

Class: IX
SESSION : 2022-2023
SUBJECT: Mathematics
SAMPLE QUESTION PAPER - 9
with SOLUTION

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section A

1. The base of an isosceles triangle is 6 cm and each of its equal sides is 5 cm. The height of the triangle is [1]

a) $\sqrt{11}$ cm b) 8 cm

c) $\sqrt{30}$ cm d) 4 cm
2. PQRS is a cyclic quadrilateral such that PR is a diameter of the circle. If $\angle QPR = 67^\circ$ and $\angle SPR = 72^\circ$, then $\angle QRS =$ [1]

a) 23° b) 41°

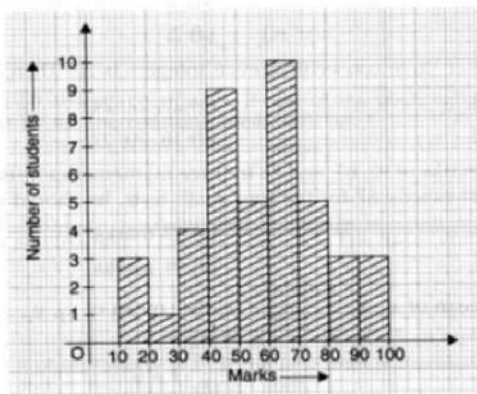
c) 67° d) 18°
3. The perpendicular distance of the point P(-2, -3) from the y-axis is [1]

a) 3 units b) -2

c) 2 units d) -3
4. The simplest form of $0.\overline{57}$ is [1]

a) $\frac{26}{45}$ b) $\frac{57}{99}$

c) $\frac{57}{100}$ d) none of these
5. In the given graph, the number of students who scored 60 or more marks is [1]



a) 22

b) 20

c) 21

d) 19

6. The equation $x = 7$ in two variables can be written as [1]

a) $1.x + 1.y = 7$

b) $1.x + 0.y = 7$

c) $0.x + 1.y = 7$

d) $0.x + 0.y = 7$

7. If $\triangle PQR \equiv \triangle EFD$, then $\angle E =$ [1]

a) None of these

b) $\angle P$

c) $\angle Q$

d) $\angle R$

8. Which of the following is a polynomial? [1]

i. $x^2 - 5x + 4\sqrt{x} + 3$

ii. $x^{3/2} - x + x^{1/2} + 1$

iii. $\sqrt{x} + \frac{1}{\sqrt{x}}$

iv. $\sqrt{2}x^2 - 3\sqrt{3}x + \sqrt{6}$

a) Option (iv)

b) Option (ii)

c) Option (i)

d) Option (iii)

9. In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 70^\circ$ and $\angle D = 30^\circ$. Then, $\angle AOB = ?$ [1]

a) 100°

b) 50°

c) 80°

d) 40°

10. If $10^{2y} = 25$, then 10^{-y} equals [1]

a) $-\frac{1}{5}$

b) $\frac{1}{5}$

c) $\frac{1}{625}$

d) $\frac{1}{50}$

c) (5, 1)

d) (1, 5)

18. **Assertion (A):** If the angles of a quadrilateral are in the ratio 2 : 3 : 7 : 6, then the measure of angles are 40° , 60° , 140° , 120° , respectively. [1]

Reason (R): The sum of the angles of a quadrilateral is 360° .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. If $x - 2$ is a factor of $x^2 + 3ax - 2a$, then $a =$ [1]

a) 1

b) -1

c) 2

d) -2

20. **Assertion (A):** $5 - \sqrt{2} = 5 - 1.414 = 3.586$ is an irrational number. [1]

Reason (R): The difference of a rational number and an irrational number is an irrational number.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

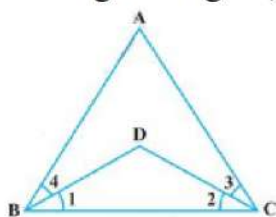
c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve the equation $u - 5 = 15$ and state the axiom that you use here. [2]

22. In the given figure, we have $\angle ABC = \angle ACB$, $\angle 4 = \angle 3$. Show that $\angle 1 = \angle 2$. [2]



23. Write the quadrant in which it lies: $(-7, -4)$ [2]

24. The ratio of the volumes of the two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. Find the ratio of their vertical heights. [2]

OR

Find the length of cloth used in making a conical pandal of height 100 m and base radius 240 m, if the cloth is 100π m wide.

25. Simplify: $(3\sqrt{5} - 5\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$ [2]

OR

Evaluate after rationalising the denominator of $\frac{25}{\sqrt{40}-\sqrt{80}}$ it being given that $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Section C

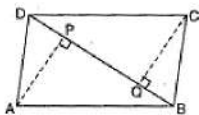
26. The following table gives the number of students in Class IX in a school during academic years 1996-1997 to 2000-2001. [3]

Academic Year	1996-97	1997-98	1998-99	1999-2000	2000-2001
Number of students	50	75	125	150	200

Represent the above data by a bar graph.

