CHAPTER 2

H.C.F & L.C.M

FACTORS

A number may be made by multiplying two or more other numbers together. The numbers that are multiplied together are called factors of the final number.

Factors of 12 = 1, 2, 3, 4, 6 and 12

All the numbers have one (1) as a factor.

Common Factor

A common factor of two or more given numbers is a number which divides each given number completely.

Common factors of 12 and 18 are 1, 2, 3, 6.

HIGHEST COMMON FACTOR (H.C.F)

The highest common factor (H.C.F.) of two or more numbers is the greatest number which divides each of them exactly. It is also known as greatest common divisor (G.C.D.).

H.C.F. can be calculated by :

(i) Prime factorisation method

(ii) Division method

(i) H.C.F. by Prime Factorisation Method

Example 1 : Find the H.C.F. of 40 and 60 by prime factorisation method.

Solution :

Prime factors of 40

```
2 | 40
2 | 20
2 | 10
5 | 5
  | 1
```

$\therefore\ 40 = 2 \times 2 \times 2 \times 5$

Prime factors of 60

```
2 | 60
2 | 30
3 | 15
5 | 5
  | 1
```

$\therefore\ 60 = 2 \times 2 \times 3 \times 5$

Hence H.C.F. = 2 × 2 × 5 = 20.

(ii) H.C.F by Division Method

- **Finding the H.C.F of two numbers :** Divide the greater number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till no remainder is left. The last divisor is the required H.C.F.
- **Finding the H.C.F of more than two numbers:** We find the H.C.F of any two numbers say P. Now, find the H.C.F of P and the thrid number and so on. The last H.C.F will be the required H.C.F.

H.C.F. of two numbers by division method:

Example 2 : Find the H.C.F. of 140 and 200 by division method.

Solution :

```
140) 200 (1
     140
      60) 140 (2
          120
           20) 60 (3
               60
                ×
```

∴ H.C.F. of 140 and 200 = 20.

H.C.F. of three numbers by division method :

Example 3 : Find the H.C.F. of 324, 630 and 342 by division method.

Solution :

```
324) 630 (1
     324
     306) 324 (1
          306
           18) 306 (17
               306
                ×

18) 342 (19
    342
     ×
```

∴ H.C.F. of 324, 630 and 342 is 18.

Co-Primes

Tow natural numbers a and b are said to be co-prime if their HCF is 1.

Ex. (2, 3), (7, 9) etc. are pairs of co-primes.

H.C.F of Polynomials

When two or more polynomials are factorised, the product of common factors is known as HCF of these polynomials.

Lets find the HCF of $16x^3 (x-1)^3 (x+1)$

and $4xy (x+1)^2 (x-1)$

Now, $16x^3 (x-1)^3 (x+1)$

$= 2 \times 2 \times 2 \times 2 \times x \times x \times x \times (x-1) \times (x-1) \times (x-1) \times (x+1)$

and $4xy (x+1)^2 (x-1) = 2 \times 2xy (x+1)(x+1)(x-1)$

$\therefore$ H.C.F. $= 2 \times 2 \times x \times (x+1)(x-1) = 4x (x^2 - 1)$

MULTIPLES

Multiples of a number are all those numbers which can be divided completely by the given number.

Ex. Multiples of 5 are 5, 10, 15, 20 etc.

Common Multiples

Common multiples of two or more numbers are the numbers which can be exactly divided by each of the given number.

Ex. Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 etc. and
Multiples of 4 are 4, 8, 12, 16, 20, 24, 28 etc.
$\therefore$ Common multiples of 3 and 4 are 12, 24 etc.

LEAST COMMON MULTIPLE (L.C.M)

The least common multiple (L.C.M.) of two or more numbers is the smallest number which is exactly divisible by each of them.
L.C.M. can be calculated by :
(i) Prime factorisation method
(ii) Division method

(i) L.C.M. by Prime Factorisation Method

Example 4. Find the L.C.M. of 12 and 20 by prime factorization method.

Solution : $12 = 2 \times 2 \times 3$ and $20 = 2 \times 2 \times 5$
$\therefore$ L.C.M $= 2 \times 2 \times 3 \times 5 = 60.$

(ii) L.C.M. by Division Method :

Example 5 : Find the L.C.M. of 14, 56, 91 and 84.

Solution :

2	14, 56, 91, 84
2	7, 28, 91, 42
7	7, 14, 91, 21
	1, 2, 13, 3

$\therefore$ L.C.M $= 2 \times 2 \times 7 \times 2 \times 13 \times 3 = 2184.$

L.C.M of Polynomials

When two or more polynomials are factorised, the product of the factors with highest powers is the lowest common multiple (LCM) of the polynomials.

Ex. Consider the polynomials $(x^3 - 8)$ and $(x^2 - 4)$
$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$
and $x^2 - 4 = (x + 2)(x - 2)$
$\therefore$ L.C.M $= (x - 2)(x + 2)(x^2 + 2x + 4)$
$= (x + 2)(x^3 - 8)$

H.C.F. AND L.C.M. OF FRACTIONS

First express the given fractions in their lowest terms. Then,

$$\text{H.C.F.} = \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

$$\text{L.C.M.} = \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

Example 6 : Find the H.C.F. and L.C.M. of $4\frac{1}{2}, \frac{6}{2}, 10\frac{1}{2}$.

Solution : Here, $4\frac{1}{2} = \frac{9}{2}, \frac{6}{2} = 3, 10\frac{1}{2} = \frac{21}{2}$.

$$\text{H.C.F.} = \frac{\text{H.C.F. of } 9, 3, 21}{\text{L.C.M. of } 2, 1, 2} = \frac{3}{2} = 1\frac{1}{2}.$$

$$\text{L.C.M.} = \frac{\text{L.C.M. of } 9, 3, 21}{\text{H.C.F. of } 2, 1, 2} = \frac{63}{1} = 63.$$

H.C.F. AND L.C.M. OF DECIMAL NUMBERS

H.C.F of decimal numbers :

STEP I : In the given numbers, make the same number of decimal places by annexing years in some numbers, if needed.

STEP II : Find the H.C.F or L.C.M as the case may be, by considering the numbers without decimal point.

STEP III : Now, in the resulting H.C.F. or L.C.M, mark off as many decimal places as are there in each of the given numbers.

Example 7 : Find the H.C.F. and L.C.M. of 0.6, 9.6 and 0.36.

Solution : H.C.F of 60, 960 and 36 = 12
$\therefore$ Required HCF = 0.12
L.C.M of 60, 960 and 36 = 2880
$\therefore$ Required L.C.M = 28.8

Product of Two Numbers = Product of their HCF and LCM

Example 8 : If H.C.F. and L.C.M. of two numbers are 3 and 60 respectively and one number is 12 then find the other number.

