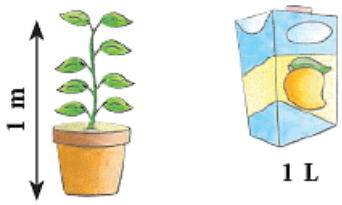
Measurement of Length, Mass and Capacity

Units of Measurement

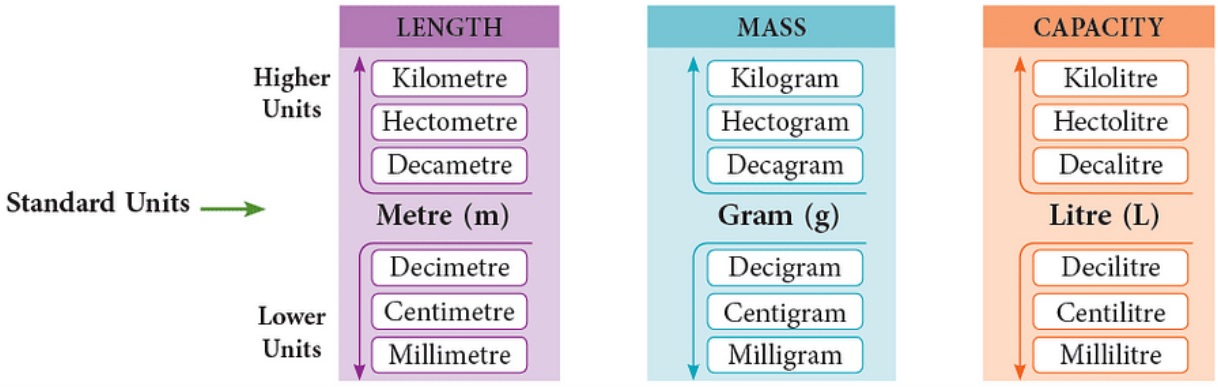
The set of standard units of measurement that we use is known as the **metric system**. It uses numbers that are multiples and sub-multiples of 10.



1 g

- The **standard unit of length** in the metric system is a **metre (m)**.
- The **standard unit of mass** in the metric system is a **gram (g).**
- The **standard unit of capacity** in the metric system is a **litre (L)**.

These standard units are too small for measuring certain quantities and also too large for some.
Gram is too small a unit to measure the weight of a human being, whereas metre becomes too large to measure the length of a paper clip. Litre is too large to measure the amount of liquid in a dropper and too small to measure the capacity of a petrol tanker.
So, we introduce some more units of measures.
Some are higher than the standard units and some are lower. We name them by adding prefixes to the standard unit and the prefix shows how big or small the unit is compared to the standard unit (metre, gram, litre).



Meaning of Prefix						
Prefix	**kilo**	**Hecto**	**Deca**	**Deci**	**Centi**	**Milli**
Meaning	thousand 1000	hundred 100	ten 10	one-tenth $\frac{1}{10}$	One-hundredth $\frac{1}{100}$	One-thousandth $\frac{1}{1000}$

To name metric units other than the standard units (metre, gram, litre) a prefix precedes the standard unit.
Since **kilo** means **1000**, therefore, 1 kilometre (km) = 1000 metres.
As **milli** means 1 / 1000 , therefore, 1 millilitre (mL) = 1 / 1000 litre = 0.001 litre.
Now, we deal with all measures, i.e., length, mass and capacity and study how the various units relate with the standard unit and with each other.

Length

Your height, the distance between two cities, the distance around a park, etc. are all quantities which measure length.
The standard unit of length in the metric system is **metre (m).**

The chart given below shows the units of length in the metric system with metre as the reference unit.

1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
kilometre (km)	hectometre (hm)	decametre (dam)	metre (m)	decimetre (dm)	centimetre (cm)	millimetre (mm)

This shows that the metric system is like the decimal system.

