Verify the Algebraic Identity $(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$

OBJECTIVE

To verify the algebraic identity $(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$.

Materials Required

1. Acrylic sheets
2. Adhesive/Adhesive tape
3. Scissors
4. Geometry Box
5. Cutter

Prerequisite Knowledge

1. Concept of cuboid and its volume.
2. Concept of cube and its volume.

Theory

1. Cuboid A cuboid is a solid bounded by six rectangular plane surfaces, e.g. Match box, brick, box, etc., are cuboid, (see Fig. 7.1)

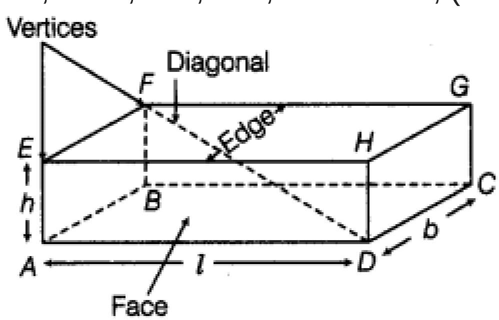

Fig. 7.1

Properties of cuboid are

1. In a cuboid, there are 6 faces, 12 edges and 8 corners (four at bottom and four at top face) which are called vertices.
2. Opposite faces of a cuboid are equal and parallel.
3. The line segment joining the opposite vertices of cuboid is called the diagonal of a cuboid.
4. There are four diagonals in a cuboid which are equal in length.
   Volume of cuboid = lbh
   where, l = length, b = breadth and h = height

2. Cube A cuboid whose length, breadth and height are same, is called a cube, (see Fig. 7.2)

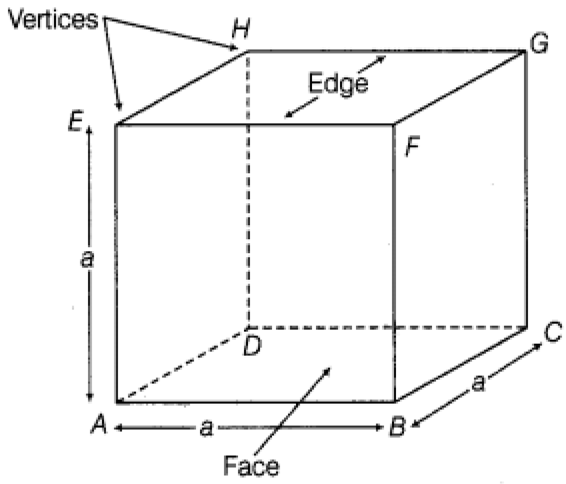

Fig. 7.2

Properties of cube are

1. In a cube, there are 6 faces, 12 edges and 8 corners (four at bottom and four at top face) which are called vertices.
2. All the six faces of a cube are congruent square faces.
3. Each edge of a cube have same length.
   Volume of cube = a^3
   where, a is side of cube.

Procedure

1. Cut six squares of equal side a units from acrylic sheet. Paste all of them to form a cube by using adhesive tape/adhesive, (see Fig. 7.3)

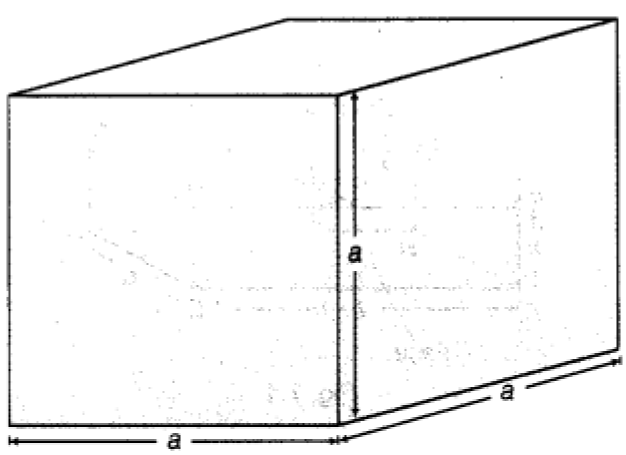

Fig. 7.3

2. Cut six squares of equal side b units (b < a) from acrylic sheet. Paste all of them to form a cube by using adhesive tape/adhesive, (see Fig. 7.4)