Solution : Let the other number be x.
Product of numbers = H.C.F. × L.C.M.
$x \times 12 = 3 \times 60$

$$x = \frac{3 \times 60}{12} = 15.$$

Comparison of Fractions: Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

Remember

- ✧ The greatest number that will exactly divide x, y, z = H.C.F of x, y and z.
- ✧ The greatest number that will divide x, y and z leaving remainders a, b and c respectively = H.C.F of (x – a), (y – b) and (z – c).
- ✧ The least number which is exactly divisible by x, y and z = L.C.M of x, y and z.
- ✧ The least number which when divided by x, y and z leaves the remainder a, b and c respectively = L.C.M of(x, y and z) – R where R = (x – a) = (y – b) = (z – c)
- ✧ The least number which when divided by x, y and z leaves

the same remainder r in each case = L.C.M of (x, y and z) + r

- ✧ The greatest number that will divide x, y and z leaving the same remainder in each case = H.C.F of (x – y), (y – z) and (z – x).
- ✧ When two numbers P and Q are exactly divisible by a third number R. Then P + Q, P – Q and PQ is also divisible by R.

Example 9 : The H.C.F of two numbers, each having three digits, is 17 and their L.C.M is 714.

Find the sum of the numbers :

Solution :

Let the numbers be 17x and 17y where x and y are co-prime.

L.C.M of 17 x and 17 y = 17 xy

According to question, $17xy = 714 \Rightarrow xy = 42 = 6 \times 7$

$\therefore x = 6$ and $y = 7$

or $x = 7$ and $y = 6$

First number = $17x = 17 \times 6 = 102$

Second number = $17y = 17 \times 7 = 119$

Required sum = 102 + 119 = 221

Example 10 : Find the greatest number of six digits which on being divided by 6,7, 8, 9 and 10 leaves 4, 5, 6, 7 and 8 as remainders respectively.

Solution : The L.C.M. of 6, 7, 8, 9 and 10 = 2520

The greatest number of 6 digits = 999999

Dividing 999999 by 2520, we get 2079 as remainder.

Hence the 6 digit number divisible by 2520 is

999999 – 2079 = 997920

Since 6 – 4 = 2, 7 – 5 = 2, 8 – 6 = 2, 9 – 7 = 2, 10 – 8 = 2, the remainder in each case is less than the divisor by 2.

$\therefore$ Required number = 997920 – 2 = 997918.

Example 11 : What least number must be subtracted from 1936 so that the remainder when divided by 9, 10, 15 will leave in each case the same remainder 7 ?

Solution : The L.C.M. of 9, 10 and 15 is 90.

On dividing 1936 by 90, the remainder = 46.

But 7 is also a part of this remainder.

$\therefore$ Required number = 46 – 7 = 39

Example 12 : Find the greatest number which will divide 410, 751 and 1030 leaving a remainder 7 in each case.

Solution : Required number

= H.C.F. of (410 – 7), (751 – 7) and (1030 – 7)

= 31.

Example 13 : Find the H.C.F and L.C.M of 1.75, 5.6 and 7

Solution :

Making the same number of decimal places, the numbers may be written as 1.75, 5.60 and 7.00.

Without decimal point, these numbers are 175, 560 and 700.

Now, H.C.F of 175, 560 and 700 is 35.

$\therefore$ H.C.F of 1.75, 5.6 and 7 is 0.35.

L.C.M of 175, 560 and 700 is 2800.

$\therefore$ L.C.M of 1.75, 5.6 and 7 is 28.00 i.e. 28.

Example 14 : The H.C.F of two polynomials is $x^2 - 1$ and their L.C.M is $x^4 - 10x^2 + 9$. If one of the polynomials is $x^3 - 3x^2 - x + 3$, find the other.

Solution :

Given that H.C.F of p (x) and q (x) = $x^2 - 1 = (x + 1)(x - 1)$

Also, LCM of p (x) and q (x) = $x^4 - 10x^2 + 9 = x^4 - 9x^2 - x^2 + 9$

$= x^2(x^2 - 9) - (x^2 - 9) = (x^2 - 9)(x^2 - 1)$

$= (x + 3)(x - 3)(x + 1)(x - 1)$

and $p(x) = x^3 - 3x^2 - x + 3 = x^2(x - 3) - (x - 3)$

$= (x - 3)(x^2 - 1) = (x - 3)(x + 1)(x - 1)$

p(x).q(x) = (H.C.F).(L.C.M)

$\therefore q(x) = \frac{(H.C.F)\,(L.C.M)}{p(x)} = \frac{(x+1)(x-1)(x+3)(x-3)(x+1)(x-1)}{(x-3)(x+1)(x-1)}$

$= (x + 3)(x + 1)(x - 1)$

$= (x + 3)(x^2 - 1) = x^3 + 3x^2 - x - 3$

Example 15 : Find the H.C.F and L.C.M of 6, 72 and 120, using the prime factorisation method.

Solution : We have : $6 = 2 \times 3$, $72 = 2^3 \times 3^2$, $120 = 2^3 \times 3 \times 5$

Here, 2^1 and 3^1 are the smallest powers of the common factors 2 and 3 respectively.

So, H.C.F (6, 72, 120) = $2^1 \times 3^1 = 2 \times 3 = 6$

2^3, 3^2 and 5^1 are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the three numbers.

So, L.C.M (6, 72, 120) = $2^3 \times 3^2 \times 5^1 = 360$

Example 16 : Find the GCD of : $14x^3 + 14$, $42(x^2 + 4x + 3)(x^2 - x + 1)$

Solution : $p(x) = 14x^3 + 14 = 14(x^3 + 1) = 2 \times 7(x + 1)(x^2 - x + 1)$

$q(x) = 42(x^2 + 4x + 3)(x^2 - x + 1)$

$= 42(x^2 + 3x + x + 3)(x^2 - x + 1)$

$= 42[x(x + 3) + (x + 3)](x^2 - x + 1)$

$= 2 \times 3 \times 7(x + 3)(x + 1)(x^2 - x + 1)$

$\therefore$ GCD of p (x) and q (x) = $14(x + 1)(x^2 - x + 1) = 14(x^3 + 1)$

Example 17 : Two bills of ₹6075 and ₹8505 respectively are to be paid separately by cheques of same amount. Find the largest possible amount of each cheque.

Solution : Largest possible amount of each cheque will be (6075, 8505).

We can write $8505 = 6075 \times 1 + 2430$

Since, remainder $2430 \neq 0$ again applying division concept we can write

$6075 = 2430 \times 2 + 1215$

Again remainder $1215 \neq 0$

So, again applying the division concept we can write

$2430 = 1215 \times 2 + 0$

Here the remainder is zero

So, H.C.F = 1215

Therefore, the largest possible amount of each cheque will be ₹1215.

