 3

**Step-2:** solve equations 1 and 2 to eliminate $b$

$$0.25a - 0.5b + c = -1 \quad \times 4$$
$$4a + 2b + c = 8 \quad \times 1$$

$5a + 5c = 4$ Equation 4

Solve the equation 2 and 3 to eliminate $b$

$$4a + 2b + c = 8 \quad \times 11$$
$$121a + 11b + c = 25 \quad \times 2$$

$-198a + 9c = 38$ Equation 5

**Step-3:** solve equations 4 and 5 to get values of $a$ and $c$

$$\begin{array}{r} 5a+5c=4 \ \times 9 \\ -198a+9c=38 \ \times 5 \\ \hline 1035a=-154 \\ a=-\frac{154}{1035} \end{array}$$

Substitute $a=-154/1035$ in equation 4 to get value of $c$

$$5\left(-\frac{154}{1035}\right)+5c=4$$

$$c=\frac{674}{1035}$$

Thus, $a=-154/1035$ and $c=674/1035$

Substitute values of $a$ and $c$ in equation 3 to get value of $b$

$$4\left(-\frac{154}{1035}\right)+2b+\frac{675}{1035}=8$$

$$-4(154)+2b(1035)+675=8(1035)$$

$$2b(1035)=8280-59$$

$$b=\frac{8221}{2(1035)}$$

Therefore, $a=-154/1035, b=8221/2(1035)$ and $c=674/1035$

Hence, a quadratic function for the parabola passing through the given points is

$$y=\frac{1}{1035}\left(-154x^2+\frac{8221}{2}x+674\right)$$

## Answer 41e.

We know that the intercept form of a quadratic function is $y=a(x-p)(x-q)$, where $p$ and $q$ are the intercepts.

It is given that the $x$-intercepts are $-11$ and 3. Substitute $-11$ for $p$, and 3 for $q$ in the intercept form.
$y=a[x-(-11)](x-3)$

Simplify.
$y=a\,(x+11)(x-3)$

The parabola passes through the point $(1, -192)$. In order to find the value of $a$, substitute 1 for $x$, and $-192$ for $y$ in the equation.
$-192 = a(1+11)(1-3)$

Solve for $a$.

$$\begin{aligned} -192 &= a(12)(-2) \\ -192 &= -24a \\ 8 &= a \end{aligned}$$

Substitute 8 for $a$ in $y = a\ (x+1)(x-3)$.
$y = 8(x+11)(x-3)$

Expand.

$$\begin{aligned} y &= 8\left(x^2 - 3x + 11x - 33\right) \\ &= 8\left(x^2 + 8x - 33\right) \\ &= 8x^2 + 64x - 264 \end{aligned}$$

The quadratic function of the graph with given characteristics is $8x^3 + 64x - 264$.

## Answer 42e.

Write a quadratic function with vertex $(4.5, 7.25)$ and passing through the point $(7, -3)$.

Use vertex form because the vertex is given.

$y = a(x-h)^2 + k$ Vertex form

$y = a(x-4.5)^2 + 7.25$ Substitute 4.5 for $h$ and 7.25 for $k$

Use the other given point, $(7, -3)$, to find $a$.

$-3 = a(7-4.5)^2 + 7.25$ Substitute 7 for $x$ and -3 for $y$

$-3 = a(6.25) + 7.25$

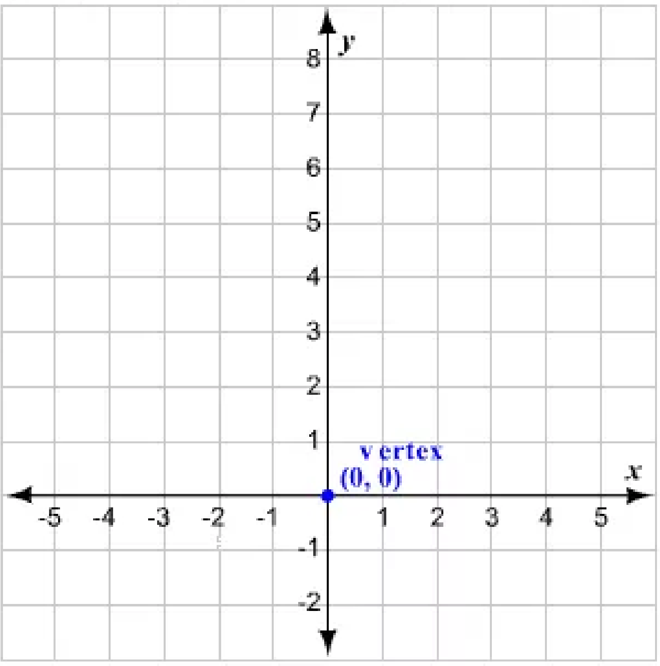
$a(6.25) = -10.25$

$a = -\dfrac{41}{24}$ Solve for $a$

Therefore $y = -\dfrac{41}{25}(x-4.5)^2 + 7.25$ is the required quadratic function.

## Answer 43e.

Assume that the vertex of the parabola is (0, 0). To draw a parabola that passes through (−2, 3), first plot the vertex (0, 0) on a coordinate plane.




We know that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where the vertex is $(h, k)$.

Substitute 0 for $h$ and $k$.

$y = a(x - 0)^2 + 0$

Simplify.

$y = ax^2$

In order to find $a$, use the given point (−2, 3). Substitute −2 for $x$, and 3 for $y$.

$3 = a(-2)^2$

Solve for $a$.

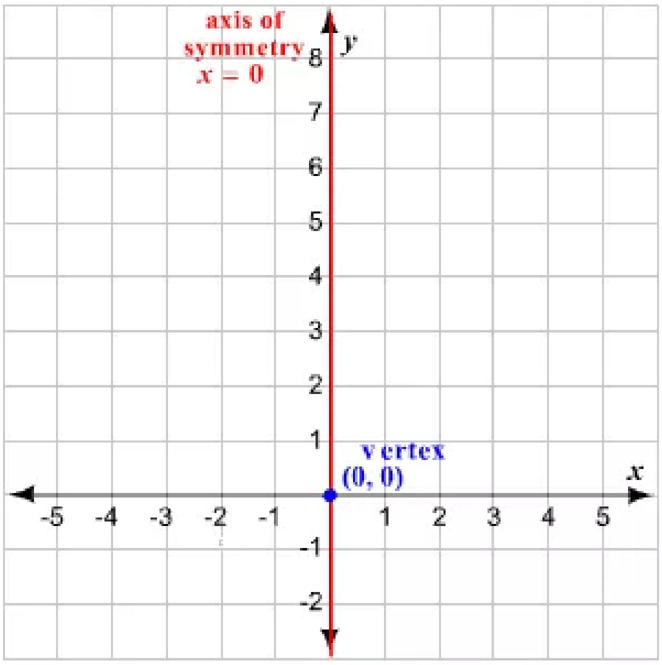
$3 = 4a$

$\frac{3}{4} = a$

The graph of the parabola opens up if $a > 0$. Since the value of $a$ is $\frac{3}{4}$, which is greater than 0, the graph of the parabola opens up.

