WAVES

Waves

Propagation of disturbance from one place to another is called wave motion i.e., a wave is a form of disturbance which travels through a material medium due to the repeated periodic motion of the particles of the medium about their mean position, the disturbance is handed over from one particle to the next particle. There is no transference of the medium but it is only the energy of the disturbance which is propagated forward by means of the waves.

Mechanical waves : Mechanical waves are formed by the simple harmonic vibrations of the particles of the medium about their mean position. The medium does not move from one place to another. Sound waves and water waves are mechanical in nature.

It should be noted that, a medium is necessary for the propagation of mechanical waves. The medium should be continuous and elastic.

Electromagnetic waves : Electromagnetic waves are formed by mutually perpendicularly vibrating electric and magnetic fields. They do not require medium for their propagation. Examples are, light waves and radio waves.

Difference between mechanical waves and electromagnetic waves (or between sound waves and light waves)

Mechanical waves (Sound waves)	Electromagnetic waves (Light waves)
1. They require material medium (solid, liquid or a gas) for their propagation.	1. They do not require any material medium for their propagation.
2. They are caused due to the vibrations of the particles of the medium.	2. They are caused due to varying electric and magnetic fields.
3. They have low speed.	3. They have high speed.

4. Their speed varies from medium to medium.	4. Their speed is always constant and its value is $3 \times 10^8\ ms^{-1}$.
5. Generally, they have low frequency and large wavelength	5. They have high frequency and short wavelength.
6. They can be transv-erse or longitudinal.	6. They are only transverse.
7. Their examples are : sound waves matter waves.	7. Their examples are : light waves radio waves.

Types of wave motion :

(i) Transverse wave : A wave motion in which the particles of the medium execute simple harmonic motion about their mean positions at right angles to the direction of propagation of the wave, is called transverse wave. These waves can propagate through solids and liquids but not through gases, because gases do not passes elastic properties. For examples, vibrations in strings, ripples on water surface, electromagnetic waves etc. They travel in the form of crests and troughs.

(ii) *Longitudinal waves* : A wave motion in which the particles of the medium execute simple harmonic motion about their mean positions in the direction of propagation of the wave, is called longitudinal wave. These wave can propagate through solids, liquids and gases. For example, waves produced by compressing a spring or rubber cord, sound waves, etc. They travel in the form of compressions and rarefactions.

Distinction between transverse wave and longitudinal waves

Transverse wave	Longitudinal wave
1. In this wave, displacement of the particles of the medium is at right angles to the direction of propagation of the waves.	1. In this wave, displacement of the particles of the medium is in the direction of propagation of the waves.
2. These waves travel in the form of crests and troughs.	2. These waves travel in the form of compressions and rarefactions.

3. The distance between two consecutive crests or troughs is called wavelength.	3. The distance between two consecutive compressions or rarefactions is called wavelength.
4. These waves can travel in solids and liquids only.	4. These waves can travel in solids, liquids and gases.
5. These waves can be polarised.	5. These waves can't be polarised.

Amplitude : The maximum displacement of the vibrating particle on either side of its mean position is called the amplitude (A). Its unit is metre.

Time-period : Time taken by a vibrating particle to make one complete vibration is called time-period (T). Its unit is second.

Frequency : The number of vibrations completed by a particle in one second is called the frequency (ν), Its unit is Hz.

Also, $T = \frac{1}{\nu}$

or $\nu.T. = 1$

$\therefore$ Frequency × time period = 1.

Wavelength : Wavelength can be defined in the following different ways :

(i) It is the distance travelled by a wave during one complete vibration of the vibrating particle.

(ii) It is the distance between two nearest particles in the same phase.

(iii) It is the distance between two consecutive crests or troughs.

(iv) It is the length of one complete wave.

Wavelength is represented by λ. Its unit is metre.

Wave number : Number of waves in unit distance is known as wave number. It is equal to reciprocal of wavelength, i.e.,

$$\text{Wave number} = \frac{1}{\lambda}.$$

Relation between wave velocity, frequency and wavelength

Wave velocity is defined as the distance travelled by the wave in one second.

