4. Second Degree Equations

Questions Pg-79

1. Question

When each side of a square was reduced by 2 metres, the area became 49 square metres. What was the length of a side of the original square?

Answer

Given area of reduced square = 49 square metres

We know that area of a square = a^2, where a is the side of square.

⇒ √49 = 7 (Side of reduced square)

Given, the original square's side was reduced by 2 metres.

⇒ 7 + 2 = 9 metres

∴ The length of a side of the original square is 9 metres.

2. Question

A square ground has 2 metre wide path all around it. The total area of the ground and path is 1225 square metres. What is the area of the ground alone?

Answer

Given the square ground has 2 metre wide path.

We know that area of a square = a^2, where a is the side of square.

⇒ Area of path = 2^2 = 4 square metres

Given the total area of the ground and path = 1225 square metres

So, the Area of the ground = Total area of ground and path – Area of path

⇒ Area of ground = 1225 – 4 = 1221 square metres

∴ The area of the ground alone = 1221 square metres

3. Question

The square of a term in the arithmetic sequence 2, 5, 8, …, is 2500. What is its position?

Answer

The square of a term is 2500.

⇒ Term = √2500 = 50

Given, arithmetic sequence 2, 5, 8 …

Here first term = 2 = a_1

Common difference = 5 – 2 = 3 = d

We know that the expression on nth term in an arithmetic sequence is $t_n = a_1 + (n – 1) d$ where n is the number of term.

⇒ 50 = 2 + (n – 1) (3)

⇒ 50 = 2 + 3n – 3

⇒ 50 = 3n – 1

⇒ 50 + 1 = 3n

⇒ 51 = 3n

⇒ n = 51/ 3

⇒ n = 17

∴ The position of the term 50 is 17.

4. Question

2000 rupees was deposited in a scheme in which interest is compounded annually. After two years the amount in the account was 2200 rupees. What is the rate of interest?

Answer

We know that when interest is compounded annually,

Amount = P $(1 + \frac{R}{100})^n$

Where P = principal, R = rate of interest and n = time in years

Given Amount = 2200 rupees, P = 2000, R = R and n = 2 years

⇒ 2200 = 2000 $(1 + \frac{R}{100})^2$

⇒ $\frac{2200}{2000} = (1 + \frac{R}{100})^2$

⇒ $(\frac{\sqrt{11}}{\sqrt{10}})^2 = (1 + \frac{R}{100})^2$

⇒ $\frac{\sqrt{11}}{\sqrt{10}} = 1 + \frac{R}{100}$

⇒ $\frac{\sqrt{11}}{\sqrt{10}} - 1 = \frac{R}{100}$

⇒ $\frac{\sqrt{11}-\sqrt{10}}{\sqrt{10}} = \frac{R}{100}$

We know that √11 ≈ 3.316 and √10 ≈ 3.162.

⇒ $\frac{3.316-3.162}{3.162} = \frac{R}{100}$

⇒ $\frac{0.154}{3.162} = \frac{R}{100}$

⇒ 0.04881 = $\frac{R}{100}$

⇒ 100 × 0.04881 = R

⇒ R = 4.881%

∴ The rate of interest is 4.881%.

Questions Pg-84

1. Question

1 Added to the product of two consecutive even numbers gives 289. What are the numbers?

Answer

Let the number be 2x. Therefore, the consecutive even number will be (2x + 2).

Now the product of these two numbers will be 2×(2x + 2)

Therefore, 1 added to the product will be x(x + 2) + 1 which is equal to 289

⇒ 2x (2x + 2) + 1 = 289

⇒ $4x^2 + 4x + 1 = 289$

$\Rightarrow (2x + 1)^2 = 17^2$

$\Rightarrow \sqrt{((2x + 1)^2)} = \sqrt{(17^2)}$

$\Rightarrow (2x + 1) = 17$

$\Rightarrow 2x = 17 - 1$

$\Rightarrow x = \frac{16}{2}$

$\Rightarrow x = 8$

One number is 16 and the other number 18

2. Question

9 added to the product of two consecutive multiples of 6 gives 729. What are the numbers?