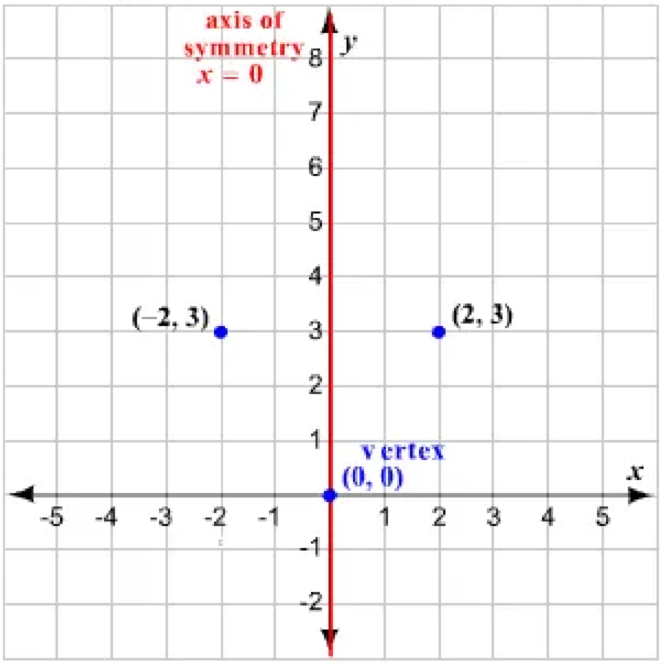
The axis of symmetry of the graph of the function $y = a(x - h)^2 + k$ is $x = h$. Since the value of $h$ is 0, the axis of symmetry of the graph of the given function is $x = 0$.

Draw the axis of symmetry $x = 0$ on the coordinate plane.

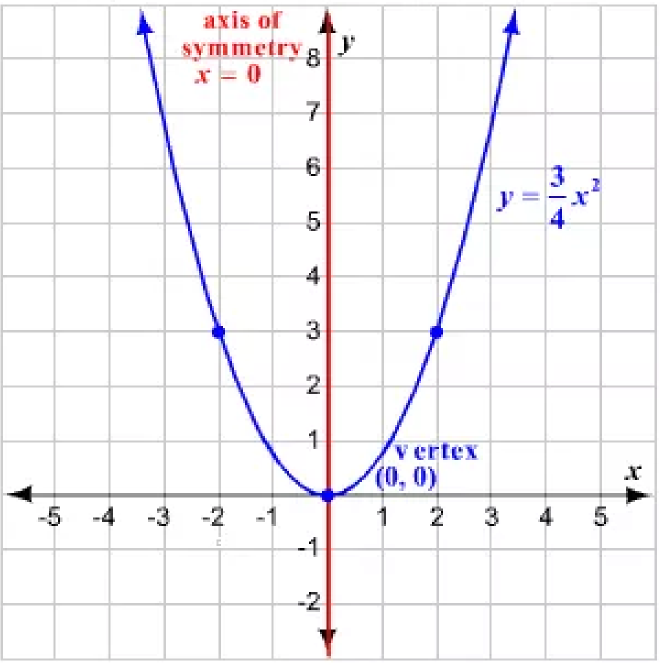



Now, evaluate the reflection of (–2, 3). Since the point (–2, 3) is 2 units left of the axis of symmetry, the reflection of (–2, 3) is 2 units right of the axis of symmetry. Thus, the reflection of (–2, 3) is (2, 3).

Plot the point and its reflection in the axis of symmetry.



Draw a parabola through the plotted points.




Substitute $\frac{3}{4}$ for $a$ in $y = ax^2$.

$$y = \frac{3}{4}x^2$$

Thus, the quadratic function in vertex form is

$$y = \frac{3}{4}x^2.$$

The above equation is both in the standard form and in the intercept form.

## Answer 44e.

Given a set of data pairs $(x, y)$.

We can determine whether the data can be modeled by a quadratic function of the form $y = ax^2$ by checking the ratio $\frac{y}{x^2}$ is a constant always.

## Answer 45e.

*Step 1* **Substitute** the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations.

Substitute 1 for $x$, and –4 for $y$ in $y = ax^2 + bx + c$.

$-4 = a(1)^2 + b(1) + c$

Simplify.

$-4 = a + b + c$ Equation 1

Replace $x$ with 3, and $y$ with $-16$ in $y = ax^2 + bx + c$ and simplify.

$$-16 = a(-3)^2 + b(-3) + c$$

$$-16 = 9a - 3b + c \qquad \text{Equation 2}$$

Substitute 7 for $x$, and 14 for $y$ in $y = ax^2 + bx + c$ and simplify.

$$14 = a(7)^2 + b(7) + c$$

$$14 = 49a + 7b + c \qquad \text{Equation 3}$$

***Step 2*** **Rewrite** the system of three equations as a system of two equations.

Add 3 times Equation 1 to Equation 2 to eliminate $b$.

$$\begin{array}{rcl} -16 &=& 9a - 3b + c \\ -4 &=& a + b + c \quad \times \; 3 \end{array} \qquad \begin{array}{rcl} -16 &=& 9a - 3b + c \\ -12 &=& 3a + 3b + 3c \\ \hline -28 &=& 12a \quad + 4c \end{array} \quad \text{Equation 4}$$

Add 7 times Equation 2 to three times Equation 3 to eliminate $b$ again.

$$\begin{array}{rcl} -16 &=& 9a - 3b + c \quad \times \; 7 \\ 14 &=& 49a + 7b + c \quad \times \; 3 \end{array} \qquad \begin{array}{rcl} -112 &=& 63a - 21b + 7c \\ 42 &=& 147a + 21b + 3c \\ \hline -70 &=& 210a \quad + 10c \end{array} \quad \text{Equation 5}$$

***Step 3*** **Solve** the system consisting Equations 4 and 5.

Add $-10$ times Equation 4 to four times Equation 5.

$$\begin{array}{rcl} -28 &=& 12a + 4c \quad \times \; -10 \\ -70 &=& 210a + 10c \quad \times \; 4 \end{array} \qquad \begin{array}{rcl} 280 &=& -120a \quad - 40c \\ -280 &=& 840a \quad + 40c \\ \hline 0 &=& 720a \end{array} \quad \text{Equation 6}$$

Solve for $a$. For this, divide both the sides by 720..

$$\frac{0}{720} = \frac{720a}{720}$$

$$0 = a$$

Now, substitute the value of $a$ in Equation 5 and simplify to find the value of $c$.

$$-28 = 12(0) + 4c$$

$$-28 = 4c$$

$$-7 = c$$

In order to find the value of $b$, substitute 0 for $a$, and $-7$ for $c$ in Equation 1.

