Arithmetic Progression

Ex 1.1

Answer 1-i.

The given sequence is 1, 2, 4, 7, 11, ...

Here,

$t_1 = 1$

$t_2 = 2 = t_1 + 1$

$t_3 = 4 = 2 + 2 = t_2 + 2$

$t_4 = 7 = 4 + 3 = t_3 + 3$

$t_5 = 11 = 7 + 4 = t_4 + 4$

Hence, $t_n = t_{n-1} + (n - 1)$

Thus, we have

$t_6 = t_5 + (6 - 1) = 11 + 5 = 16$

$t_7 = t_6 + (7 - 1) = 16 + 6 = 22$

$t_8 = t_7 + (8 - 1) = 22 + 7 = 29$

$t_9 = t_8 + (9 - 1) = 29 + 8 = 37$

Therefore, the required terms of the sequence are 16, 22, 29 and 37.

Answer 1-ii.

The given sequence is 3, 9, 27, 81, ...

Here,

$t_1 = 3 = 3^1$

$t_2 = 9 = 3^2$

$t_3 = 27 = 3^3$

$t_4 = 81 = 3^4$

Hence, $t_n = 3^n$

Thus, we have

$t_5 = 3^5 = 243$

$t_6 = 3^6 = 729$

$t_7 = 3^7 = 2187$

$t_8 = 3^8 = 6561$

Therefore, the required terms of the sequence are 243, 729, 2187 and 6561.

Answer 1-iii.

The given sequence is 1, 3, 7, 15, 31, ...

Here,

$t_1 = 1 = 2 - 1 = 2^1 - 1$

$t_2 = 3 = 4 - 1 = 2^2 - 1$

$t_3 = 7 = 8 - 1 = 2^3 - 1$

Hence, $t_n = 2^n - 1$

Thus, we have

$t_6 = 2^6 - 1 = 64 - 1 = 63$

$t_7 = 2^7 - 1 = 128 - 1 = 127$

$t_8 = 2^8 - 1 = 256 - 1 = 255$

$t_9 = 2^9 - 1 = 512 - 1 = 511$

Therefore, the required terms of the sequence are 63, 127, 255 and 511.

Answer 1-iv.

The given sequence is 192, −96, 48, −24, ...

Here,

$t_1 = 192$

$t_2 = -96 = \frac{192}{-2} = \frac{t_1}{-2}$

$t_3 = 48 = \frac{-96}{-2} = \frac{t_2}{-2}$

Hence, $t_n = \frac{t_{n-1}}{-2}$

Thus, we have

$t_5 = \frac{t_4}{-2} = \frac{-24}{-2} = 12$

$t_6 = \frac{t_5}{-2} = \frac{12}{-2} = -6$

$t_7 = \frac{t_6}{-2} = \frac{-6}{-2} = 3$

$t_8 = \frac{t_7}{-2} = \frac{3}{-2} = -\frac{3}{2}$

Therefore, the required terms of the sequence are 12, −6, 3 and $-\frac{3}{2}$.

Answer 1-v.

The given sequence is 2, 6, 12, 20, 30, ...

Here,

$t_1 = 2$

$t_2 = 6 = 2 + 4 = 2 + (2 \times 2) = t_1 + 2 \times 2$

$t_3 = 12 = 6 + 6 = 6 + (2 \times 3) = t_2 + 2 \times 3$

$t_4 = 20 = 12 + 8 = 12 + (2 \times 4) = t_3 + 2 \times 4$

$t_5 = 30 = 20 + 10 = 20 + (2 \times 5) = t_4 + 2 \times 5$

Hence, $t_n = t_{n-1} + 2n$

Thus, we have

$t_6 = t_5 + 2n = 30 + 2 \times 6 = 30 + 12 = 42$

$t_7 = t_6 + 2n = 42 + 2 \times 7 = 42 + 14 = 56$

$t_8 = t_7 + 2n = 56 + 2 \times 8 = 56 + 16 = 72$

$t_9 = t_8 + 2n = 72 + 2 \times 9 = 72 + 18 = 90$

Therefore, the required terms of the sequence are 42, 56, 72 and 90.

Answer 1-vi.

The given sequence is 0.1, 0.01, 0.001, 0.0001, ...

Here,

$t_1 = 0.1$

$t_2 = 0.01 = \frac{0.1}{10} = \frac{t_1}{10}$

$t_3 = 0.001 = \frac{0.01}{10} = \frac{t_2}{10}$

$t_4 = 0.0001 = \frac{0.001}{10} = \frac{t_3}{10}$

Hence, $t_n = \frac{t_{n-1}}{10}$

Thus, we have

$t_5 = \frac{t_4}{10} = \frac{0.0001}{10} = 0.00001$

$t_6 = \frac{t_5}{10} = \frac{0.00001}{10} = 0.000001$

$t_7 = \frac{t_6}{10} = \frac{0.000001}{10} = 0.0000001$

$t_8 = \frac{t_7}{10} = \frac{0.0000001}{10} = 0.00000001$

Therefore, the required terms of the sequence are 0.00001, 0.000001, 0.0000001 and 0.00000001.

Answer 1-vii.

The given sequence is 2, 5, 8, 11, ...