27. Simplify $\left\{ \left[625^{\frac{-1}{2}} \right]^{-\frac{1}{4}} \right\}^2$ [3]

28. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively. [3]



Show that :

- $\triangle APB \cong \triangle CQD$
 - $AP = CQ$.
29. The following table gives the quantity of goods (in crore tonnes) [3]

Year	1950-51	1960-61	1965-66	1970-71	1980-81	1982-83
Quantity of Goods (in crore tonnes)	9	16	20	20	22	26

Represent this information with the help of a bar graph. Explain through the bar graph if the quantity of goods carried by the Indian Railways in 1965-66 is more than double the quantity of goods carried in the year 1950-51.

OR

Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political party	A	B	C	D	E	F
Seats won	65	52	34	28	10	31

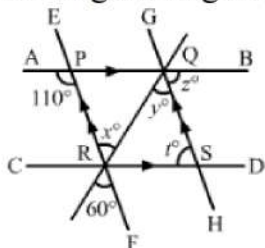
Draw a bar graph to represent the polling results.

30. Find at least 3 solutions for the following linear equation in two variables: [3]
 $2x + 5y = 13$

31. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $x + \pi$ [3]

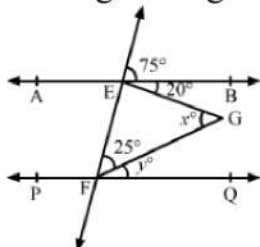
Section D

32. A right angled triangle with sides 3 cm and 4 cm is revolved around its hypotenuse. Find the volume of the double cone thus generated. [5]
33. In the given figure, $AB \parallel CD$ and $EF \parallel GH$. Find the values of x , y , z and t . [5]



OR

In the given figure, $AB \parallel PQ$. Find the value of x and y .



34. The base of a triangular field is three times its altitude. If the cost of sowing the field at Rs.58 per hectare is Rs.783, find its base and height. [5]

OR

Find the percentage increase in the area of a triangle if its each side is doubled.

35. Using factor theorem, factorize the polynomial: $x^4 + 10x^3 + 35x^2 + 50x + 24$ [5]

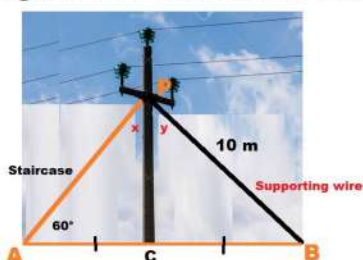
Section E

36. **Read the text carefully and answer the questions:** [4]

As shown In the village of Surya there was a big pole PC. This pole was tied with a strong wire of 10 m length.

Once there was a big spark on this pole, thus wires got damaged very badly. Any small fault was usually repaired with the help of a rope which normal board electricians were carrying on bicycles.

This time electricians need a staircase of 10 m so that it can reach at point P on the pole and this should make 60° with line AC.



- (i) Show that $\triangle APC$ and $\triangle BPC$ are congruent.

- (ii) Find the value of $\angle x$.

OR

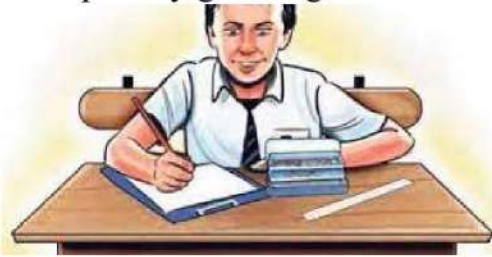
Find the value of $\angle y$.

- (iii) What is the value of $\angle PBC$?

37. Read the text carefully and answer the questions:

[4]

Ajay is writing a test which consists of 'True' or 'False' questions. One mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. Ajay knew answers to some of the questions. Rest of the questions he attempted by guessing.



- (i) If he answered 110 questions and got 80 marks and answer to all questions, he attempted by guessing were wrong, then how many questions did he answer correctly?
- (ii) If he answered 110 questions and got 80 marks and answer to all questions, he attempted by guessing were wrong, then how many questions did he guess?
- (iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?

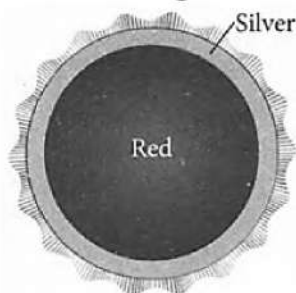
OR

If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

38. Read the text carefully and answer the questions:

[4]

The principal of a school decided to give badges to students who are chosen for the post of Head boy, Head girl, Prefect and Vice Prefect. Badges are circular in shape with two colour area, red and silver, as shown in figure. The diameter of the region representing red colour is 22 cm and the silver colour is filled in 10.5 cm wide ring.



- (i) Find the radius of circle representing the red region.

(ii) Find the area of the red region.

OR

Find the area of the silver region.

(iii) Find the radius of the circle formed by combining the red and silver region.

SOLUTION

Section A

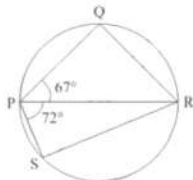
1. (d) 4 cm

Explanation: Height of isosceles triangle $= \frac{1}{2}\sqrt{4a^2 - b^2}$
 $= \frac{1}{2}\sqrt{4(5)^2 - 6^2}$ (a = 5 cm and b = 6 cm)
 $= \frac{1}{2} \times \sqrt{100 - 36}$
 $= \frac{1}{2} \times \sqrt{64}$
 $= \frac{1}{2} \times 8$
 $= 4 \text{ cm}$

2. (b) 41°

Explanation: Here we have a cyclic quadrilateral PQRS with PR being a diameter of the circle. Let the centre of this circle be O.