$-4 = 0 + b - 7$

Solve for $b$.

$b = 3$

Substitute the values for $a$, $b$, and $c$ in $y = ax^2 + bx + c$.
$y = 0x^2 + 3x - 7$ or $y = 3x - 7$.

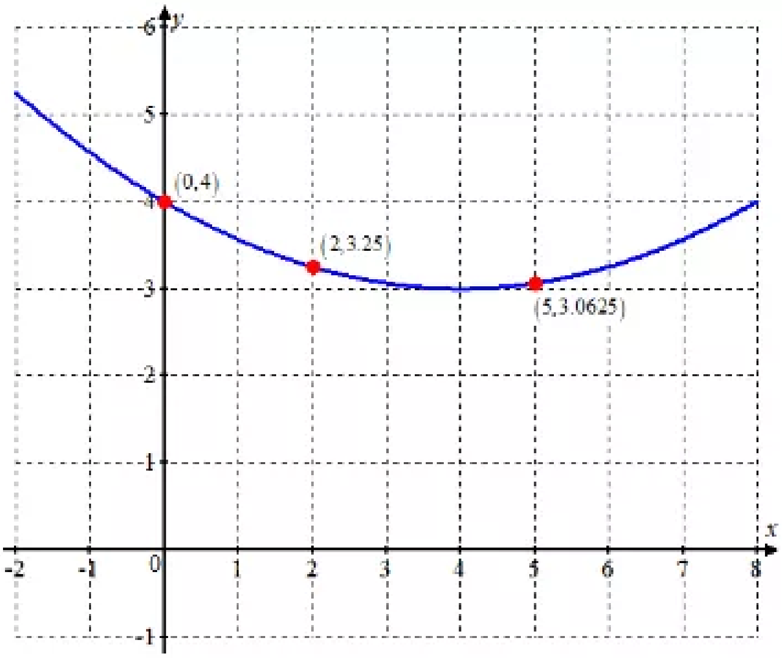
The quadratic equation is $y = 3x - 7$.

A linear function is a function that can be written in the form $y = mx + b$ where $m$ and $b$ are constants. Since $y = 3x - 7$ is of the form $y = mx + b$, the graph of the equation is a straight line.

Therefore, the given points lie on a straight line.

**Answer 46e.**

Suppose that, three points on the parabola formed by the cross section of an antenna dish are $(0,4)$, $(2,3.25)$ and $(5,3.0625)$




It is need to write a quadratic function that passes through the points $(0,4)$, $(2,3.25)$ and $(5,3.0625)$.

**Step-1:** substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain the system of three linear equations shown below.

$4 = a(0)^2 + b(0) + c$ Substitute 0 for $x$ and 4 for $y$.

$4 = c$ Equation 1

$3.25 = a(2)^2 + b(2) + c$ Substitute 2 for $x$ and 3.25 for $y$.

$3.25 = 4a + 2b + c$ Equation 2

$3.0625 = a(5)^2 + b(5) + c$ Substitute 5 for $x$ and 3.0625 for $y$.

$3.0625 = 25a + 5b + c$ Equation 3

**Step-2:** Rewrite the system of three equations in step 1as a system of two equations by substituting 4 for $c$ in equations 2 and 3.

$4a+2b+c=3.25$ Equation 2

$4a+2b+4=3.25$ Substitute 4 for $c$

$4a+2b=0.75$ Revised equation 2.

$25a+5b+c=3.0625$ Equation 3

$25a+5b+4=3.0625$ Substitute 4 for $c$

$25a+5b=-0.9375$ Revised equation 3.

**Step-3:** solve the system consisting of revised equations 1 and 3.

Use the elimination method.

$$\begin{array}{rl} 4a+2b=0.75 & \times 25 \\ 25a+5b=-0.9375 & \times 4 \\ \hline 30b=22.5 & \\ b=0.75 & \end{array}$$

So, $4a+2(0.75)=0.75$, which means $a=-0.1875$

The solution is $a=-0.1875$, $b=0.75$ and $c=4$

A quadratic for the parabola is $y=-0.1875x^2+0.75x+4$

## Answer 47e.

In order to find the quadratic function that models the path of the football, substitute the values in the vertex form of a quadratic function.

We know that the vertex form of a quadratic function is $y=a(x-h)^2+k$ with vertex at $(h, k)$. Thus,
$y=a(x-20)^2+15$.

Use the given point (0, 0), to find $a$. Substitute 20 for $x$ and 15 for $y$ in equation (1).
$0=a(0-20)^2+15$

Solve for $a$.

$$\begin{aligned} 0 &= 400a+15 \\ -15 &= 400a \\ -\frac{3}{80} &= a \end{aligned}$$

Substitute the value of $a$ in equation $y = a(x-20)^2 + 15$.