Here,

$t_1 = 2$

$t_2 = 5 = 2 + 3 = t_1 + 3$

$t_3 = 8 = 5 + 3 = t_2 + 3$

$t_4 = 11 = 8 + 3 = t_3 + 3$

Hence, $t_n = t_{n-1} + 3$

Thus, we have

$t_5 = t_4 + 3 = 11 + 3 = 14$

$t_6 = t_5 + 3 = 14 + 3 = 17$

$t_7 = t_6 + 3 = 17 + 3 = 20$

$t_8 = t_7 + 3 = 20 + 3 = 23$

Therefore, the required terms of the sequence are 14, 17, 20 and 23.

Answer 1-viii.

The given sequence is -25, -23, -21, -19, ...

Here,

$t_1 = -25$

$t_2 = -23 = -25 + 2 = t_1 + 2$

$t_3 = -21 = -23 + 2 = t_2 + 2$

$t_4 = -19 = -21 + 2 = -19$

Hence, $t_n = t_{n-1} + 2$

Thus, we have

$t_5 = t_4 + 2 = -19 + 2 = -17$

$t_6 = t_5 + 2 = -17 + 2 = -15$

$t_7 = t_6 + 2 = -15 + 2 = -13$

$t_8 = t_7 + 2 = -13 + 2 = -11$

Therefore, the required terms of the sequence are -17,

-15, -13, and -11.

Answer 1-ix.

The given sequence is 2, 4, 8, 16, ...

Here,

$t_1 = 2$

$t_2 = 4 = 2 \times 2 = t_1 \times 2$

$t_3 = 8 = 4 \times 2 = t_2 \times 2$

$t_4 = 16 = 8 \times 2 = t_3 \times 2$

Hence, $t_n = 2t_{n-1}$

Thus, we have

$t_5 = 2t_{5-1} = 2t_4 = 2 \times 16 = 32$

$t_6 = 2t_{6-1} = 2t_5 = 2 \times 32 = 64$

$t_7 = 2t_{7-1} = 2t_6 = 2 \times 64 = 128$

$t_8 = 2t_{8-1} = 2t_7 = 2 \times 128 = 256$

Therefore, the required terms of the sequence are 32, 64, 128 and 256.

Answer 1-x.

The given sequence is $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots$

Here,

$t_1 = \frac{1}{2}$

$t_2 = \frac{1}{6} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{3} \times t_1$

$t_3 = \frac{1}{18} = \frac{1}{3} \times \frac{1}{6} = \frac{1}{3} \times t_2$

$t_4 = \frac{1}{54} = \frac{1}{3} \times \frac{1}{18} = \frac{1}{3} \times t_3$

Hence, $t_n = \frac{1}{3} \times t_{n-1}$

Thus, we have

$t_5 = \frac{1}{3} \times t_4 = \frac{1}{3} \times \frac{1}{54} = \frac{1}{162}$

$t_6 = \frac{1}{3} \times t_5 = \frac{1}{3} \times \frac{1}{162} = \frac{1}{486}$

$t_7 = \frac{1}{3} \times t_6 = \frac{1}{3} \times \frac{1}{486} = \frac{1}{1458}$

$t_8 = \frac{1}{3} \times t_7 = \frac{1}{3} \times \frac{1}{1458} = \frac{1}{4374}$

Therefore, the required terms of the sequence are

$\frac{1}{162}, \frac{1}{486}, \frac{1}{1458}$ and $\frac{1}{4374}$.

Answer 2-i.

$t_n = 4n - 3$

Thus,

$t_1 = (4 \times 1) - 3 = 4 - 3 = 1$

$t_2 = (4 \times 2) - 3 = 8 - 3 = 5$

$t_3 = (4 \times 3) - 3 = 12 - 3 = 9$

$t_4 = (4 \times 4) - 3 = 16 - 3 = 13$

$t_5 = (4 \times 5) - 3 = 20 - 3 = 17$

Hence, the first five terms are 1, 5, 9, 13 and 17.

Answer 2-ii.

$t_n - 2n - 5$

Thus,

$t_1 = (2 \times 1) - 5 = 2 - 5 = -3$

$t_2 = (2 \times 2) - 5 = 4 - 5 = -1$

$t_3 = (2 \times 3) - 5 = 6 - 5 = 1$

$t_4 = (2 \times 4) - 5 = 8 - 5 = 3$

$t_5 = (2 \times 5) - 5 = 10 - 5 = 5$

Hence, the first five terms are -3, -1, 1, 3 and 5.

Answer 2-iii.

$t_n = n + 2$

Thus,

$t_1 = 1 + 2 = 3$

$t_2 = 2 + 2 = 4$

$t_3 = 3 + 2 = 5$ $t_4 = 4 + 2 = 6$

$t_5 = 5 + 2 = 7$

Hence, the first five terms are 3, 4, 5, 6 and 7.

Answer 2-iv.

$t_n = n^2 - 2n$

Thus,

$t_1 = (1)^2 - (2 \times 1) = 1 - 2 = -1$

$t_2 = (2)^2 - (2 \times 2) = 4 - 4 = 0$

$t_3 = (3)^2 - (2 \times 3) = 9 - 6 = 3$

$t_4 = (4)^2 - (2 \times 4) = 16 - 8 = 8$

$t_5 = (5)^2 - (2 \times 5) = 25 - 10 = 15$

Hence, the first five terms are -1, 0, 3, 8 and 15.

Answer 2-v.

$t_n = n^3$

Thus,

$t_1 = 1^3 = 1$

$t_2 = 2^3 = 8$

$t_3 = 3^3 = 27$

$t_4 = 4^3 = 64$ $t_5 = 5^3 = 125$

Hence, the first five terms are 1, 8, 27, 64 and 125.