We are given that $\angle QPR = 67^\circ$ and $\angle SPR = 72^\circ$.



So, we see that,

$$\begin{aligned}\angle QPS &= \angle QPR + \angle RPS \\ &= 67^\circ + 72^\circ \\ &= 139^\circ\end{aligned}$$

In a cyclic quadrilateral, it is known that the opposite angles are supplementary.

$$\begin{aligned}\angle QPS + \angle QRS &= 180^\circ \\ \angle QRS &= 180^\circ - \angle QPS\end{aligned}$$

$$\angle QRS = 180^\circ - 139^\circ = 41^\circ$$

$$\angle QRS = 41^\circ$$

3. (c) 2 units

Explanation: Perpendicular distance of any point from y-axis is the given x-coordinate of point,

So distance = 2 unit

4. (a) $\frac{26}{45}$

Explanation: $0.\overline{57} = \frac{57-5}{90}$
 $= \frac{52}{90} = \frac{26}{45}$

5. (c) 21

Explanation: Add the values corresponding to the height of the bar from 60 to 100

$$10 + 5 + 3 + 3 = 21$$

6. (b) $1.x + 0.y = 7$

Explanation: The equation $x = 7$ in two variables can be written as exactly $1.x + 0.y = 7$

because it contains two variables x and y and coefficient of y is zero as there is no term containing y
in equation $x = 7$

7. (b) $\angle P$

Explanation: Since, by corresponding part of congruent $\angle E$ of $\triangle EFD$ is equal to the $\angle P$ of $\triangle PQR$.

8. (a) Option (iv)

Explanation: Clearly, $\sqrt{2}x^2 - 3\sqrt{3}x + \sqrt{6}$ is a polynomial.

9. (b) 50°

Explanation: It is given in the question that, ABCD is a quadrilateral where AO and BO are the bisectors of $\angle A$ and $\angle B$

We know that, sum of all angles of a quadrilateral is equal to 360°

$$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\angle A + \angle B + 70^\circ + 30^\circ = 360^\circ$$

$$\angle A + \angle B = 360^\circ - 100^\circ$$

$$\angle A + \angle B = 260^\circ$$

$$\frac{1}{2} (\angle A + \angle B) = \frac{1}{2} \times 260^\circ$$

$$\frac{1}{2} (\angle A + \angle B) = 130^\circ$$

Now, in triangle AOB

$$\frac{1}{2} (\angle A + \angle B) + \angle AOB = 180^\circ$$

$$130^\circ + \angle AOB = 180^\circ$$

$$\angle AOB = 180^\circ - 130^\circ$$

$$\angle AOB = 50^\circ$$

10. (b) $\frac{1}{5}$

Explanation: $10^{2y} = 25$

$$10^{2y} = 5^2$$

$$(10^y)^2 = (5)^2$$

$$\Rightarrow 10^y = 5$$

Now 10^{-y}

$$= \frac{1}{10^y}$$

$$= \frac{1}{5}$$

11. (b) a real number

Explanation: Every rational number (1, 4.5, 10, $\frac{1}{2}$, -27, $\frac{75}{5}$, 0) is a real number.

However, not every real number, is a rational number.

Although some numbers that appear to be irrational are actually rational because they can be reduced i.e. $\sqrt{25} = 5$

12. (d) 95°

Explanation: Given,

AOB = Straight line

$$\angle AOC + \angle BOD = 85^\circ$$

$$\angle AOC + \angle COD + \angle BOD = 180^\circ \text{ (Straight line)}$$

$$85^{\circ} + \angle \text{COD} = 180^{\circ}$$

$$\angle \text{COD} = 95^{\circ}$$

13. (b) $x = \frac{3y+5}{2}$

$$2x - 3y - 5 = 0$$

Explanation: $2x = 3y + 5$

$$x = \frac{3y+5}{2}$$

14. (a) 194

Explanation: Given $x = 7 + 4\sqrt{3}$ and $xy = 1$

$$\frac{1}{x} = \frac{1}{7+4\sqrt{3}}$$

$$= \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{1 \cdot (7-4\sqrt{3})}{(7)^2 - (4\sqrt{3})^2}$$

$$= \frac{7-4\sqrt{3}}{49-48}$$

$$= 7-4\sqrt{3}$$

$$\Rightarrow \frac{1}{x} = 7-4\sqrt{3}$$

$$\Rightarrow \left(\frac{1}{x}\right)^2 = (7-4\sqrt{3})^2$$

$$\Rightarrow \frac{1}{x^2} = 49 + 48 - 56\sqrt{3}$$

$$\Rightarrow \frac{1}{x^2} = 97-56\sqrt{3}$$

Also given. $x \cdot y = 1$

$$\therefore y = \frac{1}{x}$$

$$\Rightarrow y = 7-4\sqrt{3}$$

$$\Rightarrow \frac{1}{y} = x$$

$$\Rightarrow \frac{1}{y} = 7+4\sqrt{3}$$

$$\Rightarrow \left(\frac{1}{y}\right)^2 = (7+4\sqrt{3})^2$$

$$\Rightarrow \frac{1}{y^2} = 49+48+56\sqrt{3}$$

$$\Rightarrow \frac{1}{y^2} = 97+56\sqrt{3}$$

Now

$$\therefore \frac{1}{x^2} + \frac{1}{y^2} = 97-56\sqrt{3} + 97+56\sqrt{3}$$

$$= 194$$

15. (c) (3,9)

