Algebra

Chapter 11

11.1 Introduction

Our study so far has been with numbers and shapes. We have learnt numbers, operations on numbers and properties of numbers. We applied our knowledge of numbers to various problems in our life. The branch of mathematics in which we studied numbers is **arithmetic**. We have also learnt about figures in two and three dimensions and their properties. The branch of mathematics in which we studied shapes is **geometry**. Now we begin the study of another branch of mathematics. It is called **algebra**.

The main feature of the new branch which we are going to study is the use of letters. Use of letters will allow us to write rules and formulas in a general way. By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life. Thirdly, since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find algebra interesting and useful. It is very useful in solving problems. Let us begin our study with simple examples.

11.2 Matchstick Patterns

Ameena and Sarita are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Ameena takes two matchsticks and forms the letter L as shown in Fig 11.1 (a).

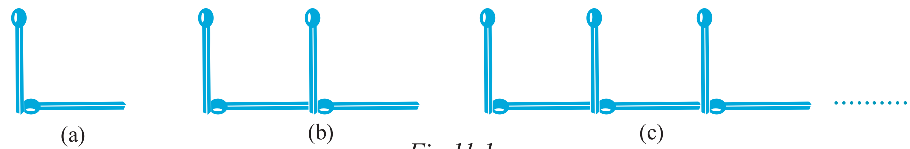


Fig 11.1

Then Sarita also picks two sticks, forms another letter L and puts it next to the one made by Ameena [Fig 11.1 (b)].

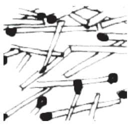
Then Ameena adds one more L and this goes on as shown by the dots in Fig 11.1 (c).

Their friend Appu comes in. He looks at the pattern. Appu always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Ameena and Sarita are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Appu gets the answer to his question from the Table 1; 7Ls require 14 matchsticks.



While writing the table, Ameena realises that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = 2 × number of Ls.

For convenience, let us write the letter n for the number of Ls. If one L is made, $n = 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5, ... We then write,

Number of matchsticks required = $2 \times n$.

Instead of writing $2 \times n$, we write $2n$. Note that $2n$ is same as $2 \times n$.

Ameena tells her friends that her rule gives the number of matchsticks required for forming any number of Ls.

Thus, For $n = 1$, the number of matchsticks required $= 2 \times 1 = 2$

For $n = 2$, the number of matchsticks required $= 2 \times 2 = 4$

For $n = 3$, the number of matchsticks required $= 2 \times 3 = 6$ etc.

These numbers agree with those from Table 1.

Sarita says, "The rule is very powerful! Using the rule, I can say how many matchsticks are required to form even 100 Ls. I do not need to draw the pattern or make a table, once the rule is known".
Do you agree with Sarita?

11.3 The Idea of a Variable

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was :

Number of matchsticks required = 2*n*

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4,.... Let us look at Table 1 once again. In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

***n* is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4, ... We wrote the rule for the number of matchsticks required using the variable *n*.**

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

We shall look at another example of matchstick patterns to learn more about variables.

11.4 More Matchstick Patterns

Ameena and Sarita have become quite interested in matchstick patterns. They now want to try a pattern of the letter C. To make one C, they use three matchsticks as shown in Fig. 11.2(a).

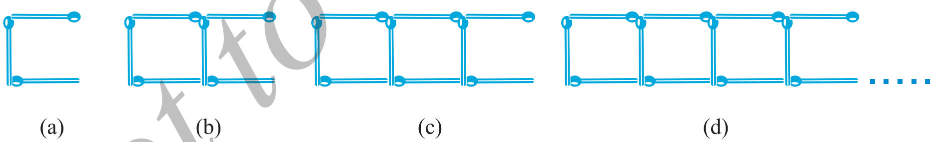


Fig 11.2

Table 2 gives the number of matchsticks required to make a pattern of Cs.

Table 2

Number of Cs formed	1	2	3	4	5	6	7	8	....	....	....
Number of matchsticks required	3	6	9	12	15	18	21	24	....	....	....

Can you complete the entries left blank in the table?

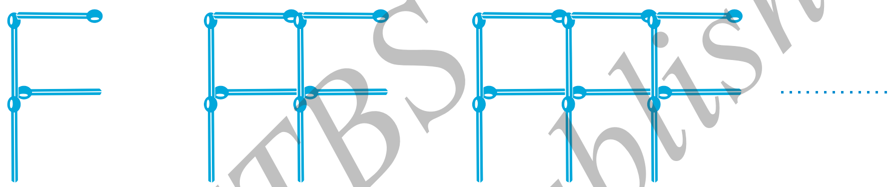
Sarita comes up with the rule :
Number of matchsticks required = 3*n*

She has used the letter n for the number of Cs; n is a variable taking on values 1, 2, 3, 4, ...

Do you agree with Sarita ?

Remember $3n$ is the same as $3 \times n$.

Next, Ameena and Sarita wish to make a pattern of Fs. They make one F using 4 matchsticks as shown in Fig 11.3(a).



(a) (b) (c)

Fig 11.3

Can you now write the rule for making patterns of F?

Think of other letters of the alphabet and other shapes that can be made from matchsticks. For example, U (⊔), V (\/), triangle (△), square (□) etc. Choose any five and write the rules for making matchstick patterns with them.

11.5 More Examples of Variables

We have used the letter n to show a variable. Raju asks, "Why not m"? There is nothing special about n, any letter can be used.

> **One may use any letter as *m, l, p, x, y, z* etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3. It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But *n* in the examples we have looked is a variable. It takes on various values 1, 2, 3, 4, ...**

Let us now consider variables in a more familiar situation.

Students went to buy notebooks from the school bookstore. Price of one notebook is ₹ 5. Munnu wants to buy 5 notebooks, Appu wants to buy 7 notebooks, Sara wants to buy 4 notebooks and so on. How much money should a student carry when she or he goes to the bookstore to buy notebooks?

This will depend on how many notebooks the student wants to buy. The students work together to prepare a table.

Table 3

Number of notebooks required	1	2	3	4	5	.....	m	.....
Total cost in rupees	5	10	15	20	25	.....	$5m$	.....

The letter m stands for the number of notebooks a student wants to buy; m is a variable, which can take any value 1, 2, 3, 4, ... The total cost of m notebooks is given by the rule :

The total cost in rupees = 5 × number of note books required

= $5m$

If Munnu wants to buy 5 notebooks, then taking $m = 5$, we say that Munnu should carry ₹ 5 × 5 or ₹ 25 with him to the school bookstore.

Let us take one more example. For the Republic Day celebration in the school, children are going to perform mass drill in the presence of the chief guest. They stand 10 in a row (Fig 11.4). How many children can there be in the drill?

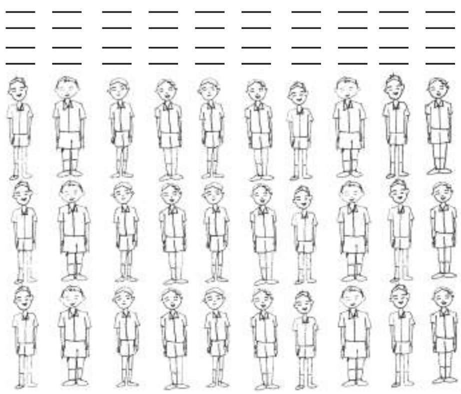


Fig 11.4

The number of children will depend on the number of rows. If there is 1 row, there will be 10 children. If there are 2 rows, there will be 2 × 10 or 20 children and so on. If there are r rows, there will be $10r$ children in the drill; here, r is a variable which stands for the number of rows and so takes on values 1, 2, 3, 4, ...

In all the examples seen so far, the variable was multiplied by a number. There can be different situations as well in which numbers are added to or subtracted from the variable as seen below.

Sarita says that she has 10 more marbles in her collection than Ameena. If Ameena has 20 marbles, then Sarita has 30. If Ameena has 30 marbles, then Sarita has 40 and so on. We do not know exactly how many marbles Ameena has. She may have any number of marbles.

But we know that, Sarita's marbles = Ameena's marbles + 10.

We shall denote Ameena's marbles by the letter x. Here, x is a variable, which can take any value 1, 2, 3, 4,... ,10,... ,20,... ,30,... . Using x, we write Sarita's marbles = $x + 10$. The expression $(x + 10)$ is read as 'x plus ten'. It means 10 added to x. If x is 20, $(x + 10)$ is 30. If x is 30, $(x + 10)$ is 40 and so on.

> The expression $(x + 10)$ cannot be simplified further.
> Do not confuse $x + 10$ with $10x$, they are different.
> In $10x$, x is multiplied by 10. In $(x + 10)$, 10 is added to x.
> We may check this for some values of x.
> For example,
> If $x = 2$, $10x = 10 \times 2 = 20$ and $x + 10 = 2 + 10 = 12$.
> If $x = 10$, $10x = 10 \times 10 = 100$ and $x + 10 = 10 + 10 = 20$.

Raju and Balu are brothers. Balu is younger than Raju by 3 years. When Raju is 12 years old, Balu is 9 years old. When Raju is 15 years old, Balu is 12 years old. We do not know Raju's age exactly. It may have any value. Let x denote Raju's age in years, x is a variable. If Raju's age in years is x, then Balu's age in years is $(x - 3)$. The expression $(x - 3)$ is read as x minus three. As you would expect, when x is 12, $(x - 3)$ is 9 and when x is 15, $(x - 3)$ is 12.

EXERCISE 11.1

1. Find the rule which gives the number of matchsticks required to make the following matchstick patterns. Use a variable to write the rule.

   (a) A pattern of letter T as 

   (b) A pattern of letter Z as 

(c) A pattern of letter U as 

(d) A pattern of letter V as 

(e) A pattern of letter E as 

(f) A pattern of letter S as 

(g) A pattern of letter A as 

2. We already know the rule for the pattern of letters L, C and F. Some of the letters from Q.1 (given above) give us the same rule as that given by L. Which are these? Why does this happen?

3. Cadets are marching in a parade. There are 5 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)

4. If there are 50 mangoes in a box, how will you write the total number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)

5. The teacher distributes 5 pencils per student. Can you tell how many pencils are needed, given the number of students? (Use s for the number of students.)

6. A bird flies 1 kilometer in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (Use t for flying time in minutes.)

7. Radha is drawing a dot Rangoli (a beautiful pattern of lines joining dots) with chalk powder. She has 9 dots in a row. How many dots will her Rangoli have for r rows? How many dots are there if there are 8 rows? If there are 10 rows?

