Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

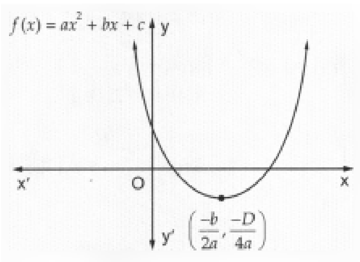
Section A

1. (HCF $\times$ LCM) for the numbers 30 and 70 is: **[1]**

a) 21 b) 70

c) 2100 d) 210

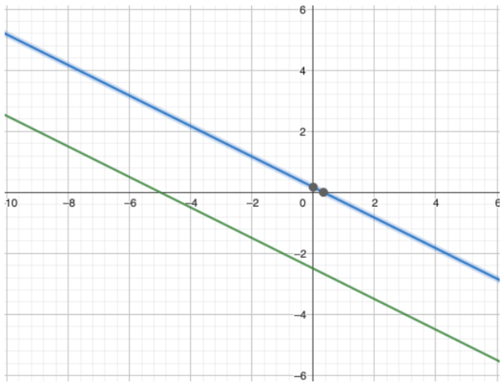
2. Figure show the graph of the polynomial f(x) = ax^2 + bx + c for which **[1]**



a) a > 0, b < 0 and c > 0 b) a < 0, b < 0 and c < 0

c) a < 0, b > 0 and c > 0 d) a > 0, b > 0 and c < 0

3. The pair of equations x + 2y + 5 = 0 and –3x – 6y + 1 = 0 have **[1]**



a) a unique solution

b) infinitely many solutions

c) no solution

d) exactly two solutions

4. In the Maths Olympiad of 2020 at Animal Planet, two representatives from the donkey's side, while solving a quadratic equation, committed the following mistakes. **[1]**

i. One of them made a mistake in the constant term and got the roots as 5 and 9.

ii. Another one committed an error in the coefficient of x and he got the roots as 12 and 4.

But in the meantime, they realised that they are wrong and they managed to get it right jointly. Find the quadratic equation.

a) $2x^2 + 7x - 24 = 0$

b) $x^2 + 4x + 14 = 0$

c) $3x^2 - 17x + 52 = 0$

d) $x^2 - 14x + 48 = 0$

5. What is the common difference of an AP in which $a_{18} - a_{14} = 32$? **[1]**

a) -8

b) 4

c) -4

d) 8

6. ABCD is a rectangle whose three vertices are B (4,0), C (4,3) and D (0,3). The length of one of its diagonals is **[1]**

a) 5

b) 3

c) 4

d) 25

7. In what ratio, does x-axis divide the line segment joining the points A(3, 6) and B(-12, -3)? **[1]**

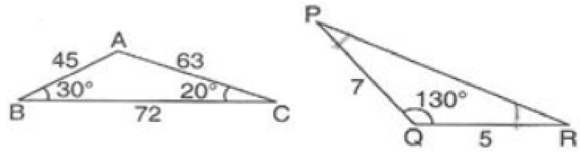
a) 2 : 1

b) 1 : 2

c) 4 : 1

d) 1 : 4

8. In the figures find the measures of $\angle P$ and $\angle R$ **[1]**



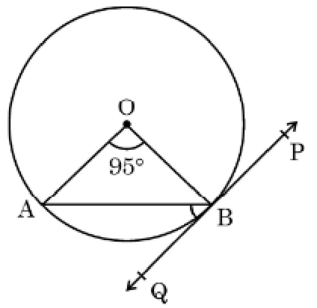
a) $20^o, 30^o$.

b) $50^o, 40^o$.

c) $30^o, 20^o$.

d) $40^o, 50^o$.

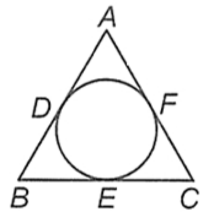
9. In the given figure, PQ is tangent to the circle centred at O. If $\angle AOB = 95^o$, then the measure of $\angle ABQ$ will be **[1]**



a) 85^{o} b) 47.5^{o}

c) 95^{o} d) 42.5^{o}

10. A circle inscribed in $\triangle$ABC having AB = 10 cm, BC = 12 cm, CA = 28 cm touching sides at D, E, F (respectively). Then AD + BE + CF = ________. **[1]**



a) 22 cm b) 25 cm

c) 18 cm d) 20 cm

11. $1 + \frac{\cot^2\alpha}{1+\cos ec\alpha} =$ **[1]**

a) $\sin\alpha$ b) $\sec\alpha$

c) $\text{cosec}\alpha$ d) $\tan\alpha$

12. If $\cos\theta = \frac{4}{5}$ then $\tan\theta$ = ? **[1]**

a) $\frac{3}{4}$ b) $\frac{5}{3}$

c) $\frac{4}{3}$ d) $\frac{3}{5}$

13. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder (in metres) is **[1]**

a) $2\sqrt{2}$ b) $4\sqrt{3}$

c) 4 d) $\frac{4}{\sqrt{3}}$

14. In a circle of radius 14 cm, an arc subtends an angle of 120^0 at the centre. If $\sqrt{3}$= 1.73 then the area of the segment of the circle is **[1]**

a) 124.63 cm^2 b) 130.57 cm^2

c) 120.56 cm^2 d) 118.24 cm^2

15. A pendulum swings through an angle of 30^{o} and describes an arc 8.8 cm in length. Find the length of the pendulum. **[1]**

a) 8.8 cm b) 17 cm

c) 15.8 cm d) 16.8 cm

16. A die is thrown once. The probability of getting an even number is **[1]**

a) $\frac{1}{3}$

b) $\frac{5}{6}$

c) $\frac{1}{6}$

d) $\frac{1}{2}$

17. 3 rotten eggs are mixed with 12 good ones. One egg is chosen at random. The probability of choosing a rotten egg is **[1]**

a) $\frac{1}{15}$

b) $\frac{4}{5}$

c) $\frac{1}{5}$

d) $\frac{2}{5}$

18. The distribution below gives the marks obtained by 80 students on a test: **[1]**

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60
Number of Students	3	12	27	57	75	80