Explanation: Here, $y = 2x + 3$

So, for $x = 3$, we have

$$y = 2 \times 3 + 3$$

$$= 6 + 3$$

$$= 9$$

So, (3, 9) lies on the given line

16. (c) 90°

Explanation: The angle made by an arc at the centre is double the angle made by it on any other point on the circumference.

17. (b) (1, -2)

Explanation: Solution of the equation $3x - 2y = 7$ is (1, -2) as it satisfy the given equation

$$3x - 2y = 7$$

$$\Rightarrow 3(1) - 2(-2) = 7$$

$$\Rightarrow 3 + 4 = 7$$

$$\text{LHS} = \text{RHS}$$

18. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

19. (b) -1

Explanation: As $(x - 2)$ is a factor of $f(x) = x^2 + 3ax - 2a$

$$\text{i.e. } f(2) = 0$$

$$(2)^2 + 3a(2) - 2a = 0$$

$$4 + 6a - 2a = 0$$

$$= -1$$

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. In the given equation, we have

$$u - 5 = 15$$

On adding 5 to both sides, we have

$$u - 5 + 5 = 15 + 5 \text{ \{ on applying Euclid's axiom \}}$$

we get, $u = 20$.

22. We have

$$\Rightarrow \angle ABC = \angle ACB \dots (1) \text{ [(Given)]}$$

$$\text{And } \angle 4 = \angle 3 \dots (2) \text{ [(Given)]}$$

Now, subtracting (2) from (1), we get

Now, by Euclid's axiom 3, if equals are subtracted from equals, the remainders are equal.

$$\angle ABC - \angle 4 = \angle ACB - \angle 3$$

$$\text{Hence, } \angle 1 = \angle 2.$$

23. Recall that (+,+) lies in I quadrant, (-,+) lies in II quadrant, (-,-) lies in III quadrant (+,-) lies in IV quadrant

Hence, (-7, -4) lies in the 3rd Quadrant.

24. Let the radii of bases, vertical heights and volumes of the two cones be r_1 , h_1 , v_1 and r_2 , h_2 , v_2 respectively.

According to the question,

$$\frac{v_1}{v_2} = \frac{4}{5} \dots (1)$$

$$\frac{r_1}{r_2} = \frac{2}{3} \dots (2)$$

From (1), we have

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{4}{5}$$

$$\Rightarrow \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{4}{5} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 \frac{h_1}{h_2} = \frac{4}{5}$$

$$\Rightarrow \left(\frac{2}{3}\right)^2 \frac{h_1}{h_2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{4}{5} \left(\frac{3}{2}\right)^2 \dots [\text{Using (2)}]$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{9}{5}$$

∴ The ratio of their vertical height is 9 : 5

OR

Given that,

Height of conical pardal (A) = 100 m

Base radius (r) = 240 m

$$\begin{aligned}\text{Therefore Slant height (l)} &= \sqrt{r^2 + h^2} \\ &= \sqrt{(240)^2 + (100)^2} = \sqrt{57600 + 10000} \\ &= \sqrt{67600} = 260 \text{ m}\end{aligned}$$

Now area of curved surface = $\pi r l$

$$= \pi \times 240 \times 260 \text{ m}^2 = 62400\pi \text{ m}^2$$

Width of canvas cloth = 100 π m

$$\begin{aligned}\text{Therefore Length of cloth} &= \frac{\text{Area}}{\text{Width}} = \frac{62400\pi}{100\pi} \\ &= 624 \text{ m}\end{aligned}$$

$$\begin{aligned}25. (3\sqrt{5} - 5\sqrt{2})(4\sqrt{5} + 3\sqrt{2}) \\ &= 3\sqrt{5}(4\sqrt{5} + 3\sqrt{2}) - 5\sqrt{2}(4\sqrt{5} + 3\sqrt{2}) \\ &= 12 \times 5 + 9\sqrt{10} - 20\sqrt{10} - 15 \times 2 \\ &= 60 + 9\sqrt{10} - 20\sqrt{10} - 30 \\ &= 30 - 11\sqrt{10}\end{aligned}$$