$$y = -\frac{3}{80}(x-20)^2 + 15$$

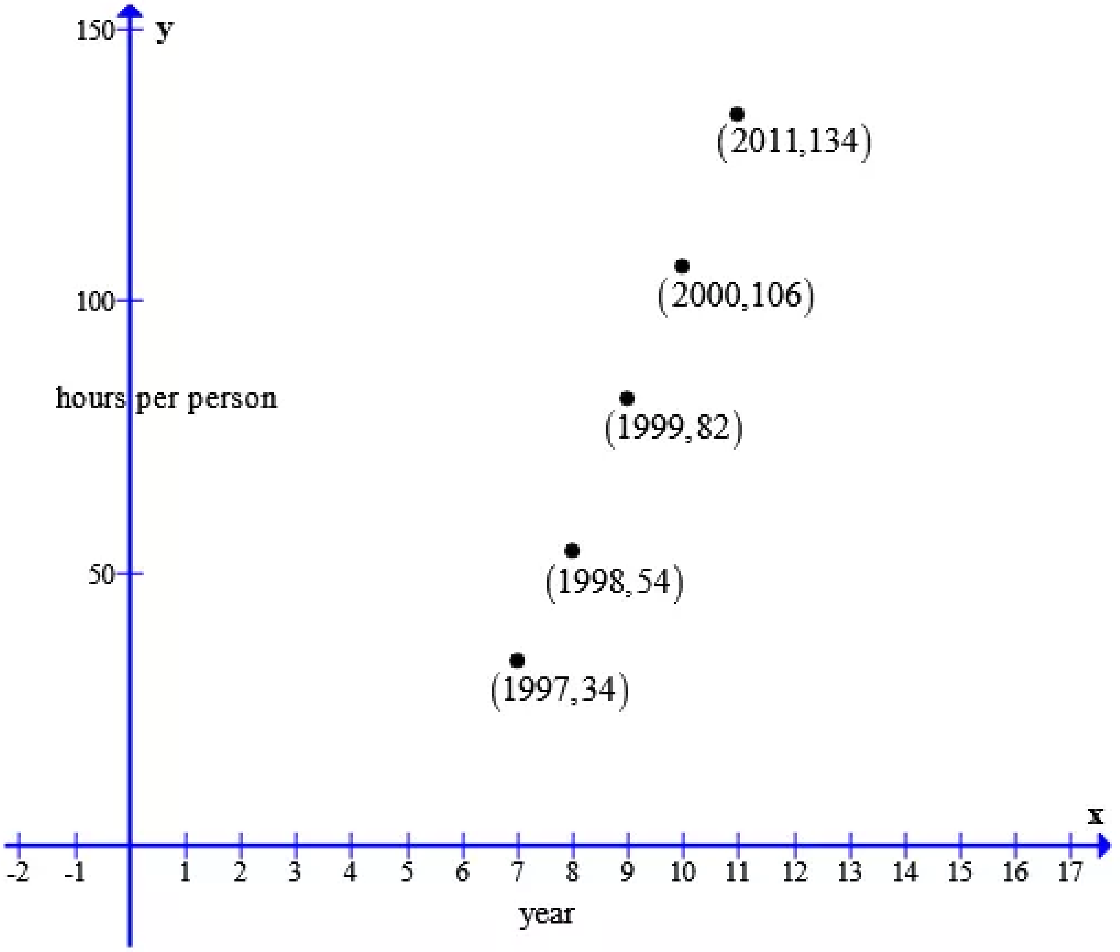
Thus, a quadratic function that models the path is $y = -\frac{3}{80}(x-20)^2 + 15$.

## Answer 48e.

For a bar graph of average number of hours per person per year spent on the internet in the United States for the years 1997-2001, we proceed as follows:

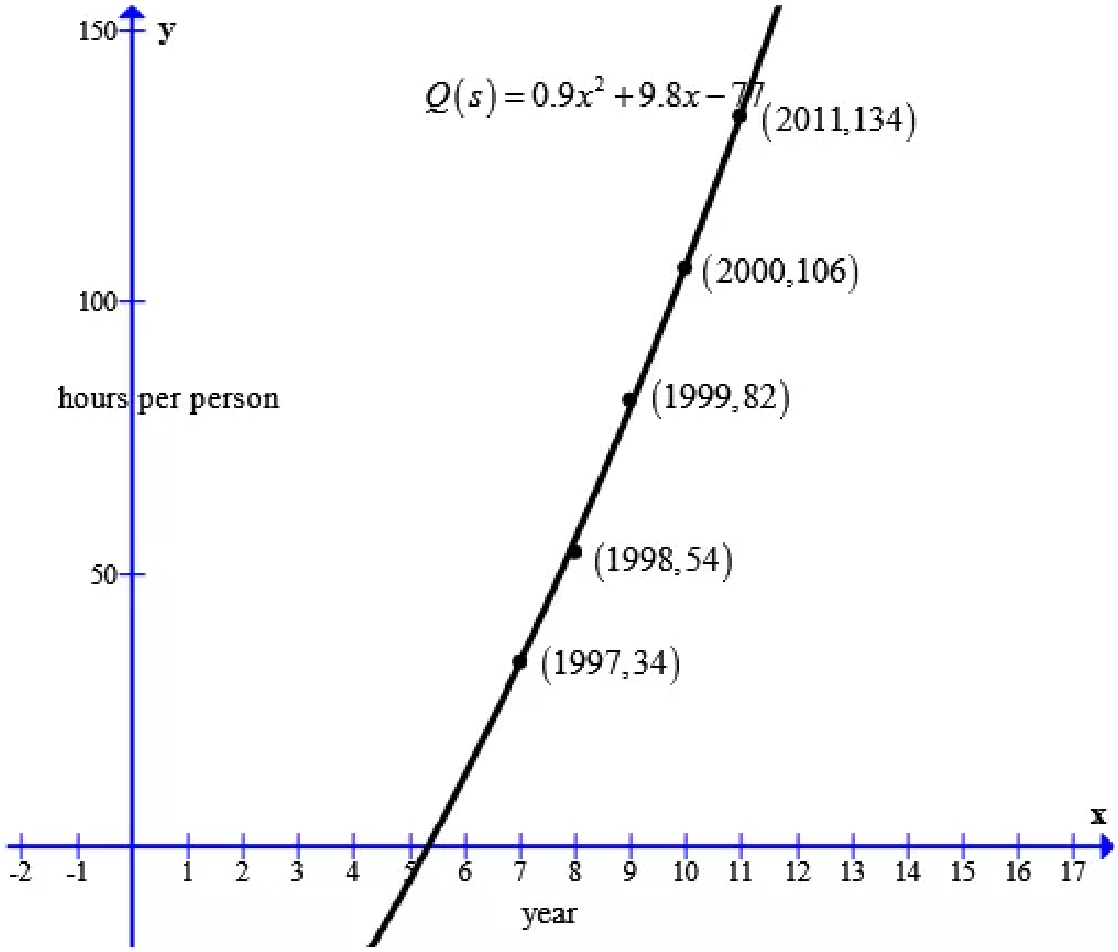
(a)

The scatter plot of the given model is shown below:



(b)

The best fitting quadratic model of the data is given by the quadratic equation is $Q(x)=0.9x^2+9.8x-77$ .And the model is shown below:




(c)

Since, the quadratic model is $Q(s)=0.9x^2+9.8x-77$ ,where

$x$ stands for years, like $x=1$ represents 1991.

Therefore, the average numbers of years a person will spend on internet in 2010 is given by:

When $x=20$ then

$$\begin{aligned} Q(x) &= 0.9x^2+9.8x-77 \\ &= 0.9(20)^2+9.8(20)-77 \\ &= 479 \end{aligned}$$

That is the average numbers of years a person will spend on internet in 2010 is $\boxed{479 \text{ years}}$ .