Answer 2-vi.

$t_n = \frac{1}{n+1}$

Hence,

$t_1 = \frac{1}{1+1} = \frac{1}{2}$

$t_2 = \frac{1}{2+1} = \frac{1}{3}$

$t_3 = \frac{1}{3+1} = \frac{1}{4}$

$t_4 = \frac{1}{4+1} = \frac{1}{5}$

$t_5 = \frac{1}{5+1} = \frac{1}{6}$

Hence, the first five terms are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$.

Answer 3-i.

$S_n = n^2(n + 1)$

$\therefore S_1 = (1)^2(1 + 1) = 1(2) = 2 = t_1$

$S_2 = (2)^2(2 + 1) = 4(3) = 12$

$S_3 = (3)^2(3 + 1) = 9(4) = 36$

Now, $t_n = S_n - S_{n-1}$

$\therefore t_2 = S_2 - S_1 = 12 - 2 = 10$

$t_3 = S_3 - S_2 = 36 - 12 = 24$

Hence, the first three terms are 2, 10 and 24.

Answer 3-ii.

$$S_n = \frac{n^2(n+1)^2}{4}$$

$$\therefore S_1 = \frac{(1)^2(1+1)^2}{4} = \frac{(1)(2)^2}{4} = \frac{4}{4} = 1 = t_1$$

$$S_2 = \frac{(2)^2(2+1)^2}{4} = \frac{4(3)^2}{4} = 9$$

$$S_3 = \frac{(3)^2(3+1)^2}{4} = \frac{9(4)^2}{4} = 9 \times 4 = 36$$

Now, $t_n = S_n - S_{n-1}$

$\therefore t_2 = S_2 - S_1 = 9 - 1 = 8$

$t_3 = S_3 - S_2 = 36 - 9 = 27$

Hence, the first three terms are 1, 8 and 27.

Answer 3-iii.

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

$$\therefore\ S_1 = \frac{1(1+1)(2+1)}{6} = \frac{1 \times 2 \times 3}{6} = \frac{6}{6} = 1 = t_1$$

$$S_2 = \frac{2(2+1)(4+1)}{6} = \frac{2 \times 3 \times 5}{6} = 5$$

$$S_3 = \frac{3(3+1)(6+1)}{6} = \frac{3 \times 4 \times 7}{6} = 2 \times 7 = 14$$

Now, $tn = S_n - S_{n-1}$

$$\therefore\ t_2 = S_2 - S_1 = 5 - 1 = 4$$

$$t_3 = S_3 - S_2 = 14 - 5 = 9$$

Hence, the first three terms are 1, 4 and 9.

Ex. 1.2

Answer 1-i.

1, 3, 6, 10 ....

Here, t_1= 1, t_2= 3, t_3 = 6, t_4 = 10 ....

Now,

$t_2 - t_1 = 3 - 1 = 2$

$t_3 - t_2 = 6 - 3 = 3$

$t_4 - t_3 = 10 - 6 = 4$ and so on

Since the difference between two consecutive terms is not constant, the given sequence is not an A.P.

Answer 1-ii.

3, 5, 7, 9, 11, ...

Here, t_1= 3, t_2= 5, t_3= 7, t_4= 9, t_5 = 11, ...

Now,

$t_2 - t_1 = 5 - 3 = 2$

$t_3 - t_2 = 7 - 5 = 2$

$t_4 - t_3 = 9 - 7 = 2$

$t_5 - t_4 = 11 - 9 = 2$ and so on

Since the difference between any two consecutive terms is constant, the given sequence is an A.P.

Answer 1-iii.

1, 4, 7, 10, ...

Here, t_1= 1, t_2= 4, t_3= 7, t_4= 10, ...

Now,

$t_2 - t_1 = 4 - 1 = 3$

$t_3 - t_2 = 7 - 4 = 3$ $t_4 - t_3 = 10 - 7 = 3$ and so on

Since the difference between any two consecutive terms is constant, the given sequence is an A.P.

Answer 1-iv.

3, 6, 12, 24, ...

Here, t_1 = 3, t_2= 6, t_3 = 12, t_4 = 24, ...

Now,

$t_2 - t_1 = 6 - 3 = 3$

$t_3 - t_2 = 12 - 6 = 6$

$t_4 - t_3 = 24 - 12 = 12$ and so on

Since the difference between any two consecutive terms is not constant, the given sequence is not an A.P.

Answer 1-v.

22, 26, 28, 31, ...

Here, $t_1 = 22, t_2 = 26, t_3 = 28, t_4 = 31, \ldots$

Now,

$t_2 - t_1 = 26 - 22 = 4$

$t_3 - t_2 = 28 - 26 = 2$

$t_4 - t_3 = 31 - 28 = 3$ and so on

Since the difference between any two consecutive terms is not constant, the given sequence is not an A.P.

Answer 1-vi.

0.5, 2, 3.5, 5, ...

Here, $t_1 = 0.5, t_2 = 2, t_3 = 3.5, t_4 = 5$

Now,

$t_2 - t_1 = 2 - 0.5 = 1.5$

$t_3 - t_2 = 3.5 - 2 = 1.5$

$t_4 - t_3 = 5 - 3.5 = 1.5$ and so on

Since the difference between any two consecutive terms is constant, the given sequence is an A.P.

Answer 1-vii.

4, 3, 2, 1, ...

