Motion

Improve your learning

Q. 1. As shown in figure 23, a point traverses the curved path. Draw the displacement vector from given points A to B

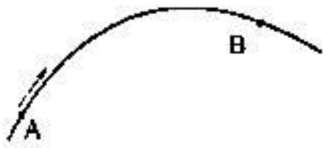
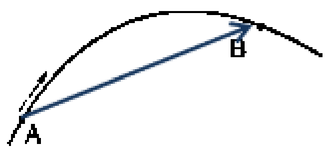


Fig-23

Answer :



Displacement is shortest distance between initial point to final point and its direction will be from A to B. So it will be a straight line from point A to point B.

Q. 2. "She moves at a constant speed in a constant direction.". Rephrase the same sentence in fewer words using concepts related to motion.

Answer : Sentence can be rephrased as "She moves at constant velocity." or you can also write it as "She moves at zero acceleration." as there is no change in its speed or direction resulting in no change in velocity.

Q. 3. What is the average speed of a Cheetah that sprints 100m in 4sec.? What if it sprints 50m in 2sec?

Answer : Given, distance = 100m and time = 4sec.

Using formula, $\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time taken}}$

$$\text{Average speed} = \frac{100\text{m}}{4\text{sec}}$$

$$= 25\ \text{m/s}$$

If distance = 50m and time = 2 sec, then

$$\text{Average speed} = \frac{50\text{m}}{2\text{sec}}$$

= 25 m/s

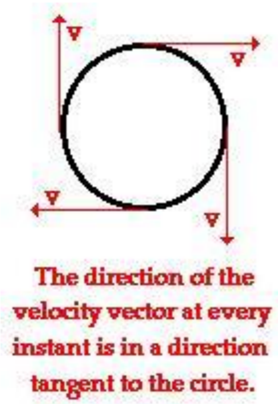
So speed remains same for both the cases.

Q. 4. Correct your friend who says, "The car rounded the curve at a constant velocity of 70 km/h".

Answer : Correct statement will be: "The car rounded the curve at a constant speed of 70 km/h."

Explanation:

Because velocity denotes speed (magnitude) with direction. And, we know that in circular motion, the direction changes, as shown below:



So velocity will not remain constant but speed will remain constant.

Q. 5. Suppose that the three ball's shown in figure start simultaneously from the tops of the hills. Which one reaches the bottom first? Explain. See figure 24.

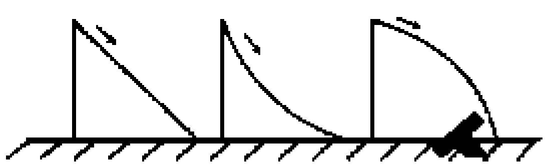


Fig-24

Answer : First ball will reach the bottom first because all the balls will have same initial speed but path covered by the first ball is the shortest one i.e. straight line between top of the hill and bottom.

So, as distance is shorter and initial speed is same, so first ball will take less time than other two balls.

Q. 6. Distance vs time graphs showing motion of two cars A and B are given. Which car moves fast? See figure 25.

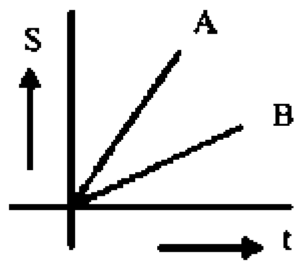


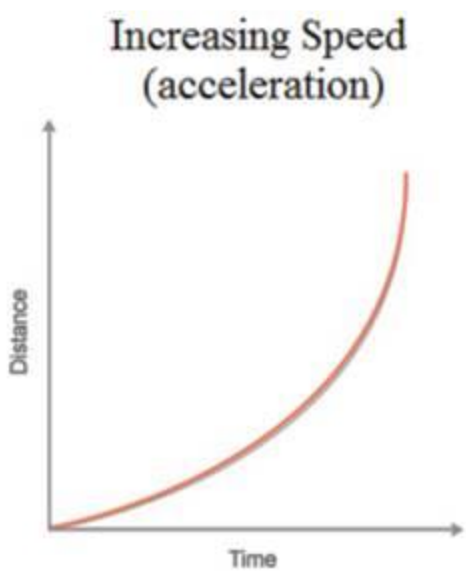
Fig-25

Answer : Car A moves faster than car B because slope of s-t graph denotes speed.