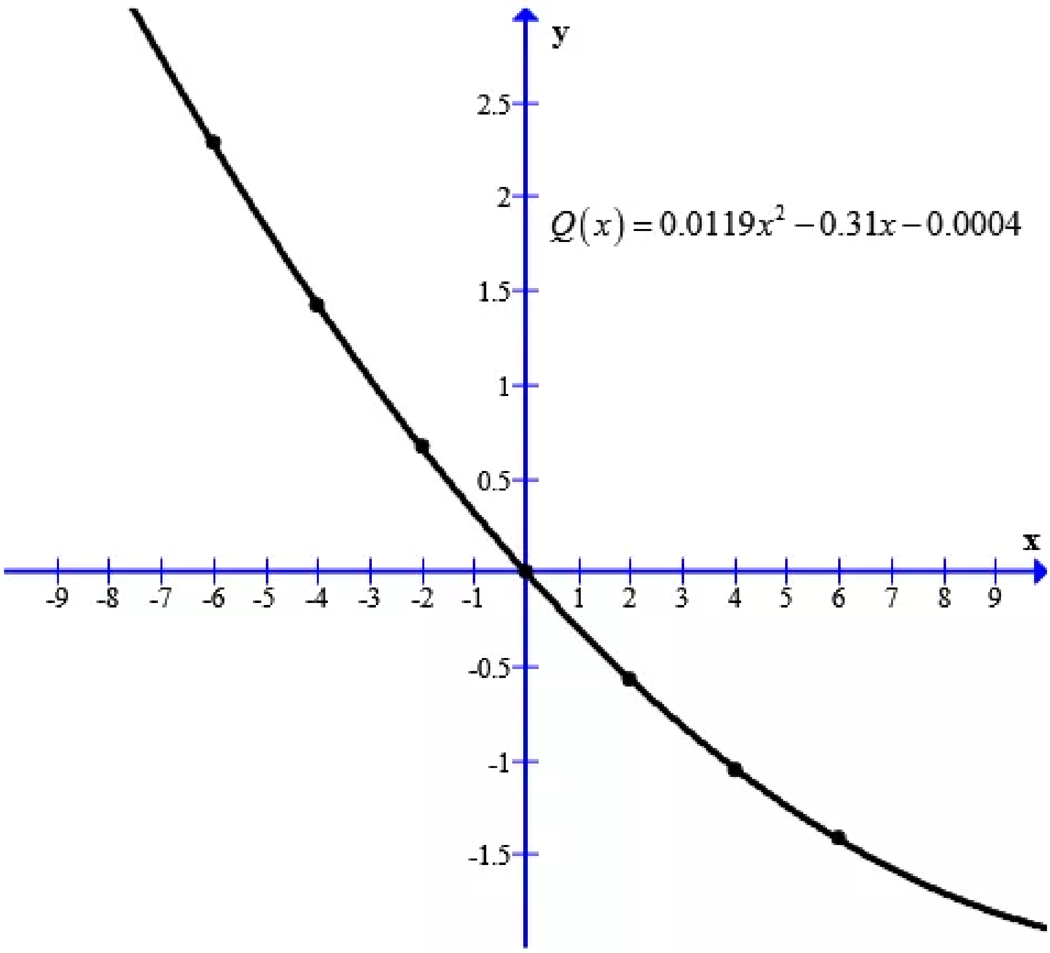
## Answer 49e.

Below is the table which shows how wind affects a runners performance in the 200 meter dash.

| Wind speed(m/s) $s$ | $-6$ | $-4$ | $-2$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|---|---|---|
| Change in finishing time(sec) $t$ | 2.28 | 1.42 | 0.67 | 0 | $-0.57$ | $-1.05$ | $-1.42$ |

(a)

The best fitting quadratic model of the data is given by the quadratic equation is $Q(x)=0.0119x^2-0.31x-0.0004$ .And the model is shown below:




(b)

Since, the quadratic model is $Q(x)=0.0119x^2-0.31x-0.0004$ ,where $x$ stands for wind speed.

Therefore, the finishing time when the wind speed is 10 $m/\sec$ is given by:

When $x=10$ then

$$Q(x)=0.0119x^2-0.31x-0.0004$$
$$=0.0119(10)^2-0.31(10)-0.0004$$
$$=-1.9104$$

That is the finishing time when the wind speed is 10 $m/\sec$ is $\boxed{-1.9104\text{secs}}$.

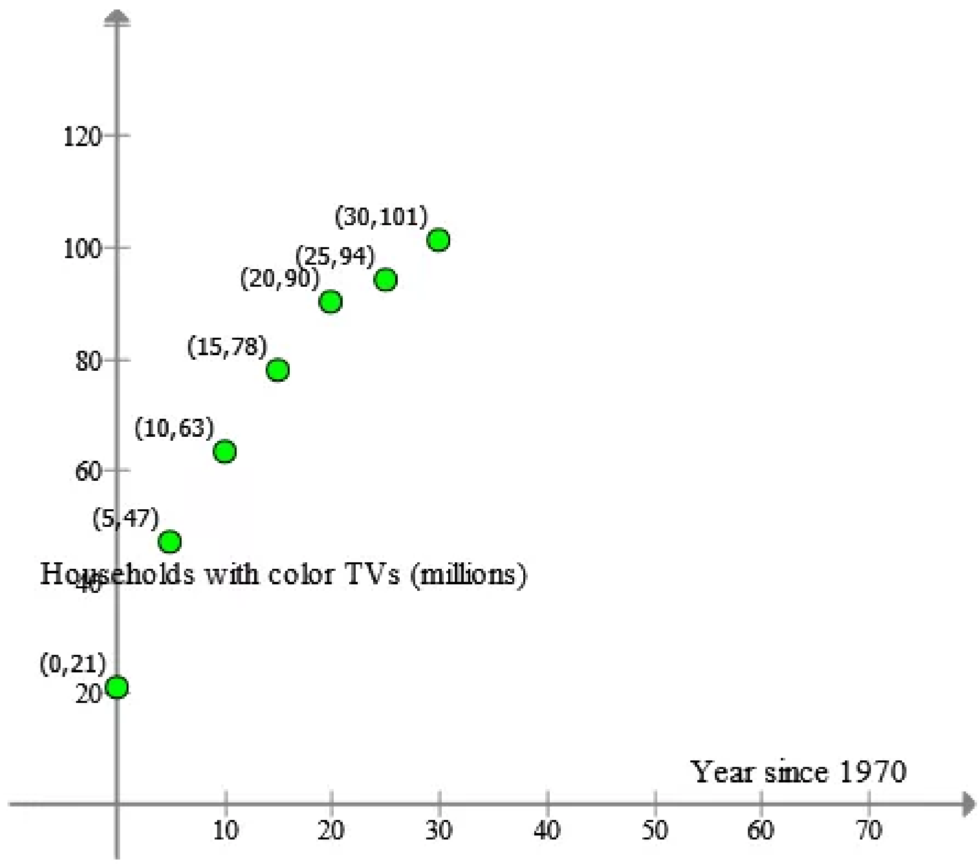
## Answer 50e.

The table given below shows the number of U.S. households (in millions) with color televisions from 1970 through 2000.