From the above given chart, you can conclude the following:

10 millimetres = 1 centimetre

10 centimetres = 1 decimetre

10 decimetres = 1 metre

100 centimetres = 1 metre

$1\text{ dm} = \frac{1}{10}\text{ m} = 0.1\text{ m}$

$1\text{ cm} = \frac{1}{100}\text{ m} = 0.01\text{ m}$

$1\text{ mm} = \frac{1}{1000}\text{ m} = 0.001\text{ m}$

1000 millimetre = 1 metre

10 metres = 1 decametre

10 decametres = 1 hectometre

10 hectometres = 1 kilometre

100 metres = 1 hectometre

1000 metres = 1 kilometre

1 km = 1000 m

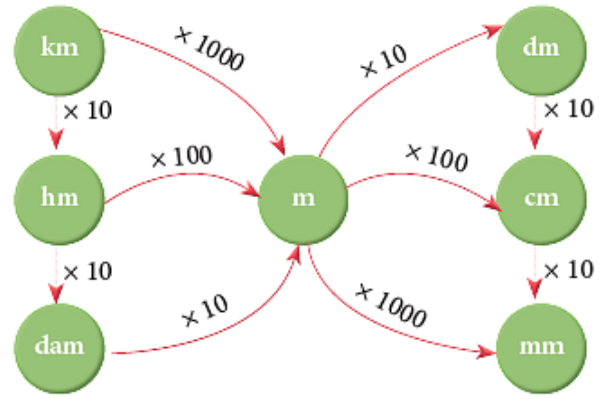
1 hm = 100 m

1 dam = 10 m

Conversion of Units

The **conversion of various units to metre and metre to various units** can be shown diagrammatically as shown below.

To change from higher unit to lower unit, you **multiply.**

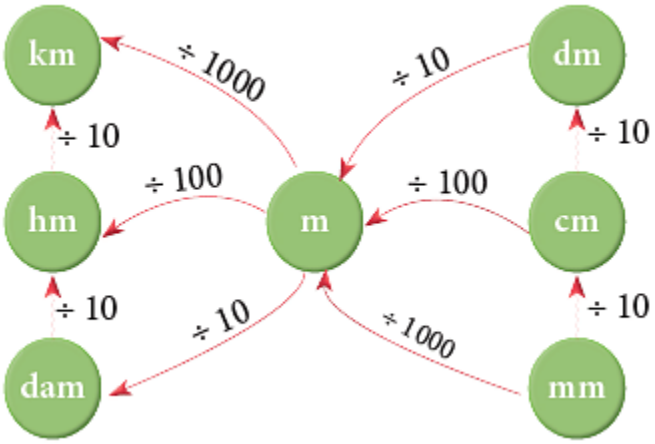


Examples:

- 3 km = (3 × 1000) m = 3000 m
- 8 hm = (8 × 100) m = 800 m

- 5 m = (5 × 100) cm = 500 cm
- 1.03 m = (1.03 × 1000) mm = 1030 mm

To change from lower unit to higher unit, you **divide.**

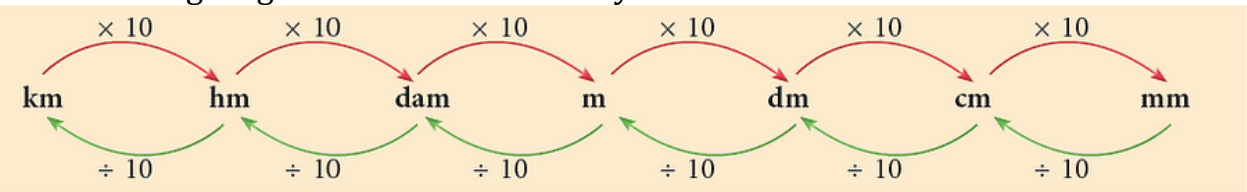


Examples:

- 8400 m = (8400 ÷ 1000) km = 8.4 km
- 370 m = (370 ÷ 100) hm = 3.7 hm
- 2000 mm = (2000 ÷ 1000) m = 2 m
- 73 cm = (73 ÷ 100) m = 0.73 m

Conversion from One Unit to Other Units

The following diagram shows how to carry out these conversions.



Rule:

1. Moving from left to right, you multiply by 10 at each step. Move one step right, multiply by 10, move two steps right, multiply by 10 × 10 = 100 and so on.
2. Moving from right to left, you divide by 10 at each step. Move one step left, divide by 10, move two steps left, divide by 100 and so on.