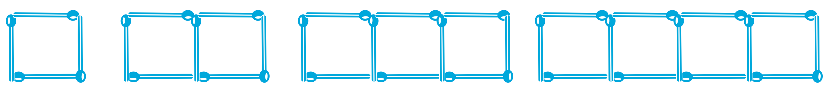
8. Leela is Radha's younger sister. Leela is 4 years younger than Radha. Can you write Leela's age in terms of Radha's age? Take Radha's age to be x years.

Fig 11.5

9. Mother has made laddus. She gives some laddus to guests and family members; still 5 laddus remain. If the number of laddus mother gave away is l, how many laddus did she make?

10. Oranges are to be transferred from larger boxes into smaller boxes. When a large box is emptied, the oranges from it fill two smaller boxes and still 10 oranges remain outside. If the number of oranges in a small box are taken to be x, what is the number of oranges in the larger box?

11. (a) Look at the following matchstick pattern of squares (Fig 11.6). The squares are not separate. Two neighbouring squares have a common matchstick. Observe the patterns and find the rule that gives the number of matchsticks



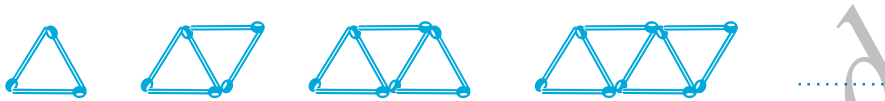
.....

(a) (b) (c) (d)

Fig 11.6

in terms of the number of squares. (Hint : If you remove the vertical stick at the end, you will get a pattern of Cs.)

(b) Fig 11.7 gives a matchstick pattern of triangles. As in Exercise 11 (a) above, find the general rule that gives the number of matchsticks in terms of the number of triangles.



(a) (b) (c) (d)

Fig 11.7

11.6 Use of Variables in Common Rules

Let us now see how certain common rules in mathematics that we have already learnt are expressed using variables.

Rules from geometry

We have already learnt about the perimeter of a square and of a rectangle in the chapter on Mensuration. Here, we go back to them to write them in the form of a rule.

1. **Perimeter of a square** We know that perimeter of any polygon (a closed figure made up of 3 or more line segments) is the sum of the lengths of its sides.
   A square has 4 sides and they are equal in length (Fig 11.8). Therefore,
   The perimeter of a square = Sum of the lengths of the sides of the square
   = 4 times the length of a side of the square
   $= 4 \times l = 4l$.

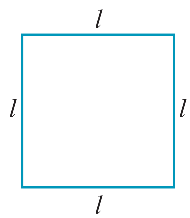


Fig 11.8

Thus, we get the rule for the perimeter of a square. The use of the variable l allows us to write the general rule in a way that is concise and easy to remember.

We may take the perimeter also to be represented by a variable, say p. Then the rule for the perimeter of a square is expressed as a relation between the perimeter and the length of the square, $p = 4l$

2. **Perimeter of a rectangle** We know that a rectangle has four sides. For example, the rectangle ABCD has four sides AB, BC, CD and DA. The opposite sides of any rectangle are always equal in length. Thus, in the rectangle ABCD, let us denote by l, the length of the sides AB or CD and, by b, the length

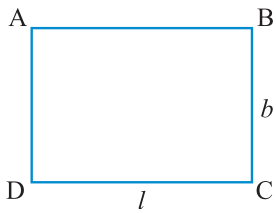


Fig 11.9

of the sides AD or BC. Therefore,

Perimeter of a rectangle = length of AB + length of BC + length of CD + length of AD

= 2 × length of CD + 2 × length of BC = $2l + 2b$

The rule, therefore, is that the perimeter of a rectangle = $2l + 2b$

where, l and b are respectively the length and breadth of the rectangle.

Discuss what happens if $l = b$.

If we denote the perimeter of the rectangle by the variable p, the rule for perimeter of a rectangle becomes $\boldsymbol{p = 2l + 2b}$

> **Note :** Here, both l and b are variables. They take on values independent of each other. i.e. the value one variable takes does not depend on what value the other variable has taken.

In your studies of geometry you will come across several rules and formulas dealing with perimeters and areas of plane figures, and surface areas and volumes of three-dimensional figures. Also, you may obtain formulas for the sum of internal angles of a polygon, the number of diagonals of a polygon and so on. The concept of variables which you have learnt will prove very useful in writing all such general rules and formulas.

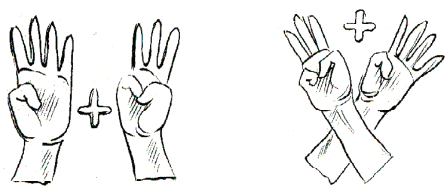
Rules from arithmetic

3. Commutativity of addition of two numbers

We know that

4 + 3 = 7 and 3 + 4 = 7

i.e. 4 + 3 = 3 + 4



As we have seen in the chapter on whole numbers, this is true for any two numbers. This property of numbers is known as the **commutativity of addition of numbers**. Commuting means interchanging. Commuting the order of numbers in addition does not change the sum. The use of variables allows us to express the generality of this property in a concise way. Let a and b be two variables which can take any number value.

Then, $\boldsymbol{a + b = b + a}$

Once we write the rule this way, all special cases are included in it. If $a = 4$ and $b = 3$, we get 4 + 3 = 3 + 4. If $a = 37$ and $b = 73$, we get 37 + 73 = 73 + 37 and so on.

4. Commutativity of multiplication of two numbers

We have seen in the chapter on whole numbers that for multiplication of two numbers, the order of the two numbers being multiplied does not matter.

For example,

$4 \times 3 = 12$, $3 \times 4 = 12$

Hence, $4 \times 3 = 3 \times 4$

This property of numbers is known as **commutativity of multiplication of numbers.** Commuting (interchanging) the order of numbers in multiplication does not change the product. Using variables a and b as in the case of addition, we can express the commutativity of multiplication of two numbers as $\boldsymbol{a \times b = b \times a}$

Note that a and b can take any number value. They are variables. All the special cases like

$4 \times 3 = 3 \times 4$ or $37 \times 73 = 73 \times 37$ follow from the general rule.

5. Distributivity of numbers

Suppose we are asked to calculate 7×38. We obviously do not know the table of 38. So, we do the following:

$7 \times 38 = 7 \times (30 + 8) = 7 \times 30 + 7 \times 8 = 210 + 56 = 266$

This is always true for any three numbers like 7, 30 and 8. This property is known as **distributivity of multiplication over addition of numbers.**

By using variables, we can write this property of numbers also in a general and concise way. Let a, b and c be three variables, each of which can take any number. Then, $\boldsymbol{a \times (b + c) = a \times b + a \times c}$

Properties of numbers are fascinating. You will learn many of them in your study of numbers this year and in your later study of mathematics. Use of variables allows us to express these properties in a very general and concise way. One more property of numbers is given in question 5 of Exercise 11.2. Try to find more such properties of numbers and learn to express them using variables.

EXERCISE 11.2

1. The side of an equilateral triangle is shown by l. Express the perimeter of the equilateral triangle using l.
2. The side of a regular hexagon (Fig 11.10) is denoted by l. Express the perimeter of the hexagon using l.

   (**Hint :** A regular hexagon has all its six sides equal in length.)

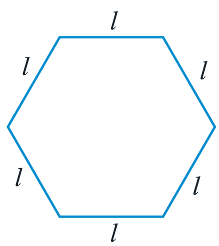


Fig 11.10

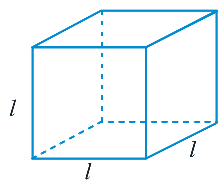


Fig 11.11

3. A cube is a three-dimensional figure as shown in Fig 11.11. It has six faces and all of them are identical squares. The length of an edge of the cube is given by l. Find the formula for the total length of the edges of a cube.

4. The diameter of a circle is a line which joins two points on the circle and also passes through the centre of the circle. (In the adjoining figure (Fig 11.12) AB is a diameter of the circle; C is its centre.) Express the diameter of the circle (d) in terms of its radius (r).

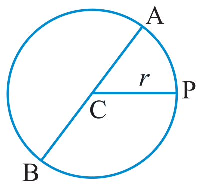


Fig 11.12

5. To find sum of three numbers 14, 27 and 13, we can have two ways:
   (a) We may first add 14 and 27 to get 41 and then add 13 to it to get the total sum 54 or
   (b) We may add 27 and 13 to get 40 and then add 14 to get the sum 54. Thus, (14 + 27) + 13 = 14 + (27 + 13)

This can be done for any three numbers. This property is known as the **associativity of addition of numbers**. Express this property which we have already studied in the chapter on Whole Numbers, in a general way, by using variables a, b and c.

11.7 Expressions with Variables

Recall that in arithmetic we have come across expressions like (2 × 10) + 3, 3 × 100 + (2 × 10) + 4 etc. These expressions are formed from numbers like 2, 3, 4, 10, 100 and so on. To form expressions we use all the four number operations of addition, subtraction, multiplication and division. For example, to form (2 × 10) + 3, we have multiplied 2 by 10 and then added 3 to the product. Examples of some of the other arithmetic expressions are :

3 + (4 × 5),	(– 3 × 40) + 5,
8 – (7 × 2),	14 – (5 – 2),
(6 × 2) – 5,	(5 × 7) – (3 × 4),
7 + (8 × 2)	(5 × 7) – (3 × 4 – 7) etc.

Expressions can be formed from variables too. In fact, we already have seen expressions with variables, for example: $2n$, $5m$, $x + 10$, $x – 3$ etc. These expressions with variables are obtained by operations of addition, subtraction, multiplication and division on variables. For example, the expression $2n$ is formed by multiplying the variable n by 2; the expression $(x + 10)$ is formed by adding 10 to the variable x and so on.

We know that variables can take different values; they have no fixed value. But they are numbers. That is why as in the case of numbers, operations of addition, subtraction, multiplication and division can be done on them.

One important point must be noted regarding the expressions containing variables. A number expression like (4 × 3) + 5 can be immediately evaluated as (4 × 3) + 5 = 12 + 5 = 17

But an expression like $(4x + 5)$, which contains the variable x, cannot be evaluated. Only if x is given some value, an expression like $(4x + 5)$ can be evaluated. For example,

when $x = 3$, $4x + 5 = (4 \times 3) + 5 = 17$ as found above.

Expression	**How formed?**
(a) $y + 5$	5 added to y
(b) $t - 7$	7 subtracted from t
(c) $10\,a$	a multiplied by 10
(d) $\frac{x}{3}$	x divided by 3
(e) $-5\,q$	q multiplied by -5
(f) $3\,x + 2$	first x multiplied by 3, then 2 added to the product
(g) $2\,y - 5$	first y multiplied by 2, then 5 subtracted from the product

Write 10 other such simple expressions and tell how they have been formed.