Here slope of A is greater than slope of B so car A is moving with faster speed than car B.

Q. 7. Draw the distance vs time graph when the speed of a body increases uniformly.

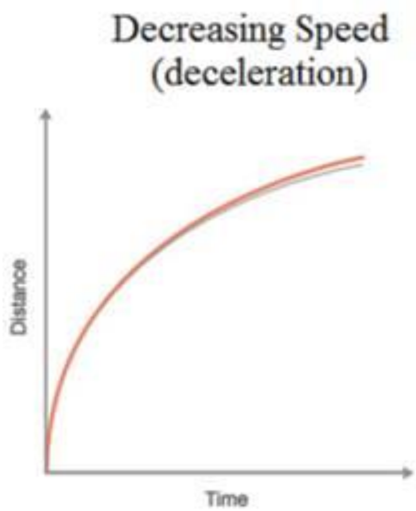
Answer :



Slope of Distance-time graph denotes speed and change in slope of the graph is acceleration. So it must be constant and positive for this graph as speed is increasing.

Q. 8. Draw the distance – time graph when its speed decreases uniformly.

Answer :



Slope of Distance-time graph denotes speed and change in slope of the graph is acceleration. So it must be constant and negative for this graph as speed is decreasing.

Q. 9. A car travels at a velocity of 80 km/h during the first half of its running time and at 40 km/h during the other half. Find the average speed of the car.

Answer : Let the total running time be t h.

Distance covered in first $\frac{t}{2}$ h $= \text{velocity} \times \text{time}$

$= 80\text{ km/h} \times \frac{t}{2}\text{ h}$

$= 40t\text{ km}$

Distance covered in other $\frac{t}{2}$ h $= 40\text{ km/h} \times \frac{t}{2}\text{ h}$

$= 20t\text{ km}$

Total distance $= 40t + 20t = 60t$

So, Average speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{60t}{t}\text{ km/h}$ $= 60$ km/h

Q. 10. A car covers half the distance at a speed of 50 km/h and the other half at 40km/h. Find the average speed of the car.

Answer : Let the total distance covered by the car be ‘d’ km.

Time taken to cover first $\frac{d}{2}$ distance, $t_1 = \frac{\text{distance}}{\text{speed}}$

$$= \frac{d \text{ km}}{2 \times 50 \text{ km/h}}$$

$$= \frac{d}{100} \text{ h}$$

Time taken to cover second $\frac{d}{2}$ distance, $t_2 = \frac{d \text{ km}}{2 \times 40 \text{ km/h}}$

$$= \frac{d}{80} \text{ h}$$

Total time = $t_1 + t_2$

$$= \frac{d}{100} \text{ h} + \frac{d}{80} \text{ h}$$

$$= \frac{4d + 5d}{400} \text{ h} = \frac{9d}{400} \text{ h}$$

Therefore, average speed $= \frac{d}{\frac{9d}{400}} \text{ km/h} = \frac{400}{9} \text{ km/h}$

= 44.44 km/h

Q. 11. Derive the equation for uniform accelerated motion for the displacement covered in its nth second of its motion.

Answer : Let the total displacement covered after n sec = S_n

Let the initial velocity be u and acceleration be a.

Displacement covered in n^{th} sec = $S_n - S_{n-1}$

From the equation of motion, we know that $s = ut + \frac{1}{2}at^2$

$$S_n = un + \frac{1}{2}an^2$$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

Displacement covered in n^{th} sec = $S_n - S_{n-1}$

$$= (un + \frac{1}{2}an^2) - (u(n-1) + \frac{1}{2}a(n-1)^2)$$

$$= \left(un + \frac{1}{2}an^2\right) - \left(un - u + \frac{1}{2}a(n^2 + 1 - 2n)\right)$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 + 1 - 2n)$$

$$= u + a\left(n - \frac{1}{2}\right)$$

Q. 12. A particle covers 10m in first 5s and 10m in next 3s. Assuming constant acceleration. Find initial speed, acceleration and distance covered in next 2s.