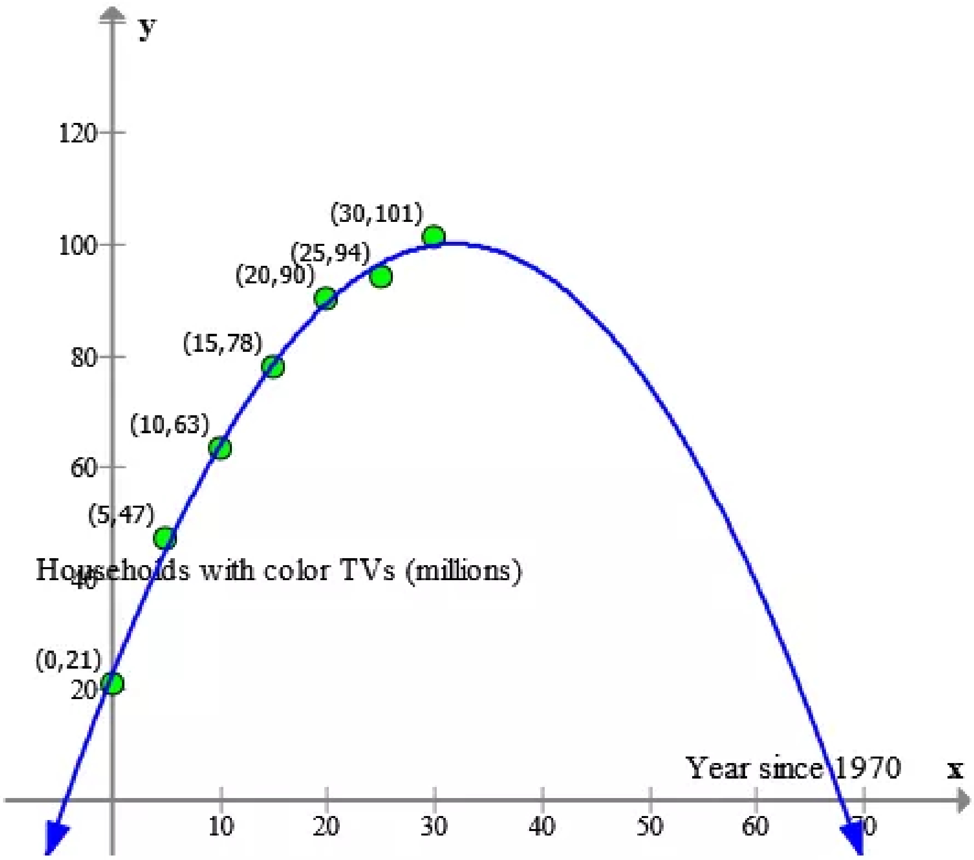
| Year since 1970 | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| Households with color TVs (millions) | 21 | 47 | 63 | 78 | 90 | 94 | 101 |

a.

We can draw a scatter plot of the data given above in the table by plotting the ordered pairs $(0,21),(5,47),(10,63),(15,78),(20,90),(25,94),(30,101)$ where years since 1970 along the horizontal axis and households with color TVs (millions) along the vertical axis as shown below.



We can draw the parabola that we think best fits the data as given below.




b.

We estimate the coordinates of three points $\boxed{(0,23),(15,78),(68,0)}$ on the parabola.

We substitute the coordinates of each point into the quadratic function $y = ax^2 + bx + c$ to obtain the system of three linear equations shown below.

$23 = a(0)^2 + b(0) + c$ [Substitute 0 for $x$ and 23 for $y$]

$23 = c$ [Equation (A)]

$78 = a(15)^2 + b(15) + c$ [Substitute 15 for $x$ and 78 for $y$]

$78 = 225a + 15b + c$ [Equation (B)]

$0 = a(68)^2 + b(68) + c$ [Substitute 68 for $x$ and 0 for $y$]

$0 = 4624a + 68b + c$ [Equation (C)]

We rewrite the system of three equations as a system of two equations by substituting 23 for $c$ in equations (B) and (C).

$$225a+15b+c=78 \quad \text{[Equation (B)]}$$
$$225a+15b+23=78 \quad \text{[Substitute 23 for } c\text{]}$$
$$225a+15b=55 \quad \text{[Subtract 23 from each side]}$$
$$45a+3b=11 \quad \text{[Divide each side by 5]}$$
$$4624a+68b+c=0 \quad \text{[Equation (C)]}$$
$$4624a+68b+23=0 \quad \text{[Substitute 23 for } c\text{]}$$
$$4624a+68b=-23 \quad \text{[Subtract 23 from each side]}$$

We are going to solve the system of the equations $45a+3b=11$ and $4624a+68b=-23$.

We multiply each side of the equation $45a+3b=11$ by 4624.

$$208080a+13872b=50864 \qquad \text{...... (D)}$$

We multiply each side of the equation $4624a+68b=-23$ by 45.

$$208080a+3060b=-1035 \qquad \text{...... (E)}$$

We subtract each side of the equations (E) from each side of the equation (D).

$$208080a+13872b=50864$$
$$\underline{208080a+3060b=-1035}$$
$$10812b=51899$$
$$b=4.8$$

We substitute 4.8 for $b$ in the equation $45a+3b=11$ .

$$45a+3(4.8)=11$$
$$45a+14.4=11$$
$$45a=-3.4$$
$$a=-0.076$$

The solution is

$$a=-0.076, b=4.8, c=23$$

Hence, the quadratic function for the parabola is

$$\boxed{y=-0.076x^2+4.8x+23}$$

c.

The table given below shows the number of U.S. households (in millions) with color televisions from 1970 through 2000 using the quadratic function $y=-0.076x^2+4.8x+23$ for the parabola.

| Year since 1970 ($x$) | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| Households with color TVs (millions) ($y$) where $y=-0.076x^2+4.8x+23$ | 23 | 45.1 | 63.4 | 77.9 | 88.6 | 95.5 | 98.6 |

Hence, the numbers of households given by given by our new function $y=-0.076x^2+4.8x+23$ $\boxed{\text{are almost same}}$ with the numbers in the original table.

**Answer 51e.**

Obtain the system of three linear equations. For this, substitute 0 for $x$ and $y$ in $y = ax^2 + bx + c$ first.

$0 = a(0)^2 + b(0) + c$

Simplify.

$0 = c$ Equation 1

Now, substitute 40 for $x$, and 38.2 for $y$ in $y = ax^2 + bx + c$ and simplify.

$38.2 = a(40)^2 + 40b + c$

$38.2 = 1600a + 40b + c$ Equation 2

Replace $x$ with 165, and $y$ with 0 in $y = ax^2 + bx + c$ and simplify.