OR

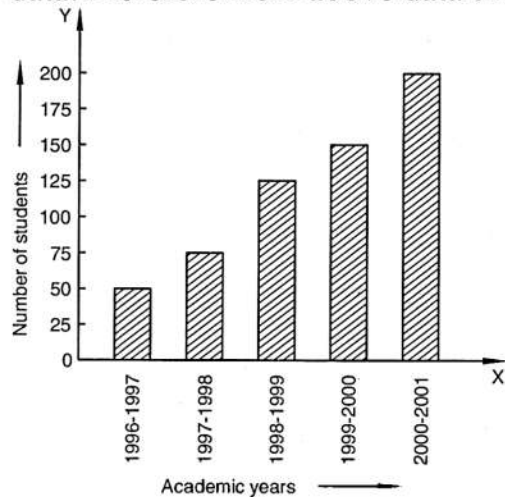
We have

$$\begin{aligned}&\frac{25}{\sqrt{40} - \sqrt{80}} \\ &= \frac{25}{2\sqrt{10} - 4\sqrt{5}} \\ &= \frac{25}{2(\sqrt{10} - 2\sqrt{5})} \times \frac{(\sqrt{10} + 2\sqrt{5})}{(\sqrt{10} + 2\sqrt{5})} \\ &= \frac{25(\sqrt{10} + 2\sqrt{5})}{2(\sqrt{10} - 2\sqrt{5})(\sqrt{10} + 2\sqrt{5})} \\ &= \frac{25(\sqrt{10} + 2\sqrt{5})}{2[(10)^2 - (2\sqrt{5})^2]} \\ &= \frac{25(\sqrt{10} + 2\sqrt{5})}{2(100 - 20)} \\ &= \frac{25(\sqrt{10} + 2\sqrt{5})}{160} \\ &= \frac{5(\sqrt{10} + 2\sqrt{5})}{32} \\ &= \frac{5(3.162 + 2 \times 2.236)}{32} \\ &= \frac{5 \times 7.634}{32} \\ &= \frac{38.17}{32} \\ &= 1.1928 \approx 1.193\end{aligned}$$

Section C

26. To represent the above data by a bar graph, we first draw a horizontal and a vertical line. Since five values of the numerical data are given. So, we mark five points on the horizontal line at equal distances and erect rectangles of the same width at these points. The heights of the rectangles are proportional to the numerical values of the

data. Therefore from above data the bar graph is given below.



$$\begin{aligned}
 27. & \left\{ \left(625^{-\frac{1}{2}} \right)^{-\frac{1}{4}} \right\}^2 \\
 &= \left\{ \left(\frac{1}{625^{\frac{1}{2}}} \right)^{-\frac{1}{4}} \right\}^2 = \left\{ \left(\frac{1}{(25^2)^{\frac{1}{2}}} \right)^{-\frac{1}{4}} \right\}^2 \quad (a^{-m} = \frac{1}{a^m}) \\
 &= \left\{ \left(\frac{1}{25} \right)^{-\frac{1}{4} \times 2} \right\} \\
 &= \left(\frac{1}{25^{-\frac{1}{2}}} \right) = \frac{1}{(5^2)^{-\frac{1}{2}}} = \frac{1}{5^{-1}} = 5
 \end{aligned}$$

28. Given: ABCD is a parallelogram and AP and CQ are perpendicular from vertices A and C on diagonal BD respectively.

To Prove :

- i. $\triangle APB \cong \triangle CQD$
- ii. $AP = CQ$.

Proof :

i. In $\triangle APB$ and $\triangle CQD$

$AB = CD \dots$ [Opp. sides of || gm ABCD]

$\angle ABP = \angle CDQ \dots$ [Alternate interior angles for $AB \parallel CD$]

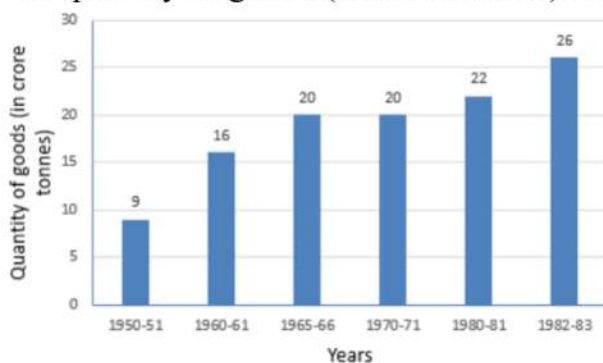
$\therefore \angle APB = \angle CQD \dots$ [Each 90°]

$\triangle APB \cong \triangle CQD \dots$ [By AAS rule]

ii. As $\triangle APB \cong \triangle CQD \dots$ [As proved above]

$\therefore AP = CQ \dots$ [c.p.c.t.]

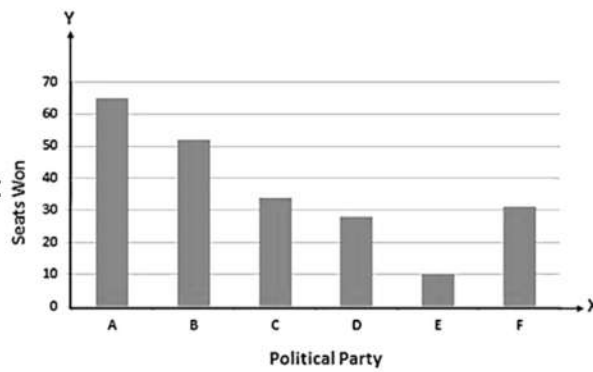
29. the quantity of goods (in crore tonnes) in different years



Yes, the quantity of goods carried by the Indian Railway in 1965 - 66 is more than double the quantity of goods carried in the year 1950 - 51.