Answer : Let the initial speed be u m/s and speed after 5s be v_1m/s.

Let acceleration of the particle be a m/s^2.

$v_1 = u + 5a$ …(using v = u + at) …(1)

Now for first 5s, distance $s_1 = 5u + \frac{1}{2} a (5)^2 \left(\text{using } s = ut + \frac{1}{2}at^2\right)$

$\Rightarrow 10 = 5u + \frac{25}{2}a$ ...(2)

For next 3s, v_1 will be its initial velocity

and distance $s_2 = 3v_1 + \frac{1}{2}a(3)^2 \left(\text{using } s = ut + \frac{1}{2}at^2\right)$

$\Rightarrow 10 = 3v_1 + \frac{9}{2}a$ ...(3)

Now s_1-s_2 = 0

$$\Rightarrow 5u + \frac{25}{2}a - \left(3v_1 + \frac{9}{2}a\right) = 0$$

$$\Rightarrow 5u + \frac{25}{2}a - 3v_1 - \frac{9}{2}a = 0$$

$$\Rightarrow 5u + \frac{16}{2}a - 3v_1 = 0$$

$$\Rightarrow 5u + 8a - 3(u + 5a) = 0 \text{ (from equation 1)}$$

$$\Rightarrow 2u + 8a - 15a = 0$$

$$\Rightarrow u = \frac{7}{2}a \; \ldots(4)$$

Putting value of u in equation (2), we get

$$10 = 5 \times (\frac{7}{2}a) + \frac{25}{2}a$$

$\Rightarrow 10 = \frac{35}{2}a + \frac{25}{2}a$

$\Rightarrow 10 = 30a$

$\Rightarrow a = \frac{1}{3}\ ms^2$ ...(5)

So $u = \frac{7}{2} \times \frac{1}{3}\ m/s$ (from equation (4))

$\Rightarrow u = \frac{7}{6}\ ms^2$

Given, Distance in 8sec = 10m + 10m = 20m

Distance in 10sec $= \frac{7}{6} \times 10 + \frac{1}{2} \times \frac{1}{3} \times 10^2\ m \left(\text{using } s = ut + \frac{1}{2}at^2\right)$

$$= \frac{70}{6} + \frac{100}{6} \text{ m}$$

$$= \frac{85}{3} \text{ m}$$

Distance covered in next 2sec $=$ Distance in 10sec $-$ Distance in 8 sec

$$= \frac{85}{3} \text{ m} - 20 \text{ m}$$

$$= \frac{85}{3} - \frac{60}{3} \text{ m}$$

$$= \frac{25}{3} \text{ m}$$

$= 8.33$ m

Q. 13. A car starts from rest and travels with uniform acceleration "α" for some time and then with uniform retardation " β" and comes to rest. The time of motion is " t". Find the maximum velocity attained by it.

Answer : Given, initial velocity = 0, acceleration = α and retardation = β

Let the time of accelerated motion be t' and time of retarded motion be t", so that t' + t" = t … (1)

Velocity after time t', $v_1 = 0 + \alpha t'$ (using v = u + at) …(2)

Now, v_1 will be initial velocity for retarded motion and

final velocity = 0 and acceleration $= -\beta$

$0 = v_1 - \beta t''$ …(using v = u + at)

Or, $t'' = \frac{v_1}{\beta}$ ...(3)

Now, in equation (2)

$$v_1 = \alpha t'$$

$$\Rightarrow v_1 = \alpha(t - t'') \text{ (from equation(1))}$$

$$\Rightarrow v_1 = \alpha(t - \frac{v_1}{\beta}) \text{ (from equation(3))}$$

$$\Rightarrow v_1 = \alpha t - \frac{\alpha v_1}{\beta}$$

$$\Rightarrow \beta v_1 = \alpha\beta t - \alpha v_1$$

$$\Rightarrow (\beta + \alpha)v_1 = \alpha\beta t$$

$$\Rightarrow v_1 = \frac{\alpha\beta t}{\alpha + \beta}$$

As, after time t', retardation will start so maximum velocity will

be at time t' which is $v_1 = \frac{\alpha\beta t}{\alpha + \beta}$.