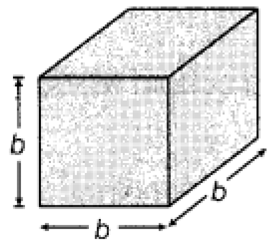

Fig. 7.4

3. Also, cut 12 rectangles of length b units and breadth a units and 6 squares of side a units. Paste all of them to form a cuboid, (see Fig. 7.5)

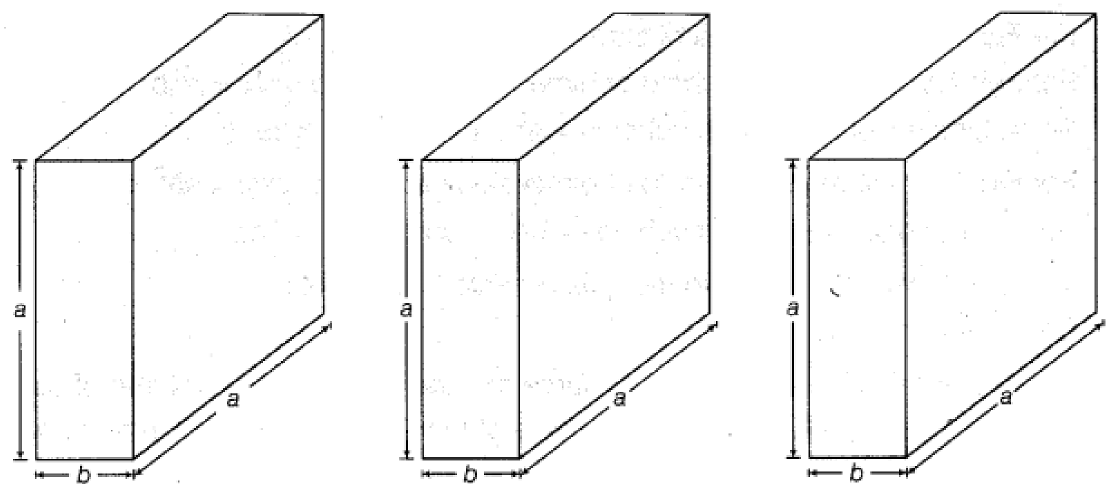

Fig. 7.5

4. Cut 12 rectangles of length a units and breadth b units and 6 squares of side b units. Paste all of them to form a cuboid, (see Fig. 7.6)