Example 18: A garden consists of 135 rose plants planted in certain number of columns. There are another set of 225 marigold plants which is to be planted in the same number of

columns. What is the maximum number of columns in which they can be planted?

Solution : To find the maximum number of columns we need to find the H.C.F(135, 225)

We can write, $225 = 135 \times 1 + 90$

Since, remainder $90 \neq 0$

So, again applying division concept, we can write,

$135 = 90 \times 1 + 45$

Remainder $45 \neq 0$ again using division concept, we have,

$90 = 45 \times 2 + 0$

Since, remainder is 0

So, H.C.F = 45

Therefore, 45 is the maximum number of columns in which the plants can be planted.

Example 19 : A watch ticks 90 times in 95 seconds and another watch ticks 315 times in 323 seconds. If both the watches are started together, how many times will they tick together in the first hour ?

Solution : The first watch ticks every $\frac{95}{90}$ seconds and another ticks every 323/315 seconds.

They will tick together after (L.C.M of $\frac{95}{90}$ & $\frac{323}{315}$) seconds.

Now, L.C.M of $\frac{95}{90}$ and $\frac{323}{315} = \frac{\text{L.C.M of } 95, 323}{\text{H.C.F of } 90, 315} = \frac{19 \times 5 \times 17}{45}$

The number of times they will tick in the first 3600 seconds

$= 3600 \div \frac{19 \times 5 \times 17}{45} = \frac{3600 \times 45}{19 \times 5 \times 17} = 100\frac{100}{323}$

Once they have already ticked in the beginning; so in 1 hour they will tick $100 + 1 = 101$ times.

Example 20 : Find the H.C.F and L.C.M of $14xy^3$, $22x^2y$ and $26x^3y^4$.

Solution : $14xy^3 = 2 \times 7 \times x \times y^3$

$\Rightarrow 22x^2y = 2 \times 11 \times x^2 \times y$

$26x^3y^4 = 2 \times 13 \times x^3 \times y^4$

H.C.F $= 2 \times x \times y = 2xy$

L.C.M $= 2 \times 7 \times 11 \times 13 \times x^3 \times y^4 = 2002x^3y^4$.

Example 21 : Find the HCF of following pair of polynomials:

$P(x) = (x^2 - 9)(x - 3)$

$q(x) = x^2 + 6x + 9$

Solution : $P(x) = (x^2 - 9)(x - 3)$

$= (x + 3)(x - 3)(x - 3)$

$= (x + 3)(x - 3)^2$

$q(x) = x^2 + 6x + 9$

$= x^2 + 3x + 3x + 9$

$= x(x + 3) + 3(x + 3) = (x + 3)(x + 3)$

$= (x + 3)^2$

HCF of p(x) and q(x) = x + 3

Example 22 : Find the L.C.M of $P(x) = (x + 3)(x - 2)^2$ and $q(x) = (x-2)(x-6)$.

Solution : $P(x) = (x + 3)(x - 2)^2$

$q(x) = (x - 2)(x - 6)$

HCF of p (x) and q (x) = (x – 2)

LCM of p (x) and q (x) $= \frac{p(x) \times q(x)}{\text{HCF}}$

$= \frac{(x+3)(x-2)^2 \times (x-2)(x-6)}{(x-2)}$

$= (x+3)(x-2)^2(x-6)$

Example 23 : Find the LCM of $7x^3 + 2x^2 - 16x - 32$ and $3x^2 - 2x - 8$.

Solution : P9 (x) $= 7x^3 + 2x^2 - 16x - 32$

$\therefore p(2) = 7(2)^3 + 2(2)^2 - 16(2) - 32$

$= 56 + 8 - 32 - 32 = 0$

$\therefore$ (x – 2), is a factor of q (x).

$x - 2)\ 7x^3 + 2x^2 - 16x - 32\ (7x^2 + 16x + 16$

$7x^3 - 14x^2$

$\quad - \quad +$

$16x^2 - 16x$

$16x^2 - 32x$

$\quad - \quad +$

$16x - 32$

$16x - 32$

$\quad - \quad +$

$\times$

$\therefore p(x) = (x - 2)(7x^2 + 16x + 16)$

$q(x) = 3x^2 - 2x - 8 = 3x^2 + 4x - 6x - 8$

$= x(3x + 4) - 2(3x + 4)$

$= (x - 2)(3x + 4)$

LCM of p(x) and q(x)

$= (x - 2)(3x + 4)(7x^2 + 16x + 16)$

Example 24 : The HCF of two polynomials is (x+3) and their LCM is $x^3 - 7x + 6$. If one polynomial is $x^2 + 2x - 3$, then find the second polynomial.

Solution : HCF = (x + 3)

LCM $= x^3 - 7x + 6$

$p(x) = x^2 + 2x - 3$

q (x) = ?

We know that p(x) × q(x) = HCF × LCM

$\Rightarrow p(x) = \frac{\text{HCF} \times \text{LCM}}{p(x)} = \frac{(x+3)(x^3 - 7x + 6)}{x^2 + 2x - 3}$