The modal class of this distribution is:

a) 30 - 40

b) 20 - 30

c) 50 - 60

d) 10 - 20

19. **Assertion (A):** A piece of cloth is required to completely cover a solid object. The solid object is composed of a hemisphere and a cone surmounted on it. If the common radius is 7 m and height of the cone is 1 m, 463.39 cm^2 is the area of cloth required. **[1]**

Reason (R): Surface area of hemisphere = $2\pi r^2$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Sum of first n terms in an A.P. is given by the formula: $S_n = 2n \times [2a + (n - 1)d]$ **[1]**

Reason (R): Sum of first 15 terms of 2 , 5 , 8 ... is 345.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

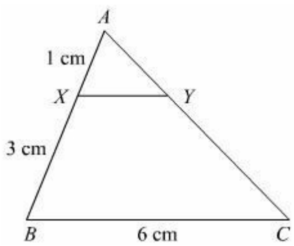
c) A is true but R is false.

d) A is false but R is true.

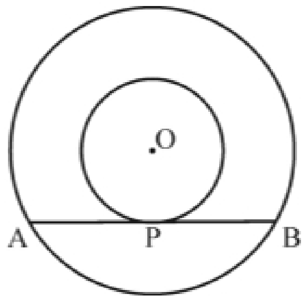
Section B

21. Find the largest number which divides 320 and 457 leaving remainder 5 and 7 respectively. **[2]**

22. In the given figure $XY \| BC$. Find the length of XY. **[2]**



23. Two concentric circles with centre O are of radii 3 cm and 5 cm. Find the length of chord AB of the larger circle which touches the smaller circle at P. **[2]**



24. Show that: $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$ for $0^\circ \leq \theta \geq 90^\circ$ **[2]**

OR

Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$

25. Find the length of the arc of a circle of diameter 42 cm which subtends an angle of 60° at the centre. **[2]**

OR

A horse is tethered to one corner of a rectangular field of dimensions 70 m $\times$ 52 m, by a rope of length 21 m. How much area of the field can it graze?

Section C

26. Prove that 3 + $\sqrt{5}$ is an irrational number. **[3]**

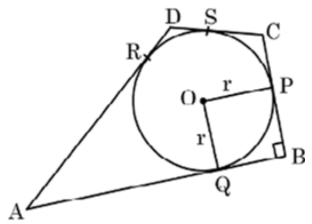
27. If α and β are the zeros of the polynomial f(x) = $6x^2 + x - 2$, find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$ **[3]**

28. A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number. **[3]**

OR

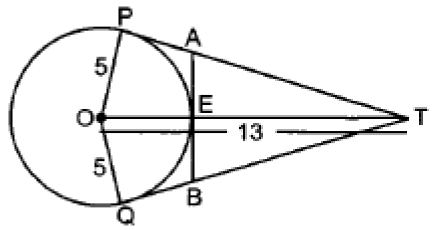
A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

29. In the given figure, a circle is inscribed in a quadrilateral ABCD in which $\angle$B = 90°. If AD = 17 cm, AB = 20 cm and DS = 3 cm, then find the radius of the circle. **[3]**



OR

In figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB. where TP and TQ are two tangents to the circle.



30. Prove that: **[3]**

$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \frac{1+\sin\theta}{\cos\theta}$

31. The percentage of marks obtained by 100 students in an examination are given below: **[3]**

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	14	16	18	23	18	8	3

Determine the median percentage of marks.

Section D

32. Solve: $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2, x \neq -\frac{1}{2}, 1$ **[5]**

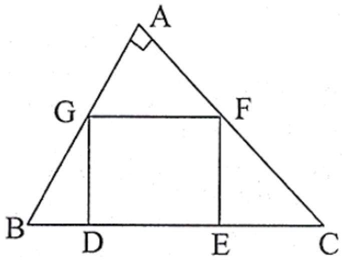
OR

₹ 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got ₹ 160 less. Find the original number of persons.

33. In Fig., DEFG is a square in a triangle ABC right angled at A. **[5]**

Prove that

i. $\triangle AGF \sim \triangle DBG$
ii. $\triangle AGF \sim \triangle EFC$



34. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of ₹10 per dm^2. **[5]**

OR

A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid (Use π = 22/7)

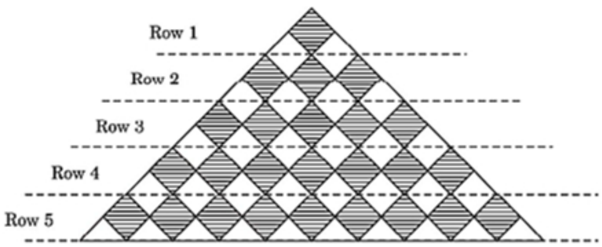
35. Find the mean from the following frequency distribution of marks at a test in statistics: **[5]**

Marks (x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A fashion designer is designing a fabric pattern. In each row, there are some shaded squares and unshaded triangles.