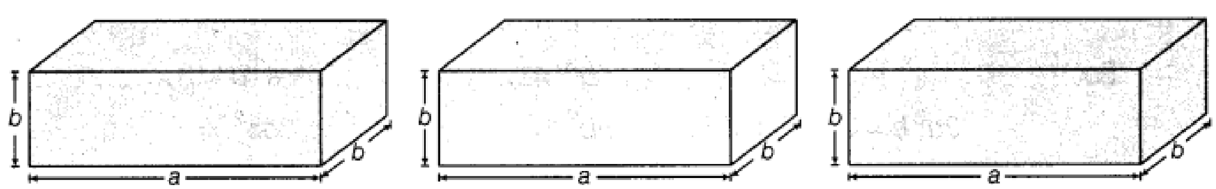

Fig. 7.6

5. Arrange the cubes obtained in Fig 7.3 and Fig 7.4 and the cuboids obtained in Fig 7.5 and Fig 7.6 as shown in Fig 7.7

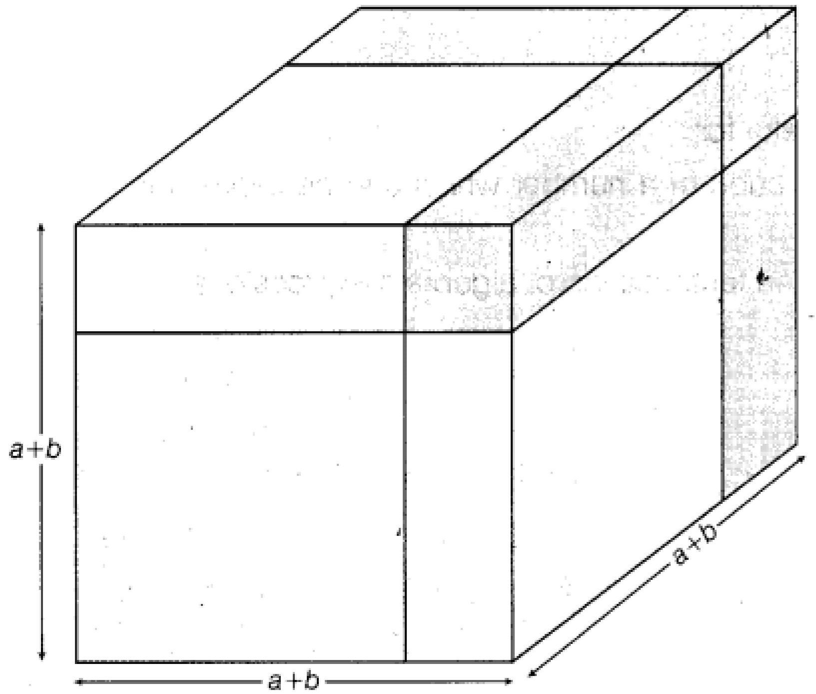

Fig. 7.7

Demonstration

For Fig. 7.3, volume of cube of side a units = a^3
For Fig. 7.4, volume of cube of side b units = b^3
For Fig. 7.5, volume of a cuboid of dimensions a x a x b units = a^2b
So, volume of all three such cuboids = $a^2b + a^2b + a^2b = 3a^2b$
For Fig. 7.6, volume of a cuboid of dimensions a x b x b units = ab^2
So, volume of all three such cuboids = $ab^2 + ab^2 + ab^2 = 3ab^2$
In Fig. 7.7, we have obtained the cube of side (a + b) units.
So, volume of cube = $(a + b)^3$
As, volume of cube of Fig. 7.7 = (Volume of cube of Fig. 7.3) + (Volume of cube of Fig. 7.4) + (Volume of three cuboids of Fig. 7.5) + (Volume of three cuboids of Fig. 7.6)
=> $(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$
Here, volume is in cubic units.

Observation

On actual measurement, we get
a =…….. , b = …….. ,
So, a^3 =…….. , b^3 = …….. ,
a2b = …….. , $3a^2b$ = …….. ,
ab^2 = …….. , $3ab^2$ = …….. ,

$(a + b)^3 = \ldots\ldots\ldots$,
Hence, $(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$

Result

The algebraic identity $(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$ has been verified.

Application

The identity is useful for

1. calculating the cube of a number which can be expressed as the sum of two convenient numbers.
2. simplification and factorization of algebraic expressions.

Viva Voce

Question 1:
Is $(a + b)^3$ a trinomial?
Answer:
No, because $(a + b)^3$ has four terms.

Question 2:
What is the degree of polynomial $(x + 2y)^3$?
Answer:
3, because the highest power of variable in the expansion of $(x + 2y)^3$ will be 3.

Question 3:
In the identity of $(a + b)^3$, what do you mean by a^3 and $3a^2b$?
Answer:
a^3 means volume of cube of side a and $3a^2b$ means volume of three cuboids of dimensions a, a and b.

Question 4:
What is the maximum number of zeroes that a cubic polynomial can have?
Answer:
Three

Question 5:
What is the expanded form of $(a + b)^3$?
Answer:
$(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$

Question 6:
For evaluating $(101)^3$, which formula we should use?
Answer:
We should use $(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$ by taking a = 100 and b = 1

Suggested Activity

Verify that $(a+b)^3 = a^3+b^3+ 3a^2b + 3ab^2$ by taking x = 11 units, y- 3 units.