We should also be able to write an expression through given instruction about how to form it. Look at the following example :

Give expressions for the following :

(a) 12 subtracted from z	$z - 12$
(b) 25 added to r	$r + 25$
(c) p multiplied by 16	$16\,p$
(d) y divided by 8	$\frac{y}{8}$
(e) m multiplied by -9	$-9\,m$
(f) y multiplied by 10 and then 7 added to the product	$10\,y + 7$
(g) n multiplied by 2 and 1 subtracted from the product	$2\,n - 1$

Sarita and Ameena decide to play a game of expressions. They take the variable x and the number 3 and see how many expressions they can make. The condition is that they should use not more than one out of the four number operations and every expression must have x in it. Can you help them?

Is $(3x + 5)$ allowed ?
Is $(3x + 3)$ allowed ?

Sarita thinks of $(x + 3)$.

Then, Ameena comes up with $(x - 3)$.

Next she suggests $3x$. Sarita then immediately makes $\frac{x}{3}$.

Are these the only four expressions that they can get under the given condition?

Next they try combinations of y, 3 and 5. The condition is that they should use not more than one operation of addition or subtraction and one operation of multiplication or division. Every expression must have y in it. Check, if their answers are right.

In the following exercise we shall look at how few simple expressions have been formed.

$y + 5, y + 3, y - 5, y - 3, 3y, 5y, \frac{y}{3}, \frac{y}{5}, 3y + 5,$
$3y - 5, 5y + 3, 5y - 3$
Can you make some more expressions?

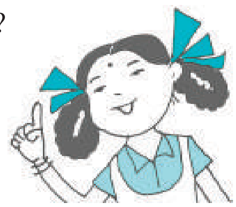
> Is $\frac{y}{3}$ 5 allowed ?
> Is $(y + 8)$ allowed ?
> Is $15y$ allowed ?

EXERCISE 11.3

1. Make up as many expressions with numbers (no variables) as you can from three numbers 5, 7 and 8. Every number should be used not more than once. Use only addition, subtraction and multiplication.

> (**Hint :** Three possible expressions are $5 + (8 - 7)$, $5 - (8 - 7)$, $(5 \times 8) + 7$; make the other expressions.)

2. Which out of the following are expressions with numbers only?
   (a) $y + 3$ (b) $(7 \times 20) - 8z$
   (c) $5 (21 - 7) + 7 \times 2$ (d) 5
   (e) $3x$ (f) $5 - 5n$
   (g) $(7 \times 20) - (5 \times 10) - 45 + p$



3. Identify the operations (addition, subtraction, division, multiplication) in forming the following expressions and tell how the expressions have been formed.
   (a) $z + 1, z - 1, y + 17, y - 17$ (b) $17y, \frac{y}{17}, 5z$
   (c) $2y + 17, 2y - 17$ (d) $7m, -7m + 3, -7m - 3$
4. Give expressions for the following cases.
   (a) 7 added to p (b) 7 subtracted from p
   (c) p multiplied by 7 (d) p divided by 7
   (e) 7 subtracted from $-m$ (f) $-p$ multiplied by 5
   (g) $-p$ divided by 5 (h) p multiplied by -5

5. Give expressions in the following cases.
   (a) 11 added to $2m$ (b) 11 subtracted from $2m$
   (c) 5 times y to which 3 is added (d) 5 times y from which 3 is subtracted
   (e) y is multiplied by -8
   (f) y is multiplied by -8 and then 5 is added to the result
   (g) y is multiplied by 5 and the result is subtracted from 16
   (h) y is multiplied by -5 and the result is added to 16.
6. (a) Form expressions using t and 4. Use not more than one number operation. Every expression must have t in it.
   (b) Form expressions using y, 2 and 7. Every expression must have y in it. Use only two number operations. These should be different.

11.8 Using Expressions Practically

We have already come across practical situations in which expressions are useful. Let us remember some of them.

	Situation (described in ordinary language)	Variable	Statements using expressions
1.	Sarita has 10 more marbles than Ameena.	Let Ameena have x marbles.	Sarita has $(x + 10)$ marbles.
2.	Balu is 3 years younger than Raju.	Let Raju's age be x years.	Balu's age is $(x - 3)$ years.
3.	Bikash is twice as old as Raju.	Let Raju's age be x years.	Bikash's age is $2x$ years.
4.	Raju's father's age is 2 years more than 3 times Raju's age.	Let Raju's age be x years.	Raju's father's age is $(3x + 2)$ years.

Let us look at some other such situations.

	Situation (described in ordinary language)	Variable	Statements using expressions
5.	How old will Susan be 5 years from now?	Let y be Susan's present age in years.	Five years from now Susan will be $(y + 5)$ years old.
6.	How old was Susan 4 years ago?	Let y be Susan's present age in years.	Four years ago, Susan was $(y - 4)$ years old.
7.	Price of wheat per kg is ₹ 5 less than price of rice per kg.	Let price of rice per kg be ₹ p.	Price of wheat per kg is ₹ $(p - 5)$.

8. Price of oil per litre is 5 times the price of rice per kg.	Let price of rice per kg be ₹ p.	Price of oil per litre is ₹ $5p$.
9. The speed of a bus is 10 km/hour more than the speed of a truck going on the same road.	Let the speed of the truck be y km/hour.	The speed of the bus is $(y + 10)$ km/hour.

Try to find more such situations. You will realise that there are many statements in ordinary language, which you will be able to change to statements using expressions with variables. In the next section, we shall see how we use these statements using expressions for our purpose.

EXERCISE 11.4

1. Answer the following:
   (a) Take Sarita's present age to be y years
      (i) What will be her age 5 years from now?
      (ii) What was her age 3 years back?
      (iii) Sarita's grandfather is 6 times her age. What is the age of her grandfather?
      (iv) Grandmother is 2 years younger than grandfather. What is grandmother's age?
      (v) Sarita's father's age is 5 years more than 3 times Sarita's age. What is her father's age?
   (b) The length of a rectangular hall is 4 meters less than 3 times the breadth of the hall. What is the length, if the breadth is b meters?
   (c) A rectangular box has height h cm. Its length is 5 times the height and breadth is 10 cm less than the length. Express the length and the breadth of the box in terms of the height.
   (d) Meena, Beena and Leena are climbing the steps to the hill top. Meena is at step s, Beena is 8 steps ahead and Leena 7 steps behind. Where are Beena and Meena? The total number of steps to the hill top is 10 less than 4 times what Meena has reached. Express the total number of steps using s.

   (e) A bus travels at v km per hour. It is going from Daspur to Beespur. After the bus has travelled 5 hours, Beespur is still 20 km away. What is the distance from Daspur to Beespur? Express it using v.

2. Change the following statements using expressions into statements in ordinary language.
(For example, Given Salim scores r runs in a cricket match, Nalin scores $(r + 15)$ runs. In ordinary language – Nalin scores 15 runs more than Salim.)
   (a) A notebook costs ₹ p. A book costs ₹ $3\,p$.
   (b) Tony puts q marbles on the table. He has $8\,q$ marbles in his box.
   (c) Our class has n students. The school has $20\,n$ students.
   (d) Jaggu is z years old. His uncle is $4\,z$ years old and his aunt is $(4z – 3)$ years old.
   (e) In an arrangement of dots there are r rows. Each row contains 5 dots.
3. (a) Given Munnu's age to be x years, can you guess what $(x – 2)$ may show? (**Hint :** Think of Munnu's younger brother.)
   Can you guess what $(x + 4)$ may show? What $(3\,x + 7)$ may show?
   (b) Given Sara's age today to be y years. Think of her age in the future or in the past.
   What will the following expression indicate? $y + 7, y – 3, y + 4\frac{1}{2}, \; y – 2\frac{1}{2}$.
   (c) Given n students in the class like football, what may $2n$ show? What may $\frac{n}{2}$ show? (**Hint :** Think of games other than football).

11.9 What is an Equation?

Let us recall the matchstick pattern of the letter L given in Fig 11.1. For our convenience, we have the Fig 11.1 redrawn here.

(a) (b) (c)

The number of matchsticks required for different number of Ls formed was given in Table 1. We repeat the table here.

Table 1

Number of L's formed	1	2	3	4	5	6	7	8	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....

We know that the number of matchsticks required is given by the rule, $2n$, if n is taken to be the number of Ls formed.

Appu always thinks differently. He asks, "We know how to find the number of matchsticks required for a given number of Ls. What about the other way round? How does one find the number of Ls formed, given the number of matchsticks"?

We ask ourselves a definite question.

How many Ls are formed if the number of matchsticks given is 10?

This means we have to find the number of Ls (*i.e.* n), given the number of matchsticks 10. So, $2n = 10$ (1)

Here, we have a condition to be satisfied by the variable n. This condition is an example of an equation.

Our question can be answered by looking at Table 1. Look at various values of n. If $n = 1$, the number of matchsticks is 2. Clearly, the condition is not satisfied, because 2 is not 10. We go on checking.

n	$2n$	Condition satisfied? Yes/No
2	4	No
3	6	No
4	8	No
5	10	Yes
6	12	No
7	14	No

We find that only if $n = 5$, the condition, i.e. the equation $2n = 10$ is satisfied. For any value of n other than 5, the equation is not satisfied.

Let us look at another equation.

Balu is 3 years younger than Raju. Taking Raju's age to be x years, Balu's age is $(x – 3)$ years. Suppose, Balu is 11 years old. Then, let us see how our method gives Raju's age.

We have Balu's age, $x – 3 = 11$ (2)

This is an equation in the variable x. We shall prepare a table of values of $(x – 3)$ for various values of x.

x	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$x – 3$	0	1	–	–	–	–	–	–	–	9	10	11	12	13	–	–

Complete the entries which are left blank. From the table, we find that only for $x = 14$, the condition $x – 3 = 11$ is satisfied. For other values, for example for $x = 16$ or for $x = 12$, the condition is not satisfied. Raju's age, therefore, is 14 years.

To summarise, **any equation like the above, is a condition on a variable. It is satisfied only for a definite value of the variable.** For example, the equation $2n = 10$ is satisfied only by the value 5 of the variable n. Similarly, the equation $x - 3 = 11$ is satisfied only by the value 14 of the variable x.

Note that an equation has an **equal sign** (=) between its two sides. The equation says that the value of the left hand side (LHS) is equal to the value of the right hand side (RHS). If the LHS is not equal to the RHS, we do not get an equation.