Here, $t_1 = 4, t_2 = 3, t_3 = 2, t_4 = 1,$

Now,

$t_2 - t_1 = 3 - 4 = -1$

$t_3 - t_2 = 2 - 3 = -1$

$t_4 - t_3 = 1 - 2 = -1$ and so on

Since the difference between any two consecutive terms is constant, the given sequence is an A.P.

Answer 1-viii.

-10, -13, -16, -19, ...

Here, $t_1 = -10, t_2 = -13, t_3 = -16, t_4 = -19$

Now,

$t_2 - t_1 = -13 - (-10) = -13 + 10 = -3$

$t_3 - t_2 = -16 - (-13) = -16 + 13 = -3$

$t_4 - t_3 = -19 - (-16) = -19 + 16 = -3$ and so on

Since the difference between any two consecutive terms is constant, the given sequence is an A.P.

Answer 2-i.

$a = 2, d = 2.5$

First term = $a = t_1 = 2$

$t_2 = t_1 + d = 2 + 2.5 = 4.5$ $t_3 = t_2 + d = 4.5 + 2.5 = 7$ $t_4 = t_3 + d = 7 + 2.5 = 9.5$ $t_5 = t_4 + d = 9.5 + 2.5 = 12$

Thus, the first five terms are 2, 4.5, 7, 9.5 and 12.

Answer 2-ii.

$a = 10, d = -3$

First term = $a = t_1 = 10$

$t_2 = t_1 + d = 10 + (-3) = 10 - 3 = 7$

$t_3 = t_2 + d = 7 + (-3) = 7 - 3 = 4$

$t_4 = t_3 + d = 4 + (-3) = 4 - 3 = 1$

$t_5 = t_4 + d = 1 + (-3) = 1 - 3 = -2$

Thus, the first five terms are 10, 7, 4, 1 and -2.

Answer 2-iii.

$a = 4, d = 0$

First term = $a = t_1 = 4$

Here, d = 0, i.e. difference between any two consecutive term is 0.

Hence, all the terms in the sequence are the same as the first term.

Thus, the first five terms are 4, 4, 4, 4 and 4.

Answer 2-vi.

$a = 5, d = 2$

First term = $a = t_1 = 5$

$t_2 = t_1 + d = 5 + 2 = 7$

$t_3 = t_2 + d = 7 + 2 = 9$ $t_4 = t_3 + d = 9 + 2 = 11$ $t_5 = t_4 + d = 11 + 2 = 13$

Thus, the first five terms are 5, 7, 9, 11 and 13.

Answer 2-v.

$a = 3, d = 4$

First term $= a = t_1 = 3$

$t_1 = a = 3$

$t_2 = t_1 + d = 3 + 4 = 7$

$t_3 = t_2 + d = 7 + 4 = 11$

$t_4 = t_3 + d = 11 + 4 = 15$

$t_5 = t_4 + d = 15 + 4 = 19$

Thus, the first five terms are 3, 7, 11, 15 and 19.

Answer 2-vi.

$a = 6, d = 6$

First term $= a = t_1 = 6$

$t_2 = t_1 + d = 6 + 6 = 12$ $t_3 = t_2 + d = 12 + 6 = 18$ $t_4 = t_3 + d = 18 + 6 = 24$ $t_5 = t_4 + d = 24 + 6 = 30$

Thus, the first five terms are 6, 12, 18, 24 and 30.

Ex. 1.3

Answer 1.

Given sequence is 12, 16, 20, 24, ...
Here,
$a = 12$
$d = 16 - 12 = 4$

Now, $t_n = a + (n - 1)d$

Thus, for $n = 25$, $d = 4$, $a = 12$, we have

$t_{25} = 12 + (25 - 1)4$

$= 12 + 24 \times 4$

$= 12 + 96$

$= 108$

Thus, the twenty-fifth term of the given sequence is 108.

Answer 2.

Given sequence is 1, 7, 13, 19, ...
Here,
$a = 1$
$d = 7 - 1 = 6$
Now, $t_n = a + (n - 1)d$
Thus, for $n = 18$, $d = 6$, $a = 1$, we have
$t_{18} = 1 + (18 - 1)6$
$= 1 + 17 \times 6$
$= 1 + 102$
$= 103$
Thus, the eighteenth term of the given sequence is 103.

Answer 3.

Let 'a' be the first term and 'd' be the common difference of the A.P.

Now, $t_n = a + (n - 1)d$

Thus, for $t_3 = 22$, $n = 3$, we have

$22 = a + (3 - 1)d$

$\therefore 22 = a + 2d$

i.e. $a + 2d = 22$ ...(1)

For $t_{17} = -20$, $n = 17$, we have

$-20 = a + (17 - 1)d$

$\therefore -20 = a + 16d$

i.e. $a + 16d = -20$ ...(2)

Subtracting equation (2) from (1), we get

$-14d = 42$

$\therefore d = -3$

Substituting $d = -3$ in equation (1), we get

$a + 2(-3) = 22 \therefore a - 6 = 22$

$\therefore a = 22 + 6$

$\therefore a = 28$

Now,

$t_n = a + (n - 1)d$

$= 28 + (n - 1)(-3)$

$= 28 - 3n + 3$

$= 31 - 3n$

$\therefore t_n = -3n + 31$

Answer 4.

$t_n = a + (n - 1)$

For $t_4 = 12$, $n = 4$, $d = -10$, we have

$t_4 = a + (4 - 1)(-10)$

$\therefore 12 = a + 3(-10)$

$\therefore 12 = a - 30$

$\therefore a - 30 = 12$

$\therefore a = 42$

Now,

$t_n = a + (n - 1)d$

$= 42 + (n - 1)(-10)$

$= 42 - 10n + 10$

$= 52 - 10n$

$\therefore t_n = -10n + 52$

Answer 5.