(a) Identify A.P. for the number of squares in each row.

(b) Identify A.P. for the number of triangles in each row.

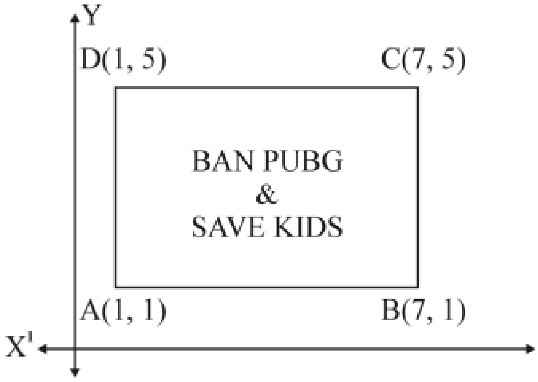
OR

Write a formula for finding total number of triangles in n number of rows. Hence, find S_{10}.

(c) If each shaded square is of side 2 cm, then find the shaded area when 15 rows have been designed.

37. **Read the text carefully and answer the questions:** **[4]**

Use of mobile screen for long hours makes your eye sight weak and give you headaches. Children who are addicted to play "PUBG'' can get easily stressed out. To raise social awareness about ill effects of playing PUBG, a school decided to start 'BAN PUBG' campaign, in which students are asked to prepare campaign board in the shape of a rectangle. One such campaign board made by class X student of the school is shown in the figure.



(a) Find the coordinates of the point of intersection of diagonals AC and BD.

(b) Find the length of the diagonal AC.

OR

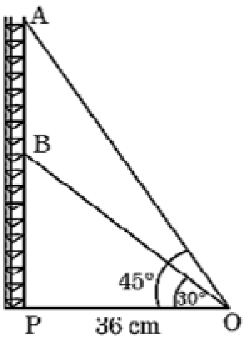
Find the ratio of the length of side AB to the length of the diagonal AC.

(c) Find the area of the campaign Board ABCD.

38. **Read the text carefully and answer the questions:** **[4]**

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30^{o} and the angle of elevation of the top of Section A is 45^{o}.



(a) Find the length of the wire from the point O to the top of Section B.

(b) Find the distance AB.

OR

Find the height of the Section A from the base of the tower.

(c) Find the area of $\triangle$OPB.

Solution

Section A

1.

(c) 2100

Explanation: As we know HCF $\times$ LCM = Product of the Numbers

Hence HCF $\times$ LCM (30,70) = 30 $\times$ 70 = 2100

2. **(a)** a > 0, b < 0 and c > 0

Explanation: Clearly, $f(x) = ax^2 + bx + c$ represent a parabola opening upwards.

Therefore, $a > 0$

The vertex of the parabola is in the fourth quadrant, therefore b < 0

$y = ax^2 + bx + c$ cuts Y axis at P which lies on OY.

Putting x = 0 in $y = ax^2 + bx + c$, we get y = c.

So the coordinates of P is (0, c).

Clearly, P lies on OY. $\Rightarrow$ c>0

Hence, a>0, b<0 and c>0

3.

(c) no solution

Explanation: Given, equations are

x + 2y + 5 = 0, and

- 3x - 6y + 1 = 0.

Comparing the equations with general form:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$

And $a_2 = -3$, $b_2 = -6$, $c_2 = 1$

Taking the ratio of coefficients to compare

$\frac{a_1}{a_2} = \frac{-1}{3}, \frac{b_1}{b_2} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

So $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

This represents a pair of parallel lines.

Hence, the pair of equations has no solution.

4.

(d) $x^2 - 14x + 48 = 0$

Explanation: For 1st one,

Let the equation be $x^2 + ax + b = 0$

Since roots are 5 and 9

$\therefore$a = -14 and b = 45

For 2nd one,

Let the equation be $x^2 + px + q = 0$

Since roots are 12 and 4.

$\therefore$p = -16 and q = 48

Now, according to the question, b and p both are wrong.

Therefore, the correct equation would be

$x^2 - 14x + 48 = 0$

5.

(d) 8

Explanation: $a_{18} - a_{14} = 32$

$(18 - 14)d = 32$

$\Rightarrow 4d = 32$

$\Rightarrow d = 8.$

6. **(a)** 5

Explanation: Three vertices of a rectangle ABCD are B (4,0), C (4, 3) and D (0, 3) length of one of its diagonals

$\text{BD} = \sqrt{(4-0)^2 + (0-3)^2} = \sqrt{4^2 + 3^2}$

$= \sqrt{16+9} = \sqrt{25}$ = 5

7. **(a)** 2 : 1

Explanation: 2 : 1

8. **(a)** 20^o, 30^o.

Explanation: In triangle ABC, $\angle\text{A} + \angle\text{B} + \angle\text{C} = 180^\circ$

$\Rightarrow \angle\text{A} + 30^\circ + 20^\circ = 180^\circ$

$\Rightarrow \angle\text{A} = 130^\circ$

In triangle ABC and QRP, $\frac{AB}{QR} = \frac{AC}{PQ}$

$\Rightarrow \frac{45}{5} = \frac{63}{7}$ => $\frac{9}{1} = \frac{9}{1}$

Since sides of triangles ABC and QRP are proportional, and included angles are equal, therefore by SAS similarity criteria ,

$\Delta\text{ABC} \sim \Delta\text{QRP}$

$\angle\text{A} = \angle\text{Q}, \angle\text{B} = \angle\text{R}, \angle\text{C} = \angle\text{P}$

$\Rightarrow \angle\text{P} = 20^\circ, \angle\text{R} = 30^\circ$

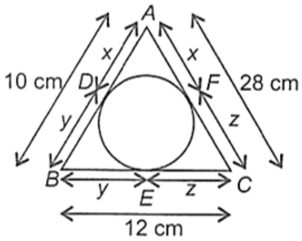
9.