$\therefore$ Wave-velocity

$$= \frac{\text{Distance travelled by the wave}}{\text{Time taken by the wave}}$$

$$= \frac{\text{Wave length}}{\text{Time period}}$$

or $c = \frac{\lambda}{T}$

But, $\nu = \frac{1}{T}$

$\therefore \quad c = \nu\lambda.$

or wave-velocity = frequency × wave length

Velocities of different waves

(i) Velocity of longitudinal waves in an elastic medium

$$c = \sqrt{\frac{E}{\rho}}$$

where E = volume of elasticity of the medium

ρ = density of the medium

(ii) Velocity of longitudinal waves in a gas (or sound waves in air)

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

where γ = ratio of two specific heats of the gas

P = pressure of a given mass of a gas

ρ = density of the gas

(iii) Velocity of sound waves in a liquid medium

$$c = \sqrt{\frac{K}{\rho}}$$

Where, K = bulk modulus of elasticity of the liquid

ρ = density of the liquid.

(iv) Velocity of sound waves in a solid medium

$$c = \sqrt{\frac{Y}{\rho}}$$

where, Y = Young's modulus of elasticity of the solid

ρ = density of the solid.

(v) Velocity of transverse waves in a string

$$c = \sqrt{\frac{T}{m}}$$

where, T = tension in the string

m = mass per unit length of the string.

Energy transmission waves :

Total energy per unit volume is given by,

$$E = 2\pi^2 \rho \nu^2 A^2$$

And energy transferred per unit area per second (called an intensity) is given by

$$I = Ec = 2\pi^2 c \rho \nu^2 A^2$$

Equation of a plane simple harmonic wave

$$\xi = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{cT} \right)$$

or $$\xi = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

or $$\xi = A \sin \frac{2\pi}{\lambda} (ct - x)$$

These are the various forms of the equation of a plane simple harmonic wave travelling along the X-axis with velocity c.

Differential equation of wave motion

$$\frac{d^2\xi}{dt^2} = c^2 \frac{d^2\xi}{dx^2}$$

Principle of Superposition of wave : It states that when more than one wave travel in the same medium at the same time, they behave as if it alone were present and each wave-train produces the same effect. The resultant displacement at any instant of a particle will now be the vector sum of the individual displacement. If $\xi_1, \xi_2, \xi_3 ...$ be the displacement vectors due to individual waves acting separately, then the resultant displacement is given by their vector sum, i.e.,

$$\xi = \xi_1 + \xi_2 + \xi_3 + ...$$

Superposition of waves may give rise to following important cases :

(i) Superposition of two waves of the same frequency moving in the same direction gives rise to interference of waves.

(ii) Superposition of two waves of the same frequency moving in the opposite direction gives rise to stationary waves.

(iii) Superposition of two waves of slightly different frequencies moving in the same direction gives rise to beats.

Interference of waves : Superposition of two waves of the same frequency moving in the same direction giving rise to variation of intensity is called as interference of waves.

For constructive interference, intensity of the wave

$$I_{max} \propto (A_1 + A_2)^2$$

For destructive interference, intensity of the wave

$$I_{min} \propto (A_1 - A_2)^2$$

Where A_1 and A_2 are the amplitudes of the two individual waves.

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2} = \frac{(r+1)^2}{(r-1)^2}$$

where $r = \frac{A_1}{A_2}$ = amplitude ratio.

Conditions for interference : In order to get interference due to two wave-trains, the following conditions must be fulfilled :

(a) The two waves must travel along the same path so that their displacement are collinear.

(b) To get a better contrast between two successive maxima and minima of intensity, the amplitudes of the two waves must be approximately equal. The ideal case is when two amplitudes are exactly equal.

(c) The frequencies of two sources must be exactly equal so that there is no change in their relative phase with time. To achieve this condition, the two sources must vibrate either in the same phase or with a constant phase difference between them. Such type of sources are called as coherent sources.

Nodes and antinodes : Nodes are those points in stationary waves where displacement and velocity are zero but strain is maximum. They are separated from each other by a distance $\frac{\lambda}{2}$.

Antinodes are those points in stationary waves where displacement and velocity are the maximum but strain is zero. They are also separated from each other by a distance $\frac{\lambda}{2}$.

Characteristics of stationary waves :

(i) The distance between two consecutive nodes or antinodes is $\frac{\lambda}{2}$, whereas the distance between a node and the adjacent antinode is $\frac{\lambda}{4}$.

(ii) All the particles, except all the nodes, oscillate in simple harmonic motion with the same period as that of the component waves.

(iii) All the particles have their maximum and minimum velocities at the same time.

(iv) There is no transference of energy across any plane.

(v) The amplitude of vibrations of the particles gradually increases from zero at the nodes to the maximum at the antinodes.

(vi) The medium splits up into segments. All the particles in the same segment are in phase but in opposite phase with the particles in the neighbouring segment.