Given sequence is -5, 2, 9, 16, 23, 30, ...

Here,

$t_1 = -5$

$t_2 = 2$

$\therefore t_2 - t_1 = 2 - (-5) = 2 + 5 = 7$

$t_3 = 9$

$\therefore t_3 - t_2 = 9 - 2 = 7$

$t_4 = 16$

$\therefore t_4 - t_3 = 16 - 9 = 7$

$t_5 = 23$

$\therefore t_5 - t_4 = 23 - 16 = 7$

$t_6 = 30$

$\therefore t_6 - t_5 = 30 - 23 = 7$

Since, the difference between any two consecutive terms of this sequence is constant, i.e. d = 7, the given sequence is an A.P.

Thus, in the given sequence, a = -5 and d = 7

$\therefore$ General term,
$t_n = a + (n - 1)d$
$= -5 + (n - 1)7$
$= -5 + 7n - 7$
$\therefore t_n = 7n - 12$

Thus, the given sequence is an A.P. with general term 7n - 12.

Answer 6.

Given sequence is 5, 2, -2, -6, -11, ...
Here, $t_1 = 5$
$t_2 = 2$
$\therefore t_2 - t_1 = 2 - 5 = -3$
$t_3 = -2$
$\therefore t_3 - t_2 = -2 - 2 = -4$
$t_4 = -6$
$\therefore t_4 - t_3 = -6 - (-2) = -6 + 2 = -4$
$t_5 = -11$
$\therefore t_5 - t_4 = -11 - (-6) = -11 + 6 = -5$
Since the difference between any two consecutive terms is not constant, the given sequence is not an A.P.

Answer 7.

The smallest three - digit number divisible by 4 is 100.
And, the biggest three - digit number divisible by 4 is 996.
This is an A.P. with first term, $a = 100$ and common difference, $d = 4$
Then, $t_n = 996$

$t_n = a + (n-1)d$

$\therefore 996 = 100 + (n-1)4$

$\therefore 996 - 100 = (n-1)4$

$\therefore (n-1)4 = 896$

$\therefore n - 1 = \frac{896}{4}$

$\therefore n - 1 = 224$

$\therefore n = 224 + 1 = 225$

Thus, there are 225 three - digit natural numbers which are divisible by 4.

Answer 8.

(i) Let 'a' be the first term and 'd' the common difference of the given A.P.

For $t_{11} = 16$, $n = 11$, we have

$t_n = a + (n-1)d$

$\therefore t_{11} = a + (11-1)d$

$\therefore 16 = a + 10d \quad \ldots (1)$

For $t_{21} = 29$, $n = 21$, we have

$t_{21} = a + (21-1)d$

$\therefore 29 = a + 20d \quad \ldots (2)$

Subtracting (1) from (2), we get

$13 = 10d$

$\therefore d = 1.3$

Substituting $d = 1.3$ in (1), we get

$16 = a + 10(1.3)$

$\therefore 16 = a + 13$

$\therefore a = 16 - 13$

$\therefore a = 3$

Thus, the first term is 3 and the common difference is 1.3.

(ii) For 34th term, $n = 34$, $a = 3$, $d = 1.3$

$$t_n = a + (n-1)d$$

$$\therefore\ t_{34} = 3 + (34-1)(1.3)$$

$$= 3 + 33 \times 1.3$$

$$= 3 + 42.9$$

$$= 45.9$$

Thus, the 34th term is 45.9.

(iii) $t_n = 55$, $a = 3$, $d = 1.3$

$$\therefore\ t_n = a + (n-1)d$$

$$\therefore\ 55 = 3 + (n-1)(1.3)$$

$$\therefore\ 55 - 3 = (n-1)(1.3)$$

$$\therefore\ (n-1)(1.3) = 52$$

$$\therefore\ n - 1 = \frac{52}{1.3}$$

$$\therefore\ n - 1 = 40$$

$$\therefore\ n = 40 + 1$$

$$\therefore\ n = 41$$

Ex. 1.4

Answer 1.

First n natural numbers are 1, 2, 3, ..., n.

Here, $t_1 = a = 1,\ d = 1,\ t_n = n$

Now, $S_n = \frac{n}{2}[t_1 + t_n]$

$= \frac{n}{2}[1 + n]$

$\therefore S_n = \frac{n(n+1)}{2}$

Thus, the sum of the first 'n' natural numbers is $\frac{n(n+1)}{2}$.

The sum of the first 20 natural numbers is given by

$S_{20} = \frac{20(20+1)}{2}$

$= \frac{20 \times 21}{2}$

$= 10 \times 21$

$\therefore S_{20} = 210$

Thus, the sum of the first 20 natural numbers is 210.

Answer 2.

1, 3, 5, ..., 149 are the odd natural numbers from 1 to 150.

Here, $t_1 = a = 1,\ d = 2,\ t_n = 149$

$t_n = a + (n-1)d$

$\therefore 149 = 1 + (n-1)2$

$\therefore 149 - 1 = (n-1)2$

$\therefore 148 = (n-1)2$

$\therefore n - 1 = \frac{148}{2} = 74$

$\therefore n = 74 + 1$

$\therefore n = 75$

Now, $S_n = \frac{n}{2}(t_1 + t_n)$

$\therefore S_{75} = \frac{75}{2}(1 + 149)$

$= \frac{75}{2} \times 150$

$= 75 \times 75$

$\therefore S_{75} = 5625$

Thus, the sum of all odd natural numbers from 1 to 150 is 5625.