(b) 47.5^o

Explanation: 47.5^o

10.

(b) 25 cm

Explanation: 

x + y = 10 cm ...(i)

y + z = 12 cm ...(ii)

x + z = 28 cm ...(iii)

Adding (i), (ii) and (iii), we get

2(x + y + z) = 50

$\Rightarrow$ x + y + z = 25 cm

11.

(c) cosecα

Explanation: 1 + $\frac{\cot^2\alpha}{1+\cos ec\alpha}$

= 1 + $\frac{cosec^2\alpha - 1}{1+\cos ec\alpha}$

= 1 + $\frac{(cosec\,\alpha - 1)(cosec\,\alpha + 1)}{1+cosec\alpha}$

= 1 + cosecα - 1 = cosec α

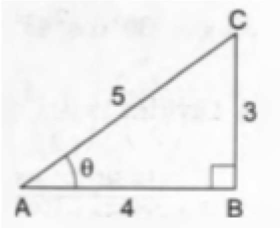
12. **(a)** $\frac{3}{4}$

Explanation: $\cos\theta = \frac{4}{5} = \frac{AB}{AC}$

$\therefore \quad BC^2 = AC^2 - AB^2 = 25 - 16 = 9$

$\Rightarrow$ BC = 3

$\therefore \tan\theta = \frac{BC}{AB} = \frac{3}{4}$

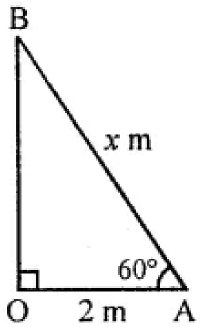


13.

(c) 4

Explanation: Suppose AB is the ladder of length x m

$\therefore$ OA = 2m, $\angle$OAB = 60^o



In right $\triangle$AOB, sec 60^o = $\frac{x}{2}$

$\Rightarrow$ 2 = $\frac{x}{2} \Rightarrow$ = 4 m

14.

(c) 120.56 cm^2

Explanation: ar(segment) = $\left(\frac{\pi r^2\theta}{360} - r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$

$= \left(\frac{22}{7} \times 14 \times 14 \times \frac{120}{360}\right) - (14 \times 14 \times \sin 60^\circ \cos 60^\circ)$

$= \left(\frac{616}{3} - \frac{\sqrt{3}}{2} \times \frac{1}{2} \times 14 \times 14\right) \text{cm}^2$

=(205.33 -49$\times$1.73) cm^2

=(205.33 - 84.77) cm^2

= 120.56 cm^2

15.

(d) 16.8 cm

Explanation: Length of the pendulum = Radius of a sector of the circle

Arc length = 8.8

$\frac{\theta}{360}(2\pi r)$ = 8.8

$\frac{30}{360} \times 2 \times \frac{22}{7} \times r$ = 8.8

r = 16.8 cm

16.

(d) $\frac{1}{2}$

Explanation: Number of all possible outcomes = 6.

Even numbers are 2,4, 6. Their number is 3.

$\therefore$ P (getting an even number) = $\frac{3}{6} = \frac{1}{2}$

17.

(c) $\frac{1}{5}$

Explanation: Number of possible outcomes = 3

Number of Total outcomes = 15

$\therefore$ Required Probability = $\frac{3}{15} = \frac{1}{5}$

18. **(a)** 30 - 40

Explanation: 30 - 40

19.

(c) A is true but R is false.

Explanation: A is true but R is false.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. The given numbers are 320 and 457

Now as 5 and 7 are remainders on division of 320 and 457 by said number

On subtracting the reminders 5 and 7 from 320 and 457 respectively we get:

320 - 5 = 315,

457 - 7 = 450

The prime factorizations of 315 and 405 are

$315 = 3 \times 3 \times 5 \times 7$

$= 3^2 \times 5 \times 7$

$450 = 2 \times 3 \times 3 \times 5 \times 5$

$= 2 \times 3^2 \times 5^2$

$\therefore$ H.C.F. of 315 and 450 = $3^2 \times 5 = 9 \times 5 = 45$

Hence the said number = 45

22. Given XY $\|$ BC

AX = 1 cm, XB = 3 cm, and BC = 6 cm

AB = AX + XB

= 1 + 3 = 4 cm

In ΔAXY and ΔABC

$\angle A = \angle A$ [Common]

$\angle AXY = \angle ABC$ [Corresponding angles]

Then, $\triangle$AXY ~ $\triangle$ABC [By AA similarity]

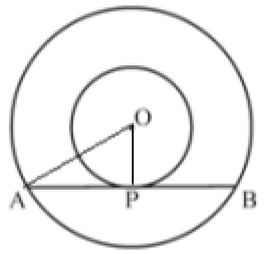
$\therefore \frac{AX}{AB} = \frac{XY}{BC}$ [Corresponding parts of similar $\triangle$are proportional]

$\Rightarrow \frac{1}{4} = \frac{XY}{6}$

$\Rightarrow XY = \frac{6}{4} = 1.5cm$

23. Join OA and OP

OP $\perp$ AB (radius $\perp$ tangent at the point of contact)



OP is the radius of smaller circle and AB is tangent at P.