Example 1: Convert:
(a) 64 hm to dam

(b) 12 km to m
(c) 12 dam to cm
(d) 0.4 cm to mm
(e) 7.2 dm to mm

(a) 64 hm = (64 × 10) dam = **640 dam**
dam is one step to the right of hm, so multiply by 10.

(b) 12 km = (12 × 1000) m = **12000 m**
m is three steps to the right of km, so multiply by 10 × 10 × 10 = 1000.

(c) 12 dam = (12 × 1000) cm = **12000 cm**
cm is three steps to the right of dam, so multiply by 10 × 10 × 10 = 1000.

(d) 0.4 cm = (0.4 × 10) mm = **40 mm**
mm is one step to the right of cm, so multiply by 10.

(e) 7.2 dm = (7.2 × 100) mm = **720 mm**
mm is two steps to the right of dm, so multiply by 10 × 10 = 100.

Tips: All these are conversions from higher units to lower units, so multiply.

Example 2: Convert:
(a) 60 mm to cm
(b) 720 dam to km
(c) 85 hm to km
(d) 7340 dm to hm
(e) 12000 cm to dam

(a) 60 mm = (60 ÷ 10) cm = **6 cm**
cm is one step to the left of mm, so divide by 10.

(b) 720 dam = (720 ÷ 100) km = **7.2 km**
km is two steps to the left of dam, so divide by 100.

(c) 85 hm = (85 ÷ 10) km = **8.5 km**

km is one step to the left of hm, so divide by 10.

(d) 7340 dm = (7340 ÷ 1000) hm = **7.340 hm**
hm is three steps to the left of dm, so divide by 1000.

(e) 12000 cm = (12000 ÷ 1000) dam = **12 dam**
dam is three steps to the left of cm, so divide by 1000.

Writing Metric Measures in Decimal Notation
Since 100 centimetres = 1 metre, we can write
$264 \text{ cm} = \frac{264}{100} \text{ m} = 2.64 \text{ m}$
2.64 m = 2 metres 64 centimetres
= 2 metres 6 decimetres 4 centimetres
2.64 m = 2 m + 0.6 m + 0.04 m
= 2 m + 0.6 × 10 dm + 0.04 × 100 cm
= 2 m + 6 dm + 4 cm

Again, 3174 metres = 3000 metres + 100 metres + 70 metres + 4 metres
= 3 km + 1 hm + 7 dam + 4 m

$$= 3 \text{ km} + \frac{1}{10} \text{ km} + \frac{7}{100} \text{ km} + \frac{4}{1000} \text{ km}$$

= 3 km + 0.1 km + 0.07 km + 0.004 km = 3.174 km.

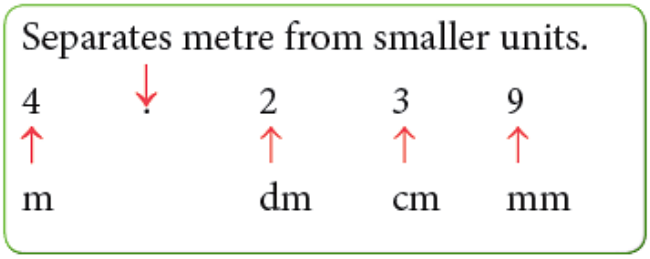
3	.	1	7	4	km
↑		↑	↑	↑	
km		hm	dam	m	

The decimal point separates the kilometres from the smaller units.

More Examples:

1. 0.408 km = 0 km 4 hm 0 dam 8 m
2. 3.015 m = 3 m 0 dm 1 cm 5 mm

3. 6 dam 3 m 5 dm 7 cm 2 mm can be written as 63.572 m.
   6 dam 3 m = 6 × 10 m + 3 m = 63 m



   Therefore, write 63, put the decimal point and after the decimal point write the figures for dm, cm and mm respectively.

4. 4 metres and 3 centimetres can be written as 4.030 m, which means 4 metres 0 decimetres 3 centimetres 0 millimetres. You can simply write it as 4.03 m as there are no millimetres. Since there is no decimetre, we put a zero at the decimetres place.

Now, we can use the above concept to devise a shortcut to make conversion between units easier.