$= \frac{(x+3)(x^3 - 7x + 6)}{(x-1)(x+3)} = \frac{x^3 - 7x + 6}{x - 1}$

Now, $x - 1)\ x^3 - 7x + 6\ (x^2 + x - 6$

$x^3 - x^2$

$\quad - \quad +$

$x^2 - 7x + 6$

$x^2 - x$

$\quad - \quad +$

$-6x + 6$

$-6x + 6$

$\quad + \quad -$

$\times$

$\therefore$ Second polynomial $q(x) = x^2 + x - 6$

EXERCISE

1. Three bells chime at an interval of 18, 24 and 32 minutes respectively. At a certain time they begin to chime together. What length of time will elapse before they chime together again ?
   (a) 2 hours 24 minutes (b) 4 hours 48 minutes
   (c) 1 hour 36 minutes (d) 5 hours
2. The L.C.M and H.C.F of two numbers are 84 and 21, respectively. If the ratio of two numbers be 1 : 4, then the larger of the two numbers is
   (a) 21 (b) 48
   (c) 84 (d) 108
3. The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is
   (a) 91 (b) 910
   (c) 1001 (d) 1011
4. The L.C.M. of two number is 630 and their H.C.F. is 9. If the sum of numbers is 153, their difference is
   (a) 17 (b) 23
   (c) 27 (d) 33
5. Find the greatest number that will divide 55, 127 and 175, so as to leave the same remainder in each case.
   (a) 11 (b) 16
   (c) 18 (d) 24
6. The least number, which when divided by 2, 3, 4, 5 and 6, leaves in each case, a remainder 1, but when divided by 7 leaves no remainder. The number is
   (a) 121 (b) 181
   (c) 241 (d) 301
7. The L.C.M. of two numbers is 45 times their H.C.F. If one of the numbers is 125 and the sum of H.C.F. and L.C.M. is 1150, the other number is:
   (a) 215 (b) 220
   (c) 225 (d) 235
8. One pendulum ticks 57 times in 58 seconds and another 608 times in 609 seconds. If they started simultaneously, find the time after which they will tick together.
   (a) $\frac{211}{19}$ s (b) $\frac{1217}{19}$ s
   (c) $\frac{1218}{19}$ s (d) $\frac{1018}{19}$ s
9. Three men start together to travel the same way around a circular track of 11 kms. Their speeds are 4, $5\frac{1}{2}$, and 8 kms per hour respectively. When will they meet at the starting point?
   (a) 22 hrs (b) 12 hrs
   (c) 11 hrs (d) 44 hrs
10. A shopkeeper has three kinds of sugar 184 kg; 230 kg and 276kg. He wants to store it into minimum number of bags to equal size without mixing. Find the size of the bag and the number of bags required to do the needful.
    (a) 23 kg; 30 (b) 38 kg; 23
    (c) 46 kg; 15 (d) 46 kg; 25
11. The traffic light at three different road crossing changes after 24 second, 36 second, 54 second respectively. If they all changes simultaneously. at 10 : 15 : 00 AM then at what time will they again changes, simultanously
    (a) 10 : 16 : 54 AM
    (b) 10 : 18 : 36 AM
    (c) 10 : 17 : 02 AM
    (d) 10 : 22 : 12 AM
12. The least number of five digits which is exactly divisible by 12, 15 and 18, is:
    (a) 10010 (b) 10051
    (c) 10020 (d) 10080
13. The sum of two numbers is 462 and their highest common factor is 22. What is the maximum number of pairs that satisfy these conditions?
    (a) 1 (b) 3
    (c) 5 (d) 6
14. Three wheels can complete respectively 60,36,24 revolutions per minute. There is a red spot on each wheel that touches the ground at time zero. After how much time, all these spots will simultaneously touch the ground again?
    (a) 5/2 seconds (b) 5/3 seconds
    (c) 5 seconds (d) 7.5 seconds
15. Number of students who have opted the subjects A, B, C are 60, 84, 108 respectively. The examination is to be conducted for these students such that only the students of the same subject are allowed in one room. Also the number of students in each room must be same. What is the minimum number of rooms that should be arranged to meet all these conditions?
    (a) 28 (b) 60
    (c) 12 (d) 21
16. Product of two co-prime numbers is 117. Their L.C.M. should be
    (a) 1 (b) 117
    (c) equal to their H.C.F. (d) cannot be calculated
17. Three numbers are in the ratio of 3 : 4 : 5 and their L.C.M is 2400. Their H.C.F is
    (a) 40 (b) 80
    (c) 120 (d) 200

18. From 3 drums of milk, 279, 341 and 465 respectively are to be drawn out. To do it in a minimum time, the capacity of the measuring can be
(a) 271 (b) 61
(c) 111 (d) 31

19. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is
(a) 101 (b) 107
(c) 111 (d) 185

20. About the number of pairs which have 16 as their H.C.F. and 136 as their L.C.M., we can definitely say that
(a) no such pair exists
(b) only one such pair exists
(c) only two such pairs exist
(d) many such pairs exist

21. Five persons fire bullets at a target at an interval of 6, 7, 8, 9 and 12 seconds respectively. The number of times they would fire the bullets together at the target in an hour is
(a) 6 (b) 7
(c) 8 (d) 9

22. A bell rings every 18 minutes. A second bell rings every 24 minutes. A third bell rings every 32 minutes. Ifall the three bells ring at the same time at 8 o'clock in the morning, at what other time will they all ring together ?
(a) 12 : 40 hrs (b) 12 : 48 hrs
(c) 12 : 56 hrs (d) 13 : 04 hrs

23. If h * k denotes the H.C.F. of h and k, and h Δ k denotes the L.C.M. of h and k, where h and k are positive integers, then what is the value of [231 Δ (12 * 42)] * 49 ?
(a) 6 (b) 7
(c) 4 (d) 5

24. Four metal rods of lengths 78 cm, 104 cm 117 cm and 169 cm are to be cut into parts of equal length. Each part must be as long as possible. What is the maximum number of pieces that can be cut?
(a) 27 (b) 36
(c) 43 (d) 13

25. The LCM of $x^3 - 1$, $x^4 + x^2 + 1$ and $x^4 - 5x^2 + 4$ is
(a) $(x-1)(x+1)(x-2)$
(b) $(x-1)(x+1)(x+2)$
(c) $(x^2-1)(x^2-4)$
(d) $(x^2-1)(x^2-4)\ (x^2+x+1)(x^2+1-x)$

26. What is the LCM of
$x^2 - 1$, $x^2 + 4x + 3$ and $x^2 - 3x + 2$?
(a) $(x+3)(x+1)$
(b) $(x^2-1)(x-2)(x+3)$
(c) $(x-1)(x-2)(x-3)$
(d) $(x^2-1)(x+2)(x+3)$

27. What is the HCF of $x^2 - x - 12$, $x^2 - 7x + 12$ and $2x^2 - 11x + 15$?
(a) 1 (b) $(x-3)$
(c) $2x-5$ (d) $x-4$

28. If the LCM and HCF of two quadratic polynomials are $x^3 - 7x + 6$ and $(x - 1)$ respectively, find the polynomials.
(a) $(x^2-3x+2), (x^2+2x+3)$
(b) $(x^2+3x-2), (x^2-2x+3)$
(c) $(x^2-3x+2), (x^2+2x-3)$
(d) $(x^2+3x+2), (x^2+2x+3)$

29. The HCF of two expressions is $x - 7$ and their LCM is $x^3 - 10x^2 + 11x + 70$. If one of them is $x^2 - 5x - 14$, find the other.
(a) $x^2 - 12x + 35$
(b) $x^2 + 12x - 35$
(c) $x^2 - 14x + 35$
(d) $x^2 + 14x - 35$

30. The HCF and LCM of two expressions are $(x + 3)$ and $(x^3 - 7x + 6)$ respectively. If one of the polynomials is $(x^3 + 2x - 3$, the other polynomial is
(a) $x^2 - x + 6$
(b) $x^2 - 2x + 6$
(c) $x^2 - 2x + 8$
(d) $x^2 + x - 6$

31. If HCF and LCM of two terms x and y are a and b respectively and $x + y = a + b$, then $x^2 + y^2 = ?$
(a) $a^2 - b^2$ (b) $2a^2 + b^2$
(c) $a^2 + b^2$ (d) $a^2 + 2b^2$

32. If common factor of $x^2 + bx + c$ and $x^2 + mx + n$ is $(x + a)$, then the value of a is
(a) $\frac{c-n}{b-m}$ (b) $\frac{c-n}{b+m}$
(c) $\frac{c+1}{b-m}$ (d) $\frac{c-n}{m-b}$