For example : The statement $2n$ is greater than 10, i.e. $2n > 10$ is not an equation. Similarly, the statement $2n$ is smaller than 10 i.e. $2n < 10$ is not an equation. Also, the statements

$(x - 3) > 11$ or $(x - 3) < 11$ are not equations.

Now, let us consider $8 - 3 = 5$

There is an equal sign between the LHS and RHS. Neither of the two sides contain a variable. Both contain numbers. We may call this a numerical equation. Usually, the word equation is used only for equations with one or more variables.

Let us do an exercise. State which of the following are equations with a variable. In the case of equations with a variable, identify the variable.

(a) $x + 20 = 70$ (Yes, x)

(b) $8 \times 3 = 24$ (No, this a numerical equation)

(c) $2p > 30$ (No)

(d) $n - 4 = 100$ (Yes, n)

(e) $20b = 80$ (Yes, b)

(f) $\frac{y}{8} < 50$ (No)

Following are some examples of an equation. (The variable in the equation is also identified).

Fill in the blanks as required :

$x + 10 = 30$	(variable x)	(3)
$p - 3 = 7$	(variable p)	(4)
$3n = 21$	(variable ____)	(5)
$\frac{t}{5} = 4$	(variable ____)	(6)
$2l + 3 = 7$	(variable ____)	(7)
$2m - 3 = 5$	(variable ____)	(8)

11.10 Solution of an Equation

We saw in the earlier section that the equation

$$2n = 10 \quad (1)$$

was satisfied by $n = 5$. No other value of n satisfies the equation. **The value of the variable in an equation which satisfies the equation is called a solution to the equation**. Thus, $n = 5$ is a solution to the equation $2\,n = 10$.

Note, $n = 6$ is not a solution to the equation $2n = 10$; because for $n = 6$, $2n = 2 \times 6 = 12$ and not 10.

Also, $n = 4$ is not a solution. Tell, why not?

Let us take the equation $\quad x - 3 = 11 \quad$ (2)

This equation is satisfied by $x = 14$, because for $x = 14$,

LHS of the equation = 14 – 3 = 11 = RHS

It is not satisfied by $x = 16$, because for $x = 16$,

LHS of the equation = 16 – 3 = 13, which is not equal to RHS.

Thus, $x = 14$ is a solution to the equation $x - 3 = 11$ and $x = 16$ is not a solution to the equation. Also, $x = 12$ is not a solution to the equation. Explain, why not?

Now complete the entries in the following table and explain why your answer is Yes/No.

In finding the solution to the equation $2n = 10$, we prepared a table for various values of n and from the table, we picked up the value of n which was the solution to the equation (i.e. which satisfies the equation). What we used is **a trial and error method**. It is not a **direct** and **practical** way of finding a

Equation	Value of the variable	Solution (Yes/No)
1. $x + 10 = 30$	$x = 10$	No
2. $x + 10 = 30$	$x = 30$	No
3. $x + 10 = 30$	$x = 20$	Yes
4. $p - 3 = 7$	$p = 5$	No
5. $p - 3 = 7$	$p = 15$	–
6. $p - 3 = 7$	$p = 10$	–
7. $3n = 21$	$n = 9$	–
8. $3n = 21$	$n = 7$	–
9. $\frac{t}{5} \quad 4$	$t = 25$	–
10. $\frac{t}{5} \quad 4$	$t = 20$	–
11. $2l + 3 = 7$	$l = 5$	–
12. $2l + 3 = 7$	$l = 1$	–
13. $2l + 3 = 7$	$l = 2$	–

solution. We need a direct way of solving an equation, i.e. finding the solution of the equation. We shall learn a more systematic method of solving equations only next year.

Beginning of Algebra

It is said that algebra as a branch of Mathematics began about 1550 BC, i.e. more than 3500 years ago, when people in Egypt started using symbols to denote unknown numbers.

Around 300 BC, use of letters to denote unknowns and forming expressions from them was quite common in India. Many great Indian mathematicians, **Aryabhatt** (born 476AD), **Brahmagupta** (born 598AD), **Mahavira** (who lived around 850AD) and **Bhaskara II** (born 1114AD) and others, contributed a lot to the study of algebra. They gave names such as *Beeja*, *Varna* etc. to unknowns and used first letters of colour names [e.g., ka from *kala* (black), nee from *neela* (blue)] to denote them. The Indian name for algebra, *Beejaganit*, dates back to these ancient Indian mathematicians.

The word 'algebra' is derived from the title of the book, **'Aljebar w'al almugabalah'**, written about 825AD by an Arab mathematician, Mohammed Ibn Al Khowarizmi of Baghdad.

EXERCISE 11.5

1. State which of the following are equations (with a variable). Give reason for your answer. Identify the variable from the equations with a variable.

   (a) $17 = x + 7$ (b) $(t - 7) > 5$ (c) $\frac{4}{2} = 2$

   (d) $(7 \times 3) - 19 = 8$ (e) $5 \times 4 - 8 = 2x$ (f) $x - 2 = 0$

   (g) $2m < 30$ (h) $2n + 1 = 11$ (i) $7 = (11 \times 5) - (12 \times 4)$

   (j) $7 = (11 \times 2) + p$ (k) $20 = 5y$ (l) $\frac{3q}{2} < 5$

   (m) $z + 12 > 24$ (n) $20 - (10 - 5) = 3 \times 5$

   (o) $7 - x = 5$

2. Complete the entries in the third column of the table.

S.No.	Equation	Value of variable	Equation satisfied Yes/No
(a)	$10y = 80$	$y = 10$	
(b)	$10y = 80$	$y = 8$	
(c)	$10y = 80$	$y = 5$	
(d)	$4l = 20$	$l = 20$	
(e)	$4l = 20$	$l = 80$	
(f)	$4l = 20$	$l = 5$	
(g)	$b + 5 = 9$	$b = 5$	
(h)	$b + 5 = 9$	$b = 9$	
(i)	$b + 5 = 9$	$b = 4$	
(j)	$h - 8 = 5$	$h = 13$	
(k)	$h - 8 = 5$	$h = 8$	
(l)	$h - 8 = 5$	$h = 0$	
(m)	$p + 3 = 1$	$p = 3$	
(n)	$p + 3 = 1$	$p = 1$	
(o)	$p + 3 = 1$	$p = 0$	
(p)	$p + 3 = 1$	$p = -1$	
(q)	$p + 3 = 1$	$p = -2$	

3. Pick out the solution from the values given in the bracket next to each equation. Show that the other values do not satisfy the equation.

(a) $5m = 60$ (10, 5, 12, 15)

(b) $n + 12 = 20$ (12, 8, 20, 0)

(c) $p - 5 = 5$ (0, 10, 5 – 5)

(d) $\frac{q}{2} = 7$ (7, 2, 10, 14)

(e) $r - 4 = 0$ (4, – 4, 8, 0)

(f) $x + 4 = 2$ (– 2, 0, 2, 4)

4. (a) Complete the table and by inspection of the table find the solution to the equation $m + 10 = 16$.

m	1	2	3	4	5	6	7	8	9	10	__	__	__
$m + 10$	__	__	__	__	__	__	__	__	__	__	__	__	__

(b) Complete the table and by inspection of the table, find the solution to the equation $5t = 35$.

t	3	4	5	6	7	8	9	10	11	__	__	__	__	__
$5t$	__	__	__	__	__	__	__	__	__	__	__	__	__	__

(c) Complete the table and find the solution of the equation $z/3 = 4$ using the table.

z	8	9	10	11	12	13	14	15	16	__	__	__	__
$\frac{z}{3}$	$2\frac{2}{3}$	3	$3\frac{1}{3}$	__	__	__	__	__	__	__	__	__	__

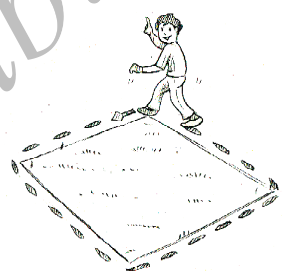
(d) Complete the table and find the solution to the equation $m - 7 = 3$.

m	5	6	7	8	9	10	11	12	13	__	__
$m - 7$	__	__	__	__	__	__	__	__	__	__	__

5. Solve the following riddles, you may yourself construct such riddles.

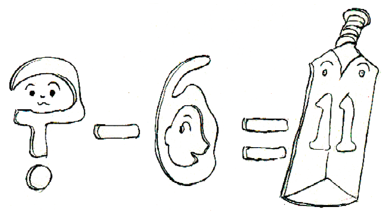
Who am I?

(i) Go round a square
Counting every corner
Thrice and no more!
Add the count to me
To get exactly thirty four!



(ii) For each day of the week
Make an upcount from me
If you make no mistake
You will get twenty three!

(iii) I am a special number
Take away from me a six!
A whole cricket team
You will still be able to fix!



(iv) Tell me who I am
I shall give a pretty clue!
You will get me back
If you take me out of twenty two!

What have we discussed?

1. We looked at patterns of making letters and other shapes using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1,2,3,... . It is a variable, denoted by some letter like n.

2. A variable takes on different values, its value is not fixed. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable.
3. We may use any letter $n, l, m, p, x, y, z,$ etc. to show a variable.
4. A variable allows us to express relations in any practical situation.
5. Variables are numbers, although their value is not fixed. We can do the operations of addition, subtraction, multiplication and division on them just as in the case of fixed numbers. Using different operations we can form expressions with variables like $x - 3$, $x + 3$, $2n$, $5m$, $\frac{p}{3}$, $2y + 3$, $3l - 5$, etc.

6. Variables allow us to express many common rules in both geometry and arithmetic in a general way. For example, the rule that the sum of two numbers remains the same if the order in which the numbers are taken is reversed can be expressed as $a + b = b + a$. Here, the variables a and b stand for any number, 1, 32, 1000 – 7, – 20, etc.
7. An equation is a condition on a variable. It is expressed by saying that an expression with a variable is equal to a fixed number, e.g. $x - 3 = 10$.
8. An equation has two sides, LHS and RHS, between them is the equal (=) sign.
9. The LHS of an equation is equal to its RHS only for a definite value of the variable in the equation. We say that this definite value of the variable satisfies the equation. This value itself is called the solution of the equation.
10. For getting the solution of an equation, one method is the trial and error method. In this method, we give some value to the variable and check whether it satisfies the equation. We go on giving this way different values to the variable` until we find the right value which satisfies the equation.