Shortcut for Conversion between Units

Example 3: Convert 3.18 m to cm.
1 m = 100 cm
To multiply by 100, shift the decimal point 2 places to the right.
∴ 3.18 m = 3.18 × 100 cm = 318 cm

Example 4: Convert 240 dm to dam.

$$240 \text{ dm} = \frac{240}{100} \text{ dam}$$

To divide by 100, shift the decimal point 2 places to the left.
= 2.40 dam.

Reference chart:

$$\text{dam} \xleftarrow{\div 10} \text{m} \xleftarrow{\div 10} \text{dm}$$

$$\therefore \text{ dm} = \frac{1}{100} \text{ dam.}$$

Example 5: Convert 3.2 m to mm.
1 m = 1000 mm
1. To multiply by 1000, shift the decimal point 3 places to the right.
2. As there are no digits after 2, add two zeros.
∴ 3.2 m = (3.2 × 1000) mm
= 3200 mm.

The same principle of conversions is used in case of units of measurement of mass and capacity. The only change is in the standard unit.

Mass
The amount of matter in an object is its mass. The standard unit that is used to measure mass in the metric system is **gram (g).**
The chart given below shows the units of mass in the metric system with gram as the reference unit.

1000	100	10	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$
kilogram (kg)	**hectogram** (hg)	**decagram** (dag)	**gram** (g)	**decigram** (dg)	**centigram** (cg)	**milligram** (mg)

Important Relationships:

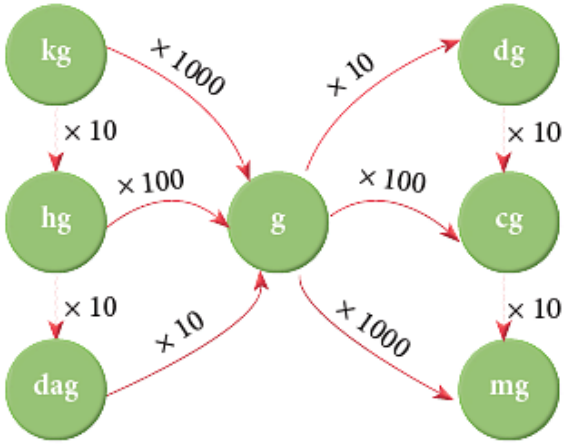
$1\ kg = 1000\ g$ $\qquad$ $1\ mg = \frac{1}{1000}\ g = 0.001\ g$

$1\ hg = 100\ g$ $\qquad$ $1\ cg = \frac{1}{100}\ g = 0.01\ g$

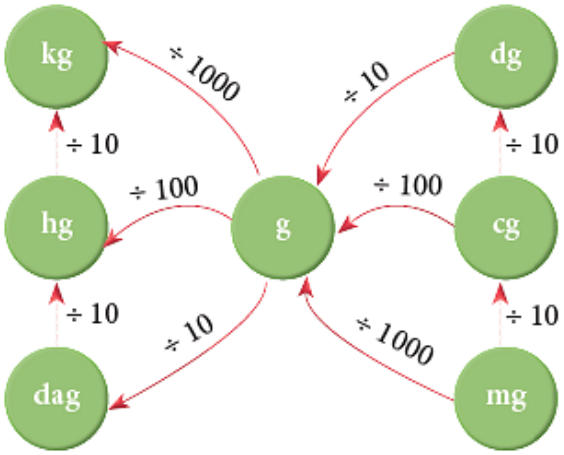
$1\ dag = 10\ g$ $\qquad$ $1\ dg = \frac{1}{10}\ g = 0.1\ g$

1. Conversions of Units
The given diagrams show conversions from the various higher and lower units to gram and vice versa. They also depict the conversions between the various units.