Answer 3.

Here, $a = 6,\ d = 3$ and $n = 10$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore S_{10} = \frac{10}{2}\left[2 \times 6 + (10-1) \times 3\right]$$

$$= 5\left[12 + 9 \times 3\right]$$

$$= 5\left[12 + 27\right]$$

$$= 5 \times 39$$

$$\therefore S_{10} = 195$$

Answer 4.

The numbers from 1 to 140 which are divisible by 4 are 4, 8, 12,..., 140.

$\therefore$ First term $= a = t_1 = 4,\ d = 4$ and $t_n = 140$

$$t_n = a + (n-1)d$$

$$\therefore 140 = 4 + (n-1)4$$

$$\therefore 140 - 4 = (n-1)4$$

$$\therefore n - 1 = \frac{136}{4} = 34$$

$$\therefore n = 34 + 1$$

$$\therefore n = 35$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$\therefore S_{35} = \frac{35}{2}(4 + 140)$$

$$= \frac{35}{2} \times 144$$

$$= 35 \times 72$$

$$\therefore S_{35} = 2520$$

Thus, the sum of all numbers from 1 to 140, which are divisible by 4, is 2520.

Answer 5.

Odd natural numbers are 1, 3, 5, ...

$\therefore$ First term $= a = t_1 = 1,\ d = 2$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 \times 1 + (n-1)2\right]$$

$$= \frac{n}{2}\left[2 + (n-1)2\right]$$

$$= \frac{n}{2} \times 2\left[1 + n - 1\right]$$

$$= \frac{n}{2} \times 2n$$

$$\therefore S_n = n^2$$

Now, we have

$1 + 3 + 5 + \dots\dots\dots + 101$

Here, $a = 1$, $d = 2$, $t_n = 101$

$t_n = a + (n - 1)d$

$\therefore 101 = 1 + (n - 1)2$

$\therefore 101 - 1 = (n - 1)2$

$\therefore 100 = 2n - 2$

$\therefore 2n = 102$

$\therefore n = 51$

Now, $S_n = n^2$

$\therefore S_{51} = (51)^2 = 2601$

Thus, the sum of the first n odd natural numbers is n^2 and $S_{51} = 2601$.

Answer 6.

Let the first term be 'a' and the common difference be 'd'.

19^{th} term $= t_{19} = 52$ and 38^{th} term $= t_{38} = 148$ ....(Given)

Now, $t_n = a + (n - 1)d$

$\therefore t_{19} = a + (19 - 1)d$

$\therefore 52 = a + 18d$

i.e. $a + 18d = 52$ ....(1)

And, $t_{38} = a + (38 - 1)d$

$\therefore 148 = a + 37d$

i.e. $a + 37d = 148$ ....(2)

Adding (1) and (2), we get

$2a + 55d = 200$ ....(3)

Now,

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$$

$$\therefore S_{56} = \frac{56}{2}\left[2a + (56 - 1)d\right]$$

$$= 28\left[2a + 55d\right]$$

$$= 28 \times 200 \quad ....[\text{From } (3)]$$

$\therefore S_{56} = 5600$

Thus, the sum of the first 56 terms is 5600.

Answer 7.

Let the first term be 'a' and the common difference 'd'.

$S_{55} = 3300$ ....(Given)

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ ...(Formula)

$\therefore S_{55} = \frac{55}{2}\left[2a + (55-1)d\right]$

$= \frac{55}{2}\left[2a + 54d\right]$

$\therefore S_{55} = 55(a + 27d)$

$\therefore 3300 = 55(a + 27d)$

$\therefore a + 27d = \frac{3300}{55}$

$\therefore a + 27d = 60$ ....(1)

Now, $t_n = a + (n-1)d$

$\therefore t_{28} = a + (28-1)d$

$= a + 27d$

$= 60$ ....[From (1)]

Thus, the 28^{th} term is 60.

Answer 8.

First n even natural numbers are 2, 4, 6, ... , n.

Here, $a = 2$ and $d = 2$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[2 \times 2 + (n-1)2\right]$

$= \frac{n}{2}\left[4 + 2(n-1)\right]$

$= \frac{n}{2} \times 2\left[2 + (n-1)\right]$

$= n\left[2 + (n-1)\right]$

$= n(n+1)$

$\therefore S_n = n(n+1)$

$\therefore S_{20} = 20(20+1)$

$= 20 \times 21$

$\therefore S_{20} = 420$

Thus, the sum of the first n even natural numbers is $n(n+1)$ and the sum of first 20 even natural numbers is 420.

Ex. 1.5

Answer 1.

Let a - 3d, a - d, a + d and a + 3d be the four consecutive terms in the A.P.
The sum of these four terms is 12.
∴ (a - 3d) + (a - d) + (a + d) + (a + 3d) = 12
∴ 4a = 12
∴ a = 3
The sum of the 3rd and the 4th term is 14
∴ (a + d) + (a + 3d) = 14
∴ 2a + 4d = 14
∴ a + 2d = 7
∴ 3 + 2d = 7 (Substituting a = 3)∴ 2d = 7 - 3
∴ 2d = 4
∴ d = 2
Substituting a = 3 and d = 2 in the four consecutive terms, we get a - 3d = 3 - 3(2) = 3 - 6 = -3
a - d = 3 - 2 = 1
a + d = 3 + 2 = 5
a + 3d = 3 + 3(2) = 3 + 6 = 9
Hence, the four consecutive terms are -3, 1, 5 and 9.