33. If the HCF of three numbers 144, x and 192 is 12, then the number x cannot be
(a) 180 (b) 84
(c) 60 (d) 48

34. The sum of two numbers is 232 and their HCF is 29. What is the number of such pairs of numbers satisfying the above condition?
(a) One (b) Two
(c) Four (d) None of these

35. What is the HCF of $36(3x^4 + 5x^3 - 2x^2)$, $9(6x^3 + 4x^2 - 2x)$ and $54(27x^4 - x)$?
(a) $9x(x + 1)$ (b) $9x(3x - 1)$
(c) $18x(3x - 1)$ (d) $18x(x + 1)$

36. If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$, then what is the value of k ?
(a) 3 (b) 5
(c) 6 (d) 8

37. Three planets revolve round the Sun once in 200, 250 and 300 days, respectively in their own orbits. When do they all come relatively to the same position as at a certain point of time in their orbits?
(a) After 3000 days (b) After 2000 days
(c) After 1500 days (d) After 1200 days

38. What is the HCF of the polynomials $x^3 + 8$, $x^2 + 5x + 6$ and $x^3 + 2x^2 + 4x + 8$?
(a) $x + 2$ (b) $x + 3$
(c) $(x + 2)^2$ (d) None of these

39. The LCM of $(x^3 - x^2 - 2x)$ and $(x^3 + x^2)$ is
(a) $x^3 - x^2 - 2x$ (b) $x^2 + x$
(c) $x^4 - x^3 - 2x^2$ (d) $x - 2$

40. The HCF of $(x^4 - y^4)$ and $(x^6 - y^6)$ is
(a) $x^2 - y^2$ (b) $x - y$
(c) $x^3 - y^3$ (d) $x^4 - y^4$

41. What is the HCF of $a^2b^4 + 2a^2b^2$ and $(ab)^7 - 4a^2b^9$?
(a) ab (b) a^2b^3
(c) a^2b^2 (d) a^3b^3

42. The HCF of two numbers is 98 and their LCM is 2352. The sum of the numbers may be
(a) 1372 (b) 1398
(c) 1426 (d) 1484

43. If for integers a, b and c, if HCF $(a, b) = 1$ and HCF $(a, c) = 1$, then which one of the following is correct?
(a) HCF $(a, bc) = 1$ (b) HCF $(a, bc) = a$
(c) HCF $(a, bc) = b$ (d) None of these

44. What is the HCF of $8(x^5 - x^3)$ and $28(x^6 + 1)$?
(a) $4(x^4 - x^2 + 1)$ (b) $2(x^4 - x^2 + 1)$
(c) $(x^4 - x^2 + 1)$ (d) None of these

45. For any integer n, what is HCF $(22n + 7, 33n + 10)$ equal to?
(a) n (b) 1
(c) 11 (d) None of these

46. For any integers 'a' and 'b' with HCF $(a, b) = 1$ what is HCF $(a + b, a - b)$ equal to ?
(a) It is always 1 (b) It is always 2
(c) Either 1 or 2 (d) None of these

47. If a and b be positive integers, then HCF $\left(\frac{a}{\text{HCF}(a,b)}, \frac{b}{\text{HCF}(a,b)}\right)$ equal to ?
(a) a (b) b
(c) 1 (d) $\frac{a}{\text{HCF}(a,b)}$

48. What is the number of integral solutions of the equations HCF $(a, b) = 5$ and $a + b = 65$?
(a) None (b) Infinitely many
(c) Less than 65 (d) Exactly one

49. The HCF of two natural numbers m and n is 24 and their product is 552. How many sets of values of m and n are possible ?
(a) 1
(b) 2
(c) 4
(d) No set of m and n is possible satisfying the given conditions

50. The HCF and LCM of two polynomials are $(x + y)$ and $(3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5)$ respectively. If one of the polynomials is $(x^2 - y^2)$, then the other polynomial is
(a) $3x^4 - 8x^3y + 10x^2y^2 + 7xy^3 - 2y^4$
(b) $3x^4 - 8x^3y - 10x^2y^2 + 7xy^3 + 2y^4$
(c) $3x^4 + 8x^3y + 10x^2y^2 + 7xy^3 + 2y^4$
(d) $3x^4 + 8x^3y - 10x^2y^2 + 7xy^3 + 2y^4$

51. The LCM of two intergers is 1237. What is their HCF ?
(a) 37 (b) 19
(c) 1 (d) Cannot be determined

52. What is the LCM of $\frac{2}{3}, \frac{7}{9}$ and $\frac{14}{15}$?
(a) $\frac{7}{3}$ (b) $\frac{14}{3}$
(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

53. What is the HCF of 3.0, 1.2 and 0.06 ? **(CDS)**
(a) 0.6 (b) 0.06
(c) 6.0 (d) 6.06

54. Consider the following in respect of natural numbers a, b and c: **(CDS)**
1. LCM $(ab, ac) = a$ LCM (b, c)
2. HCF $(ab, ac) = a$ HCF (b, c)
3. HCF $(a, b) <$ LCM (a, b)
4. HCF (a, b) divides LCM (a, b)

Which of the above are correct?
(a) 1 and 2 only (b) 3 and 4 only
(c) 1, 2 and 4 only (d) 1, 2, 3 and 4

HINTS & SOLUTIONS

1. (b) L.C.M of 18, 24 & 32 = 288
Hence they would chime after every 288 min. or 4 hrs 48min

2. (c) Let the numbers be x and 4x.
Then, $84 \times 21 = x \times 4x$
or $4x^2 = 1764$
or $x^2 = 441$ or $x = 21$
$\Rightarrow 4x = 4 \times 21 = 84$
Thus the larger number = 84

3. (a) Required number of students = H.C.F. of 1001 and 910 = 91

4. (c) Let numbers be x and y.
$\because$ Product of two numbers = their (L.C.M × H.C.F)
$\Rightarrow xy = 630 \times 9$
Also, $x + y = 153$ (given)
since $x - y = \sqrt{(x+y)^2 - 4xy}$
$\Rightarrow x - y = \sqrt{(153)^2 - 4(630 \times 9)}$
$= \sqrt{23409 - 22680} = \sqrt{729} = 27$

5. (d) Required number = H.C.F of (127 – 55), (175 – 127) and (175 – 55)
= H.C.F of 72, 48 and 120 = 24

6. (d) L.C.M (2, 3, 4, 5, 6) = 60
$\therefore$ Required number is of the form 60 k + 1
Least value of k for which 60k + 1 is divisible by 7 is k=5
$\therefore$ Required number = $60 \times 5 + 1 = 301$