- To convert higher units to lower units, **multi ply.**



- To convert lower units to higher units, **divide.**

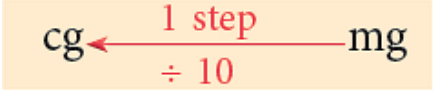


2. Conversion of various units to grams and vice versa.
Examples:

- 24 kg = (24 × 1000) g = 24000 g;
- 1.217 g = (1.217 × 1000) mg = 1217 mg
- 3200 mg = (3200 ÷ 1000) g = 3.2 g;
- 315 cg = (315 ÷ 100) g = 3.15 g

3. Conversion between Units

- 14 mg = (14 ÷ 10) cg = 1.4 cg



- 300 dg = (300 ÷ 100) dag = 3 dag

dag ← (2 steps, ÷ 100) dg

- 2417 cg = (2417 ÷ 1000) dag = 2.417 dag

dag ← (3 steps, ÷ 1000) cg

- 38 kg = (38 × 100) dag = 3800 dag

kg → (2 steps, × 100) dag

- 23.7 dg = (23.7 × 10) cg = 237 cg

dg → (1 step, × 10) cg

- 4 hg = (4 × 1000) dg = 4000 dg

hg → (3 steps, × 1000) dg

Capacity

The standard unit of capacity in the metric system is **litre (L)**. The chart given below shows the units of capacity in the metric system with litre as the reference unit.

1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
kilolitre (kL)	**hectolitre** (hL)	**decalitre** (daL)	**litre** (L)	**decilitre** (dL)	**centilitre** (cL)	**millilitre** (mL)

Important Relationships:

1 kL = 1000 L

1 hL = 100 L

1 daL = 10 L

$1\ \text{mL} = \frac{1}{1000}\ \text{L} = 0.001\ \text{L}$

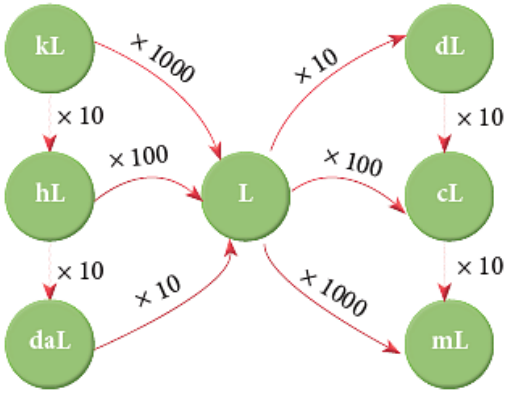
$1\ \text{cL} = \frac{1}{100}\ \text{L} = 0.01\ \text{L}$

$1\ \text{dL} = \frac{1}{10}\ \text{L} = 0.1\ \text{L}$

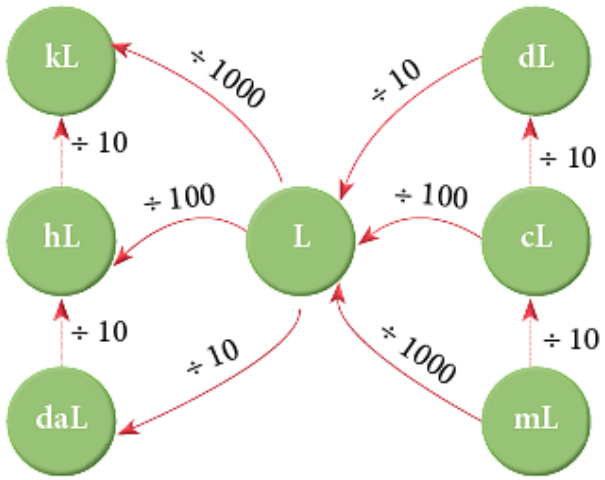
1. Conversion of Units

The conversion from higher and lower units to litres and conversion of litres to various units as well as the interconversions between the various units are shown in the following diagrams.

- To convert higher units to lower units, **multiply**



- To convert lower units to higher units, **divide.**



2. Conversion of various units to litres and litres to various units
Examples:

- 2.3 kL = (2.3 × 1000) L = 2300 L;
- 1.8 cL = (1.8 ÷ 100) L = 0.018 L;
- 64 hL = (64 × 100) L = 6400 L;
- 600 L = (600 ÷ 100) hL = 6 hL;
- 7315 mL = (7315 ÷ 1000) L = 7.315 L;
- 11.9 L = (11.9 × 1000) mL = 11900 mL