Answer

Let the numbers be 6x and (6x + 6)

Product of the two numbers is $6x \times (6x + 6)$

Now adding 9 to the product gives 729

Therefore we can write it as

$\Rightarrow 6x(6x + 6) + 9 = 729$

$\Rightarrow 36x^2 + 36x + 9 = 729$

$\Rightarrow (6x + 3)^2 = 27^2$

$\Rightarrow \sqrt{(6x + 3)^2} = \sqrt{((27)^2}$

$\Rightarrow 6x + 3 = 27$

$\Rightarrow 6x = 27 - 3$

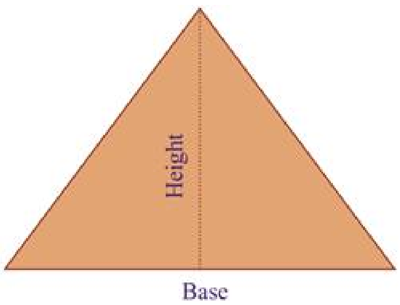
$\Rightarrow x = \frac{24}{6}$

$\Rightarrow x = 4$

therefore the number is 24 and 30

3. Question

An isosceles triangle has to be made like this



The height should be 2 metres less than the base. What should be the length of its sides?

Answer

If height is 2 m less than base

Let Base be x m

Then;

Height is (x–2)

In half of the triangle

⇒ By Pythagoras theorem:-

$Base^2 + Height^2 = Side^2$

As in isosceles triangle

Altitude and median are the same

∴ in half of triangle

Base = $\frac{base}{2} = \frac{x}{2}$

Height = (x–2)

Side is same as base because it is isosceles triangle

Side = x

$Base^2 + Height^2 = Side^2$

$\left(\frac{x}{2}\right)^2 + (x-2)^2 = x^2$

$\frac{x^2}{4} + x^2 + 4–4x = x^2$

$\frac{x^2}{4} + 4–4x = x^2–x^2 = 0$

$\frac{x^2 + 16–16x}{4} = 0$

$x^2 + 16–16x = 0$

As comparing eq to $ax^2 + bx + c = 0$

$x = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{-16+\sqrt{16^2-4\times1\times(-16)}}{2\times1} = \frac{-16+\sqrt{320}}{2} = \frac{-16+8\sqrt{5}}{2}$

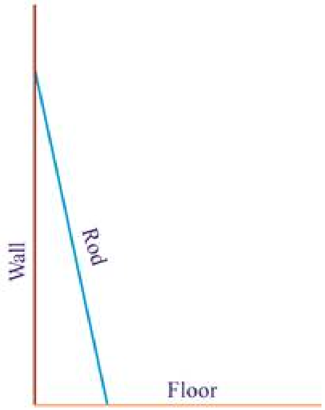
$x = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-16-\sqrt{16^2-4\times1\times(-16)}}{2\times1} = \frac{-16-\sqrt{320}}{2} = \frac{-16-8\sqrt{5}}{2}$

The length cannot be negative

Hence Base is $\frac{-16+8\sqrt{5}}{2}$

4. Question

A 2.6 metres long rod leans against a wall, its foot 1 metre from the wall. When the foot is moved a little away from the wall, its upper end slides the same length down. How much farther is the foot moved?



Answer

Length of rod = 2.6 m

Floor Base = 1 m

Height of wall = ?

Now using Pythagoras Theorem

$(\text{Floor Base})^2 + (\text{Height of wall})^2 = (\text{Length of rod})^2$

$(\text{Height of wall})^2 = (\text{Length of rod})^2 – (\text{Floor Base})^2$

⇒ Height of the wall = $\sqrt{((\text{Length of rod})^2 – (\text{Floor Base})^2)}$

⇒ Height of the wall = $\sqrt{((2.6)^2 + 1^2)}$

using the formula $(a + b)(a - b) = a^2 - b^2$

⇒ Height of the wall = $\sqrt{((2.6 + 1)(2.6 – 1)}$

= $\sqrt{(3.6 \times 1.6)}$

⇒ $\sqrt{5.76}$

= 2.4

now we got the height of the wall but in the question it is given that it is further slided down, therefore we consider that as x.