AB is chord of larger circle and OP $\perp$ AB

$\therefore$ AP = PB($\perp$ from centre bisects the chord)

In right $\triangle$AOP, $AP^2 = OA^2 - OP^2$

$= (5)^2 - (3)^2 = 16$

AP = 4 cm = PB

$\therefore$ AB = 8 cm

24. $\text{L. HS} = \tan^4\theta + \tan^2\theta$

$= \tan^2\theta\left(\tan^2\theta + 1\right)$

$= \tan^2\theta \quad \sec^2\theta$

$= \left(\sec^2\theta - 1\right)\sec^2\theta \left[\because \tan^2\theta = \sec^2\theta - 1\right]$

$= \sec^4\theta - \sec^2\theta$

= R.H.S.

OR

$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$

L.H.S. $=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right)$

$=\frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$=\tan\theta+\cot\theta+1=1+\tan\theta+\cot\theta=RHS$

Hence proved.

25. Diameter of a circle = 42 cm

$\Rightarrow$ Radius of a circle = r $=\frac{42}{2}$

= 21 cm

Central angle $=\theta=60^\circ$

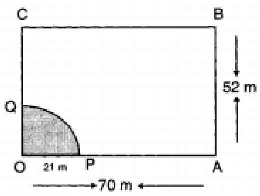
$\therefore$ Length of the arc $=\frac{2\pi r\theta}{360}$

$=\frac{2\times\frac{22}{7}\times21\times60^\circ}{360^\circ}\mathrm{cm}$

= 22 cm

OR

Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius r - 21 m.



$\therefore$Required Area = $\frac{1}{4}\pi r^2$

$\therefore$Required Area = $\left\{\frac{1}{4}\times\frac{22}{7}\times(21)^2\right\}\mathrm{cm}^2=\frac{693}{2}\mathrm{cm}^2$ = 346.5 cm^2

Section C

26. Let 3 + $\sqrt{5}$ is a rational number.

$3+\sqrt{5}=\frac{p}{q},q\neq0$

$3+\sqrt{5}=\frac{p}{q}$

$\Rightarrow\quad\sqrt{5}=\frac{p}{q}-3$

$\Rightarrow\quad\sqrt{5}=\frac{p-3q}{q}$

Now in RHS $\frac{p-3q}{p}$ is rational

This shows that $\sqrt{5}$ is rational

But this contradict the fact that $\sqrt{5}$ is irrational, This is because we assumed that 3 + $\sqrt{5}$ is a rational number.

$\therefore 3+\sqrt{5}$ is an irrational number.

27. Let f(x) = 6x^2 + x - 2

a = 6, b = 1 and c = -2

And α and β are the zeros of polynomial,

α + β = $-\frac{b}{a}=-\frac{1}{6}$

$\alpha\beta=\frac{c}{a}=\frac{-2}{6}=\frac{-1}{3}$

$\therefore\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$=\frac{\left(-\frac{1}{6}\right)^2-2\left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)}$

$=-\frac{\frac{1}{36}+\frac{2}{3}}{\frac{1}{3}}$

$= -\frac{\frac{25}{36}}{\frac{1}{3}}$

$= -\frac{25}{36} \times \frac{3}{1}$

$= -\frac{25}{12}$

28. Let us suppose that the digit at unit place be x

Suppose the digit at tens place be y.

Thus, the number is 10y + x.

According to question it is given that the number is 4 times the sum of the two digits.

Therefore, we have

10y + x = 4(x + y)

$\Rightarrow$ 10y + x = 4x + 4y

$\Rightarrow$ 4x + 4y - 10y - x = 0

$\Rightarrow$ 3x - 6y = 0

$\Rightarrow$ 3(x - 2y) = 0

$\Rightarrow$ x - 2y = 0

After interchanging the digits, the number becomes 10x + y.

Again according to question If 18 is added to the number, the digits are reversed.

Thus, we have

(10y + x) + 18 = 10x + y

$\Rightarrow$ 10x + y - 10y - x = 18

$\Rightarrow$ 9x - 9y = 18

$\Rightarrow$ 9(x - y) = 18

$\Rightarrow x - y = \frac{18}{9}$

$\Rightarrow$ x - y = 2

Therefore, we have the following systems of equations

x - 2y = 0 ....(1)

x - y = 2...............(2)

Here x and y are unknowns. Now let us solve the above systems of equations for x and y.

Subtracting the equation (1) from the (2), we get

(x - y) - (x - 2y) = 2 - 0

$\Rightarrow$ x - y - x + 2y = 2

$\Rightarrow$ y = 2

Now, substitute the value of y in equation (1), we get

x - 2 $\times$ 2 = 0

$\Rightarrow$ x - 4 = 0

$\Rightarrow$ x = 4

Therefore the number is 10 $\times$ 2 + 4 = 24

Thus the number is 24

OR

Let the usual speed of the plane = x km/hr.