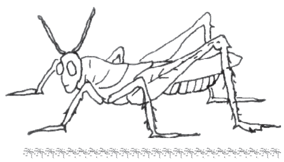
Ratio and Proportion

Chapter 12

12.1 Introduction

In our daily life, many a times we compare two quantities of the same type. For example, Avnee and Shari collected flowers for scrap notebook. Avnee collected 30 flowers and Shari collected 45 flowers. So, we may say that Shari collected 45 – 30 = 15 flowers more than Avnee.

Also, if height of Rahim is 150 cm and that of Avnee is 140 cm then, we may say that the height of Rahim is 150 cm – 140 cm = 10 cm more than Avnee. This is one way of comparison by taking difference.



If we wish to compare the lengths of an ant and a grasshopper, taking the difference does not express the comparison. The grasshopper's length, typically 4 cm to 5 cm is too long as compared to the ant's length which is a few mm. Comparison will be better if we try to find that how many ants can be placed one behind the other to match the length of grasshopper. So, we can say that 20 to 30 ants have the same length as a grasshopper.

Consider another example.

Cost of a car is ₹ 2,50,000 and that of a motorbike is ₹ 50,000. If we calculate the difference between the costs, it is ₹ 2,00,000 and if we compare by division;

i.e. $\frac{2,50,000}{50,000} = \frac{5}{1}$

We can say that the cost of the car is five times the cost of the motorbike. Thus, in certain situations, comparison by division makes better sense than comparison by taking the difference. The comparison by division is the Ratio. In the next section, we shall learn more about 'Ratios'.

12.2 Ratio

Consider the following:

Isha's weight is 25 kg and her father's weight is 75 kg. How many times Father's weight is of Isha's weight? It is three times.

Cost of a pen is ₹ 10 and cost of a pencil is ₹ 2. How many times the cost of a pen that of a pencil? Obviously it is five times.

In the above examples, we compared the two quantities in terms of 'how many times'. This comparison is known as the Ratio. We denote ratio using symbol ':'

Consider the earlier examples again. We can say,

The ratio of father's weight to Isha's weight $= \frac{75}{25} = \frac{3}{1} = 3{:}1$

The ratio of the cost of a pen to the cost of a pencil $= \frac{10}{2} = \frac{5}{1} = 5{:}1$

Let us look at this problem.

In a class, there are 20 boys and 40 girls. What is the ratio of

(a) Number of girls to the total number of students.

(b) Number of boys to the total number of students.

First we need to find the total number of students, which is,

Number of girls + Number of boys = 20 + 40 = 60.

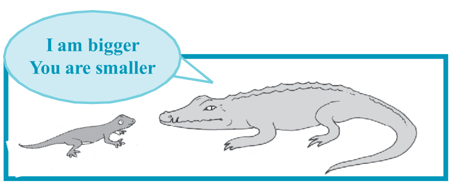
Then, the ratio of number of girls to the total number of students is $\frac{40}{60} = \frac{2}{3} = 2 : 3$

Find the answer of part (b) in the similar manner.

Now consider the following example.

Length of a house lizard is 20 cm and the length of a crocodile is 4 m.

"I am 5 times bigger than you", says the lizard. As we can see this



Try These

1. In a class, there are 20 boys and 40 girls. What is the ratio of the number of boys to the number of girls?
2. Ravi walks 6 km in an hour while Roshan walks 4 km in an hour. What is the ratio of the distance covered by Ravi to the distance covered by Roshan?

is really absurd. A lizard's length cannot be 5 times of the length of a crocodile. So, what is wrong? Observe that the length of the lizard is in centimetres and length of the crocodile is in metres. So, we have to convert their lengths into the same unit.

Length of the crocodile = 4 m = 4 × 100 = 400 cm.

Therefore, ratio of the length of the crocodile to the length of the lizard

$= \frac{400}{20} = \frac{20}{1} = 20:1.$

Two quantities can be compared only if they are in the same unit.

Now what is the ratio of the length of the lizard to the length of the crocodile?

It is $\frac{20}{400} = \frac{1}{20} = 1:20.$

Observe that the two ratios 1 : 20 and 20 : 1 are different from each other. The ratio 1 : 20 is the ratio of the length of the lizard to the length of the crocodile whereas, 20 : 1 is the ratio of the length of the crocodile to the length of the lizard.

Now consider another example.

Length of a pencil is 18 cm and its diameter is 8 mm. What is the ratio of the diameter of the pencil to that of its length? Since the length and the diameter of the pencil are given in different units, we first need to convert them into same unit.

Thus, length of the pencil = 18 cm = 18 × 10 mm = 180 mm.

The ratio of the diameter of the pencil to that of the length of the pencil

$= \frac{8}{180} = \frac{2}{45} = 2:45.$

Try These

1. Saurabh takes 15 minutes to reach school from his house and Sachin takes one hour to reach school from his house. Find the ratio of the time taken by Saurabh to the time taken by Sachin.
2. Cost of a toffee is 50 paise and cost of a chocolate is ₹ 10. Find the ratio of the cost of a toffee to the cost of a chocolate.
3. In a school, there were 73 holidays in one year. What is the ratio of the number of holidays to the number of days in one year?

A B

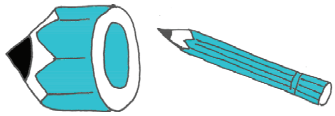
Think of some more situations where you compare two quantities of same type in different units.

We use the concept of ratio in many situations of our daily life without realising that we do so.

Compare the drawings A and B. B looks more natural than A. Why?

The legs in the picture A are too long in comparison to the other body parts. This is because we normally expect a certain ratio of the length of legs to the length of whole body.

Compare the two pictures of a pencil. Is the first one looking like a full pencil? No.



Why not? The reason is that the thickness and the length of the pencil are not in the correct ratio.

Same ratio in different situations :

Consider the following :

- Length of a room is 30 m and its breadth is 20 m. So, the ratio of length of the room to the breadth of the room $= \frac{30}{20} = \frac{3}{2} = 3:2$
- There are 24 girls and 16 boys going for a picnic. Ratio of the number of girls to the number of boys $= \frac{24}{16} = \frac{3}{2} = 3:2$

  The ratio in both the examples is 3 : 2.
- Note the ratios 30 : 20 and 24 : 16 in lowest form are same as 3 : 2. These are equivalent ratios.
- Can you think of some more examples having the ratio 3 : 2?

  It is fun to write situations that give rise to a certain ratio. For example, write situations that give the ratio 2 : 3.
- Ratio of the breadth of a table to the length of the table is 2 : 3.
- Sheena has 2 marbles and her friend Shabnam has 3 marbles.

Then, the ratio of marbles that Sheena and Shabnam have is 2 : 3.

Can you write some more situations for this ratio? Give any ratio to your friends and ask them to frame situations.

Ravi and Rani started a business and invested money in the ratio 2 : 3. After one year the total profit was ₹ 4,00,000.

Ravi said "we would divide it equally", Rani said "I should get more as I have invested more".

It was then decided that profit will be divided in the ratio of their investment.

Here, the two terms of the ratio 2 : 3 are 2 and 3.

Sum of these terms = 2 + 3 = 5

What does this mean?

This means if the profit is ₹ 5 then Ravi should get ₹ 2 and Rani should get ₹ 3. Or, we can say that Ravi gets 2 parts and Rani gets 3 parts out of the 5 parts.

i.e., Ravi should get $\frac{2}{5}$ of the total profit and Rani should get $\frac{3}{5}$ of the total profit.

If the total profit were ₹ 500

Ravi would get ₹ $\frac{2}{5} \times 500 =$ ₹ 200

and Rani would get $\frac{3}{5} \times 500 =$ ₹ 300

Now, if the profit were ₹ 4,00,000 could you find the share of each?

Ravi's share = ₹ $\frac{2}{5} \times 4,00,000 =$ ₹ 1,60,000

And Rani's share = ₹ $\frac{3}{5} \times 4,00,000 =$ ₹ 2,40,000

Can you think of some more examples where you have to divide a number of things in some ratio? Frame three such examples and ask your friends to solve them.

Let us look at the kind of problems we have solved so far.

Try These

1. Find the ratio of number of notebooks to the number of books in your bag.
2. Find the ratio of number of desks and chairs in your classroom.
3. Find the number of students above twelve years of age in your class. Then, find the ratio of number of students with age above twelve years and the remaining students.
4. Find the ratio of number of doors and the number of windows in your classroom.
5. Draw any rectangle and find the ratio of its length to its breadth.

Example 1 : Length and breadth of a rectangular field are 50 m and 15 m respectively. Find the ratio of the length to the breadth of the field.

Solution : Length of the rectangular field = 50 m

Breadth of the rectangular field = 15 m

The ratio of the length to the breadth is 50 : 15

The ratio can be written as $\frac{50}{15} = \frac{50 \div 5}{15 \div 5} = \frac{10}{3} = 10 : 3$

Thus, the required ratio is 10 : 3.

Example 2 : Find the ratio of 90 cm to 1.5 m.

Solution : The two quantities are not in the same units. Therefore, we have to convert them into same units.

1.5 m = 1.5 × 100 cm = 150 cm.

Therefore, the required ratio is 90 : 150.

$$= \frac{90}{150} = \frac{90 \times 30}{150 \times 30} = \frac{3}{5}$$

Required ratio is 3 : 5.

Example 3 : There are 45 persons working in an office. If the number of females is 25 and the remaining are males, find the ratio of:

(a) The number of females to number of males.

(b) The number of males to number of females.

Solution : Number of females = 25

Total number of workers = 45

Number of males = 45 – 25 = 20

Therefore, the ratio of number of females to the number of males

= 25 : 20 = 5 : 4

And the ratio of number of males to the number of females

= 20 : 25 = 4 : 5.

(Notice that there is a difference between the two ratios 5 : 4 and 4 : 5).

Example 4 : Give two equivalent ratios of 6 : 4.

Solution : Ratio 6 : 4 $= \frac{6}{4} = \frac{6 \times 2}{4 \times 2} = \frac{12}{8}$.

Therefore, 12 : 8 is an equivalent ratio of 6 : 4

Similarly, the ratio 6 : 4 $= \frac{6}{4} = \frac{6 \times 2}{4 \times 2} = \frac{3}{2}$

So, 3:2 is another equivalent ratio of 6 : 4.

Therefore, we can get equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

Write two more equivalent ratios of 6 : 4.

Example 5 : Fill in the missing numbers :

$$\frac{14}{21} = \frac{\square}{3} = \frac{6}{\square}$$

Solution : In order to get the first missing number, we consider the fact that $21 = 3 \times 7$. i.e. when we divide 21 by 7 we get 3. This indicates that to get the missing number of second ratio, 14 must also be divided by 7.