3. Conversions between Units
Examples:

- $\text{hL} \xrightarrow[\times\ 10]{1\ \text{step}} \text{daL}$

- 13.8 dL = (13.8 × 100) mL = 1380 mL

$\text{dL} \xrightarrow[\times\ 100]{2\ \text{steps}} \text{mL}$

- 500 daL = (500 ÷ 100) kL = 5 kL

$\text{kL} \xleftarrow[\div\ 100]{2\ \text{steps}} \text{daL}$

- 3.117 hL = (3.117 × 1000) dL = 3117 dL

$\text{hL} \xrightarrow[\times\ 1000]{3\ \text{steps}} \text{dL}$

The Four Basic Operations on Metric Measures

1. Addition and Subtraction

Example 6: Ritu drew a line segment of length 15 cm 4 mm. Then, she erased a portion of it. The remaining line segment measured 7 cm 6 mm. What is the length in mm of the erased line segment?

Length of the line segment in the beginning = 15 cm 4 mm

Length of line segment left after erasing = 7 cm 6 mm

Length of line segment erased = 15 cm 4 mm – 7 cm 6 mm

= 7 cm 8 m m

$$= 7 + \frac{8}{10}\ \text{cm} = \mathbf{7.8\ cm.}$$

	cm	mm
	(14)	(14)
	~~15~~	~~4~~
–	7	6
	7	8

Example 7: Add:

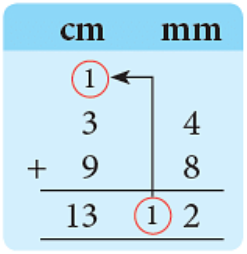
(a) 3 cm 4 mm and 9 cm 8 mm

(b) 14 dm 4 cm and 23 dm 7 cm

(c) 53 kg 305 g and 7 kg 828 g

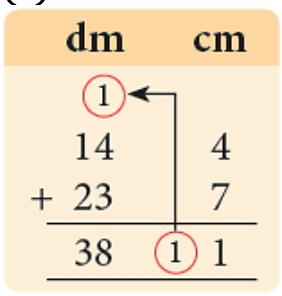
(d) 8 L 718 mL and 7 L 732 mL

(a)



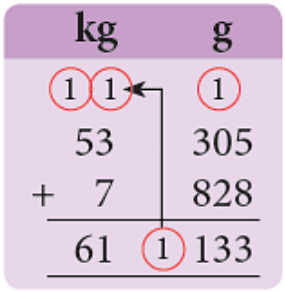
12 mm
= 10 mm + 2 mm
= 1 cm + 2 mm
= 13 cm 2 mm = **13.2 cm.**

(b)



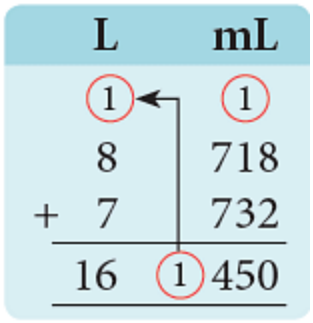
11 cm
= 10 cm + 1 cm
= 1 dm + 1 cm
= 38 dm 1 cm = **38.1 dm.**

(c)



1133 g
= 1000 g + 133 g
= 1 kg + 133 g
= 61 kg 133 g = **61.133 kg.**

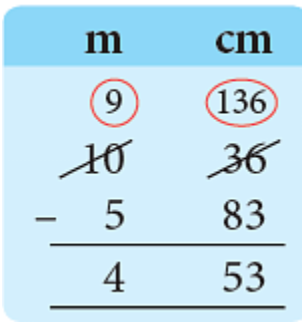
(d)



1450 mL
= 1000 mL + 450 mL
= 1 L + 450 mL
= 16 L 450 mL = **16.450 L.**

Example 8: Subtract:
(a) 10 m 36 cm – 5 m 83 cm
(b) 28 cm 4 mm – 13 cm 8 mm
(c) 40 kg 353 g – 17 kg 500 g
(d) 8 kL 150 L – 4 kL 850 L

(a)



36 cm < 83 cm
Borrow 1 m = 100 cm
100 cm + 36 cm
= 136 cm
= 4.53 m.