Distance to the destination = 1500 km

Case (i):

we know that, $Speed = \frac{Distance}{Time}$

$\Rightarrow Time = \frac{Distance}{speed}$

So, in case(i) Time = $\frac{1500}{x}$Hrs

Case (iI)

Distance to the destination = 1500 km

Increased speed = 100 km/hr

So, speed = x+100

So, in case(ii) Time = $\frac{1500}{x+100}$Hrs

So, according to the question

$\therefore \frac{1500}{x}$ - $\frac{1500}{x+100}$ = $\frac{30}{60}$

$\Rightarrow x^2 + 100x - 300000 = 0$

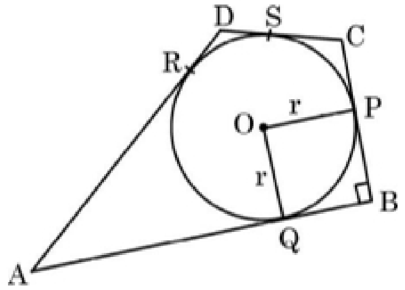
$\Rightarrow x^2 + 600x - 500x - 300000 = 0$

$\Rightarrow (x + 600)(x - 500) = 0$

$\Rightarrow x = 500$ or $x = -600$

Since, speed can not be negative, x = 500

Therefore, Speed of plane = 500 km/hr.

29. 

DR = DS = 3 cm

$\therefore$ AR = AD - DR = 17 - 3 = 14 cm

$\Rightarrow$ AQ = AR = 14 cm

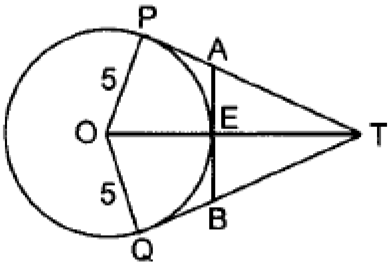
$\therefore$ QB = AB - AQ = 20 - 14 = 6 cm

Since QB = OP = r $\therefore$ radius = 6 cm

OR

According to the question,

O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E.



$\because$ OP $\perp$ TP [Radius from point of contact of the tangent]

$\therefore \angle$OPT = 90°

In right $\triangle$OPT *

$OT^2 = OP^2 + PT^2$

$\Rightarrow (13)^2 = (5)^2 + PT^2 \Rightarrow$ PT = 12 cm

Let AP = x cm AE = AP $\Rightarrow$ AE = x cm

and AT = (12 - x)cm

TE = OT - OE = 13 - 5 = 8 cm

$\because$OE $\perp$ AB [Radius from the point of contact]

$\therefore \angle$AEO = 90° $\Rightarrow \angle$AET = 90°

In right $\triangle$AET,

$\mathrm{AT}^2 = \mathrm{AE}^2 + \mathrm{ET}^2$

$(12 - x)^2 = x^2 + 8^2$

$\Rightarrow 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow \quad 24x = 80 \Rightarrow x = \frac{80}{24} = \frac{10}{3}$ cm

Also BE = AE = $\frac{10}{3}$ cm

$\Rightarrow \quad AB = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$ cm

30. We have,

$\mathrm{LHS} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$\Rightarrow \quad \mathrm{LHS} = \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$

$\Rightarrow \quad \mathrm{LHS} = \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$ [$\because \sec^2\theta - \tan^2\theta = 1$]

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)[1-(\sec\theta-\tan\theta)]}{\tan\theta-\sec\theta+1}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)(1-\sec\theta+\tan\theta)}{(\tan\theta-\sec\theta+1)}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec\theta+\tan\theta)(\tan\theta-\sec\theta+1)}{(\tan\theta-\sec\theta+1)}$

$\Rightarrow \quad LHS = \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$ = RHS

31.

Marks (Class)	Number of Students (Frequency)	Cumulative frequency
30-35	14	14
35-40	16	30
40-45	18	48
45-50	23	71 (Median class)
50-55	18	89
55-60	8	97
60-65	3	100

Here, N = 100

Therefore, $\frac{N}{2}$ = 50, This observation lies in the class 45-50.

l (the lower limit of the median class) = 45

cf (the cumulative frequency of the class preceding the median class) = 48

f (the frequency of the median class) = 23

h (the class size) = 5

$\text{Median} = l + \left(\frac{\frac{n}{2}-\text{cf}}{f}\right)h$

$= 45 + \left(\frac{50-48}{23}\right) \times 5$

$= 45 + \frac{10}{23} = 45.4$

So, the median percentage of marks is 45.4.

Section D

32. Given

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 0$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

$\therefore$ Substituting this value

$y + \frac{1}{y} = 2$ or $\frac{y^2+1}{y} = 2$

or $y^2 + 1 = 2y$

or $y^2 - 2y + 1 = 0$

or $(y-1)^2 = 0$

Putting $y = \frac{x-1}{2x+1}$,

$\frac{x-1}{2x+1} = 1$ or $x - 1 = 2x + 1$

or x = -2

OR

Let the original number of persons be x.

Total amount to be divided among all people = Rs. 9000/-

So, Share of each person = Rs. $\frac{9000}{x}$

If the number of persons is increased by 20. Then,

New share of each person = Rs. $\frac{9000}{x+20}$

According to the question ;

$\frac{9000}{x} - \frac{9000}{x+20} = 160$

$\Rightarrow \frac{9000(x+20)-9000x}{x(x+20)} = 160$

$\Rightarrow \frac{9000x+180000-9000x}{x^2+20} = 160$

$\Rightarrow \frac{180000}{x^2+20} = 160$

$\Rightarrow \frac{180000}{160} = x^2 + 20$

$\Rightarrow 1125 = x^2 + 20x$

$\Rightarrow x^2 + 20x - 1125 = 0$

$\Rightarrow x^2 + 45x - 25x - 1125 = 0$

$\Rightarrow x(x + 45) - 25(x + 45) = 0$

$\Rightarrow (x + 45)(x - 25) = 0$

$\Rightarrow x - 25 = 0$ [$\because$ The number of persons cannot be negative. $\therefore x + 45 \neq 0$]

$\Rightarrow x = 25$

Hence, the original number of persons is 25.