Length of rod remains same.

Floor base becomes (1 + x)

Height of wall becomes (2.4 – x)

Now using Pythagoras theorem again

⇒ $(\text{floor base})^2 + (\text{height of wall})^2 = (\text{ lengthof rod})^2$

⇒ $(2.4 – x)^2 + (1 + x)^2 = 2.6^2$

⇒ $5.76 + x^2 – 4.8 + 1 + x^2 + 2x = 6.76$

⇒ $6.76 + 2x^2 – 2.8x = 6.76$

⇒ $2x^2 – 2.8x = 6.76 – 6.76$

⇒ $2x^2 – 2.8x = 0$

⇒ $2x(x – 1.4) = 0$

⇒ $(x – 1.4) = 0$

∴ $x = 1.4$

5. Question

16 added to the sum of the first few terms of the arithmetic sequence 9, 11, 13, … gave 256. How many terms were added?

Answer

Using the given information we conclude

$16 + S_n = 256$

Using the formula $S_n = \left(\frac{n}{2}\right)(2a + (n - 1)d)$

$\Rightarrow 16 + \left(\frac{n}{2}\right)(2a + (n-1)d) = 256$

$\Rightarrow 16 + \frac{n}{2}(2 \times 9 + (n-1)2) = 256$

$\Rightarrow 16 + \frac{n}{2}(18 + 2n - 2) = 256$

$\Rightarrow 16 + \frac{n}{2}(2n + 16) = 256$

$\Rightarrow \frac{32 + 2n^2 + 16n}{2} = 256$

$\Rightarrow 2n^2 + 16n + 32 = 256 \times 2$

$\Rightarrow 2n^2 + 16n + 32 = 512$

$\Rightarrow 2(n^2 + 8n + 16) = 512$

$\Rightarrow n^2 + 8n + 16 = \frac{512}{2}$

$\Rightarrow n^2 + 4n + 4n + 16 = 256$

$\Rightarrow n(n + 4) + 4(n + 4) = 256$

$\Rightarrow (n + 4)^2 = 16^2$

$\Rightarrow \sqrt{(n + 4)^2} = \sqrt{16^2}$

$\Rightarrow n + 4 = 16$

$\Rightarrow n = 16 - 4$

$\therefore n = 12$

6. Question

How many terms of the arithmetic sequence 5, 7, 9, ..., must be added to get 140?

Answer

Using arithmetic sequence equation

$\Rightarrow \frac{n}{2}(2a + (n-1)d = Sn$

$\Rightarrow \frac{n}{2}(2 \times 5 + (n-1)2) = 140$

$\Rightarrow \frac{n}{2}(10 + 2n - 2) = 140$

$\Rightarrow \frac{n}{2}(2n + 8) = 140$

$\Rightarrow \frac{2n^2 + 8n}{2} = 140$

$\Rightarrow 2n^2 + 8n = 140 \times 2$

$\Rightarrow 2n^2 + 8n = 280$

$\Rightarrow n^2 + 4n = \frac{280}{2}$

⇒ $n^2 + 4n - 140 = 0$

⇒ $n^2 + 14n - 10n - 140 = 0$

⇒ $n(n + 14) - 10(n + 14) = 0$

⇒ $(n + 14)(n - 10) = 0$

x = –14 as we cannot have negative number of terms.

Therefore x = 10, the number of terms is 10

7. Question

A mathematician travelled three hundred kilometres to attend a conference. During his take he said: “Had my average speed been increased by 10 kilometres per hour, I could have reached here one hour earlier.” What was the average speed?