When we divide, we have, 14 ÷ 7 = 2

Hence, the second ratio is $\frac{2}{3}$.

Similarly, to get third ratio we multiply both terms of second ratio by 3. (Why?)

Hence, the third ratio is $\frac{6}{9}$

Therefore, $\frac{14}{21} = \frac{\boxed{2}}{3} = \frac{6}{\boxed{9}}$ [These are all equivalent ratios.]

Example 6 : Ratio of distance of the school from Mary's home to the distance of the school from John's home is 2 : 1.

(a) Who lives nearer to the school?

(b) Complete the following table which shows some possible distances that Mary and John could live from the school.

Distance from Mary's home to school (in km.)	10		4		
Distance from John's home to school (in km.)	5	4		3	1

(c) If the ratio of distance of Mary's home to the distance of Kalam's home from school is 1 : 2, then who lives nearer to the school?

Solution : (a) John lives nearer to the school (As the ratio is 2 : 1).

(b)

Distance from Mary's home to school (in km.)	10	8	4	6	2
Distance from John's home to school (in km.)	5	4	2	3	1

(c) Since the ratio is 1 : 2, so Mary lives nearer to the school.

Example 7 : Divide ₹ 60 in the ratio 1 : 2 between Kriti and Kiran.

Solution : The two parts are 1 and 2.

Therefore, sum of the parts = 1 + 2 = 3.

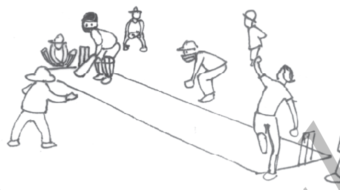
This means if there are ₹ 3, Kriti will get ₹ 1 and Kiran will get ₹ 2. Or, we can say that Kriti gets 1 part and Kiran gets 2 parts out of every 3 parts.

Therefore, Kriti's share $= \frac{1}{3} \times 60 =$ ₹ 20

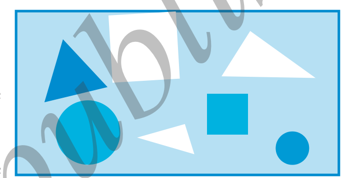
And Kiran's share $= \frac{2}{3} \times 60 =$ ₹ 40.

EXERCISE 12.1

1. There are 20 girls and 15 boys in a class.
   (a) What is the ratio of number of girls to the number of boys?
   (b) What is the ratio of number of girls to the total number of students in the class?
2. Out of 30 students in a class, 6 like football, 12 like cricket and remaining like tennis. Find the ratio of
   (a) Number of students liking football to number of students liking tennis.
   (b) Number of students liking cricket to total number of students.



3. See the figure and find the ratio of
   (a) Number of triangles to the number of circles inside the rectangle.
   (b) Number of squares to all the figures inside the rectangle.
   (c) Number of circles to all the figures inside the rectangle.



4. Distances travelled by Hamid and Akhtar in an hour are 9 km and 12 km. Find the ratio of speed of Hamid to the speed of Akhtar.
5. Fill in the following blanks :

   $$\frac{15}{18} = \frac{\square}{6} = \frac{10}{\square} = \frac{\square}{30}$$ [Are these equivalent ratios?]

6. Find the ratio of the following :
   (a) 81 to 108 (b) 98 to 63
   (c) 33 km to 121 km (d) 30 minutes to 45 minutes
7. Find the ratio of the following :
   (a) 30 minutes to 1.5 hours (b) 40 cm to 1.5 m
   (c) 55 paise to ₹ 1 (d) 500 mL to 2 litres
8. In a year, Seema earns ₹ 1,50,000 and saves ₹ 50,000. Find the ratio of
   (a) Money that Seema earns to the money she saves.
   (b) Money that she saves to the money she spends.
9. There are 102 teachers in a school of 3300 students. Find the ratio of the number of teachers to the number of students.
10. In a college, out of 4320 students, 2300 are girls. Find the ratio of
    (a) Number of girls to the total number of students.
    (b) Number of boys to the number of girls.

(c) Number of boys to the total number of students.

11. Out of 1800 students in a school, 750 opted basketball, 800 opted cricket and remaining opted table tennis. If a student can opt only one game, find the ratio of
    (a) Number of students who opted basketball to the number of students who opted table tennis.
    (b) Number of students who opted cricket to the number of students opting basketball.
    (c) Number of students who opted basketball to the total number of students.
12. Cost of a dozen pens is ₹ 180 and cost of 8 ball pens is ₹ 56. Find the ratio of the cost of a pen to the cost of a ball pen.
13. Consider the statement: Ratio of breadth and length of a hall is 2 : 5. Complete the following table that shows some possible breadths and lengths of the hall.
14. Divide 20 pens between Sheela and Sangeeta in the ratio of 3 : 2.

Breadth of the hall (in metres)	10		40
Length of the hall (in metres)	25	50	

15. Mother wants to divide ₹ 36 between her daughters Shreya and Bhoomika in the ratio of their ages. If age of Shreya is 15 years and age of Bhoomika is 12 years, find how much Shreya and Bhoomika will get.

16. Present age of father is 42 years and that of his son is 14 years. Find the ratio of
    (a) Present age of father to the present age of son.
    (b) Age of the father to the age of son, when son was 12 years old.
    (c) Age of father after 10 years to the age of son after 10 years.
    (d) Age of father to the age of son when father was 30 years old.

12.3 Proportion

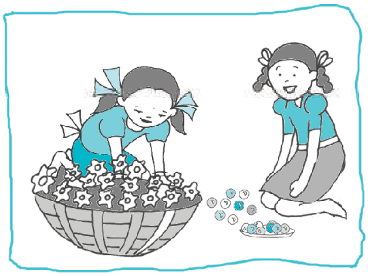
Consider this situation :

Raju went to the market to purchase tomatoes. One shopkeeper tells him that the cost of tomatoes is ₹ 40 for 5 kg. Another shopkeeper gives the cost as 6 kg for ₹ 42. Now, what should Raju do? Should he purchase tomatoes from the first shopkeeper or from the second? Will the comparison by taking the difference help him decide? No. Why not?

Think of some way to help him. Discuss with your friends.

Consider another example.

Bhavika has 28 marbles and Vini has 180 flowers. They want to share these among themselves. Bhavika gave 14 marbles to Vini and Vini gave 90 flowers to Bhavika. But Vini was not satisfied. She felt that she had given more flowers to Bhavika than the marbles given by Bhavika to her.



What do you think? Is Vini correct?

To solve this problem both went to Vini's mother Pooja.

Pooja explained that out of 28 marbles, Bhavika gave 14 marbles to Vini.

Therefore, ratio is 14 : 28 = 1 : 2.

And out of 180 flowers, Vini had given 90 flowers to Bhavika.

Therefore, ratio is 90 : 180 = 1 : 2.

Since both the ratios are the same, so the distribution is fair.

Two friends Ashma and Pankhuri went to market to purchase hair clips. They purchased 20 hair clips for ₹ 30. Ashma gave ₹ 12 and Pankhuri gave ₹ 18. After they came back home, Ashma asked Pankhuri to give 10 hair clips to her. But Pankhuri said, "since I have given more money so I should get more clips. You should get 8 hair clips and I should get 12".

Can you tell who is correct, Ashma or Pankhuri? Why?

Ratio of money given by Ashma to the money given by Pankhuri

= ₹ 12 : ₹ 18 = 2 : 3

According to Ashma's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 10 : 10 = 1 : 1

According to Pankhuri's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 8 : 12 = 2 : 3

Now, notice that according to Ashma's distribution, ratio of hair clips and the ratio of money given by them is not the same. But according to the Pankhuri's distribution the two ratios are the same.

Hence, we can say that Pankhuri's distribution is correct.

Sharing a ratio means something!

Consider the following examples :

Raj purchased 3 pens for ₹ 15 and Anu purchased 10 pens for ₹ 50. Whose pens are more expensive?

Ratio of number of pens purchased by Raj to the number of pens purchased by Anu = 3 : 10.

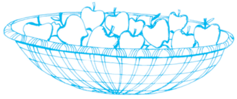
Ratio of their costs = 15 : 50 = 3 : 10

Both the ratios 3 : 10 and 15 : 50 are equal. Therefore, the pens were purchased for the same price by both.

- Rahim sells 2 kg of apples for ₹ 180 and Roshan sells 4 kg of apples for ₹ 360. Whose apples are more expensive?

  Ratio of the weight of apples = 2 kg : 4 kg = 1 : 2

  Ratio of their cost = ₹ 180 : ₹ 360 = 6 : 12 = 1 : 2



So, the ratio of weight of apples = ratio of their cost.

Since both the ratios are equal, hence, we say that they are in proportion. They are selling apples at the same rate.

If two ratios are equal, we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

For the first example, we can say 3, 10, 15 and 50 are in proportion which is written as 3 : 10 :: 15 : 50 and is read as 3 is to 10 as 15 is to 50 or it is written as 3 : 10 = 15 : 50.

For the second example, we can say 2, 4, 180 and 360 are in proportion which is written as 2 : 4 :: 180 : 360 and is read as 2 is to 4 as 180 is to 360.

Let us consider another example.

A man travels 35 km in 2 hours. With the same speed would he be able to travel 70 km in 4 hours?

Now, ratio of the two distances travelled by the man is 35 to 70 = 1 : 2 and the ratio of the time taken to cover these distances is 2 to 4 = 1 : 2 .

Hence, the two ratios are equal i.e. 35 : 70 = 2 : 4.

Therefore, we can say that the four numbers 35, 70, 2 and 4 are in proportion.

Hence, we can write it as 35 : 70 :: 2 : 4 and read it as 35 is to 70 as 2 is to 4. Hence, he can travel 70 km in 4 hours with that speed.

Now, consider this example.

Cost of 2 kg of apples is ₹ 180 and a 5 kg watermelon costs ₹ 45.

Now, ratio of the weight of apples to the weight of watermelon is 2 : 5.

And ratio of the cost of apples to the cost of the watermelon is 180 : 45 = 4 : 1.

Here, the two ratios 2 : 5 and 180 : 45 are not equal, i.e. 2 : 5 ≠ 180 : 45

Therefore, the four quantities 2, 5, 180 and 45 are not in proportion.

Try These

Check whether the given ratios are equal, i.e. they are in proportion.

If yes, then write them in the proper form.