7. (c) Let H.C.F. be h and L.C.M. be l. Then, $l = 45h$ and $l + h = 1150$.
$\therefore$ 45h + h = 1150 or h = 25. So, $l = (1150 - 25) = 1125$
Hence, other number $= \left(\frac{25 \times 1125}{125}\right) = 225$

8. (c) Time gap between two consecutive ticks
$\frac{58}{57}$ sec. and $\frac{609}{608}$ sec.
$\therefore$ Required time = L.C.M of $\frac{58}{57}$ and $\frac{609}{608}$
$= \frac{\text{L.C.M of 58 and 609}}{\text{H.C.F of 57 and 608}} = \frac{1218}{19}$ sec

9. (a) Time taken by them to complete the track
$= \frac{11}{4}, \frac{11}{11/2}, \frac{11}{8}$ hrs $= \frac{11}{4}, 2, \frac{11}{8}$ hrs
Required time = L.C.M of $\left(\frac{11}{4}, 2, \frac{11}{8}\right)$
$= \frac{\text{L.C.M of (11, 2, 11)}}{\text{H.C.F of (4,1,8)}} = 22$hrs

10. (c) Size of the bag is the H.C.F. of the numbers 184, 230, 276 which is 46.
The number of bags
$= \frac{184}{46} + \frac{230}{46} + \frac{276}{46} = 4 + 5 + 6 = 15$

11. (b) L.C.M of 24, 36 and 54 second.
= 216 second
= 3 minute + 36 second.
Required time = 10 : 15:00 + L.C.M of 24, 36 and 54
= 10 : 15 : 00 + 3 min + 36 sec.
= 10 : 18 : 36 AM

12. (d) Least number of 5 digits is 10,000. L.C.M. of 12, 15 and 18 is 180.
On dividing 10000 by 180, the remainder is 100.
$\therefore$ Required number = 10000 + (180 – 100) = 10080

13. (d) There are 6 such pairs :
(22, 440), (44, 418), (88, 374), (110, 352)
(176, 286), (220, 242)

14. (c) A makes 1 rev. per sec
B makes $\frac{6}{10}$ rev per sec
C makes $\frac{4}{10}$ rev. per sec
In other words A, B and C take 1 , $\frac{5}{3}$ & $\frac{5}{2}$ seconds to complete one revolution.
L.C.M of $1, \frac{5}{3} \& \frac{5}{2} = \frac{\text{L.C.M. of 1,5,5}}{\text{H.C.F. of 1,3,2}} = 5$
Hence, after every 5 seconds the red spots on all the three wheels touch the ground

15. (d) H.C.F of 60, 84 and 108 is 12 so each room contain 12 students at minimum
So that each room contains students of only 1 subject
$\therefore$ Number of rooms $= \frac{60}{12} + \frac{108}{12} + \frac{84}{12} = 21$ rooms

16. (b) H.C.F of co-prime numbers is 1.
So, L.C.M. = 117/1 = 117.

17. (a) Let the numbers are 3x, 4x and 5x then their L.C.M = 60x
So, 60x = 2400 (given)
x = 40
$\therefore$ The number are (3×40), (4×40), (5×40) = 120, 160, 200
Hence required H.C.F. = 40

18. (d) To find the capacity we have to take the HCF of 279, 341 and 465.

$279 = 31 \times 9$
$341 = 31 \times 11$
$465 = 31 \times 3 \times 5$
$\Rightarrow$ HCF(279, 341, 465) = 31
Capacity of the measuring can be = 31 ml.

19. (c) Let the numbers be 37a and 37b. Then, $37a \times 37b = 4107$
$\Rightarrow$ ab = 3.
Now, co-primes with product 3 are (1, 3).
So, the required numbers are $(37 \times 1, 37 \times 3)$ i.e. (1, 111)
$\therefore$ Greater number = 111

20. (a) Since 16 is not a factor of 136, it follows that there does not exist any pair of numbers with H.C.F. 16 and L.C.M. 136.

21. (b) Time gap after which they will first hit the target is given by LCM of 6, 7, 8, 9, 12.

2	6, 7, 8, 9, 12
3	3, 7, 4, 9, 6
2	1, 7, 4, 3, 2
	1, 7, 2, 3, 1

LCM = (12×42) sec.
$\therefore$ In 1 hr (= 3600 sec) no. of time they will hit together is

$= \frac{3600}{12 \times 42} = \frac{50}{7} = 7\frac{1}{7}$ times
= 7 times in an hour.

22. (b) LCM of 18, 24, 32
= 288 min

$= \frac{288}{60} = 4$ hrs 48min

$\therefore$ Bell will ring together again after
4 hrs. 48 min i.e., 12 : 48 hrs

23. (b) $[231 \Delta (12 * 42)] * 49 = [\{(3 \times 7 \times 11) \Delta (2 \times 3) * 7^2$
$= \{(2 \times 3 \times 7 \times 11) * 7^2) = 7$

24. (b) Since each rod must be cut into parts of equal length and each part must be as long as possible, so HCF should be taken.
HCF of 78, 104, 117 and 169 = 13.

No. of parts from 78cm. rod $= \frac{78}{13} = 6$

No. of parts from 104 cm. rod $= \frac{104}{13} = 8$

No. of parts from 117 cm. rol $= \frac{117}{13} = 9$

No. of parts from 169 cm. rod $= \frac{169}{13} = 13$.

$\therefore$ Maximum no. of pieces = 6 + 8 + 9 + 13 = 36

25. (d) (i) $x^3 - 1 = (x-1)(x^2 + x + 1)$
(ii) $x^4 + x^2 + 1 = x^2 + 2x^2 + 1 - x^2$
$= (x^2 + 1)^2 - x^2$
$= (x^2 + 1 - x)(x^2 + 1 + x)$
(iii) $x^4 - 5x^2 + 4 = x^4 - 4x^2 - x^2 + 4$
$= x^2(x^2 - 4) - 1(x^2 - 4)$
$= (x^2 - 4)(x^2 - 1)$
$= (x-2)(x+2)(x-1)(x+1)$
LCM
$= (x-1)(x+1)(x-2)(x+2)(x^2+x+1)(x^2+1-x)$

26. (b) (i) $x^2 - 1 = x^2 - (1)^2$
$= (x-1)(x+1)$
(ii) $x^2 + 4x + 3 = x^2 + 3x + x + 3$
$= x(x+3) + 1(x+3)$
$= (x+3)(x+1)$
(iii) $x^2 - 3x + 2 = x^2 - 2x - x + 2$
$= x(x-2) - 1(x-2)$
$= (x-2)(x-1)$
LCM $= (x-1)(x+1)(x-2)(x+3)$

27. (a) (i) $x^2 - x - 12 = x^2 - 4x + 3x - 12$
$= x(x-4) + 3(x-4)$
$= (x-4)(x+3)$
(ii) $x^2 - 7x + 12 = x^2 - 3x - 4x + 12$
$= x(x-3) - 4(x-3)$
$= (x-3)(x-4)$
(iii) $2x^2 - 11x + 15$
$= 2x^2 - 6x - 5x + 15$
$= 2x(x-3) - 5(x-3)$
$= (x-3)(2x-5)$
HCF = 1 as no term is common.