33. GF || DE (DEFG is square)

$\therefore \angle AGF = \angle ABC$ (Corresponding angles)

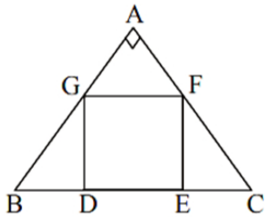
$\therefore \angle A = \angle GDB = 90^o$

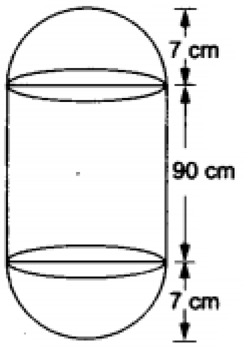
$\therefore \angle AGF \sim \angle DBG$ (By AA similarity)

Again DEFG being a square $\angle AFG = \angle ACB$ (corresponding angles)

$\therefore \angle A = \angle CEF$ (each 90^o)

$\angle AGF \sim \angle EFC$ (By AA similarity)



34. 

Radius of each hemispherical end = 7 cm.

Height of each hemispherical part = its radius = 7 cm.

Height of the cylindrical part = (104 - 2 $\times$ 7) cm = 90 cm.

Area of surface to be polished = 2(curved surface area of the hemisphere) + (curved surface area of the cylinder)

= [2 $(2\pi r^2)$ + $2\pi rh$] sq units

$= \left[\left(4 \times \frac{22}{7} \times 7 \times 7\right) + \left(2 \times \frac{22}{7} \times 7 \times 90\right)\right] \text{cm}^2$

= (616 + 3960) cm^2 = 4576 cm^2

$= \left(\frac{4576}{10\times 10}\right) \text{dm}^2$ = 45.76 dm^2 [$\because$ 10 cm = 1 dm].

$\therefore$ cost of polishing the surface of the solid

= ₹(45.76 $\times$10) = ₹ 457.60.

OR

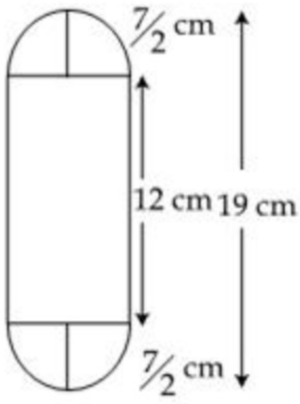
Diameter of the cylinder = 7 cm

Therefore radius of the cylinder = $\frac{7}{2}cm$

Total height of the solid = 19 cm

Therefore, Height of the cylinder portion = 19 - 7 = 12 cm

Also, radius of hemisphere = $\frac{7}{2}$cm



Let V be the volume and S be the surface area of the solid. Then,

V = Volume of the cylinder + Volume of two hemispheres

$\Rightarrow \quad V=\left\{\pi r^2 h+2\left(\frac{2}{3} \pi r^3\right)\right\} \mathrm{cm}^3$

$\Rightarrow \quad V=\pi r^2\left(h+\frac{4 r}{3}\right) \mathrm{cm}^3$

$\Rightarrow \quad V=\left\{\frac{22}{7} \times\left(\frac{7}{2}\right)^2 \times\left(12+\frac{4}{3} \times \frac{7}{2}\right)\right\} \mathrm{cm}^3=\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{50}{3} \mathrm{cm}^3=641.66 \mathrm{cm}^3$

and,

S = Curved surface area of cylinder + Surface area of two hemispheres

$\Rightarrow \quad S=\left(2 \pi r h+2 \times 2 \pi r^2\right) \mathrm{cm}^2$

$\Rightarrow \quad S=2 \pi r(h+2 r) \mathrm{cm}^2$

$\Rightarrow \quad S=2 \times \frac{22}{7} \times \frac{7}{2} \times\left(12+2 \times \frac{7}{2}\right) \mathrm{cm}^2$

$=\left(2 \times \frac{22}{7} \times \frac{7}{2} \times 19\right) \mathrm{cm}^2$

= 418 cm^2

35. Let the assumed mean be A = 25 and h = 5.

marks (x_1):	no. of students (f_1):	$d_1 = x_1 = A = x_1 - 25$	$u_1 = \frac{1}{h}(d_1)$	f_1u_1
5	15	-20	-4	-60
10	50	-15	-3	-150
15	80	-10	-2	-160
20	76	-5	-1	-76
25	72	0	0	0
30	45	5	1	45
35	39	10	2	78
30	9	15	3	27
45	8	20	4	32
50	6	25	5	30
	$\sum f_1 = 400$			$\sum f_1u_1 = -234$

We know that mean, $\overline{X}$ = A + h $\left(\frac{1}{N} \sum_{i=1}^n f_i u_i\right)$

Now, we have N = $\sum f_1$ = 400, = -234, h = 5 and A = - 234, h = 5 and A = 25.