(vii) Condensations (crests) or rarefaction (troughs) do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(viii) Nodes and antinodes are formed alternately. Nodes are the points where displacement and velocity are zero but strain is maximum. Antinodes are the points where displacement and velocity are the maximum but strain is zero.

Stationary waves in a string : Frequency of the vibrating string is given by,

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Where, l = vibrating length of the string

T = tension in the string

m = mass per unit length of the string.

Law of vibration of strings : The three laws of vibration of strings are obtained from

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

(a) ***Law of length*** **:** For a string of given mass and tension, the frequency of a string varies inversely as its length,

i.e., $\nu \propto \frac{1}{l}$ when T and m are constants.

(b) ***Law of tension*** **:** For a string of given length and material, the frequency of a string varies directly as the square root of the tension,

i.e., $\nu \propto \sqrt{T}$ when l and m are constants.

(c) ***Law of mass*** **:** For a string of given length and tension, the frequency of a string varies inversely as the square root of its mass per unit length,

i.e., $\nu \propto \frac{1}{\sqrt{m}}$ when l and T are constants.

Law of mass can be divided into following two laws :

(i) ***Law of diameter*** **:** For a string of given material, length and tension, the frequency

of a string varies inversely as its diameter,

i.e., $\nu \propto \frac{1}{D}$ when r, l, and T are constants.

(ii) ***Law of density*** **:** For a string of given diameter, length and tension, the frequency of a string varies inversely as the square root of its density.

i.e., $\nu \propto \frac{1}{\sqrt{\rho}}$ when D, l and T are constants.

Stationary waves in air columns :

(i) ***Closed end pipes*** **:** Frequency of the fundamental mode of vibrations is given by,

$$\nu_1 = \frac{c}{4l}$$

Frequencies of the overtones are given by,

$\nu_2 = 3\,\nu_1$

$\nu_3 = 5\,\nu_1$ etc.

i.e., in a closed end pipe only odd harmonics are present.

(ii) ***Open end pipes*** **:** Frequency of the fundamental mode of vibrations is given by

$$\nu_1 = \frac{c}{2l}$$

Frequencies of the overtones are given by,

$\nu_2 = 2\,\nu_1$

$\nu_3 = 3\,\nu_1$ etc.

i.e., in open end pipes all harmonics are present.

Distinction between progressive and stationary waves

Progressive waves	Stationary waves
1. It consists of only one type of waves.	1. It is combination of two progressive waves, one original and other its reflected wave, travelling in opposite directions.
2. No particle of the medium is permanently at rest.	2. Particles of the medium at nodes are permanently at rest.
3. The change in pressure and density of the medium is uniform at all parts of the medium.	3. The change of pressure and density is not uniform throughout. It is maximum at the nodes and minimum at the antinodes.

4. Different particles of the medium pass through their mean position one after the other with same maximum velocity.	4. All particles of the medium pass through their mean position at the same time with different velocity.
5. These waves appear to be travelling from one point to another.	5. These waves appear to be stationary.
6. Each particle of the medium executes simple harmonic motion with same amplitude.	6. All particles of the medium, except at nodes, execute simple harmonic motion with different amplitudes.
7. There is continuous change of phase from particle to particle.	7. All the particles in the same segment vibrate in the same phase.
8. The amplitude of vibration of each particle is the same.	8. Different particles vibrate with different amplitudes.
9. It consists of crests and troughs.	9. It consists of nodes and antinodes.

Beats : When two wave trains of nearly the same frequency travelling along the same line in the same direction superpose each other, they give rise to periodic alternations of maximum and minimum loudness of sound, which are called as beats. The time interval between two successive loud sounds is called one beat period and the number of such periodic alternations per second is called beat frequency.

Beat frequency (ν) is always equal to the difference between the frequencies of the two notes, i.e.,

$$\nu = \nu_1 - \nu_2$$

Doppler's Effect

When there is relative motion between the source, medium and the observer, the frequency of sound as received by the observer is different from the frequency of sound emitted by the source. This effect is known as Doppler's effect.

Expression for apparent frequency of sound in different cases are as follows :

Case I : Sound source moving and observer **Stationary :** Let 'n' be the frequency of the source and 'v' the velocity of sound. If the source is stationary, then the 'n' waves emitted by the source with spread in distance v, the true wavelength being $\lambda = v/n$.