28. (c) Putting $x - 1 = 0$ i.e. $x = 1$ in $(x-1)$ respectively.
Remainder $= (+1)^3 - 7(1) + 6 = 1 - 7 + 6 = 0$
$\therefore (x-1)$ is a factor of expression $x^3 - 7x + 6$
Now, $x^3 - 7x + 6 = x^2(x-1) + x(x-1) - 6(x-1)$
$= (x-1)(x^2 + x - 6)$
$= (x-1)[x^3 + 3x - 2x - 6]$
$= (x-1)[x(x+3) - 2(x+3)]$
$= (x-1)(x-2)(x+3)$
LCM $= x^3 - 7x + 6 = (x-1)(x-2)(x+3)$ and their HCF $= (x-1)$
$\because (x-1)$ is common in both.
$\therefore$ First expression $= (x-1)(x-2) = x^2 - 3x + 2$ and second expression
$= (x-1)(x+3) = x^2 + 2x - 3$

29. (a) $p(x) \times q(x) = \text{HCF} \times \text{LCM}$
$(x^2 - 5x - 14) \times q(x)$
$= (x-7)(x^3 - 10x^2 + 11x + 70)$

$$\therefore q(x) = \frac{(x-7)(x^3 - 10x^2 + 11x + 70)}{(x^2 - 5x - 14)}$$

Putting $x = 5$ in $x^3 - 10x^2 + 11x + 70$
Remainder $= (5)^3 - 10(5)^2 + 11 \times 5 + 70$
$= 125 - 250 + 55 + 70 = 0$
$\therefore (x-5)$ is a factor.
Now, $x^3 - 10x^2 + 11x + 70$

$= x^3(x-5) - 5x(x-5) - 14(x-5)$
$= (x-5)(x^2-5x-14)$

$\therefore$ Second expression $= \dfrac{(x-7)(x-5)(x^2-5x-14)}{(x^2-5x-14)}$

$= (x-7)(x-5)$
$= x^2 - 12x + 35$

30. (d) $p(x) \times q(x) = \text{HCF} \times \text{LCM}$
$\Rightarrow (x^2+2x-3) \times q(x) = (x+3) \times (x^3 - 7x + 6)$

$\therefore$ Second expression $= \dfrac{(x+3)(x^3-7x+6)}{x^2+2x-3}$

$= \dfrac{(x+3)(x-1)(x-2)(x+3)}{(x-1)(x+3)}$

$= (x+3)(x-2)$
$= x^2 + x - 6$

31. (c) $\because p(x) \times q(x) = \text{LCM} \times \text{HCF}$
$x \times y = b \times a$
$\therefore xy = ab$
and, $x + y = a + b$ (Given)
Squaring both sides,
$\therefore (x+y)^2 = (a+b)^2$
or, $x^2 + y^2 + 2xy = a^2 + b^2 + 2ab$
or, $x^2 + y^2 + 2ab = a^2 + b^2 + 2ab$ ($\because xy = ab$)
$\therefore x^2 + y^2 = a^2 + b^2$

32. (a) Since $(x + a)$ is common factor of $x^2 + bx + c$ and $x^2 + mx + n$ therefore, remainder = 0 when both polynomials are divided by $(x + a)$
$\because x + a = 0$
or, $x = -a$
For first polynomial, remainder $= (-a)^2 + b(-a) + c$
$0 = a^2 - ab + c$
or, $a^2 = ab - c$ ...(i)
For second polynomial, remainder
$= (-a)^2 + m(-a) + n$
$0 = a^2 - ma + n$
or, $a^2 = ma - n$
Comparing values of a^2 from equations (i) and (ii),
$ab - c = ma - n$
$\Rightarrow ab - ma = c - n$
$\Rightarrow a(b-m) = c - n$

$\Rightarrow a = \dfrac{c-n}{b-m}$

33. (d) Here given the HCF of 144, x, 192 is 12.

$$\begin{array}{r} 144\overline{)192}(1 \\ 144 \\ \hline \times 48\overline{)x} \end{array}$$

x can not be 48 because HCF is 12.

34. (b) Let two numbers by $29x$ and $29y$.
$\therefore$ $29x + 29y = 232 \Rightarrow x + y = 8$
$\Rightarrow$ $(x, y) = (1, 7), (3, 5)$
Since, one such pair is 87 and 145.
Hence, the other pairs is 203 and 29.

35. (c) Let $f_1(x) = 36(3x^4 + 5x^3 - 2x^2)$
$= 36x^2(3x^2 + 5x - 2)$
$= 36 \cdot x^2 \{3x^2 + 6x - x - 2\}$
$= 36 \cdot x^2 \{3x(x+2) - 1(x+2)\}$
$= 2 \times 2 \times 3 \times 3 \times x \times x \times (x+2)(3x-1)$
$f_2(x) = 9(6x^3 + 4x^2 - 2x)$
$= 9x(6x^2 + 4x - 2)$
$= 18x(3x^2 + 2x - 1)$
$= 18x(3x^2 + 3x - x - 1)$
$= 3 \times 3 \times 2 \times x(3x-1)(x+1)$
$f_3(x) = 54(27x^4 - x)$
$= 54x(27x^3 - 1)$
$= 2 \times 3 \times 3 \times 3 \times x \times (3x-1)(9x^2 + 3x + 1)$
$\therefore$ HCF of $f_1(x), f_2(x), f_3(x)$
$= 2 \times 3 \times 3 \times x \times (3x-1)$
$= 18x(3x-1)$

36. (b) Here, $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$
So, that $(x - 6)$ is a factor of both expression.
According to question,
$\Rightarrow f(x_1) = f(x_2)$ at $(x_1 = x_2 = 6)$
$\Rightarrow (6)^2 - 2(6) - 24 = (6)^2 - k(6) - 6$ (By condition)
$\Rightarrow 36 - 12 - 24 = 36 - 6k - 6$
$\Rightarrow 0 = 30 - 6k \Rightarrow 6k = 30$
$\therefore k = 5$

37. (a) Given that, three planets revolves the Sun once in 200, 250, 300 days,.
$\therefore$ Required time = LCM of (200, 250, 300)
= 3000 days
Now, after 3000 days they all come relatively to the same position as at a certain point of time in their orbits.