1. 1 : 5 and 3 : 15
2. 2 : 9 and 18 : 81
3. 15 : 45 and 5 : 25
4. 4 : 12 and 9 : 27
5. ₹ 10 to ₹ 15 and 4 to 6

If two ratios are not equal, then we say that they are not in proportion. In a statement of proportion, the four quantities involved when taken in order are known as respective terms. First and fourth terms are known as extreme terms. Second and third terms are known as middle terms.

For example, in 35 : 70 : : 2 : 4;

35, 70, 2, 4 are the four terms. 35 and 4 are the extreme terms. 70 and 2 are the middle terms.

Example 8 : Are the ratios 25g : 30g and 40 kg : 48 kg in proportion?

Solution : 25 g : 30 g = $\frac{25}{30}$ = 5 : 6

40 kg : 48 kg = $\frac{40}{48}$ = 5 : 6   So, 25 : 30 = 40 : 48.

Therefore, the ratios 25 g : 30 g and 40 kg : 48 kg are in proportion, i.e. 25 : 30 :: 40 : 48

The middle terms in this are 30, 40 and the extreme terms are 25, 48.

Example 9 : Are 30, 40, 45 and 60 in proportion?

Solution : Ratio of 30 to 40 = $\frac{30}{40}$ = 3 : 4.

Ratio of 45 to 60 = $\frac{45}{60}$ = 3 : 4.

Since, 30 : 40 = 45 : 60.

Therefore, 30, 40, 45, 60 are in proportion.

Example 10 : Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Solution : Ratio of 15 cm to 2 m = 15 : 2 × 100 (1 m = 100 cm)

= 3 : 40

Ratio of 10 sec to 3 min = 10 : 3 × 60 (1 min = 60 sec)

= 1 : 18

Since, 3 : 40   1 : 18, therefore, the given ratios do not form a proportion.

EXERCISE 12.2

1. Determine if the following are in proportion.

(a) 15, 45, 40, 120 (b) 33, 121, 9,96 (c) 24, 28, 36, 48

(d) 32, 48, 70, 210 (e) 4, 6, 8, 12 (f) 33, 44, 75, 100

2. Write True (T) or False (F) against each of the following statements :

(a) 16 : 24 :: 20 : 30 (b) 21: 6 :: 35 : 10 (c) 12 : 18 :: 28 : 12

(d) 8 : 9 :: 24 : 27 (e) 5.2 : 3.9 :: 3 : 4 (f) 0.9 : 0.36 :: 10 : 4

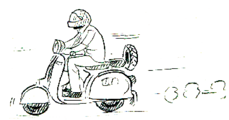
3. Are the following statements true?
   (a) 40 persons : 200 persons = ₹ 15 : ₹ 75
   (b) 7.5 litres : 15 litres = 5 kg : 10 kg
   (c) 99 kg : 45 kg = ₹ 44 : ₹ 20
   (d) 32 m : 64 m = 6 sec : 12 sec
   (e) 45 km : 60 km = 12 hours : 15 hours
4. Determine if the following ratios form a proportion. Also, write the middle terms and extreme terms where the ratios form a proportion.
   (a) 25 cm : 1 m and ₹ 40 : ₹ 160 (b) 39 litres : 65 litres and 6 bottles : 10 bottles
   (c) 2 kg : 80 kg and 25 g : 625 g (d) 200 mL : 2.5 litre and ₹ 4 : ₹ 50

12.4 Unitary Method

Consider the following situations:

- Two friends Reshma and Seema went to market to purchase notebooks. Reshma purchased 2 notebooks for ₹ 24. What is the price of one notebook?
- A scooter requires 2 litres of petrol to cover 80 km. How many litres of petrol is required to cover 1 km?

These are examples of the kind of situations that we face in our daily life. How would you solve these?



Reconsider the first example: Cost of 2 notebooks is ₹ 24.

Therefore, cost of 1 notebook = ₹ 24 ÷ 2 = ₹ 12.

Now, if you were asked to find cost of 5 such notebooks. It would be = ₹ 12 × 5 = ₹ 60

Reconsider the second example: We want to know how many litres are needed to travel 1 km.

For 80 km, petrol needed = 2 litres.

Therefore, to travel 1 km, petrol needed = $\frac{2}{80} = \frac{1}{40}$ litres.

Now, if you are asked to find how many litres of petrol are required to cover 120 km?

Then petrol needed = $\frac{1}{40} \times 120$ litres = 3 litres.

The method in which first we find the value of one unit and then the value of required number of units is known as Unitary Method.

Try These

1. Prepare five similar problems and ask your friends to solve them.
2. Read the table and fill in the boxes.

Time	Distance travelled by Karan	Distance travelled by Kriti
2 hours	8 km	6 km
1 hour	4 km	☐
4 hours	☐	☐

We see that,
Distance travelled by Karan in 2 hours = 8 km

Distance travelled by Karan in 1 hour = $\frac{8}{2}$ km = 4 km

Therefore, distance travelled by Karan in 4 hours = 4 × 4 = 16 km

Similarly, to find the distance travelled by Kriti in 4 hours, first find the distance travelled by her in 1 hour.

Example 11 : If the cost of 6 cans of juice is ₹ 210, then what will be the cost of 4 cans of juice?

Solution : Cost of 6 cans of juice = ₹ 210

Therefore, cost of one can of juice = $\frac{210}{6}$ = ₹ 35

Therefore, cost of 4 cans of juice = ₹ 35 × 4 = ₹ 140.

Thus, cost of 4 cans of juice is ₹ 140.

Example 12 : A motorbike travels 220 km in 5 litres of petrol. How much distance will it cover in 1.5 litres of petrol?

Solution : In 5 litres of petrol, motorbike can travel 220 km.

Therefore, in 1 litre of petrol, motor bike travels = $\frac{220}{5}$ km

Therefore, in 1.5 litres, motorbike travels = $\frac{220}{5} \times 1.5$ km

$= \frac{220}{5} \times \frac{15}{10}$ km = 66 km.

Thus, the motorbike can travel 66 km in 1.5 litres of petrol.

Example 13 : If the cost of a dozen soaps is ₹ 153.60, what will be the cost of 15 such soaps?

Solution : We know that 1 dozen = 12

Since, cost of 12 soaps = ₹ 153.60

Therefore, cost of 1 soap = $\frac{153.60}{12}$ = ₹ 12.80

Therefore, cost of 15 soaps = ₹ 12.80 × 15 = ₹ 192

Thus, cost of 15 soaps is ₹ 192.

Example 14 : Cost of 105 envelopes is ₹ 350. How many envelopes can be purchased for ₹ 100?

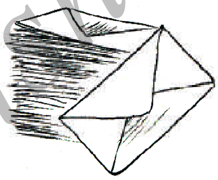
Solution : In ₹ 350, the number of envelopes that can be purchased = 105

Therefore, in ₹ 1, number of envelopes that can be purchased = $\frac{105}{350}$

Therefore, in ₹ 100, the number of envelopes that can be

purchased $= \frac{105}{350} \times 100 = 30$

Thus, 30 envelopes can be purchased for ₹ 100.



Example 15 : A car travels 90 km in $2\frac{1}{2}$ hours.

(a) How much time is required to cover 30 km with the same speed?

(b) Find the distance covered in 2 hours with the same speed.

Solution : (a) In this case, time is unknown and distance is known. Therefore, we proceed as follows :

$2\frac{1}{2}$ hours = $\frac{5}{2}$ hours = $\frac{5}{2} \times 60$ minutes = 150 minutes.

90 km is covered in 150 minutes

Therefore, 1 km can be covered in $\frac{150}{90}$ minutes

Therefore, 30 km can be covered in $\frac{150}{90} \times 30$ minutes i.e. 50 minutes

Thus, 30 km can be covered in 50 minutes.

(b) In this case, distance is unknown and time is known. Therefore, we proceed as follows :

Distance covered in $2\frac{1}{2}$ hours (i.e. $\frac{5}{2}$ hours) = 90 km

Therefore, distance covered in 1 hour = $90 \div \frac{5}{2}$ km = $90 \times \frac{2}{5}$ = 36 km

Therefore, distance covered in 2 hours = 36 × 2 = 72 km.

Thus, in 2 hours, distance covered is 72 km.

EXERCISE 12.3

1. If the cost of 7 m of cloth is ₹ 1470, find the cost of 5 m of cloth.
2. Ekta earns ₹ 3000 in 10 days. How much will she earn in 30 days?
3. If it has rained 276 mm in the last 3 days, how many cm of rain will fall in one full week (7 days)? Assume that the rain continues to fall at the same rate.
4. Cost of 5 kg of wheat is ₹ 91.50.
   (a) What will be the cost of 8 kg of wheat?
   (b) What quantity of wheat can be purchased in ₹ 183?
5. The temperature dropped 15 degree celsius in the last 30 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next ten days?
6. Shaina pays ₹ 15000 as rent for 3 months. How much does she has to pay for a whole year, if the rent per month remains same?
7. Cost of 4 dozen bananas is ₹ 180. How many bananas can be purchased for ₹ 90?
8. The weight of 72 books is 9 kg. What is the weight of 40 such books?
9. A truck requires 108 litres of diesel for covering a distance of 594 km. How much diesel will be required by the truck to cover a distance of 1650 km?
10. Raju purchases 10 pens for ₹ 150 and Manish buys 7 pens for ₹ 84. Can you say who got the pens cheaper?
11. Anish made 42 runs in 6 overs and Anup made 63 runs in 7 overs. Who made more runs per over?

What have we discussed?

1. For comparing quantities of the same type, we commonly use the method of taking difference between the quantities.
2. In many situations, a more meaningful comparison between quantities is made by using division, i.e. by seeing how many times one quantity is to the other quantity. This method is known as comparison by ratio.

   For example, Isha's weight is 25 kg and her father's weight is 75 kg. We say that Isha's father's weight and Isha's weight are in the ratio 3 : 1.
3. For comparison by ratio, the two quantities must be in the same unit. If they are not, they should be expressed in the same unit before the ratio is taken.
4. The same ratio may occur in different situations.
5. Note that the ratio 3 : 2 is different from 2 : 3. Thus, the order in which quantities are taken to express their ratio is important.

6. A ratio may be treated as a fraction, thus the ratio 10 : 3 may be treated as $\frac{10}{3}$.

7. Two ratios are equivalent, if the fractions corresponding to them are equivalent. Thus, 3 : 2 is equivalent to 6 : 4 or 12 : 8.

8. A ratio can be expressed in its lowest form. For example, ratio 50 : 15 is treated as $\frac{50}{15}$; in its lowest form $\frac{50}{15} = \frac{10}{3}$. Hence, the lowest form of the ratio 50 : 15 is 10 : 3.