$0 = a(165)^2 + 165b + c$

$0 = 27,225a + 165b + c$ Equation 3

Rewrite the system of three equations as s system of two equations. For this, substitute 0 for $c$ in Equation 2.

Simplify.

$38.2 = 1600a + 40b + 0$

$38.2 = 1600a + 40b$ Revised Equation 2

Substitute 0 for $c$ in Equation 3 and simplify.

$0 = 27,225a + 165b + 0$

$0 = 27,225a + 165b$

Divide each side by 165.

$0 = 165a + b$ Revised Equation 3

Add 40 times Revised Equation 3 to Revised Equation 2.

$$\begin{aligned} 38.2 &= 1600a + 40b \\ 0 &= 165a + b \quad \times \quad 40 \end{aligned} \qquad \begin{aligned} 38.2 &= 1600a + 40b \\ 0 &= -6600a - 40b \\ \hline 38.2 &= -5000a \end{aligned}$$

Solve for $a$.

$$-\frac{38.2}{5000} = a$$

Substitute the value for $a$ in $165a + b = 0$ and simplify to find the value of $b$.

$$165\left(-\frac{38.2}{5000}\right) + b = 0$$

$$b = \frac{6303}{5000}$$

Substitute the values for $a$, $b$, and $c$ in $y = ax^2 + bx + c$.

$$y = -\frac{38.2}{5000}x^2 + \frac{6303}{5000}x + 0$$

$$y = -\frac{38.2}{5000}x^2 + \frac{6303}{5000}x$$

Thus, the quadratic equation is $y = -\frac{38.2}{5000}x^2 + \frac{6303}{5000}x \quad \cdots\cdots(7)$.

Substitute 10 for $x$, and $-11.84$ for $y$ to check whether $(10, -11.84)$ lies on the given parabola.

$$-11.84 \stackrel{?}{=} -\frac{38.2}{5000}(10)^2 + \frac{6303}{5000}(10)$$

$$-11.84 \approx 11.84$$

The point $(10, -11.84)$ does not lie on the given parabola.

Similarly check for the remaining choices. The only points that satisfies the equation is (80, 51.95).

The correct answer is choice **C**.

## Answer 52e.

Let $R$ be the maximum number of regions into which a circle can be divided using $n$ chords.
We are going to complete the table given below.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $R$ | ? | ? | 4 | ? | ? | ? | ? |

We complete the table as given below.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $R$ | $\boxed{1}$ | $\boxed{2}$ | 4 | $\boxed{6}$ | $\boxed{8}$ | $\boxed{10}$ | $\boxed{12}$ |

We use three ordered pairs $(0,1),(1,2)$ and $(5,10)$ to write a quadratic model giving $R$ as a function of $n$.

We substitute each ordered pair into the quadratic function $R = an^2 + bn + c$ to obtain the system of three linear equations shown below.

$1 = a(0)^2 + b(0) + c$ [Substitute 0 for $n$ and 1 for $R$]

$1 = c$ [Equation (A)]

$2 = a(1)^2 + b(1) + c$ [Substitute 1 for $n$ and 2 for $R$]

$2 = a + b + c$ [Equation (B)]

$10 = a(5)^2 + b(5) + c$ [Substitute 5 for $n$ and 10 for $R$]

$10 = 25a + 5b + c$ [Equation (C)]

We rewrite the system of three equations as a system of two equations by substituting 1 for $c$ in equations (B) and (C).

$a + b + c = 2$ [Equation (B)]

$a + b + 1 = 2$ [Substitute 1 for $c$]

$a + b = 1$ [Subtract 1 from each side]

$25a + 5b + c = 10$ [Equation (C)]

$25a + 5b + 1 = 10$ [Substitute 1 for $c$]

$25a + 5b = 9$ [Subtract 1 from each side]

We are going to solve the system of the equations $a + b = 1$ and $25a + 5b = 9$.

We multiply each side of the equation $a + b = 1$ by 25.

$25a + 25b = 25$ ...... (D)

We subtract each side of the equation $25a + 5b = 9$ from each side of the equation (D).

$$\begin{aligned} 25a + 25b &= 25 \\ \underline{25a + \ \ 5b} &\underline{= 9} \\ 20b &= 16 \\ b &= 0.8 \end{aligned}$$

We substitute 0.8 for $b$ in the equation $a + b = 1$.

$$\begin{aligned} a + 0.8 &= 1 \\ a &= 0.2 \end{aligned}$$

The solution is

$$a = 0.2, b = 0.8, c = 1$$

Hence, the quadratic model is

$$\boxed{R = 0.2n^2 + 0.8n + 1}.$$

## Answer 53e.

Substitute 5 for $x$ in the given expression.
$x^2 - 3 = 5^2 - 3$

Evaluate the power.
$5^2 - 3 = 25 - 3$

Subtract.
$25 - 3 = 22$

Therefore, when $x$ is 5, the value of the given expression is 22.

## Answer 54e.

Consider the expression $3a^5 - 10$
Evaluate the expression $3a^5 - 10$ when $a = -1$.
$3a^5 - 10$

| | |
|---|---|
| $= 3(-1)^5 - 10$ | Substitute $-1$ for $a$ |
| $= 3(-1) - 10$ | Evalute power. |
| $= -3 - 10$ | Multiply. |
| $= -13$ | Add. |

Therefore, $\boxed{-13}$

## Answer 55e.

Substitute $-2$ for $x$ in the given expression.
$x^4 = (-2)^4$

Evaluate the power.
$(-2)^4 = 16$

Therefore, when $x$ is $-2$, the value of the given expression is 16.

## Answer 56e.

Consider the expression $4u^3 - 15$
Evaluate the expression $4u^3 - 15$ when $u = 3$.
$4u^3 - 15$

| | |
|---|---|
| $= 4(3^3) - 15$ | Substitute 3 for $u$. |
| $= 4(27) - 15$ | Evalute power. |
| $= 108 - 15$ | Multiply. |
| $= 93$ | Subract. |

Therefore, $\boxed{93}$

## Answer 57e.

Substitute 5 for $v$ in the given expression.
$v^2 + 3v - 5 = 5^2 + 3(5) - 5$

Evaluate the power.
$5^2 + 3(5) - 5 = 25 + 3(5) - 5$

Multiply.
$25 + 3(5) - 5 = 25 + 15 - 5$

Subtract.
$25 + 15 - 5 = 35$

Therefore, when $x$ is 5, the value of the given expression is 35.

## Answer 58e.

Consider the expression $-y^3 + 2y + 5$

Evaluate the expression $-y^3 + 2y + 5$ when $y = 2$.

$-y^3 + 2y + 5$

$= -2^3 + 2(2) + 5$ Substitute 2 for $y$

$= -8 + 4 + 5$ Evalute power.