38. (a) Let $f_1(x) = x^3 + 8$
$= x^3 + 2^3 = (x+2)(x^2 - 2x + 4)$
$= (x+2) = (x+2)(x^2 - 2x + 4)$
$f_2(x) = x^2 + 5x + 6 = x^2 + 3x + 2x + 6$
$= x(x+3) + 2(x+3)$
$= (x+3)(x+2)$
and $f_3(x) = x^3 + 2x^2 + 4x + 8$
$= x^2(x+2) + 4(x+2)$
$= (x+2)(x^2+4)$
$\therefore$ HCF of $[f_1(x), f_2(x), f_3(x)] = x + 2$

39. (c) Let $f_1(x) = x^3 - x^2 - 2x = x(x^2 - x - 2)$
$= x\{x^2 - 2x + x - 2\}$
$= x\{x(x-2) + 1(x-2)\} = x(x+1)(x-2)$
and $f_2(x) = x^3 + x^2 = x^2(x+1) = x \cdot x(x+1)$
$\therefore$ LCM of $[f_1(x), f_2(x)] = x(x+1) \cdot x(x-2)$
$= x^2(x+1)(x-2) = x^2(x^2 - x - 2)$
$= x^4 - x^3 - 2x^2$

40. (a) Let $f_1(x) = (x^4 - y^4) = [(x^2)^2 - (y^2)^2]$
$= (x^2 - y^2)(x^2 + y^2)$

$= (x - y)(x + y)(x^2 + y^2)$
and $f_2(x) = (x^6 - y^6) = (x^3)^2 - (y^3)^2$
$= (x^3 + y^3)(x^3 - y^3)$
$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$
$= (x - y)(x + y)(x^2 - xy + y^2)(x^2 + xy + y^2)$
$\therefore$ HCF of $[f_1(x), f_2(x)] = (x - y)(x + y) = x^2 - y^2$

41. (c) $a^2b^4 + 2a^2b^2 = a^2b^2(b^2 + 2)$
and $(ab)^7 - 4a^2b^9 = a^7b^7 - 4a^2b^9$
$= a^2b^2(a^5b^5 - 4b^7)$
HCF of $[(a^2b^4 + 2a^2b^2), ((ab)^7 - 4a^2b^9) = a^2b^2$

42. (a) HCF of two numbers is 98. It means that 98 is common in both the numbers. Therefore, the sum of these two numbers also be multiple of 98. So, 1372 is divided by 98.

43. (a) For integers a, b and c, if HCF $(a, b) = 1$ and HCF $(a, c) = 1$, then HCF $(a, bc) = 1$

44. (a) Let $f_1(x) = 8(x^5 - x^3 + x)$
$= 4 \times 2 \times x(x^4 - x^2 + 1)$
and $f_2(x) = 28(x^6 + 1) = 7 \times 4[(x^2)^3 + (1)^3]$
$= 4 \times 7 \times (x^2 + 1)(x^4 - x^2 + 1)$
$\therefore$ HCF of $f_1(x)$ and $f_2(x) = 4(x^4 - x^2 + 1)$

45. (b) HCF of $(22n + 7, 33n + 10)$ is always 1
Examples
For $n = 1$, HCF $(29, 43) \Rightarrow$ HCF $= 1$
For $n = 2$, HCF $(51, 76) \Rightarrow$ HCF $= 1$
For $n = 3$, HCF $(73, 109) \Rightarrow$ HCF $= 1$
since $22n$ and $33n$ are multiples of 11, therefore $22n + 7$ and $33n + 10$ are not the multiple of 11.
Hence, HCF of $22n + 7$ and $33 + 10$ will not be equal to 11 or n.

46. (c) Given that HCF $(a, b) = 1$ means that a and b are co-prime numbers.
So, HCF $(a + b, a - b)$
Let $a = 4, b = 3$
HCF $(4, 3) = 1$
Now, HCF $(3 + 4, 4 - 3) =$ HCF $(7, 1)$
HCF is equal $= 1$
Let $a = 23$ and $b = 17$
HCF $(23, 17) = 1$
HCF $(23 + 17, 23 - 17) =$ HCF $(40, 6) = 2$
So, HCF $(a + b, a - b) =$ Either 1 or 2

47. (c) HCF $\left(\frac{a}{\text{HCF}(a,b)}, \frac{b}{\text{HCF}(a,b)}\right)$

$$= \frac{\text{HCF (a,b)}}{\text{LCM (HCF (a,b), HCF (a,b))}} \frac{\text{HCF (a,b)}}{\text{HCF (a,b)}} = 1$$

Example 1: $a = 12$ and $b = 24$

HCF of $\left(\frac{12}{\text{HCF}(12,24)}, \frac{24}{\text{HCF}(12,24)}\right)$

$= \text{HCF}\left(\frac{12}{12}, \frac{24}{12}\right) = \text{HCF}(1, 2) = 1.$

Example 2: $a = 19$, $b = 23$

HCF of $\left(\frac{19}{\text{HCF}(19,23)}, \frac{23}{\text{HCF}(19,23)}\right)$

$= \text{HCF of} \left(\frac{19}{1}, \frac{23}{1}\right)$

$=$ HCF of $(19, 23) = 1$

48. (c) $\because$ HCF $(a, b) = 5$
Let $a = 5x$ and $b = 5y$
$\therefore$ $5x + 5y = 65$
$\Rightarrow$ $x + y = 13$
$\therefore$ Number of pairs of (x, y)
$= (1, 12), (2, 11) ((3, 10), (4, 9), (5, 8), (6, 7)$
Hence, total number of solution is less than 65.

49. (d) HCF of two natural numbers m and $n = 24$
$m \times n = 552$
LCM of two natural numbers

$= \frac{\text{Product of } m \text{ and } n}{\text{HCF of } m \text{ and } n}$

$= \frac{552}{24} = 23$

Therefore, no set of m and n is possible satisfying the given condition.

50. (c) HCF × LCM = 1st polynomial × 2nd polynomial

$\Rightarrow$ 2nd polynomial $= \frac{\text{HCF} \times \text{LCM}}{\text{1st Polynomial}}$

$$= \frac{(x+y) \times (3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5)}{(x^2 - y^2)}$$

$= 3x^4 + 8x^3y + 10x^2y^2 + 7xy^3 + 2y^4$

51. (c) Given, LCM of two integers is 1237, which is a prime number.
So, their HCF is 1

52. (b) LCM $\left(\frac{2}{3}, \frac{7}{9}, \frac{14}{15}\right) = \frac{\text{LCM}(2,7,14)}{\text{HCF}(3,9,15)} = \frac{14}{3}$

53. (b) Multiply by 100
3.0 = 300
1 . 2 = 120
0.06 = 6
Now, HCF of (300, 120, 6) = 6
So, HCF (3.0, 1.2, 0.06) = 0.06

54. (c) By taking the values of (a), (b) and (c), we can easily say that only 1, 2 and 4 are correct.