(b)

	cm	mm
	27	14
	~~28~~	~~4~~
−	13	8
	14	6

4 mm < 8 mm
Borrow 1 cm = 10 mm
10 mm + 4 mm
= 14 mm
= 14.6 cm.

(c)

	kg	g
	39	1353
	~~40~~	~~353~~
−	17	500
	22	853

353 g < 500 g
Borrow 1 kg = 1000 g
1000 g + 353 g
= 1353 g
= 22.853 kg.

(d)

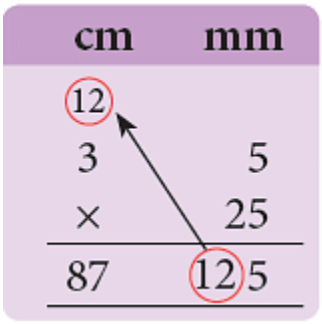
	kL	L
	7	1150
	~~8~~	~~150~~
−	4	850
	3	300

150 L < 850 L
Borrow 1 kL = 1000 L

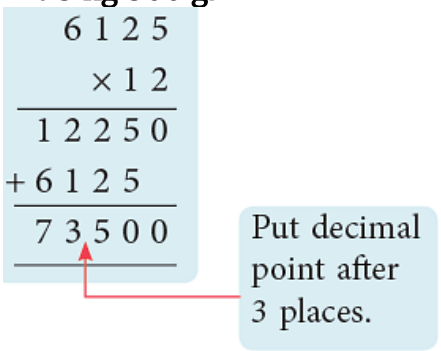
1000 L + 150 L
= 1150 L
= 3.300 kL.

2. Multiplication
Arrange numbers in columns unitwise and then multiply as you would multiply whole numbers.

Example 9: Find, in centimetres, the height of a pile of 25 books, if each book is 3 cm, 5 mm thick.
Thickness of 1 book = 3 cm 5 mm = 3.5 cm
Height of 25 books = 3.5 cm × 25
= 87.5 cm
Thus, height of the pile of 25 books
= 87.5 cm.



Example 10: A carton full of fruits weighs 6 kg 125 g. What is the weight of 12 such cartons in kg?
Weight of one carton = 6 kg 125 g
= 6.125 kg
∴ Weight of 12 cartons = (6.125 × 12) kg
= 73.500 kg
= 73 kg 500 g.



3. Division

In division also arrange the numbers in columns unitwise and then divide like whole numbers.

Example 11: Reena prepared 4 L 156 mL of orange juice. Distribute it equally among 8 children. How many mL of orange juice each child gets?

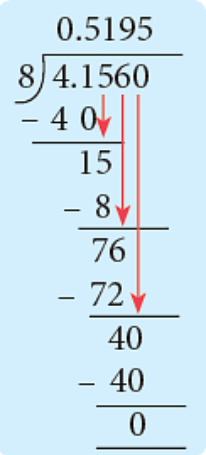
Juice Reena prepared = 4 L 156 mL = 4.156 L

When distributed among 8 children,

juice each child gets = (4.156 ÷ 8) L

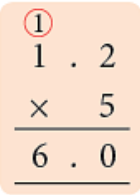
= 0.51 95 L

= 519 .5 mL.



Example 12: How many 150 mL glasses can I fill with 5 bottles of soft drinks each holding 1.2 litres?

Total soft drink in 5 bottles = 1.2 L × 5 = 6.0 = 6 L

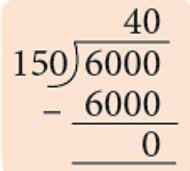


∵ 1 L = 1000 mL

Total soft drink = 6 L = 6 × 1000 = 6000 mL

Number of 150 mL of glasses that can be filled = 6000 mL ÷ 150 mL = 40

Thus, with 6 L of soft drink, I can fill 40 glasses of 150 mL.



Estimate in Measures

You cannot always be exact in measures. In our day-to-day life, we estimate the measures. To be able to estimate measures correctly, you should have a fair idea of how to relate commonplace things with the commonly used units of measures.

1. Length

- For long lengths or distances, we use **kilometres (km)** as unit.

Length of India = 3214 km Breadth of India = 2933 km

- For medium lengths, we use **metres (m).**

The length of your dining table.

- For short lengths, we use **centimetres (cm).**

Length of a small ruler = 15 cm.