$= 1$ Add.

Therefore, $\boxed{1}$

## Answer 59e.

Number the equations.

$4x + 5y = 18$ Equation 1

$-x + 2y = 15$ Equation 2

***STEP 1*** Solve Equation 2 for $x$. Subtract $2y$ from both the sides.

$$-x + 2y - 2y = 15 - 2y$$
$$-x = 15 - 2y$$

Divide both the sides by $-1$.

$$\frac{-x}{-1} = \frac{15 - 2y}{-1}$$
$$x = -15 + 2y \quad \text{Revised Equation 2}$$

***STEP 2*** Substitute $-15 + 2y$ for $x$ in Equation 1.
$4(-15 + 2y) + 5y = 18$

Clear the parentheses using the distributive property.

$$-60 + 8y + 5y = 18$$
$$-60 + 13y = 78$$

Solve for $y$. Add 60 to both the sides.

$$-60 + 13y + 60 = 18 + 60$$
$$13y = 78$$

Divide both the sides by 13.

$$\frac{13y}{13} = \frac{78}{13}$$
$$y = 6$$

***STEP 3*** Substitute 6 for $y$ in Revised Equation 2.

$$x = -15 + 2(6)$$
$$x = -15 + 12$$
$$x = -3$$

The solution is (−3, 6).

## Answer 60e.

Consider the system of linear equations

$$3x + 7y = 1 \quad \text{......(1)}$$

$$4x + 5y = 23 \quad \text{......(2)}$$

**Step 1:** Solve equation (2) for $x$.

$$x = \frac{23 - 5y}{4} \qquad \text{Revised equation } (2)$$

**Step 2:** Substitute the expression for $x$ in to (1) and solve for $y$.

$$3x + 7y = 1 \qquad \text{Write equation (1)}$$

$$3\left(\frac{23 - 5y}{4}\right) + 7y = 1 \qquad \text{Substitute } \frac{23 - 5y}{4} \text{ for } x.$$

$$y = -5 \qquad \text{Solve for } y.$$

**Step 3**: Substitute the value of $y$ into revised equation (2) and solve for $x$.

$$x = \frac{23 - 5y}{4} \qquad \text{Write Revised equation } (2)$$

$$x = \frac{23 - 5(-5)}{4} \qquad \text{Substitute } -5 \text{ for } y.$$

$$x = 12 \qquad \text{Simplify}$$

The solution is $\boxed{(12, -5)}$

**CHECK**: Check the solution by substituting into original equations.

$$3(12) + 7(-5) \stackrel{?}{=} 1 \qquad \text{Substitute for } x \text{ and } y. \qquad 4(12) + 5(-5) \stackrel{?}{=} 23$$

$$1 = 1 \qquad \text{Solution checks.} \qquad 23 = 23$$

## Answer 61e.

Number the equations.

$$3x + 4y = -1 \qquad \text{Equation 1}$$

$$2x + 6y = -31 \qquad \text{Equation 2}$$

***STEP 1*** We can eliminate one of the variables by obtaining coefficients that are opposites of each other.

In this case, multiply Equation 1 by $-2$ and Equation 2 by 3 as the first step in eliminating $x$.

$$3x + 4y = -1 \xrightarrow{\times -2} -6x - 8y = 2 \quad \text{Equation 3}$$

$$2x + 6y = -31 \xrightarrow{\times 3} 6x + 18y = -93 \quad \text{Equation 4}$$

***STEP 2*** Add Equation 3 and Equation 4.

$$\begin{array}{r} -6x - 8y = 2 \\ 6x + 18y = -93 \\ \hline 10y = -91 \end{array}$$

Divide both the sides by 10.

$$\frac{10y}{10} = \frac{-91}{10}$$

$$y = -9.1$$

***STEP 3*** Substitute $-9.1$ for $y$ in either of the original equations, say, Equation 1.

$$3x + 4(-9.1) = -1$$

Simplify.

$$3x - 36.4 = -1$$

$$3x = 35.4$$

$$x = 11.8$$

The solution is (11.8, $-9.1$).

## Answer 62e.

Solve the following system of linear equations.

$$3x + y = 10 \quad \text{.......(1)}$$

$$-x + 2y = 20 \quad \text{.......(2)}$$

Consider,

$$(1) \times 2: \quad 6x + 2y = 20$$

$$(2) \quad : \quad \underline{-x + 2y = 20} \qquad \text{Subtract}$$

$$7x = 0$$

$$x = 0 \qquad \text{Divide by 7}$$

Put $x = 0$ in (1)

$$3(0) + y = 10$$

$$y = 10$$

Therefore $\boxed{(0,10)}$ is the solution of the given system of linear equations.

## Answer 63e.

Number the equations.

$4x + 5y = 2$ Equation 1

$-3x + 2y = 33$ Equation 2

***STEP 1*** We can eliminate one of the variables by obtaining coefficients that are opposites of each other.

In this case, multiply Equation 1 by 3 and Equation 2 by 4 as the first step in eliminating $x$.

$$4x + 5y = 2 \xrightarrow{\times\ 3} 12x + 15y = 6 \quad \text{Equation 3}$$

$$-3x + 2y = 33 \xrightarrow{\times\ 4} -12x + 8y = 132 \quad \text{Equation 4}$$

***STEP 2*** Add Equation 3 and Equation 4.

$$\begin{array}{r} 12x + 15y = 6 \\ -12x + 8y = 132 \\ \hline 23y = 138 \end{array}$$

Divide both the sides by 23.

$$\frac{23y}{23} = \frac{138}{23}$$

$$y = 6$$

***STEP 3*** Substitute 6 for $y$ in either of the original equations, say, Equation 1.

$4x + 5(6) = 2$

Simplify.

$$4x + 30 = 2$$

$$4x = -28$$

$$x = -7$$

The solution is $(-7, 6)$.

## Answer 64e.

Solve the following system of linear equations.

$2x + 3y = -1$ ......(1)

$10x + 7y = -1$ ......(2)

Now,

$(1)\times 7: \quad 14x + 21y = -7$

$(2)\times 3: \quad 30x + 21y = -3$ Subtract

$-16x = -4$

$x = \frac{1}{4}$ Divide by 16

Substitute, $x = \frac{1}{4}$ in equation (1)

$$2\left(\frac{1}{4}\right) + 3y = -1$$

$$3y = -1 - \frac{1}{2}$$

$$3y = -\frac{3}{2} \quad \text{Simplify}$$

$$y = -\frac{1}{2} \quad \text{Divide by 3}$$

Therefore $\boxed{\left(\frac{1}{4}, -\frac{1}{2}\right)}$ is the solution of the given system of linear equations.