Q. 14. A man is 48m behind a bus which is at rest. The bus starts accelerating at the rate of 1 m/s^2, at the same time the man starts running with uniform velocity of 10 m/s. What is the minimum time in which the man catches the bus?

Answer : Given, initial velocity of bus, u = 0 and acceleration, a = 1 m/s^2.

When man catches the bus, distance between them = 0.

Let at time t sec, man catches the bus first.

Distance covered by man in t sec, d_1 = velocity x time

= 10 m/s x t sec

= 10t m

Distance covered by bus in t sec, $d_2 = 0 + \frac{1}{2} \times 1 \times t^2$ m

As man was 48m behind, so difference between their distance covered = 48m

$\Rightarrow d_1 - d_2 = 48$

$\Rightarrow 10t - \frac{t^2}{2} = 48$

$\Rightarrow 20t - t^2 = 96$

$\Rightarrow t^2 - 20t + 96 = 0$

$\Rightarrow t^2 - 12t - 8t - 96 = 0$

$\Rightarrow t(t - 12) - 8(t - 12) = 0$

$\Rightarrow (t - 12)(t - 8) = 0$

$\Rightarrow t = 12s \text{ or } 8s$

So, Minimum time to catch the bus will be 8sec.

Q. 15. A body leaving a certain point "O" moves with a constant acceleration. At the end of the 5th second its velocity is 1.5 m/s. At the end of the sixth second the body stops and then begins to move backwards. Find the distance traversed by the body before it stops.

Determine the velocity with which the body returns to point "O"?

Answer : Given, velocity after 5s = 1.5 m/s

Velocity after 6s = 0 m/s

Let the acceleration be 'a' and initial velocity be 'u'.

For t = 5s,

1.5 = u + 5a …(1) (using v = u + at)

For t = 6s

0 = u + 6a …(2) (using v = u + at)

Subtracting equation (2) from (1), we get

1.5 = -a

Or, a = -1.5 m/s^2

Putting value of a in equation (1), we get

u = 9 m/s

So, Distance traversed before stopping = Distance covered in 6s

$$= 9 \times 6 + \frac{1}{2}(-1.5) \times 6^2 \left(\text{using } s = ut + \frac{1}{2}at^2\right)$$

$$= 54 - 27$$

$$= 27 \text{ m}$$

Body will again move with same acceleration in opposite direction,

So here initial velocity = 0, acceleration a = -1.5 m/s^2 and

Displacement s = -27m (direction is opposite)

putting all these values in $s = ut + \frac{1}{2}at^2$, we get

$$-27 = \frac{1}{2} \times (-1.5) \times t^2$$

$$\Rightarrow t^2 = \frac{54}{1.5}$$

$$\Rightarrow t = 6s$$

Velocity at point “O” = velocity after 6s

= 0 + (-1.5) x 6 (using v = u + at)

= -9 m/s

Q. 16. Distinguish between speed and velocity.

Answer :

Speed	Velocity
1. It is defined as rate of change of distance.	1. It is defined as rate of change of displacement
2. It is a scalar quantity	2. It is a vector quantity
3. It gives the magnitude of how fast body is moving.	3. It gives magnitude and direction of how fast body is moving.
4. It cannot be negative	4. It can be negative, positive or zero.

Q. 17. What do you mean by constant acceleration?

Answer : Constant acceleration means change in velocity per unit time should be constant. i.e.,

$$a = \frac{\Delta v}{\Delta t} = \text{constant}$$

It means body should be changing its velocity with equal amount in equal time interval.

Q. 18. When the velocity is constant, can the average velocity over any time interval differ from instantaneous velocity at any instant? If so, give an example; if not explain why.