- For very short lengths, we use **millimetres (mm).**

The tip of your pencil = 2 mm.

2. Mass

- For very large masses, we use **tonne (t)** and **quintal (q)**.

Truck = 4 tonne,Sack of rice = 1 quintal.

- For medium masses, we use **kilograms (kg).**

The weight of your school bag = 5 kg.

- For small masses, we use **grams (g).**

An orange is about 100 g.

- For very small masses, we use **milligrams (mg).**

A teaspoon of salt = 5 mg.

3. Capacity

- For large capacities, we use **kilolitre**.

Capacity of a swimming pool = 600 kL.

- For medium capacities, we use **litres.**

A bathing bucket = 20 L.

- For small capacities, we use **millilitres.**

A bottle of medicine = 200 mL.

Finding Fractions of Quantities

Example 13: A basket contains 3 kg 705 g of mangoes. 23 of the mangoes are eaten by Mr Bhasin. Lata, his daughter, gets 25 of the remaining mangoes. What is her share in grams?

Total weight of mangoes = 3 kg 705 g

Mangoes eaten by Mr Bhasin

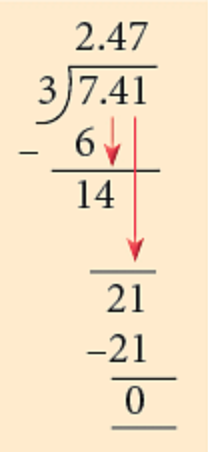
$$= \frac{2}{3} \text{ of } 3 \text{ kg } 705 \text{ g}$$

$$= \frac{2}{3} \text{ of } 3.705 \text{ kg}$$

$$= \frac{2}{3} \times 3.705 \text{ kg}$$

$$= \frac{2 \times 3.705}{3} \text{ kg}$$

$$= \frac{7.41}{3} \text{ kg} = 2.47 \text{ kg}$$



Mangoes left = 3.705 kg – 2.47 kg
= 1.235 kg

Mangoes eaten by Lata

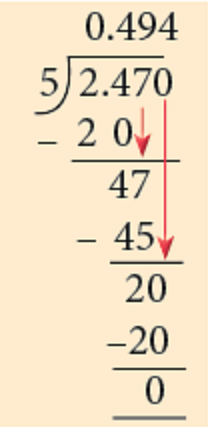
$$= \frac{2}{5} \text{ of } 1.235 \text{ kg}$$

$$= \frac{2}{5} \times 1.235 \text{ kg}$$

$$= \frac{2 \times 1.235}{5} \text{ kg}$$

$$= \frac{2.47}{5} \text{ kg} = 0.494 \text{ kg}$$

= (0.494 × 1000) g
= 494 g



So, Lata ate **494 g** of the mangoes.

Example 14: Anshul had 45 kg of wafers. He packed all the wafers equally into 5 small packets. How many grams of wafers were there in each packet?

Total wafers with Anshul = 4 / 5 kg

$$= \frac{4}{5} \times 1000 \text{ g}$$

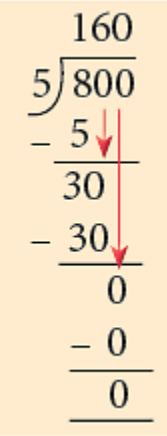
= 800 g

800 g wafers are filled in 5 small packets.

∴ Wafers in one packet = 800 ÷ 5

= 800 / 5 = 160 g

So, each packet contains **160 g** of wafers.



Example 15: Madhuri drew a line segment of length 20 cm 5 mm. She accidentally erased 2 / 5 of it. What is the length of the remaining line segment in cm?

Length of the line segment drawn = 20 cm 5 mm = 20.5 cm

Length of the erased line segment = 2 / 5 of 20.5 cm

$= (2 \times 4.1)$ cm

$= 8.2$ cm

Length of the remaining line segment $= 20.5$ cm $- 8.2$ cm

$= 12.3$ cm.

$$
\begin{array}{r}
4.1 \\
5\overline{)\,20.5} \\
-\,20 \\
\hline
05 \\
-\,5 \\
\hline
0 \\
\hline
\end{array}
$$