Answer

Given.

Distance = 100km

If average speed been increased by 10 kilometres per hour, I could have reached here one hour earlier

Formula used.

Average speed = $\frac{\text{sum of dist}}{\text{sum of time}}$

Let the sum of time taken be x

Average speed = $\frac{100}{x}$

New average speed = average speed + $10\frac{km}{h}$

$\frac{100}{x-1} = \frac{100}{x} + 10$

$$\frac{100}{x-1} = \frac{100 + 10x}{x}$$

$$\frac{100}{x-1} = \frac{10(10 + x)}{x}$$

$$\frac{10}{x-1} = \frac{10 + x}{x}$$

$10x = (x-1)(10 + x)$

$10x = 10x + x^2 - 10 - x$

$x^2 - x - 10 = 10x - 10x = 0$

$x^2 - x - 10 = 0$

As comparing eq to $ax^2 + bx + c = 0$

$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{1 + \sqrt{(-1)^2 - 4\times1\times(-10)}}{2\times1} = \frac{1 + \sqrt{41}}{2}$

$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = = \frac{1 - \sqrt{(-1)^2 - 4\times1\times(-10)}}{2\times1} = \frac{1 - \sqrt{41}}{2}$

As √41 is greater than 1

∴ time cannot get negative

Hence; time is $\frac{1 + \sqrt{41}}{2}$ hours

Average speed = $\frac{100\times2}{1+\sqrt{41}} = \frac{200}{1+\sqrt{41}}\frac{km}{h}$

8. Question

Thirty sweets were distributed equally among some kids. Sucking in the sweetness, a budding mathematician said,

“Had we been one less, each would have got one more sweet.”

How many kids were there?

Answer

Let the number of kids be x.

Let the number of sweets be y.

Total number of sweets = 30

$\frac{30}{x} = y$ ...eqn 1

$\Rightarrow \frac{30}{x-1} = y + 1$ ...eqn 2

Solving eqn 1 and eqn 2 we get

$\Rightarrow x - y - 1 = 0$

$\Rightarrow x - \frac{30}{x} - 1 = 0$ using eqn 1

$\Rightarrow x^2 - x - 30 = 0$

$\Rightarrow x^2 - 6x + 5x - 30 = 0$

$\Rightarrow x(x - 6) + 5(x - 6) = 0$

$\Rightarrow (x - 6)(x + 5) = 0$

$\therefore x = 6$ as we cannot take –5

There were 6 kids.

Questions Pg-90

1. Question

The product of a number and 2 more than that is 168. What are the numbers?

Answer

Let the number be x.

Now, According to the question

$(x)(x + 2) = 168$

$= x^2 + 2x = 168$

$= x^2 + 2x - 168 = 0$

It’s a second degree equation.

and we can use splitting the middle term method.

$= x^2 + 14x - 12x - 168 = 0$

$= x(x + 14) - 12(x + 14) = 0$

$= (x-12)(x + 14)$

There could be two values of x which are 12 and –14.

and the two number in each case would be (12,14) and (–14,–12).

2. Question

Find two numbers with sum 4 and product 2.

Answer

Let the numbers be x and y.

According to the question

x + y = 4

= y = 4 – x ---- (1)

xy = 2 ---- (2)

by (1) and (2)

x(4–x) = 2

4x – x^2 = 2

x^2 – 4x + 2 = 0

We can use the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{16 - 4 \times 2}}{2}$$

x = 2 + √(2) , 2 – √(2)

3. Question

How many terms of the arithmetic sequence 99, 97, 98, ... must be added to get 900?

Answer

sum of an AP = $\frac{n}{2}[2a + (n-1)d]$

In this question we need to find n and a = 99, d = –2

we'll put the values in the above equation.

$$\frac{n}{2}[2(99) + (n-1)(-2)] = 900$$

⇒ n[198 – 2n + 2] = 1800

⇒ $2n^2$ – 200n + 1800 = 0

⇒ n^2 – 100n + 900 = 0

⇒ n^2 – 10n – 90n + 900 = 0

⇒ n(n–10) – 90(n–10) = 0

⇒ (n–90)(n–10) = 0

⇒ n = 90,10

If we take 10 then after ten terms its sum would become 900.