9. Four quantities are said to be in proportion, if the ratio of the first and the second quantities is equal to the ratio of the third and the fourth quantities. Thus, 3, 10, 15, 50 are in proportion, since $\frac{3}{10}$ $\frac{15}{50}$. We indicate the proportion by 3:10 :: 15:50, it is read as 3 is to 10 as 15 is to 50. In the above proportion, 3 and 50 are the extreme terms and 10 and 15 are the middle terms.

10. The order of terms in the proportion is important. 3, 10, 15 and 50 are in proportion, but 3, 10, 50 and 15 are not, since $\frac{3}{10}$ is not equal to $\frac{50}{15}$.

11. The method in which we first find the value of one unit and then the value of the required number of units is known as the unitary method. Suppose the cost of 6 cans is ₹ 210. To find the cost of 4 cans, using the unitary method, we first find the cost of 1 can. It is ₹ $\frac{210}{6}$ or ₹ 35. From this, we find the price of 4 cans as ₹ 35 × 4 or ₹ 140.

Symmetry

Chapter 13

13.1 Introduction

Symmetry is quite a common term used in day to day life. When we see certain figures with evenly balanced proportions, we say, **"They are symmetrical".**

Tajmahal (U.P.)

Thiruvannamalai (Tamil Nadu)

These pictures of architectural marvel are beautiful because of their symmetry.

Suppose we could fold a picture in half such that the left and right halves match exactly then the picture is said to have line symmetry (Fig 13.1). We can see that the two halves are mirror images of each other. If we place a mirror on the fold then the image of one side of the picture will fall exactly on the other side of the picture. When it happens, the fold, which is the mirror line, is a line of symmetry (or an axis of symmetry) for the picture.

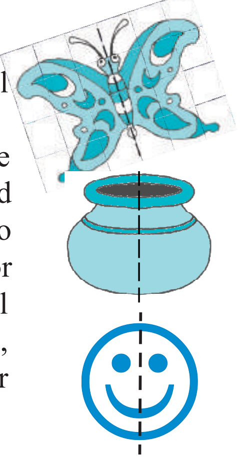


Fig 13.1

The shapes you see here are symmetrical. Why?

When you fold them along the dotted line, one half of the drawing would fit exactly over the other half.

How do you name the dotted line in the figure 13.1?

Where will you place the mirror for having the image exactly over the other half of the picture?

The adjacent figure 13.2 is not symmetrical.

Can you tell 'why not'?

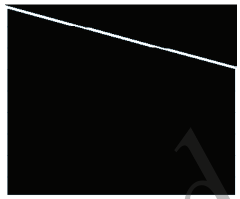


Fig 13.2

13.2 Making Symmetric Figures : Ink-blot Devils

Do This

Take a piece of paper. Fold it in half.

Spill a few drops of ink on one half side.

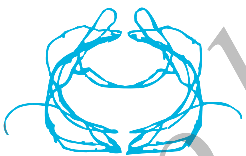
Now press the halves together.

What do you see?

Is the resulting figure symmetric? If yes, where is the line of symmetry? Is there any other line along which it can be folded to produce two identical parts?

Try more such patterns.

Inked-string patterns



Fold a paper in half. On one half-portion, arrange short lengths of string dipped in a variety of coloured inks or paints. Now press the two halves. Study the figure you obtain. Is it symmetric? In how many ways can it be folded to produce two identical halves?

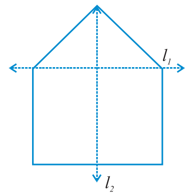
Try These

You have two set-squares in your 'mathematical instruments box'. Are they symmetric?

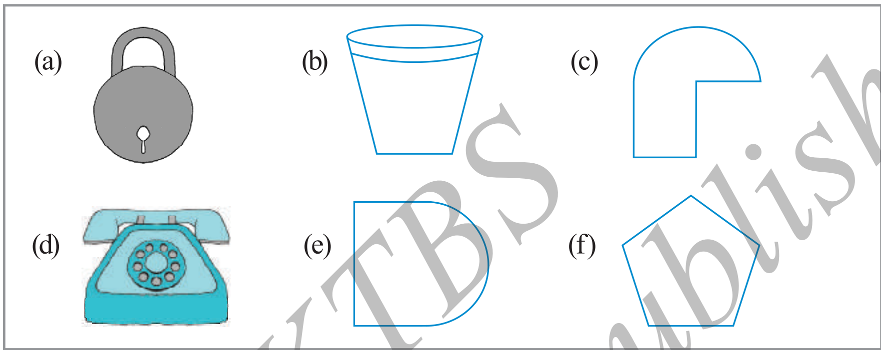
List a few objects you find in your class room such as the black board, the table, the wall, the textbook, etc. Which of them are symmetric and which are not? Can you identify the lines of symmetry for those objects which are symmetric?

EXERCISE 13.1

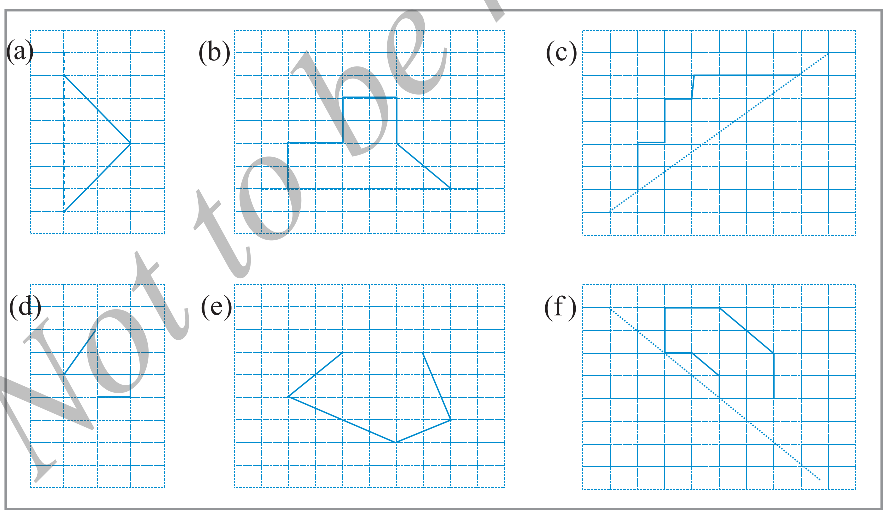
1. List any four symmetrical objects from your home or school.
2. For the given figure, which one is the mirror line, l_1 or l_2?



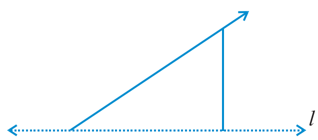
3. Identify the shapes given below. Check whether they are symmetric or not. Draw the line of symmetry as well.



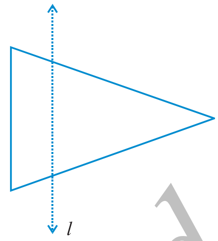
4. Copy the following on a squared paper. A square paper is what you would have used in your arithmetic notebook in earlier classes. Then complete them such that the dotted line is the line of symmetry.



5. In the figure, l is the line of symmetry.
Complete the diagram to make it symmetric.



6. In the figure, l is the line of symmetry.
Draw the image of the triangle and complete the diagram so that it becomes symmetric.



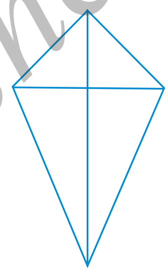
13.3 Figures with Two Lines of Symmetry

Do This

A kite

One of the two set-squares in your instrument box has angles of measure 30°, 60°, 90°.

Take two such identical set-squares. Place them side by side to form a 'kite', like the one shown here.

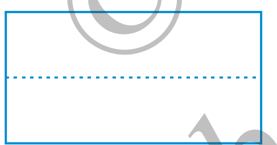


How many lines of symmetry does the shape have?

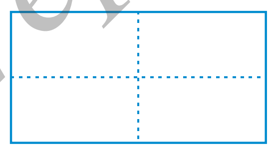
Do you think that some shapes may have more than one line of symmetry?

A rectangle

Take a rectangular sheet (like a post-card). Fold it once lengthwise so that one half fits exactly over the other half. Is this fold a line of symmetry? Why?



1st fold



2nd fold

Open it up now and again fold on its width in the same way. Is this second fold also a line of symmetry? Why?

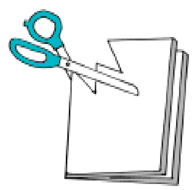
Do you find that these two lines are the lines of symmetry?

Try These

Form as many shapes as you can by combining two or more set squares. Draw them on squared paper and note their lines of symmetry.

A cut out from double fold

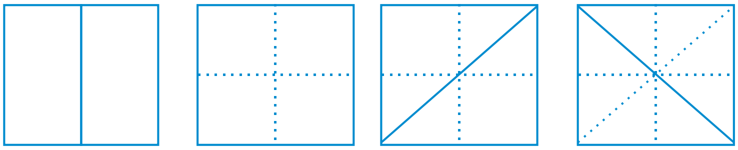
Take a rectangular piece of paper. Fold it once and then once more. Draw some design as shown. Cut the shape drawn and unfold the shape. (Before unfolding, try to guess the shape you are likely to get).



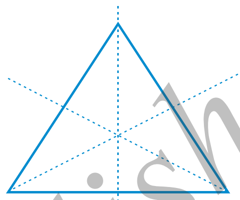
How many lines of symmetry does the shape have which has been cut out?

Create more such designs.

13.4 Figures with Multiple (more than two) Lines of Symmetry



Take a square piece of paper. Fold it into half vertically, fold it again into half horizontally. (i.e. you have folded it twice). Now open out the folds and again fold the square into half (for a third time now), but this time along a diagonal, as shown in the figure. Again open it and fold it into half (for the fourth time), but this time along the other diagonal, as shown in the figure. Open out the fold.



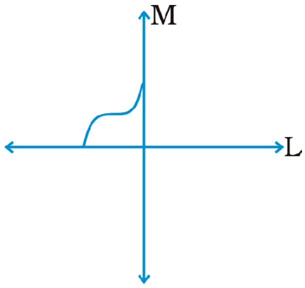
3 lines of symmetry for an equilateral triangle

How many lines of symmetry does the shape have?

We can also learn to construct figures with two lines of symmetry starting from a small part as you did in Exercise 13.1, question 4, for figures with one line of symmetry.

1. Let us have a figure as shown alongside.

2. We want to complete it so that we get a figure with two lines of symmetry. Let the two lines of symmetry be L and M.



3. We draw the part as shown to get a figure having line L as a line of symmetry.