Answer 2.

Let a − 3d, a − d, a + d and a + 3d be the four consecutive terms in the A.P.
The sum of the four consecutive terms is − 54.

$\therefore (a-3d)+(a-d)+(a+d)+(a+3d)=-54$

$\therefore 4a=-54$

$\therefore a=\frac{-54}{4}$

$\therefore a=-\frac{27}{2}$ ....(1)

The sum of the 1st and the 3rd term is − 30.

$\therefore (a-3d)+(a+d)=-30$

$\therefore 2a-2d=-30$

$\therefore -27-2d=-30$ ... [From (1)]

$\therefore 2d=3$

$\therefore d=\frac{3}{2}$ .... (2)

Substituting $a = -\frac{27}{2}$ and $d = \frac{3}{2}$ in the four terms, we get

$$a - 3d = -\frac{27}{2} - 3 \times \frac{3}{2} = -\frac{27}{2} - \frac{9}{2} = \frac{-36}{2} = -18$$

$$a - d = -\frac{27}{2} - \frac{3}{2} = -\frac{30}{2} = -15$$

$$a + d = -\frac{27}{2} + \frac{3}{2} = -\frac{24}{2} = -12$$

$$a + 3d = -\frac{27}{2} + 3 \times \frac{3}{2} = -\frac{27}{2} + \frac{9}{2} = -\frac{18}{2} = -9$$

Thus, the four consecutive terms are -18, -15, -12 and -9.

Answer 3.

Let the three consecutive terms in the A.P. be $a - d$, a and $a + d$.

The sum of three consecutive terms is -3.

$$\therefore (a - d) + a + (a + d) = -3$$

$$\therefore 3a = -3$$

$$\therefore a = -1$$

The product of their cubes is 512.

$$\therefore (a - d)^3 \times a^3 \times (a + d)^3 = 512$$

$$\therefore (-1 - d)^3 \times (-1)^3 \times (-1 + d)^3 = 512$$

$$\therefore \left[(-1)(-1 - d)^3\right] \times (-1 + d)^3 = 512$$

$$\therefore (1 + d)^3 (-1 + d)^3 = 8^3$$

Taking cube root of both the sides, we get

$$(1 + d)(-1 + d) = 8$$

$$\therefore -1 + d - d + d^2 = 8$$

$$\therefore d^2 = 9$$

$$\therefore d = \pm 3$$

Taking $a = -1$ and $d = 3$ in three terms, we get

$a - d = -1 - 3 = -4$

$a = -1$

$a + d = -1 + 3 = 2$

Thus, the three terms are $-4, -1$ and 2.

Taking $a = -1$ and $d = -3$ in three terms, we get

$a - d = -1 + 3 = 2$

$a = -1$

$a + d = -1 - 3 = -4$

Thus, the three terms are $2, -1$ and -4.

Hence, the three consecutive terms are $-4, -1$ and 2 OR $2, -1$ and -4.

Answer 4.

The temperatures from Monday to Friday are in A.P.

Hence, Monday to Friday are five consecutive days.

Let the temperatures from Monday to Friday be

a - 2d, a - d, a, a + d and a + 2d

The sum of the temperatures of Monday, Tuesday and Wednesday is zero.

$\therefore (a - 2d) + (a - d) + a = 0$

$\therefore 3a - 3d = 0$

$\therefore a - d = 0$

$\therefore a = d$ ...(1)

The sum of the temperatures of Thursday and Friday is 15.

$\therefore (a + d) + (a + 2d) = 15$

$\therefore 2a + 3d = 15$ ... (2)

Substituting a = d in equation (2), we get

2d + 3d = 15

∴ 5d = 15

∴ d = 3

From equation (1), a = d = 3.

Thus, the terms are

a - 2d = 3 - 2 × 3 = 3 - 6 = -3 a - d = 3 - 3 = 0

a = 3

a + d = 3 + 3 = 6 a + 2d = 3 + 2 × 3 = 3 + 6 = 9

Hence, the temperatures of each of the five days are -3, 0, 3, 6 and 9, respectively.

Ex. 1.6

Answer 1.

Mary's starting salary = Rs. 15000
Hence, the first term of the sequence, a = 15000
Monthly increment in salary = Rs. 100
Hence, the common difference, d = 100 To find the salary after 20 months, we need to find t_{20}
$t_n = a + (n - 1)d$
$\therefore t_{20} = 15000 + (20 - 1)10$
$= 15000 + 19 \times 100$
$= 15000 + 1900$
$\therefore t_{20} = 16900$
Thus, Mary's salary after 20 months will be Rs. 16900/-.

Answer 2.

Fare for the first kilometre = Rs. 14
Hence, a = 14
The increase in fare for each additional kilometre = Rs. 2
Hence, d = 2
To find the fare for 10 kilometres, we need to find t_{10}
$t_n = a + (n - 1)d$
$\therefore t_{10} = 14 + (10 - 1)2$
$= 14 + 9 \times 2$
$= 14 + 18$
$\therefore t_{10} = 32$
Thus, the fare for 10 kilometres will be Rs. 32.

Answer 3.