If we take 90, then after 10 terms its sum would be 900 and then it will increase until a certain point and then again the sum will start decreasing because of negative values which will continue till 90^{th} term making the sum 900 again.

4. Question

The sum of a number and its reciprocal is $2\frac{1}{6}$. What is the number?

Answer

let the number be x.

⇒ $x + \frac{1}{x} = 2\frac{1}{6}$

⇒ $\frac{x^2+1}{x} = \frac{13}{6}$

⇒ $6x^2 + 6 = 13x$

⇒ $6x^2 + 6 – 13x = 0$

⇒ (3x–2)(2x–3)

⇒ $x = \frac{2}{3}, \frac{3}{2}$

5. Question

Two taps open into a tank. If both are opened, the tank would be filled in 12 minutes. The time taken to fill the tank by the smaller tap alone is 10 minutes more than the time taken of fill it by the larger tap alone. If the smaller tap alone is opened, what would be the time taken to fill the tank?

Answer

Let the time taken by the larger tap alone to fill the tank be x minutes.

∴ Tank filled in 1 minute = $\frac{1}{x}$

time taken by smaller tap to fill the tank alone = x + 10.

∴ Tank filled in 1 minute = $\frac{1}{x+10}$

time taken by both of the taps to fill the tank = 12 minute.

Now in 1 minute they both will fill $\frac{1}{12}$th of the tank.

⇒ $\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$

⇒ $(2x + 10)12 = x^2 + 10x$

⇒ $x^2 + 10x – 24x – 120 = 0$

⇒ $x^2 – 14x – 120 = 0$

⇒ (x–20)(x + 6)

time cannot be negative so 20 minutes for larger tap

and 30 minutes for smaller tap.

Questions Pg-97

1. Question

The perimeter of a rectangle is 42 metres and its diagonal is 15 metres. What are the lengths of its sides?

Answer

Given.

Perimeter = 42 m

Diagonal = 15 m

Formula used/Theory

⇒ Pythagoras theorem:-

$Base^2 + Height^2 = Hypotenuse^2$

⇒ Perimeter of rectangle = 2(L + B)

If Perimeter of rectangle = 2(L + B)

Then;

2(L + B) = 42m

$(L + B) = \frac{42}{2}$

(L + B) = 21m

Let L be x

Then, B is (21–x)

Then by Pythagoras theorem:-

$Base^2 + Height^2 = Hypotenuse^2$

$x^2 + (21–x)^2 = 15^2$

$x^2 + (21)^2 + x^2–2×x× 21 = 225$

$2x^2 – 42x + (441 – 225) = 0$

$2x^2 – 42x + 216 = 0$

$2(x^2 – 21x + 108) = 0$

$x^2 – 21x + 108 = 0$

As comparing eq to $ax^2 + bx + c = 0$

$x = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{21+\sqrt{(-21)^2-4\times1\times108}}{2\times1} = \frac{21+\sqrt{9}}{2} = \frac{21+3}{2} = \frac{24}{2} = 12$

$x = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{21-\sqrt{(-21)^2-4\times1\times108}}{2\times1} = \frac{21-\sqrt{9}}{2} = \frac{21-3}{2} = \frac{18}{2} = 9$

if length is 12m

then, breadth = (21–12) = 9m

if length is 9m

then, breadth = (21–9) = 12m

Conclusion/Result.

Length of sides can be either (12,9) or (9,12)

2. Question

How many consecutive natural numbers starting from 1 should be added to get 300?