OR

The bar graph is given below:



30. $2x + 5y = 13$

$$\Rightarrow 5y = 13 - 2x$$

$$\Rightarrow y = \frac{13-2x}{5}$$

Put $x = 0$, then $y = \frac{13-2(0)}{5} = \frac{13}{5}$

Put $x = 1$, then $y = \frac{13-2(1)}{5} = \frac{11}{5}$

Put $x = 2$, then $y = \frac{13-2(2)}{5} = \frac{9}{5}$

Put $x = 3$, then $y = \frac{13-2(3)}{5} = \frac{7}{5}$

$\therefore (0, \frac{13}{5}), (1, \frac{11}{5}), (2, \frac{9}{5})$ and $(3, \frac{7}{5})$ are the solutions of the equation $2x + 5y = 13$.

31. $x + \pi$

We need to find the zero of the polynomial $x + \pi$

$$x + \pi = 0$$

$$\Rightarrow x = -\pi$$

While applying the remainder theorem, we need to put the zero of the polynomial $x + \pi$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

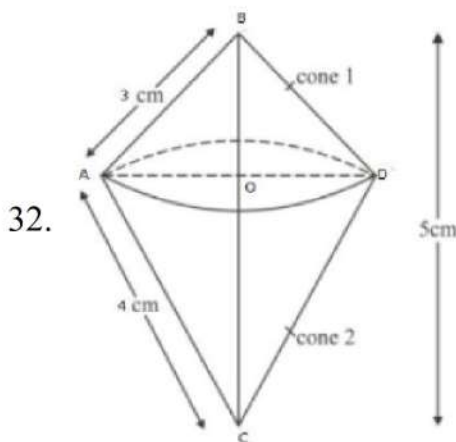
$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$$

$$= -\pi^3 + 3\pi^2 - 3\pi + 1.$$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by $x + \pi$ we will get the remainder as $-\pi^3 + 3\pi^2 - 3\pi + 1$

Section D



$$AB = 3 \text{ cm}, AC = 4 \text{ cm}$$

In $\triangle BAC$, by pythagoras theorem

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC^2 = 3^2 + 4^2$$

$$\Rightarrow BC^2 = 25$$

$$\Rightarrow BC = \sqrt{25} = 5 \text{ cm}$$

In $\triangle AOB$ and $\triangle CAB$

$$\angle ABO = \angle ABC \text{ [common]}$$

$$\angle AOB = \angle BAC \text{ [each } 90^\circ]$$

Then, $\triangle AOB \sim \triangle CAB$ [by AA similarity]

$$\therefore \frac{AO}{CA} = \frac{OB}{AB} = \frac{AB}{CB} \text{ [c.p.s.t.]}$$

$$\Rightarrow \frac{AO}{4} = \frac{OB}{3} = \frac{3}{5}$$

$$\text{Then, } AO = \frac{4 \times 3}{5} \text{ and } OB = \frac{3 \times 3}{5}$$

$$\Rightarrow AO = \frac{12}{5} \text{ cm and } OB = \frac{9}{5} \text{ cm}$$

$$\therefore OC = 5 - \frac{9}{5} = \frac{16}{5} \text{ cm}$$

\therefore Volume of double cone thus generated = volume of first cone + volume of second cone

$$= \frac{1}{3} \pi (AO)^2 \times BO + \frac{1}{3} \pi (AO)^2 \times OC$$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{12}{5}\right)^2 \times \frac{9}{5} + \frac{1}{3} \times \frac{22}{7} \times \left(\frac{12}{5}\right)^2 \times \frac{16}{5}$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{12}{5} \times \frac{12}{5} \left[\frac{9}{5} + \frac{16}{5}\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{12}{5} \times \frac{12}{5} \times 5$$

$$= \frac{1056}{35} = 30\frac{6}{35} \text{ cm}^3.$$

33. In the given figure,

$$x = 60^\circ \text{ [Vertically-Opposite Angles]}$$

$$\angle PRQ = \angle SQR \text{ [Alternate Angles]}$$

$$y = 60^\circ$$

$$\angle APR = \angle PQS \text{ [Corresponding Angles]}$$

$$\Rightarrow 110^\circ = \angle PQR + 60^\circ \text{ } [\because \angle PQS = \angle PQR + \angle RQS]$$

$$\Rightarrow \angle PQR = 50^\circ$$

$$\angle PQR + \angle RQS + \angle BQS = 180^\circ \text{ [Since AB is a straight line]}$$

$$\Rightarrow 50^\circ + 60^\circ + z = 180^\circ$$

$$\Rightarrow 110^\circ + z = 180^\circ$$

$$\Rightarrow z = 70^\circ$$

$$\angle DSH = z \text{ [Corresponding Angles]}$$

$$\Rightarrow \angle DSH = 70^\circ$$

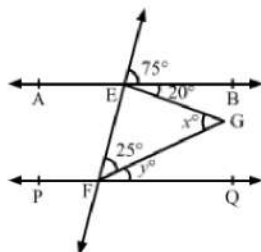
$$\therefore \angle DSH = t \text{ [Vertically-Opposite Angles]}$$

$$\Rightarrow t = 70^\circ$$

$$x = y = 60^\circ \text{ and } z = t = 70^\circ$$

$$\therefore x = 60^\circ, y = 60^\circ, z = 70^\circ \text{ and } t = 70^\circ.$$

OR



Given, $AB \parallel PQ$

Let CD be the transversal cutting AB and PQ at E and F, respectively.