Putting the values in the above formula, we get

$\bar{X}$ = A + h $\left(\frac{1}{N} \sum_{i=1}^n f_i u_i\right)$

= 25 + 5$\left(\frac{1}{400} \times(-234)\right)$

= 25 - $\frac{234}{80}$

= 25 - 2.925

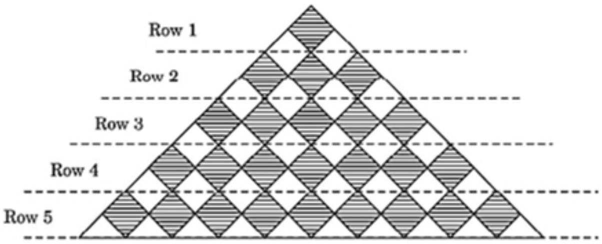
= 22.075

Hence, the mean marks is 22.075

Section E

36. **Read the text carefully and answer the questions:**

A fashion designer is designing a fabric pattern. In each row, there are some shaded squares and unshaded triangles.



(i) A.P. for the number of squares in each row is 1, 3, 5, 7, 9 …

(ii) A.P. for the number of triangles in each row is 2, 6, 10, 14 ...

OR

$S_n = \frac{n}{2}[4 + (n - 1)4] = 2n^2$

$\therefore S_{10} = 2 \times 10^2 = 200$

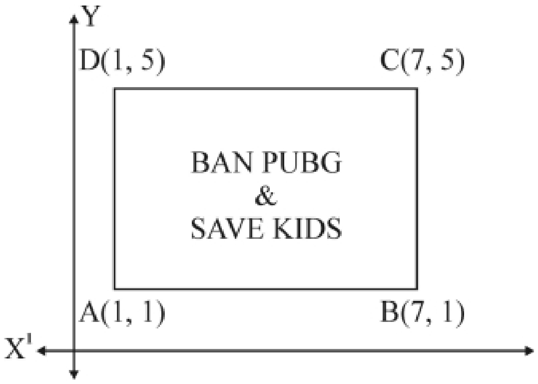
(iii) Area of each square = $2 \times 2 = 4$ cm^2

Number of squares in 15 rows = $\frac{15}{2}(2 + 14 \times 2) = 225$

Shaded area = $225 \times 4 = 900$ cm^2

37. **Read the text carefully and answer the questions:**

Use of mobile screen for long hours makes your eye sight weak and give you headaches. Children who are addicted to play “PUBG’’ can get easily stressed out. To raise social awareness about ill effects of playing PUBG, a school decided to start ‘BAN PUBG’ campaign, in which students are asked to prepare campaign board in the shape of a rectangle. One such campaign board made by class X student of the school is shown in the figure.



(i) Point of intersection of diagonals is their midpoint

So, $\left[\frac{(1+7)}{2}, \frac{(1+5)}{2}\right]$

= (4, 3)

(ii) Length of diagonal AC

AC = $\sqrt{(7-1)(7-1)+(5-1)(5-1)}$

= $\sqrt{52}$ units

OR

Ratio of lengths = $\frac{AB}{AC}$

= $\frac{6}{\sqrt{52}}$

= $6 : \sqrt{52}$

(iii) Area of campaign board

= 6×4

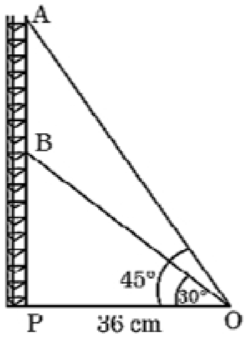
= 24 units square

38. **Read the text carefully and answer the questions:**

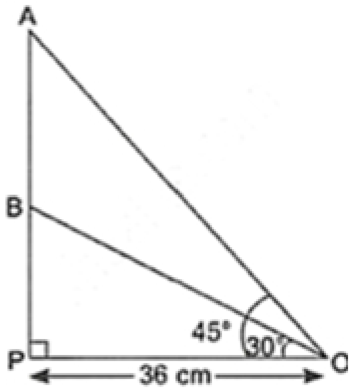
Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two

Sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30^o and the angle of elevation of the top of Section A is 45^o.



(i) Let the length of wire BO = x cm



$\therefore \cos 30^o = \frac{PO}{BO}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{x}$

$\Rightarrow x = \frac{36\times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 12 \times 2\sqrt{3}$

$= 24\sqrt{3}$ cm

(ii) In $\triangle$APO, $\tan 45^o = \frac{AP}{PO}$

$\Rightarrow 1 = \frac{AP}{36}$

$\Rightarrow$ AP = 36 cm ...(i)

Now, In $\triangle$PBO,

$\tan 30^o = \frac{BP}{PO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BP}{36}$

$\Rightarrow BP = \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{36}{3}\sqrt{3}$

$= 12\sqrt{3}$ cm

$\therefore$ AB = AP - BP

$= 36 - 12\sqrt{3}$ cm

OR

In $\triangle$APO, $\tan 45^o = \frac{AP}{36}$

$1 = \frac{AP}{36}$

A = 36 cm

Height of section A from the base of the tower = AP = 36 cm.

(iii) In $\triangle$OPB

$\tan 30^o = \frac{BP}{PO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BP}{36}$

$BP = \frac{36}{\sqrt{3}}$

$= \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 12\sqrt{3}$ cm

Now, Area of $\triangle OPB = \frac{1}{2} \times \text{height} \times \text{base}$

$= \frac{1}{2} \times BP \times OP$

$= \frac{1}{2} \times 12\sqrt{3} \times 36$

$= 216\sqrt{3}$ cm^2