Answer

Formula used/Theory

Sum of ‘n’ natural numbers = n(n + 1)/2

Sum = 300

∴ 300 = n(n + 1)/2

300×2 = n(n + 1)

600 = n(n + 1)

n^2 + n–600 = 0

As comparing eq to $ax^2 + bx + c = 0$

$x = \frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{-1+\sqrt{1^2-4\times1\times(-600)}}{2\times1} = \frac{-1+\sqrt{2401}}{2} = \frac{-1+49}{2} = \frac{48}{2} = 24$

$x = \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-1-\sqrt{1^2-4\times1\times(-600)}}{2\times1} = \frac{-1-\sqrt{2401}}{2} = \frac{-1-49}{2} = \frac{-50}{2} = -25$

Counting of numbers cannot be negative

∴ x = 24

Conclusion/Result. Sum of 24 natural numbers is 300

3. Question

The reciprocal of a positive number, subtracted from the number itself gives $1\frac{1}{2}$. What is the number?

Answer

Let the number be x

Then reciprocal of number is $\frac{1}{x}$

If reciprocal number, subtracted from the number itself is $1\frac{1}{2}$

Then;

$(x-\frac{1}{x}) = \frac{x^2-1}{x} = 1\frac{1}{2} = \frac{3}{2}$

∴ comparing both

$\frac{x^2-1}{x} = \frac{3}{2}$

x = 2

x^2–1 = 3

x^2 = 4

x = √4 = 2

∴ the number is 2

Conclusion/Result. Number comes out to be 2

4. Question

Can the sum of a number and its reciprocal be $1\frac{1}{2}$? Why?

Answer

Let the number be x

Then reciprocal of number is $\frac{1}{x}$

If reciprocal number, added to the number itself

Then;

$(x + \frac{1}{x}) = \frac{x^2 + 1}{x} = 1\frac{1}{2} = \frac{3}{2}$

∴ comparing both

$$\frac{x^2 + 1}{x} = \frac{3}{2}$$

⇒ $2x^2 + 2 = 3x$

$2x^2 – 3x + 2 = 0$

As comparing eq to $ax^2 + bx + c = 0$

$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{3 + \sqrt{(-3)^2 - 4\times2\times2}}{2\times2} = \frac{3 + \sqrt{-5}}{2}$

$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{3 - \sqrt{(-3)^2 - 4\times1\times2}}{2\times1} = \frac{3 - \sqrt{-5}}{2}$

⇒ Both values of x comes with non-real number

Their can’t be negative sign in square root

∴ this number cannot be possible

Conclusion/Result. This type of number cannot be possible

5. Question

In writing the equation to construct a rectangle of specified perimeter and area, the perimeter was wrongly written as 24 instead of 42. The length of a side was then computed as 10 metres. What is the area in the problem? What are the lengths of the rectangle in the correct problem?

Answer

Given.

Perimeter was wrongly written as 24 instead of 42.

The length of a side was then computed as 10 metres

Formula used/Theory

⇒ Perimeter of rectangle = 2(L + B)

⇒ Area of rectangle = (L×B)

If Perimeter of rectangle is taken as 24 m

And the length computed was 10 m

2(L + B) = 24m

$(L + B) = \frac{24m}{2} = 12m$

(10 + B) = 12m

B = 12m–10m = 2m

Then Area computed in problem was = (L×B)

= 10m×2m

= $20m^2$

Corrected perimeter = 42m

2(L + B) = 42m

(L + B) = $\frac{42m}{2}$ = 21m

If area computed is $20m^2$ and sum of length and breadth is 21m

Then;

The sides comes out to be 20m and 1m

Conclusion/Result. The sides comes out to be 20m and 1m

6. Question

In copying a second degree equation, the number without x was written as 24 instead of –24. The answers found were 4 and 6. What are the answers of the correct problem?

Answer

Given.

The number without x was written as 24 instead of –24

If c = 24

Means it will have factors AS 6 and 4

If c = –24

Means it will have factors either (–6,4) or (6, –4)

Conclusion/Result. Factors are either (–6,4) or (6, –4)