Then,

$$\angle CEB + \angle BEG + \angle GEF = 180^\circ \text{ [Since CD is a straight line]}$$

$$\Rightarrow 75^\circ + 20^\circ + \angle GEF = 180^\circ$$

$$\Rightarrow \angle GEF = 85^\circ$$

We know that the sum of angles of a triangle is 180° .

$$\therefore \angle GEF + \angle EGF + \angle EFG = 180$$

$$\Rightarrow 85^\circ + x + 25^\circ = 180^\circ$$

$$\Rightarrow x^\circ + 110^\circ = 180^\circ$$

$$\Rightarrow x^\circ = 180^\circ - 110^\circ$$

$$\Rightarrow x^\circ = 70^\circ$$

And

$$\angle FEG + \angle BEG = \angle DFQ \text{ [Corresponding Angles]}$$

$$\Rightarrow 85^\circ + 20^\circ = \angle DFQ$$

$$\Rightarrow \angle DFQ = 105^\circ$$

$$\angle EFG + \angle GFQ + \angle DFQ = 180^\circ \text{ [Since CD is a straight line]}$$

$$\Rightarrow 25^\circ + y^\circ + 105^\circ = 180^\circ$$

$$\Rightarrow y^\circ = 50^\circ$$

$$\therefore x^\circ = 70^\circ \text{ and } y^\circ = 50^\circ$$

34. Let the height of the triangle be h meter

$$\therefore \text{Base} = 3h \text{ meter [given]}$$

Now,

$$\text{Area of triangle} = \frac{\text{Total cost}}{\text{Rate}} = \frac{783}{58} = 13.5 \text{ ha} = 135000 \text{ m}^2$$

We have:

$$\text{Area of triangle} = 135000 \text{ m}^2$$

$$\Rightarrow \frac{1}{2} \times \text{Base} \times \text{Height} = 135000$$

$$\Rightarrow \frac{1}{2} \times 3h \times h = 135000$$

$$\Rightarrow h^2 = \frac{135000 \times 2}{3}$$

$$\Rightarrow h^2 = 90000$$

$$\Rightarrow h = 300 \text{ m [taking square root both sides]}$$

Thus, we have

$$\text{Height} = h = 300 \text{ m}$$

$$\text{Base} = 3h = 900 \text{ m.}$$

OR

Let a, b, c be the sides of the old triangle and s be its semi-perimeter. Then,

$$s = \frac{1}{2}(a + b + c)$$

The sides of the new triangle are $2a, 2b$ and $2c$.

Let s' be its semi-perimeter. Then,

$$s' = \frac{1}{2}(2a + 2b + 2c) = a + b + c = 2s$$

$$\Rightarrow s' = 2s$$

Let Δ and Δ' be the areas of the old and new triangles respectively. Then

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \dots\dots(1)$$

and

$$\Delta' = \sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$$

$$\Rightarrow \Delta' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)} \text{ [}\because s' = 2s\text{]}$$

$$\Rightarrow \Delta' = \sqrt{16s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \Delta' = 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta \text{ [from (1)]}$$

\therefore Increase in the area of the triangle $= \Delta' - \Delta = 4\Delta - \Delta = 3\Delta$

Hence, percentage increase in area $= \left(\frac{3\Delta}{\Delta} \times 100 \right) = 300\%$

35. Given, $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$

The constant term in $f(x)$ is equal to 24

The factors of 24 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

Let, $x + 1 = 0$

$\Rightarrow x = -1$

Substitute the value of x in $f(x)$

$$f(-1) = (-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24$$

$$= 1 - 10 + 35 - 50 + 24$$

$$= 0$$

$\Rightarrow (x + 1)$ is the factor of $f(x)$

Similarly, $(x + 2), (x + 3), (x + 4)$ are also the factors of $f(x)$

Since, $f(x)$ is a polynomial of degree 4, it cannot have more than four linear factors.

$\Rightarrow f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)$

$$\Rightarrow x^4 + 10x^3 + 35x^2 + 50x + 24 = k(x + 1)(x + 2)(x + 3)(x + 4)$$

Substitute $x = 0$ on both sides

$$\Rightarrow 0 + 0 + 0 + 0 + 24 = k(1)(2)(3)(4)$$

$$\Rightarrow 24 = k(24)$$

$$\Rightarrow k = 1$$

Substitute $k = 1$ in $f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)$

$$f(x) = (1)(x + 1)(x + 2)(x + 3)(x + 4)$$

$$f(x) = (x + 1)(x + 2)(x + 3)(x + 4)$$

$$\text{hence, } x^4 + 10x^3 + 35x^2 + 50x + 24 = (x + 1)(x + 2)(x + 3)(x + 4)$$

This is the required factorisation of $f(x)$.

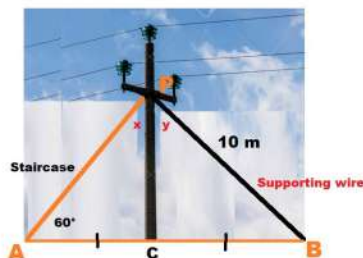
Section E

36. **Read the text carefully and answer the questions:**

As shown In the village of Surya there was a big pole PC. This pole was tied with a strong wire of 10 m length.