Answer : No. When the velocity is constant, average velocity over any time interval will not differ from instantaneous velocity at any instant because as velocity is constant, so it will not change during any time interval. It remains same for every point of time. Therefore, average velocity remains equal to the constant velocity.

Q. 19. Can the direction of velocity of an object reverse when it's acceleration is constant? If so give an example; if not, explain why.

Answer : Yes. Direction of velocity of an object can be reversed when it's acceleration is constant.

For example: When we throw a ball upward, it's acceleration will be due to gravity i.e. 9.8 m/s^2 in downward direction.

After reaching highest point, its direction will be reversed while its acceleration will remain same i.e. due to gravity.

Q. 20. A point mass starts moving in a straight line with constant acceleration “a”. At a time t after the beginning of motion, the acceleration changes sign, without change in magnitude. Determine the time t0 from the beginning of the motion in which the point mass returns to the initial position.

Answer : Given, acceleration = a, initial velocity = 0 and time = t

Velocity after time t, $v = 0 + at$ (Using $v = u + at$) ...(1)

Now it will become initial velocity.

After acceleration change its sign, let time taken by it to come to rest be t’,

$\Rightarrow 0 = v - at'$

$\Rightarrow t' = \frac{v}{a}$ …(2)

$\Rightarrow t' = t$

Total time = t + t’

Now distance covered in total time

$= \text{distance in time t} + \text{distance in time t}'$

$$= \left(0 + \frac{1}{2}at^2\right) + \left(vt' - \frac{1}{2}at'^2\right) \left(\text{using } s = ut + \frac{1}{2}at^2\right)$$

$$= \left(\frac{1}{2}at^2\right) + \left(\frac{v^2}{a} - \frac{av^2}{2a^2}\right) (\text{from (2), putting value of t}')$$

$$= \left(\frac{at^2}{2}\right) + \left(\frac{v^2}{2a}\right)$$

$$= \left(\frac{at^2}{2}\right) + \left(\frac{at^2}{2}\right) (\text{from (1), putting value of v})$$

$$= at^2 \ldots(3)$$

Now after coming to rest it will again start moving in opposite direction. To reach initial point, it has to cover same distance,

So let time taken to cover that distance be t". Here initial velocity will be 0. Putting value of distance from equation (3), we get.

$$at^2 = 0 + \frac{1}{2}at''^2$$

$$\Rightarrow t'' = t\sqrt{2}$$

Time $t_0 = t + t' + t''$

$$= t + t + t\sqrt{2}$$

$$= t(2 + \sqrt{2})$$

Q. 21. Consider a train which can accelerate with an acceleration of 20cm/s^2 and slow down with deceleration of 100cm/s^2. Find the minimum time for the train to travel between the stations 2.7 km apart.

Answer : Given, total distance need to cover, d = 2.7km = 270000 cm

Let distance covered in accelerated motion be d_1 and distance covered in decelerated motion be d_2.

So, $d_1 + d_2 = d$ …(1)

Now for d_1, initial velocity will be 0, and acceleration a = 20 cm/s^2 and let final velocity be v.

Then, $v^2 = 0 + 2ad_1$

$$\Rightarrow d_1 = \frac{v^2}{2a}$$

$$\Rightarrow d_1 = \frac{v^2}{40}\ \text{cm}$$

Now for d_1, initial velocity will be v, and acceleration a' = 100 cm/s^2 and velocity will be 0.