Initial exercise time = 10 minutes
Hence, a = 10
Increase in exercise time each day = 5 minutes
Hence, d = 5
We need to find n such that $t_n = 45$.
$t_n = a + (n - 1)d$
$\therefore 45 = 10 + (n - 1)5$
$\therefore 45 - 10 = (n - 1)5$
$\therefore 35 = (n - 1)5$
$\therefore n - 1 = \frac{35}{5} = 7$
$\therefore n = 7 + 1$
$\therefore n = 8$
Thus, 8 days are required to reach an exercise time of 45 minutes.

Answer 4.

Number of seats in the first row = 20
Hence, $a = 20$
Increase in the number of seats in consecutive rows = 2
Hence, $d = 2$
To find the number of seats in the 25^{th} row, we need to find t_{25}
$t_n = a + (n - 1)d$
$\therefore t_{25} = 20 + (25 - 1)2$
$= 20 + 24 \times 2$
$= 20 + 48$
$\therefore t_{25} = 68$
Thus, the number of seats in the twenty-fifth row is 68.

Answer 5.

Number of literate people in 2010 = 4000 Hence, $a = 4000$
Increase in the number of literate people per year = 400 Hence, $d = 400$
To find the number of literate people in 2020 i.e., from the year 2010 to the year 2020 (11 years including 2010 and 2020), we need to find t_{10}
$t_n = a + (n - 1)d$

$\therefore t_{10} = 4000 + (11 - 1)400$

$= 4000 + 10 \times 400$

$= 4000 + 4000$

$\therefore t_{10} = 8000$

Hence, the number of literate people in 2020 will be 8000.

To find the formula to know the number of literate people after nyears, we need to find t_{n+1}

$t_{n+1} = a + (n+1 - 1)d$

$= 4000 + nX400$

$= 4000 + 400n$

$\therefore t_n = 400n + 4000$

Ac

Answer 6.

Neela's savings on the first day = Rs. 2

Hence, $a = 2$

Increase in savings each day = Rs. 2

Hence, $d = 2$

Number of days in the month of February 2010 = 28

To find the total savings in 28 days, we need to find S_{28}

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore\ S_{28} = \frac{28}{2}\left[2 \times 2 + (28-1)2\right]$$

$$= 14\left[4 + 27 \times 2\right]$$

$$= 14(4 + 54)$$

$$= 14 \times 58$$

$$\therefore\ S_{28} = 812$$

Thus, Neela's savings in the month of February will be Rs. 812.

Answer 7.

Amount borrowed by Babubhai = Rs. 4000

Amount repaid by Babubhai = Rs.$(4000 + 500)$ = Rs. 4500

Number of instalments = 10

Hence, $n = 10$ and $S_n = S_{10} = 4500$

Each instalment is Rs. 10 less than the preceding one.

Hence, $d = -10$

To find the first and last instalment, we need to find a and t_{10}

Now, $n = 10,\ S_{10} = 4500,\ d = -10$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore\ S_{10} = \frac{10}{2}\left[2a + (10-1)(-10)\right]$$

$$\therefore\ 4500 = 5\left[2a + 9 \times (-10)\right]$$

$$\therefore\ \frac{4500}{5} = 2a - 90$$

$$\therefore\ 900 = 2a - 90$$

$$\therefore\ 2a = 900 + 90 = 990$$

$$\therefore\ a = 495$$

$$t_n = a + (n-1)d$$

$$\therefore\ t_{10} = 495 + (10-1)(-10)$$

$$= 495 - 90$$

$$= 405$$

Thus, the first instalment is Rs.495 and the last instalment is Rs. 405.

Answer 8.

Number of seats in the first row = 20

Hence, $a = 20$

Seats in the 1^{st} row = 20

Seats in the 2^{nd} row = 24

Seats in the 3^{rd} row = 28

Hence, the increase in the number of seats in consecutive rows = 4

Hence, $d = 4$

To find the total number of seats in the meeting hall, we need to find the number of seats in 30 rows i.e. S_{30}

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore S_{30} = \frac{30}{2}\left[2 \times 20 + (30-1)4\right]$$

$$= 15\left[40 + 29 \times 4\right]$$

$$= 15(40 + 116)$$

$$= 15 \times 156$$

$$\therefore S_{30} = 2340$$

Thus, there are 2340 seats in the meeting hall.

Answer 9.

Investment for the 1^{st} year = Rs. 500

Hence, $a = 500$

Investment for the 2^{nd} year = Rs. 700

Investment for the 3^{rd} year = Rs. 900

Hence, the investment increases by Rs. 200 in consecutive years.

Hence, $d = 200$

To find the total investment in 12 years, we need to find S_{12}

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore S_{12} = \frac{12}{2}\left[2 \times 500 + (12-1) \times 200\right]$$

$$= 6\left[1000 + (11 \times 200)\right]$$

$$= 6(1000 + 2200)$$

$$= 6 \times 3200$$

$$\therefore S_{12} = 19200$$

Thus, the total investment in 12 years is Rs. 19200.

Answer 10.

Number of trees in the first row = 1

Hence, a = 1

Increase in the number of trees in consecutive rows = 1

Hence, d = 1

Total number of rows = 25

To find the total number of plants in 25 rows, we need to find S_{25}

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore S_{25} = \frac{25}{2}\left[2 \times 1 + (25 - 1) \times 1\right]$$

$$= \frac{25}{2}(2 + 24 \times 1)$$

$$= \frac{25}{2}(2 + 24)$$

$$= \frac{25}{2} \times 26$$

$$\therefore S_{25} = 325$$

Thus, the total number of plants are 325.