Once there was a big spark on this pole, thus wires got damaged very badly. Any small fault was usually repaired with the help of a rope which normal board electricians were carrying on bicycles.

This time electricians need a staircase of 10 m so that it can reach at point P on the pole and this should make 60° with line AC.



(i) In $\triangle APC$ and $\triangle BPC$

$AP = BP$ (Given)

$CP = CP$ (common side)

$\angle ACP = \angle BCP = 90^\circ$

By RHS criteria $\triangle APC \cong \triangle BPC$

(ii) In $\triangle ACP$

$$\angle APC + \angle PAC + \angle ACP = 180^\circ$$

$$\Rightarrow x + 60^\circ + 90^\circ = 180^\circ \text{ (angle sum property of } \triangle)$$

$$\Rightarrow \angle x = 180^\circ - 150^\circ = 30^\circ$$

$$\angle x = 30^\circ$$

OR

In $\triangle APC$ and $\triangle BPC$

Corresponding part of congruent triangle

$$\angle X = \angle Y$$

$$\Rightarrow \angle Y = 30^\circ \text{ (given } \angle X = 30^\circ)$$

(iii) In $\triangle APC$ and $\triangle BPC$

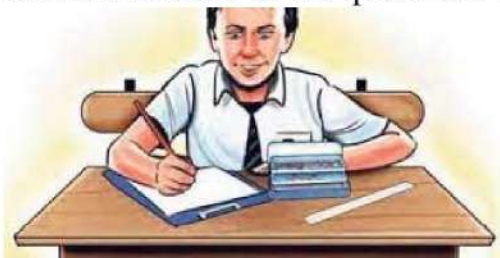
Corresponding part of congruent triangle

$$\angle PAC = \angle PBC$$

$$\Rightarrow \angle PBC = 60^\circ \text{ (given } \angle PAC = 60^\circ)$$

37. Read the text carefully and answer the questions:

Ajay is writing a test which consists of 'True' or 'False' questions. One mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. Ajay knew answers to some of the questions. Rest of the questions he attempted by guessing.



(i) Let the no of questions whose answer is known to Ajay be x and number questions attempted by guessing be y .

$$x + y = 110$$

$$x + 14y = 80 \Rightarrow 4x + y = 320 \quad x + y = 110 \dots(1)$$

$$4x + y = 320 \dots(2)$$

Solving (1) and (2)

$$x + y - 4x - y = 110 - 320 = -210$$

$$\Rightarrow -3x = -210$$

$$\Rightarrow x = 70$$

(ii) $x + y = 110$

$$x + 14y = 80 \Rightarrow 4x + y = 320$$

$$x + y = 110 \dots(1)$$

$$4x + y = 320 \dots(2)$$

Solving (1) and (2)

$$x + y - 4x - y = 110 - 320 = -210$$

$$\Rightarrow -3x = -210$$

$$\Rightarrow x = 70$$

Put $x = 70$ in (1)

$$70 + y = 110$$

$$\Rightarrow y = 40$$

40 question he answered by guessing.

$$(iii) 70 - 40 \times \frac{1}{4} = 70 - 10 = 60 \text{ marks}$$

He scored 60 marks.

OR

$$x - \frac{1}{4}(110 - x) = 95$$

$$\Rightarrow 4x - 110 + x = 380$$

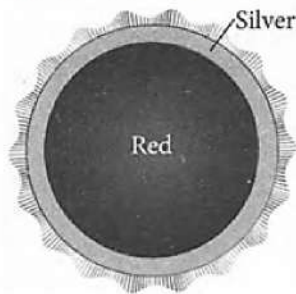
$$\Rightarrow 5x = 380 + 110 = 490$$

$$\Rightarrow x = \frac{490}{5} = 98$$

So he answered 98 correct answers 12 by guessing.

38. Read the text carefully and answer the questions:

The principal of a school decided to give badges to students who are chosen for the post of Head boy, Head girl, Prefect and Vice Prefect. Badges are circular in shape with two colour area, red and silver, as shown in figure. The diameter of the region representing red colour is 22 cm and the silver colour is filled in 10.5 cm wide ring.



(i) Radius of circle representing red region
 $= \frac{22}{2} = 11 \text{ cm} [\because \text{Diameter} = 22 \text{ cm (Given)}]$

(ii) Area of red region $= \pi r^2$
 $= \frac{22}{7} \times 11 \times 11 = 380.28 \text{ cm}^2$

OR

Radius of circle formed by combining red and silver region = Radius of red region + width of silver sign

$$= (11 + 10.5) \text{ cm} = 21.5 \text{ cm}$$

Area of silver region = Area of combined region - Area of red region

$$= \frac{22}{7} \times 21.5 \times 21.5 - 380.28$$

$$= 1452.78 - 380.28 = 1072.50 \text{ cm}^2$$

(iii) Radius of circle formed by combining red and silver region = Radius of red region + width of silver sign

$$= (11 + 10.5) \text{ cm} = 21.5 \text{ cm}$$