Suppose, the sound source is moving with a velocity v_s towards the observer, then n waves emitted by the source in one second will lie in a distance $(v - v_s)$. Then the apparant wavelength is

$$\lambda' = \frac{v - v_s}{n}$$

The apparent frequency is $n' = \frac{v}{\lambda'} = \frac{v}{v - v_s} \times n$.

If the observer is moving away from the source, then

$$n' = \left(\frac{v}{v + v_s}\right)n.$$

Case II : Observer is moving towards the stationary source : Let 'n' be the frequency and 'v' the velocity of source. If the observer were also stationary, then it would have received n waves per second. Since, the observer is moving with a speed v_0, then it will have v_0/λ waves in addition to (n) waves.

Thus, the apparent frequency is —

$$n' = n + \frac{v_0}{\lambda} = \frac{v + v_0}{\lambda} = \left(\frac{v + v_0}{v}\right) n$$

If the observer is moving away from the stationary source, then

$$n' = \left(\frac{v - v_0}{v}\right) n$$

Case III : Both the source and the observer moving :

(i) If both the observer and the source are moving in the same direction, observer ahead of the source

$$\rightarrow v_s \quad \rightarrow v_0$$

$$n' = \left(\frac{v - v_0}{v - v_s}\right) n$$

(ii) If both the observer and the source are moving in the same direction, source ahead of the observer.

$$\overset{-v_s}{\leftarrow} \quad \overset{-v_0}{\leftarrow}$$

$$n' = \frac{v - (-v_0)}{v - (-v_s)} . n$$

(*iii*) If the observer and source approach each other

$$\overset{\nu_s}{\rightarrow} \quad \overset{\nu_0}{\leftarrow}$$

$$n' = \left(\frac{\nu + \nu_0}{\nu - \nu_s}\right) n$$

(*iv*) If both the observer and the source are moving in opposite directions—

$$\underset{\text{source}}{\xleftarrow{-\nu_s}} \qquad \underset{\text{observer}}{\xrightarrow{-\nu_0}}$$

$$n' = \left(\frac{\nu + \nu_0}{\nu + \nu_s}\right) n$$

Case IV : *Effect of wind* : If the wind is also blowing with a velocity 'ω' in the direction of sound then we should use $(\nu + \omega)$ in place of 'ν' and if the sound is blowing with a velocity 'ω' in opposite direction to the speed of sound, then we should use $(\nu - \omega)$ in place of ν. Therefore, the apparent frequency becomes —

$$n' = \frac{\nu \pm \omega - \nu_0}{\nu \pm \omega - \nu_s} . n$$

Note : If $\nu_s = 0$ and $\nu_0 = 0$, then according to the above relation $n' = n$.

Similarly, if the source and observer are both moving in same direction with the same speed, i.e., $\nu_s = \nu_0$, even then $n' = n$. Hence, it is clear that if there is no relative motion between the source and observer then the wind does not affect the frequency of the source.

Case V : *Doppler effect in light* : Light waves also show Doppler effect. If a light source is moving away from a stationary observer, then the frequency of light waves appears decreased and wavelength appears increased. Therefore there is a shift in the spectrum towards the red end. This is known as 'red shift', given by

$$\Delta\lambda = \frac{\nu}{c}.\lambda$$

Where ν = speed of light source

c = speed of light

λ = actual wavelength of light

Musical sounds : A musical sound comprises of a series of harmonic waves following each other at regular interval of time with no sudden changes in amplitude. The characteristics of musical sounds are :

(i) Loudness

(ii) Pitch

(iii) Quality or timber

(i) ***Loudness :*** It is the function of the intensity of sound, where intensity of sound at any point is defined as the quantity of energy passing per unit area per unit time in a direction perpendicular to the direction of propagation of sound waves.

Its unit is bel (B).

Also 1 decibel (dB) = $\frac{1}{10}$ bel (B)

(ii) ***Pitch*** **:** It is a mare sensation in the ear depending upon the frequency of the musical sound.

(iii) ***Quality or timber*** **:** It is governed by harmonic content of the musical sound. It is measured in terms of number, distribution and relative intensities of over tones and is different for different instruments.

Noise : It is defined as the sound which produces displeasing effect on ears of the listener. Unlike musical sound, noise consists of a series of waves following each other at irregular intervals of time, without sudden changes in amplitude.

Musical Scale : It consists of a series of notes arranged in such a manner that their fundamental frequencies have definite ratios. The most common musical scale is known as diatonic scale.

The ratio of the frequency of the higher note to that of lower note in called as *interval* between them.