Then, $0 = v^2 - 2a'd_2$

$$\Rightarrow d_2 = \frac{v^2}{2a'}$$

$$\Rightarrow d_2 = \frac{v^2}{200}\ \text{cm}$$

Now putting values of d, d_1 and d_2 in equation (1), we get

$$\Rightarrow \frac{v^2}{40} + \frac{v^2}{200} = 270000$$

$$\Rightarrow 6v^2 = 270000 \times 200$$

$$\Rightarrow v^2 = 90000000$$

$$\Rightarrow v = 3000\ \text{cm/s}$$

And also for $d_{1,}$ let the time taken to travel be t_1 then

$$v = 0 + at_1$$

$$\Rightarrow t_1 = \frac{3000}{20}\ s$$

$$\Rightarrow t_1 = 150\ s$$

And also for d_2, let the time taken to travel be t_2 then

$$0 = v - a't_2$$

$$\Rightarrow t_2 = \frac{3000}{100}\ s$$

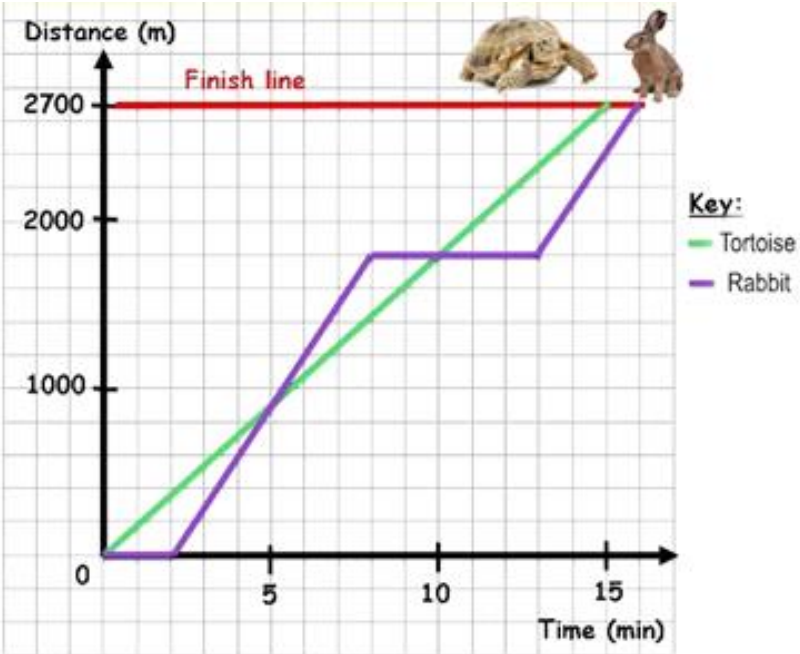
$$\Rightarrow t_2 = 30\ s$$

Now total time to travel will be $t_1 + t_2 = 150\ s + 30\ s$

$$= 180\ s$$

Q. 22. You may have heard the story of the race between the rabbit and tortoise. They started from same point simultaneously with constant speeds. During the journey, rabbit took rest somewhere along the way for a while. But the tortoise moved steadily with lesser speed and reached the finishing point before rabbit. Rabbit awoke and ran, but rabbit realized that the tortoise had won the race. Draw distance vs time graph for this story.

Answer :



Q. 23. A train of length 50m is moving with a constant speed of 10m/s. Calculate the time taken by the train to cross an electric pole and a bridge of length 250 m.

Answer : Given, speed = 10m/s

Distance needed to be covered to cross electric pole = length of the train

= 50 m

$$\text{Time taken to cross electric pole} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{50}{10}\,s = 5s$$

Distance needed to be covered to cross bridge

= length of the train + length of the bridge

= 50 + 250 m

= 300 m

$$\text{Time taken to cross electric pole} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{300}{10}\,s = 30s$$

Q. 24. Two trains, each having a speed of 30km/h, are headed at each other on the same track. A bird flies off one train to another with a constant speed of 60km/h when they are 60km apart till before they crash. Find the distance covered by the bird and how many trips the bird can make from one train to other before they crash?

Answer : Given, speed of each train = 30 km/h and

Distance between them = 60km

As they are moving in opposite direction and same speed,

So they will crash at middle of the distance between them, i.e. 30km from each train.

Time taken to travel 30 km by each train = $\frac{30}{30}$ = 1h

Speed of the bird = 60 km and time = 1h

So distance traveled by bird in 1h = speed × time

= 60 × 1

= 60 km

As trains will come closer to each other, bird can make any number of trips because distance between trains will approach to 0.

So the time taken by bird will also tends to 0. Therefore, bird can make infinite number of trips.